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Comparison of the Standard Rating Methods and the New General Rating Formula

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Relationships between standard rating methods and General Rating Formula (GRF) are established by means of convenient algebraic representations of the concepts used in general insurance ratemaking process. A new proof of GRF that is much more convenient for readers is given. Different forms of GRF that are more convenient for use in rating practice are given. It is explained that GRF is a generalization of the standard methods in the mathematical sense. The main difficulty of the standard ratemaking (the problem to obtain good quality manual rates and manual premiums as input) is solved; it is proven that we do not need any rates and premiums as input in rating process any more. Some other advantages of GRF over the standard rating methods are also mentioned.

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1. Introduction

Traditional or standard ratemaking process in general insurance is accomplished by either ***loss ratio*** or ***loss cost*** method. Loss cost method is also known as ***pure premium*** method. It was proven in 1995 that loss ratio and loss cost methods are equivalent (meaning the two methods produce the same rates) when losses are adjusted for heterogeneity in a standard way; see [3]. The main problem of the standard rating methods is to obtain good quality manual rates (and manual premiums) as input. In addition, the problem with standard rating process is that each step consists of rather complicated and tedious calculations for either of the two methods.

New general insurance rating formula, that we will call General Rating Formula (GRF), was presented first time in 2007 by M. B., see [1]. It eliminates the need for all three standard steps (*overall rate change*, *changing of risk classification differentials*, and *balancing back*). It is a relatively simple formula that calculates exactly the same indicated rates as the standard process. We will see that the new formula does not need current manual rates (and premiums) as input. Therefore it is a generalization of the standard method in the mathematical sense. The input variables for the new formula are only: *number of earned exposure units* for each risk cell, *fully developed and trended loss* (FDTL) amounts for each cell, *permissible loss ratio* (PLR) and current values of *risk classification variables* (*class, territory,…, industry)*. Note that earned exposures, losses and PLR characterize a specific product, while risk classification variables (a.k.a. rating variables, risk factors, risk parameters) characterize external forces.

General Rating Formula can be used also to calculate rates for a new product. This is easy to see because both, new product's rates and adjusted rates of an old product will be in effect for a future period of time. The only difference is that for a new product we have to assume values of input variables: exposures, losses and risk factors, while for re-rating of an old product the historic experience of the same input variables should be available to us. For this reason we can use both names ***general rating*** formula and ***general re-rating*** formula. In both cases we can use abbreviation GRF.

In order to derive GRF in [1] we needed to move away from the descriptive notation and to introduce formal algebraic notation for all rating concepts and steps. That notation is very useful tool also for revealing relationships between standard rating methods and GRF and for comparing them.

2. Algebraic Representations of the Rating Variables

In the rating model that follows, risk classification variables are assumed to act *multiplicatively*. Each risk classification variable is introduced in the multiplicative model as a vector of *differentials* or *relativities*. It is well known that uneven distribution of exposures (*heterogeneity*) affects the independence or the risk classification variables, but that discussion is beyond the scope of this paper.

In order to simplify derivations, let us limit ourselves to only three *risk classification variables (or factors*): c*lass, territory and industry*. *Class, territory* and *industry* are generic risk classification variables introduced only to ease communication and are not examples of any particular P&C insurance product nor a group insurance product.

We could not use less than three classification variables for the derivation of the new formula because that would result in a loss of generality. Namely, it is not obvious how to generalize formulae with only two variables to three or more, while it is obvious how the formulae with three variables can be extended to four or more variables.

Let $\boldsymbol{L} = \left(l_{ijk}\right)_{mxnxp}$ denote a tensor (three-dimensional array) of *fully developed and trended losses (FDTL)*. The elements of the tensor $\boldsymbol{L}$ are also called e*xpected dollar losses* (in *effective period) or projected losses*.

