Hedging Salary-Related Pension Benefits

by

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Abstract

In this paper, we consider the relationship between salary growth, stock and long bond returns. We use this to suggest that equities may have a role in hedging final salary-related benefits. Simulations from the empirical distribution for salaries, equities and interest rates are used to demonstrate the quantile hedge.

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Chapter 1

Introduction

To reduce the pension plan risk, both financial economists and traditional actuaries focus on hedging the future pension plan liabilities. They propose different models to match the future values of assets and liabilities. This paper introduces the efficiency of using stocks or bonds to hedge the future liabilities when liabilities are salary related.

The pension fund managers construct portfolios to hedge the liabilities based on financial assets such as stocks, bonds, options, and swaps. For example, to hedge their pension liabilities, the Boots Company invested all of the pension plan fund in bonds(Ralfe 2002). Since bonds are less volatile security in the financial market than stocks, the Boots Company strategy did an excellent hedging job, because there was very little uncertainty in the future pension plan liabilities. The problem with this method is that there are sometimes not enough eligible bonds to invest, since the pension plan liabilities are usually very long term. Moreover, in the defined benefit pension plan, the future liabilities are generally related to the final salary, so that the future benefits are not fixed.

This paper considers which types of financial assets are related to final salary. Chapter 2 considers the hedging problem, and discusses what kind of benefits are provided by a typical pension plan. Chapter 3 analyzes the historical data and considers the financial assets used to hedge the liability. Chapter 4 discusses the results from simulation under the static strategy. Chapter 5 introduces another model and uses it to examine the results. Chapter 6 is our conclusion.

Chapter 2

Description of Benefits

This chapter considers the benefits of the pension plan. There are two basic kinds of pension plans used by the pension plan fund managers: Defined Benefit(DB) and Defined Contribution(DC). The most important difference between these two pension plans is that the employers bear the investment risk in DB plan and the employees bear the investment risk in DC plan.

In this paper, we only consider the investment risk for a DB pension plan. In a DB pension plan, the future pension benefits are usually based on the final average salary and the years of service. We assume the normal retirement age is 65. After retirement, the pension benefits will be paid monthly. We also assume that the fund managers can purchase an appropriate annuity to pay the benefits to employees at retirement. So we only consider the actuarial present value of retirement benefits at retirement. The projected effective interest rates after NRA[1] is around 6% per year, and the mortality rate are determined using the Illustrative Life Table from Actuarial Mathematics[3]. The annuity factor $\ddot{a}_{65}^{(12)}$ is the actuarial present value of a whole life annuity of 1 payable at the beginning of each month while the employee dies.

The value of the future benefits at age 65 is defined as following:

$$FB = \alpha \times S \times n \times \ddot{a}_{65}^{(12)} \tag{2.1}$$

where:

FB: The actuarial value at age 65 of future benefits

α: The accrual rate

[1]NRA: Normal Retirement Age

S: The final salary at age 65
n: The years of service to age 65
$\ddot{a}_{65}^{(12)}$: The annuity factor

The accrual rate α, the years of service n and the annuity factor $\ddot{a}_{65}^{(12)}$ are all assumed known. In fact, these are variable. For simplicity, at this stage we treat them as constant. The uncertainty in the value of future benefits arises from the uncertainty in the final salary at retirement. The higher the final salary, the higher pension benefits.

At each annual valuation day, we assume the amount of the contribution is also determined by the salary in last year. Let annual salary growth in the t^{th} year be s_t. Under this assumed salary growth, the value of future benefits at age 65 is re-defined.

$$FB = \alpha \times s \times \prod_{t=1}^{n-1}(1 + s_t) \times n \times \ddot{a}_{65}^{(12)} \tag{2.2}$$

where:

s: The salary at the inception
s_t: The annual salary growth rate in the t^{th} year

The next chapter considers what kind of financial assets can be used to match the value of benefits at retirement.

Chapter 3

Hedging Assets

This chapter analyzes the historical data and introduces how to construct the hedging portfolio to match the value of assets and liabilities at retirement.

3.1 Data Analysis

In this paper, we consider two historical sets of data: Canadian annual data and American monthly data in the latest 10 years.

In Canadian annual data, we consider four indexes of salaries, stocks, long bonds and inflations. In American monthly data, we only considers salaries, government bonds and the S&P 500 index. This data is more frequently than the annual data.

Under the Canadian annual data, we assume the shorting values of four indexes are 100 each. So we can draw a graph to see the accumulated tendency of these four indexes in the latest 50 years which is from 1954 to 2003.

From figure 3.1, it is hard to find the correlation between the stock index, long bonds and the salary index. We can see that stocks have increased far more than bonds, and that salaries and inflations have increased the least. Also, the mean increase rate of salaries is higher than inflation.

