Suboptimality of Asian Executive Indexed Options

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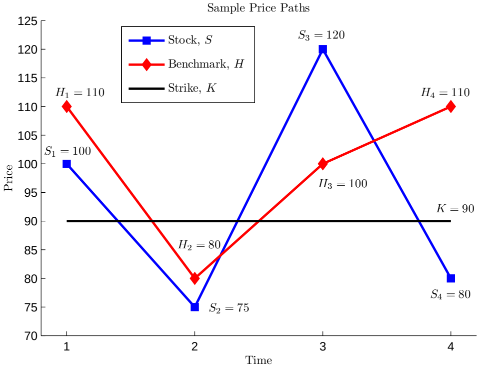
Options Preliminaries



- $\hat{S}_4 = \sqrt[4]{S_1 S_2 S_3 S_4} = 92.12, \; \hat{H}_4 = \sqrt[4]{H_1 H_2 H_3 H_4} = 99.19$
- European Call Option Payoff $= \max(S_4 - K, 0) = 0$
- Asian Option Payoff $= \max(\hat{S}_4 - K, 0) = 2.12$
- Asian Indexed Option Payoff $= \max(\hat{S}_4 - \hat{H}_4, 0) = 0$

Assumptions

1. Black-Scholes market:
   - Extension to Vasicek short rate
2. Stock S_t and benchmark H_t driven by Brownian motions
3. Existence of state-price process ξ_t
4. Agents preferences depend only on the terminal distribution of wealth

Asian Executive Indexed Option

Asian Executive Indexed Option (AIO) proposed by Tian (2011):

- *Averaging*: Prevent stock price manipulation
- *Indexing*: Only reward out-performance
- More cost-effective than traditional stock options
- Provide stronger incentives to increase stock prices

Construct a better payoff:

- Same features as the AIO
- Strictly cheaper
- Use the concept of **cost-efficiency**

Cost-Efficiency

From Bernard, Boyle and Vanduffel (2011):

Definition (1)

The **cost** of a strategy with terminal payoff X_T is given by

$$c(X_T) = E_{\mathbb{P}}[\xi_T X_T]$$

where the expectation is taken under the physical measure $\mathbb{P}$.

Intuition: ξ_T represents the price of a particular state

Definition (2)

A payoff is **cost-efficient** (CE) if any other strategy that generates the same distribution costs at least as much.

Cost-Efficiency

Theorem (1)

Let ξ_T be continuous. Define

$$Y_T^\star = F_{X_T}^{-1}(1 - F_{\xi_T}(\xi_T))$$

as the **cost-efficient counterpart** *(CEC) of the payoff X_T. Then, $Y_T^\star$ is a CE payoff with the same distribution as X_T and is almost surely unique.*

Intuition: CEC is achieved by reshuffling the outcome of X_T in each state in reverse order with ξ_T while preserving the original distribution

Constructing a Cheaper Payoff

1. Apply Theorem 1 to each term of the AIO

$$\hat{A}_T = \max(\hat{S}_T - \hat{H}_T, 0)$$

   to get

$$A^\star_T = \max\left(d_S S_T^{1/\sqrt{3}} - d_H H_T^{1/\sqrt{3}}, 0\right)$$

2. It can be shown that:
   - $\hat{A}_T \overset{d}{=} A^\star_T$
   - $A^\star_T$ costs strictly less than $\hat{A}_T$

$A^\star_T$ inherits the desired features of $\hat{A}_T$, but comes at a cheaper price

True Cost Efficient Counterpart

True CEC

$$A_T = F^{-1}_{\hat{A}_T}(1 - F_{\xi_T}(\xi_T))$$

is estimated numerically

Examples:

1. Empirical cumulative distribution functions (CDFs) for each payoff in the base case [1]
2. Reshuffling of $\hat{A}_T$ to $A^\star_T$ and A_T
3. Order of $\hat{A}_T$, $A^\star$ and A_T vs ξ_T
4. Price of each payoff and the efficiency loss