We obtain element l_{ijk} in the cell (i, j, k) by multiplying the current loss of that cell by the *development factor* and the *trend factor*. Less formally, we can say l_{ijk} is a projected loss in the c$lass\, i$, $territory\, j$, and $industry\, k$, where $i = 1,2, \dots, m$; $j = 1,2, \dots n$; $k = 1, 2, \dots, p$, and m, n, p are numbers of differentials (relativities) for c*lass, territory and industry*, respectively. Let us denote the *total projected (or expected) loss* by L, i. e.

$$L = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{p} l_{ijk} \qquad (2.1)$$

By keeping fixed one index at a time and summing loss amounts of the corresponding slice, we get:

Vector of *losses for class*, l^C is defined by:

$$l_i^C := \sum_{j=1}^{n} \sum_{k=1}^{p} l_{ijk},\ i = 1, 2, \dots, m,$$

and identically, vectors of *losses for territory*, l^T and *industry* l^I are defined by

$$l_j^T := \sum_{i=1}^{m} \sum_{k=1}^{p} l_{ijk},\ j = 1, 2, \dots, n,$$

$$l_k^I := \sum_{i=1}^{m} \sum_{j=1}^{n} l_{ijk},\ k = 1, 2, \dots, p.$$

Let us also introduce a three dimensional array, $E = (e_{ijk})_{mxnxp}$, where e_{ijk} is the *number of earned exposure units* in $class\ i,\ territory\ j,\ and\ industry\ k$. Let us denote the *total number of earned exposure units* by E, i.e.

$$E := \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{p} e_{ijk}. \qquad (2.2)$$

$\boldsymbol{R} := \left(\boldsymbol{r_{ijk}}\right)_{\boldsymbol{mxnxp}}$, where $\boldsymbol{r_{ijk}}$ represents $current\ manual\ rate$ in the cells (i, j, k).

Earned premium at current rates is denoted by P. Therefore,

$$P := \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{p} r_{ijk} e_{ijk}. \tag{2.3}$$

Then *current average rate*, denoted by R, is

$$R = \frac{P}{E}\,. \tag{2.4}$$

By summing premium of a particular slice, we get the following:

Vector of *premiums for class*, p^C, is defined by

$$p_i^C := \sum_{j=1}^{n} \sum_{k=1}^{p} r_{ijk} e_{ijk},\ i = 1, 2, \dots, m,$$

and vectors of *premiums for territory*, p^T and *industry* p^I are respectively defined by

$$p_j^T = \sum_{i=1}^{m} \sum_{k=1}^{p} r_{ijk} e_{ijk},\ j = 1, 2, \dots, n,$$

$$p_k^I = \sum_{i=1}^{m} \sum_{j=1}^{n} r_{ijk} e_{ijk},\ k = 1, 2, \dots, p.$$

The, *loss ratios* of the corresponding slices are:

$$lr_i^C = \frac{l_i^C}{p_i^C},\ i = 1, 2, \dots, m.$$

$$lr_j^T = \frac{l_j^T}{p_j^T},\ j = 1, 2, \dots, n.$$

$$lr_k^I = \frac{l_k^I}{p_k^I},\ k = 1, 2, \dots, p.$$

In this setting we will deal with two sets of differentials. Let the vectors

$$X = (x_1, x_2, \dots, x_m),\ Y = (y_1, y_2, \dots, y_n),\ Z = (z_1, z_2, \dots, z_p)\ .$$

represent **current** *class*, *territory* and *industry* differentials, respectively, and let vectors

$$\hat{X} = (\hat{x}_1, \hat{x}_2, \dots, \hat{x}_m),\ \hat{Y} = (\hat{y}_1, \hat{y}_2, \dots, \hat{y}_n),\ \hat{Z} = (\hat{z}_1, \hat{z}_2, \dots, \hat{z}_p)$$

represent **indicated** *class*, *territory* and *industry* differentials, respectively.

Let us call the cell **(1, 1, 1)** the *base cell*. The base cell is usually the cell with the largest exposure for the line of business so that it has maximal statistical credibility. Without loss of generality, we can denote: $base\ rate = r_{111}$. That means, we set $x_1 = y_1 = z_1 = 1$. We already mentioned that classification variables are applied multiplicatively. It means that the rate in the cell (i, j, k) is determined by

$$r_{ijk} = r_{111} x_i y_j z_k\ . \tag{2.5}$$

The indicated rates are denoted by $\hat{r}_{ijk}$ and it is assumed that model remains multiplicative after the rate change

$$\hat{r}_{ijk} = \hat{r}_{111}\hat{x}_i\hat{y}_j\hat{z}_k. \qquad (2.6)$$

3. Review of the Standard Rating Process

3.1 Overall Rate Change

If $\hat{R}$ denotes *indicated average rate* and if R denotes *current average rate*, then the *average rate change* (RC) is defined as

$$RC := \frac{\hat{R}}{R} - 1.$$

Let us introduce:

$$Expected\ Loss\ Ratio = \frac{Fully\ Developed\ and\ Trended\ Losses}{Earned\ Premium\ at\ Current\ Rates}.$$

Symbolically,

$$ELR = \frac{L}{P}.$$

In both *loss cost* and *loss ratio* method we need *permissible loss ratio*, PLR, which is defined as

$$PLR = 1 - Expence\ Ratio$$

It is well known that loss ratio and loss cost method produce the same overall rate change, see e.g. p.78 in [2]. It is easy to verify

$$\hat{R} = R\frac{ELR}{PLR} \qquad (3.1).$$

3.2 Changing Risk Classification Differentials

In loss ratio method, the *indicated differentials* are calculated as:

$$\hat{x}_i = x_i\frac{lr_i^C}{lr_1^C},\ i = 1, 2, \dots, m, \qquad (3.2C)$$

$$\hat{y}_j = y_j\frac{lr_j^T}{lr_1^T}, j = 1, 2, \dots, n, \qquad (3,2T)$$

$\hat{z}_k = z_k \frac{lr_k^I}{lr_1^I}, \forall k = 1, 2, \dots, p.$ (3,2I)

Formulae (3.2), the loss ratio method, better handle heterogeneity of the risks than loss cost method. Therefore we will use them for the further derivation of the new rating formula, GRF.

The focus of this paper is generalization of the most frequently used methods in ratemaking and not mitigation of effects of heterogeneity beyond what we already have with the standard methods. The goal is to make the ratemaking process easier. A discussion about concept of heterogeneity and its consequences can be found e.g. in [4], [5] and [6].

Let us introduce algebraic representations of the *adjusted exposures* E_i^C, E_j^T and E_k^I that will be used to mitigate effects of heterogeneity, equivalently to loss ratio method but without the use of the premiums.

The *class i adjusted exposure* is

$E_i^C := \sum_{j=1}^{n} \sum_{k=1}^{p} e_{ijk} y_j z_k,\ i = 1, 2, \dots, m.$

Similarly, *territory j* and *industry k adjusted exposures* are respectively

$E_j^T := \sum_{i=1}^{m} \sum_{k=1}^{p} e_{ijk} x_i z_k,\ \ j = 1, 2, \dots, n\ ,$

$E_k^I := \sum_{i=1}^{m} \sum_{j=1}^{n} e_{ijk} x_i y_j,\ \ k = 1, 2, \dots, p.$

The *loss cost adjusted for heterogeneity*, or simply *adjusted loss cost* for class i, is defined by:

$L_i^C := \frac{\sum_{j=1}^{n} \sum_{k=1}^{p} l_{ijk}}{E_i^C},\ \ i = 1, 2, \dots, m.$ (3.3C)

Similarly,

$L_j^T := \frac{\sum_{i=1}^{m} \sum_{k=1}^{p} l_{ijk}}{E_j^T}, j = 1, 2, \dots, n.$ (3.3T)

$L_k^I := \frac{\sum_{i=1}^{m} \sum_{j=1}^{n} l_{ijk}}{E_k^I}, k = 1, 2, \dots, p.$ (3.3I)

It holds

$lr_i^C = \frac{\sum_{j=1}^{n} \sum_{k=1}^{p} l_{ijk}}{\sum_{j=1}^{n} \sum_{k=1}^{p} r_{111} e_{ijk} x_i y_j z_k} = \frac{\sum_{j=1}^{n} \sum_{k=1}^{p} l_{ijk}}{x_i r_{111} E_i^C}\ .$ (3.4C)

Similarly,

$$lr_j^T = \frac{\sum_{i=1}^{m}\sum_{k=1}^{p} l_{ijk}}{y_j r_{111} E_j^T}, \qquad (3.4T)$$

$$lr_k^I = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} l_{ijk}}{z_k r_{111} E_k^I}. \qquad (3.4I)$$

3.3 Indicated Differentials without Premiums

In loss ratio method, we calculate indicated class differentials by

$$\hat{x}_i = x_i \frac{lr_i^C}{lr_1^C} = x_i \frac{\frac{\sum_{j=1}^{n}\sum_{k=1}^{p} l_{ijk}}{x_i r_{111} E_i^C}}{\frac{\sum_{j=1}^{n}\sum_{k=1}^{p} l_{1jk}}{r_{111} E_1^C}}, \; i = 1, 2, \dots, m,$$