[1] $K = 100$, $S_0 = 100$, $r = 6\%$, $\mu_S = 12\%$, $\mu_I = 10\%$, $\sigma_S = 30\%$, $\sigma_I = 20\%$, $\rho = 0.75$, $q_S = 2\%$, $q_I = 3\%$, $T = 1$

Numerical Results

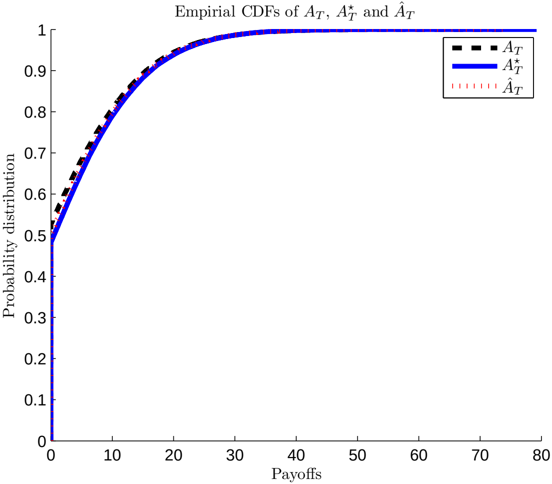


Figure: Comparison of the CDFs of A_T, $A_T^\star$ and $\hat{A}_T$.

Numerical Results

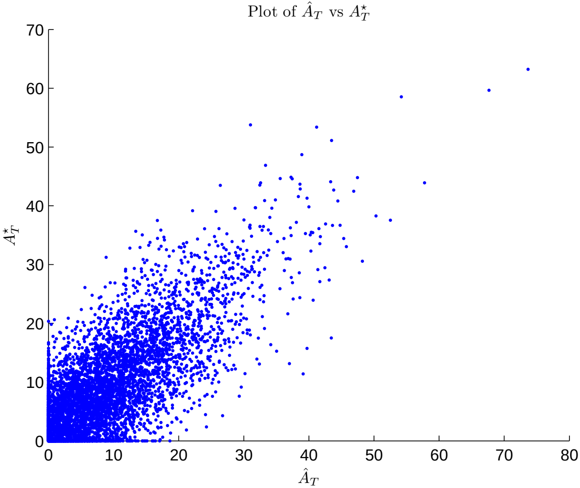


Figure: Reshuffling of outcomes of $\hat{A}_T$ to $A_T^\star$

Numerical Results

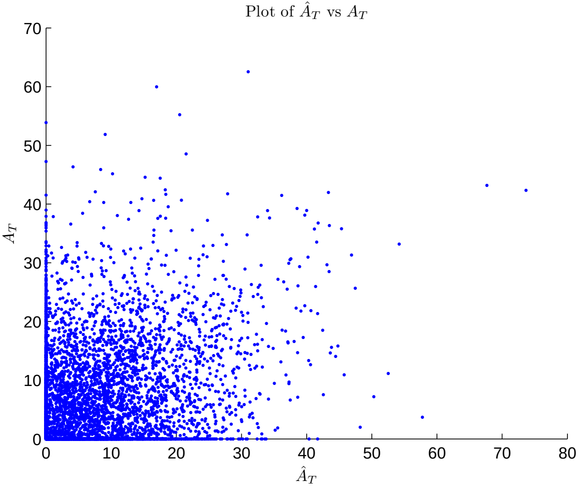


Figure: Reshuffling of outcomes of $\hat{A}_T$ to A_T

Numerical Results

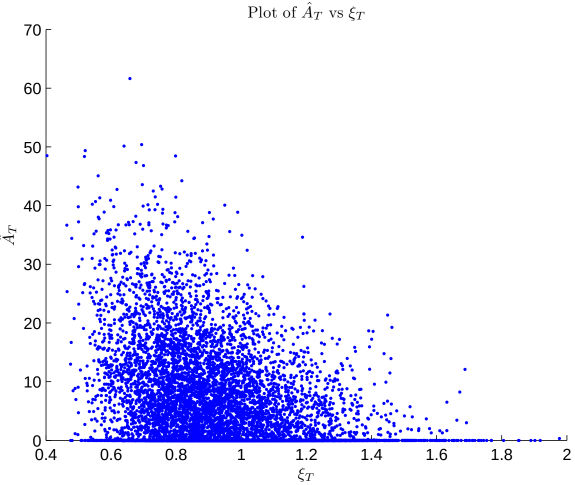


Figure: Plot of outcomes of $\hat{A}_T$ vs ξ_T

Numerical Results

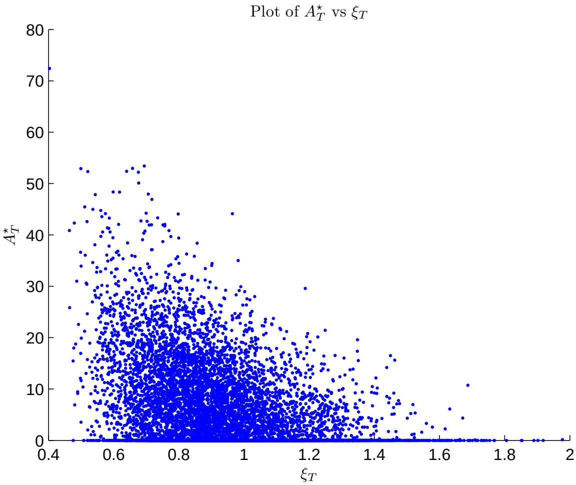


Figure: Plot of outcomes of $A_T^\star$ vs ξ_T

Numerical Results

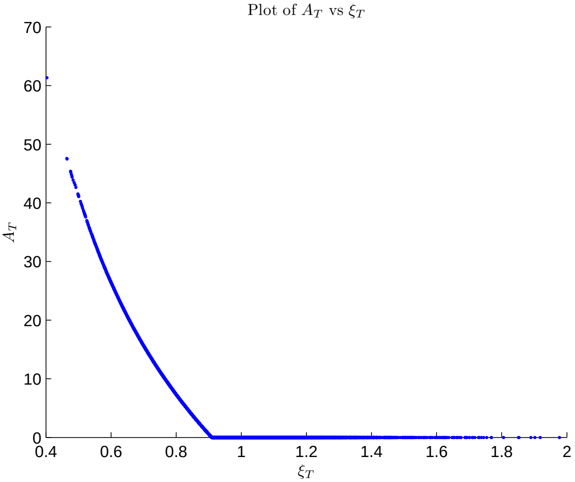


Figure: Plot of outcomes of A_T vs ξ_T

Numerical Results

Case	A_T	$A_T^\star$		$\hat{A}_T$	
	V_T	$V_T^\star$	Eff Loss	$\hat{V}_T$	Eff Loss
Base Case	3.26	4.34	33%	4.36	34%
$r = 4\%$	2.96	4.37	48%	4.40	49%
$\mu_S = 8\%$	3.97	4.35	10%	4.36	10%
$\mu_I = 13\%$	3.26	4.34	33%	4.36	34%
$\sigma_S = 35\%$	3.97	5.04	27%	5.07	28%
$\sigma_I = 15\%$	3.27	4.34	33%	4.36	33%
$\rho = 0.9$	2.28	2.86	25%	2.87	26%
$q_S = 1.5\%$	3.27	4.35	33%	4.37	34%
$q_I = 2\%$	3.25	4.34	33%	4.36	34%

Table: Prices and efficiency loss of $A_T^\star$ and $\hat{A}_T$ compared against A_T across different parameters.

Stochastic Interest Rates

Extension to a market with Vasicek short rate:

1. State price process expressed as a function of market variables
2. Pricing formula for the AIO

Summary

- Reviewed the use of *averaging* and *indexing* in the context of executive compensation
- Constructed a strictly cheaper payoff with the same features as the AIO using **cost-efficiency**
- Numerical examples that illustrate reshuffling of payoffs and loss of efficiency
- Extension to the case of stochastic interest rates