After canceling out x_i and r_{111} and substituting expressions (3.3) we get

$$\hat{x}_i = \frac{L_i^C}{L_1^C}, \; i = 1, 2, \dots, m. \qquad (3.5C)$$

Similarly,

$$\hat{y}_j = \frac{L_j^T}{L_1^T}, \; j = 1, 2, \dots, n, \qquad (3,5T)$$

$$\hat{z}_k = \frac{L_k^I}{L_1^I}, \; k = 1, 2, \dots, p. \qquad (3,5I)$$

Remark 3.1 Formulae (3.5) calculate the same indicated differentials as the loss ratio method but without premium input. Note that in [2], *Incurred loss ratios* defined as *Dollars of incurred losses* divided by *Dollars of earned premiums at current rates* were used to derive indicated differentials, while in this paper losses are fully developed and trended. However, the development and trend factors are applied on both, numerators and denominators. Therefore, they cancel out in the above expressions for $\hat{x}_i$, $\hat{y}_j$, $\hat{z}_k$ and we get correct values for indicated differentials.

3.4 Balancing Back

Typically, the indicated differentials will be different from the current ones except $\hat{x}_1 = \hat{y}_1 = \hat{z}_1 = 1$.

In addition to that, the rates calculated by means of the indicated differentials may give different rate increase than the desired increase calculated in Step 1. In order to make a correction in the process we will have to multiply those pre-indicated rates by the *balance back factor* (BBF).

$$BBF := \frac{Existing\ Average\ Differential}{Indicated\ Average\ Diffrential}.$$

In our algebraic notation it is

$$BBF = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} x_i y_j z_k\, e_{ijk}}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} \hat{x}_i \hat{y}_j \hat{z}_k\, e_{ijk}}.$$

Example 3.2 Given:

Base rate: $r_{11} = 100$

*Trend*Develop. Factor:* $TDF = 1.409$

Permissible loss ratio*:* $PLR = 0.8$

Earned exposures**:**

Class	Territory1	Territory2
Class1	12,000	3,000
Class2	4,500	2,000

Current losses:

Class	Territory1	Territory2
Class1	840,000	300,000
Class2	500,000	250,000

Class differentials*:* $X = (x_1, x_2) = (1, 1.1)$

Territory differentials*:* Y= $(y_1, y_2) = (1, 1.15)$

Calculate new rates.

Solution by standard loss cost method adjusted for heterogeneity

Step 1, Overall Rate Change:

Multiplying current losses by the Trend*Development Factor = 1.409 we get array (matrix) of fully developed and trended losses.

$$\boldsymbol{L} = \begin{pmatrix} 1{,}183{,}602.74 & 422{,}715.26 \\ 704{,}525.44 & 352{,}262.72 \end{pmatrix}$$

Then, total FDTL is the sum $L = \sum_{i=1}^{2} \sum_{j=1}^{2} l_{ij} = 2{,}663{,}106.16$.

Let us first calculate $ELR = \frac{L}{P}$. According to (2.3) we have:

$P = 2{,}293{,}000$,

$ELR = 1.1614$.

According to (3.1) we have the average rate increase:

$$\frac{\hat{R}}{R} = \frac{ELR}{PLR} = \frac{1.1614}{0.8} = 1.452$$

Step 2, Calculation of Indicated Differentials

In order to calculate indicated differentials adjusted for heterogeneity we first have to calculate:

$E_1^C = \sum_{j=1}^{2} e_{1j} y_j = 15{,}450$

$E_2^C = \sum_{j=1}^{2} e_{2j} y_j = 6{,}800.$

$E_1^T = \sum_{i=1}^{2} e_{i1} x_i = 16{,}950$

$E_2^T = \sum_{i=1}^{2} e_{i2} x_i = 5{,}200$

According to equations (3.3), simplified to two rating variables, we have:

$$L_1^C = \frac{\sum_{j=1}^{2} l_{1j}}{E_1^C} = 103.97$$

$$L_2^C = \frac{\sum_{j=1}^{2} l_{2j}}{E_2^C} = 155.41$$

$$L_1^T = \frac{\sum_{i=1}^{2} l_{i1}}{E_1^T} = 111.39$$

$$L_2^T = \frac{\sum_{i=1}^{2} l_{i2}}{E_2^T} = 149.03$$

Therefore,

indicated class differentials: $\hat{x}_1 = 1, \hat{x}_2 = \frac{L_2^C}{L_1^C} = 1.495,$

indicated territory differentials: $\hat{y}_1 = 1$, $\hat{y}_2 = \frac{L_2^T}{L_1^T} = 1.338$.

Step 3, Balance Back Factor

$$BBF = \frac{Existing\ Average\ Differential}{Indicated\ Average\ Diffrential} = 0.85745$$

Finally we can calculate the indicated rates:

$\hat{r}_{11} = 100 * 1.452 * 0.85745 = 124.5$, etc.

$\hat{r}_{12} = 166.58$,

$\hat{r}_{21} = 186.12$,

$\hat{r}_{22} = 249.03$. □

We will later solve the same task by means of GRF in order to compare GRF with standard method.

4. General Rating Formula

4.1 Alternative Proof of the GRF

In the previous sections we introduced notation, terminology and equations that will enable us to finalize the derivation of the General Rating Formula. Recall, that the goal is to find an easier way to calculate same rates that are calculated by the standard three-step process.

The following proof of GRF is shorter and easier than the original proof and we will call it the "Alternative Proof of GRF". The original proof of GRF is longer but that proof has an advantage over the following proof. Namely, ***in the original proof we also prove that GRF produces the same rates as standard loss ratio method***, while from the following short derivation of GRF this is not so visible.

Alternative proof of the GRF: By definition of indicated premium in terms of rates and exposures

$$\hat{P} = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{p} \hat{r}_{ijk} e_{ijk},$$

By definition of indicated premium in terms of losses and expenses

$$\hat{P} = \frac{L}{PLR}.$$

After we substitute first expression for $\hat{P}$ into the second equation we get

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p}\hat{r}_{ijk}e_{ijk} = \frac{L}{PLR}.$$

From the assumption (2.6), it follows

$$\hat{r}_{111}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p}\hat{x}_i\hat{y}_j\hat{z}_k e_{ijk} = \frac{L}{PLR}.$$

$$\hat{r}_{111} = \frac{L}{PLR} * \frac{1}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p}\hat{x}_i\hat{y}_j\hat{z}_k\, e_{ijk}}.$$

Then from (2.6) it follows

$$\hat{r}_{ijk} = \frac{L}{PLR}\frac{1}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p}\hat{x}_i\hat{y}_j\hat{z}_k\, e_{ijk}}\hat{x}_i\hat{y}_j\hat{z}_k. \quad (4.1)$$

Let us further simplify (4.1). Substituting $\hat{x}_i, \hat{y}_j, \hat{z}_k$ from (3.5) we get

$$\hat{r}_{ijk} = \frac{L}{PLR}\frac{1}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p}\frac{L_i^C L_j^T L_k^I}{L_1^C L_1^T L_1^I}\, e_{ijk}}\frac{L_i^C}{L_1^C}\frac{L_j^T}{L_1^T}\frac{L_k^I}{L_1^I}$$

After canceling out constant $L_1^C L_1^T L_1^I$ we finally get the formula

$$\hat{r}_{ijk} = \frac{L}{PLR}\frac{L_i^C L_j^T L_k^I}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} L_i^C L_j^T L_k^I\, e_{ijk}}. \quad (4.2)$$

Note that the number

$$M := \frac{L}{PLR\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} L_i^C L_j^T L_k^I\, e_{ijk}} \quad (4.3)$$

is a constant; therefore, we calculate it only one time. Then we calculate rates by the simple formula

$$\hat{r}_{ijk} = M L_i^C L_j^T L_k^I\ ,\ i = 1, \dots, m;\ j = 1, \dots, n;\ k = 1, \dots, p. \quad (4.4)$$

We already mentioned that formula (4.2) (i.e. (4.4)) was originally derived from the standard loss ratio method so that we were able to see that it calculates the same rates as the loss ratio method.

The formulae (4.2) and (4.4) are different forms of the same formula which we can call GRF. This formula theoretically calculates exactly the same rates as standard loss ratio method. However, this formula calculates **more accurate rates in practice** because it is difficult to obtain in practice high quality manual rate and manual premium data needed for the standard re-rating calculations. Therefore, GRF is a generalization and improvement of the standard rating methods in the mathematical sense.

Note here that good quality data needed for the new method: exposures, claims and current risk factors are readily available while this is not always the case with manual rates, premiums and loss ratios, the input data that we needed in standard ratemaking process. Sometimes only "sold" rates and premiums are recorded in the company's database, as was explained in the introduction.

The new rating formula is especially convenient for rating a new product because current rates are not available for that calculation, and the new formula does not need them anyway. It calculates indicated rates directly from the assumptions about exposures, losses, PLR and risk factors.

Remark about generalization: In general case, when there are N>3 rate classification parameters: *class, territory, …, industry*, i.e. industry is N^{th} rather than third rate classification parameter, then the formula for rates is

$$\hat{r}_{ij\ldots k} = \frac{L}{PLR} \frac{L_i^C L_j^T \ldots L_k^I}{\sum_{i=1}^m \sum_{j=1}^n \ldots \sum_{k=1}^p L_i^C L_j^T \ldots L_k^I \, e_{ij\ldots k}} .$$

As before, the formula can be written equivalently in two parts as:

$$M = \frac{L}{PLR \sum_{i=1}^m \sum_{j=1}^n \ldots \sum_{k=1}^p L_i^C L_j^T \ldots L_k^I \, e_{ij\ldots k}}, \quad (4.5)$$

$$\hat{r}_{ij\ldots k} = M L_i^C L_j^T \ldots L_k^I . \quad (4.6)$$

Example 4.2 (Re-rating by the new formula) *Given*

*Trend*Develop. Factor:* $TDF = 1.409$

Permissible loss ratio: $PLR = 0.8$

*Earned exposures***:**

	Territory1	***Territory2***
Class1	*12,000*	*3,000*
Class2	*4,500*	*2,000*

Current losses:

	Territory1	***Territory2***
Class1	*840,000*	*300,000*

Class2 *500,000* *250,000*

Class differentials: $X = (x_1, x_2) = (1, 1.1)$

Territory differentials: $Y = (y_1, y_2) = (1, 1.15)$

Calculate new rates.

Solution by new rating formula:

Note, base rate r_{11} is not given; it is not needed.

Multiplying current losses by the Trend*Development Factor = 1.409 we get

$L = \sum_{i=1}^{2} \sum_{j=1}^{2} l_{ij} = 2{,}663{,}106.16.$

$E_1^C = \sum_{j=1}^{2} e_{1j} y_j = 15{,}450$

$E_2^C = \sum_{j=1}^{2} e_{2j} y_j = 6{,}800.$

$E_1^T = \sum_{i=1}^{2} e_{i1} x_i = 16{,}950$

$E_2^T = \sum_{i=1}^{2} e_{i2} x_i = 5{,}200$

$L_1^C = \frac{\sum_{j=1}^{2} l_{1j}}{E_1^C} = 103.97,$

$L_2^C = \frac{\sum_{j=1}^{2} l_{2j}}{E_2^C} = 155.41,$

$L_1^T = \frac{\sum_{i=1}^{2} l_{i1}}{E_1^T} = 111.4,$

$L_2^T = \frac{\sum_{i=1}^{2} l_{i2}}{E_2^T} = 149.$

Now, we calculate indicated rates directly by formulae (4.5) and (4.6), for 2 risk factors.

$D := \sum_{i=1}^{2} \sum_{j=1}^{2} L_i^C L_j^T \ e_{ij} = 309{,}680{,}899$

$M = \frac{L}{PLR*D} = \frac{2{,}663{,}106.16}{0.8*309{,}680{,}899} = 0.0107494$

$\hat{r}_{ij} = M L_i^C L_j^T$

$\hat{r}_{11} = M L_1^C L_1^T = 124.50;$

$\hat{r}_{12} = M L_1^C L_2^T = 166.52;$

$\hat{r}_{21} = ML_2^C L_1^T = 186.10;$

$\hat{r}_{22} = ML_2^C L_2^T = 248.91.$ □

Note that this calculation has all the advantages mentioned after the proof of the formula (4.4) including the fact that we did not need to calculate indicated differentials. However, if we need to know indicated differentials, they are given by formulae (3.5).

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