To see the correlation easily, we compare the annual increase rate of these four indexes. Since the stock index and the long bonds are more volatile than the salary

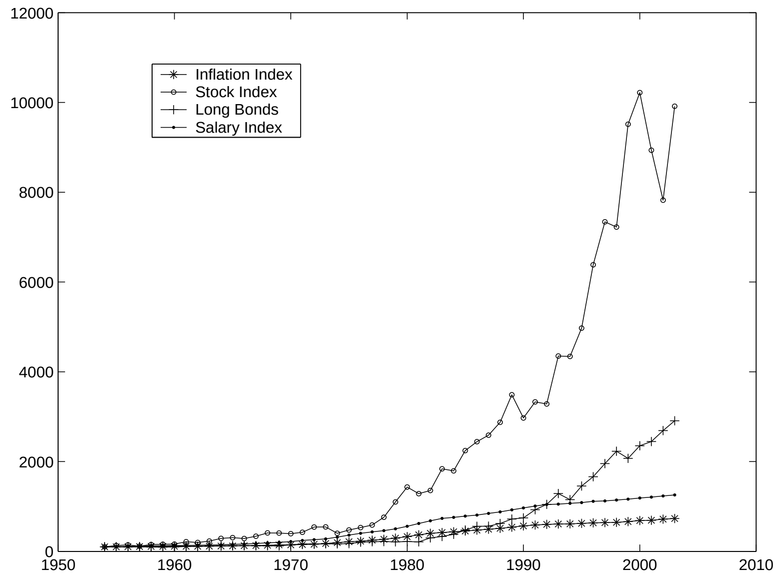

Figure 3.1: The accumulated values of four indexes in the latest 50 years

and the inflation. We use 7-year average value of the stock index and the long bonds. See figure 3.2.

The increase rates of salaries and inflation are quite similar in figure 3.2. The correlation coefficient between the increase rate of these two indexes is 0.7905. Therefore, we may be able to use the inflation-linked bonds to hedge the pension benefits which are based on the final salary. Note however that the graph plots inflation, nor the returns on inflation-linked bonds, which will exhibit more volatility than inflation in general.

Since the maturity period of liabilities is typically around 30 years, we also consider the trend of these four assets chosen in the latest 30 years. Figure 3.3 and figure 3.4 show the accumulated values and the annual increase rates of these four indexes respectively.

In these two figures, we can see the correlation between the salary and the inflation clearly. The correlation coefficient between the increase rate of these two indexes is 0.9041.

We also consider the correlation between the salary and the two assets classes: the stock index and the long bonds. We find that the stock index has a higher

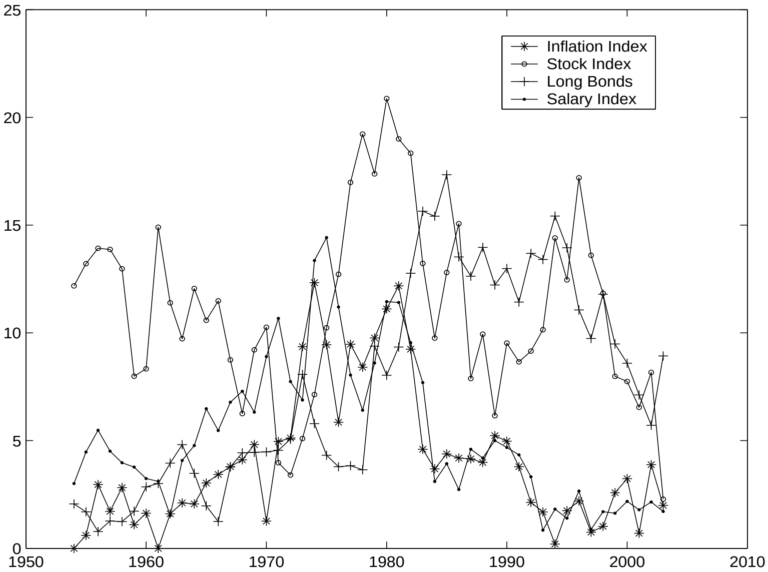

Figure 3.2: The annual increase rates of four indexes in the latest 50 years

correlation with salaries than bonds, though much lower than inflations.

Here we have the two tables:

Corr Coefficient	Inflation Index	Stock Index	Long Bonds
Salary Index	0.7905	0.1540	-0.2441

Table 3.1: The correlation between four Canadian indexes in the latest 50 years

Corr Coefficient	Inflation Index	Stock Index	Long Bonds
Salary Index	0.9041	0.3758	-0.4631

Table 3.2: The correlation between four Canadian indexes in the latest 30 years

Also, we differently consider the monthly data from 1994 to 2003, since the more frequent data provides information. However, the monthly data is more volatile than the annual data. We can't see any trend of salaries, stock indexes and long bonds from that 10-year monthly data. We have smoothed the monthly data and calculate the annual increase rates of these assets in figure 3.5.

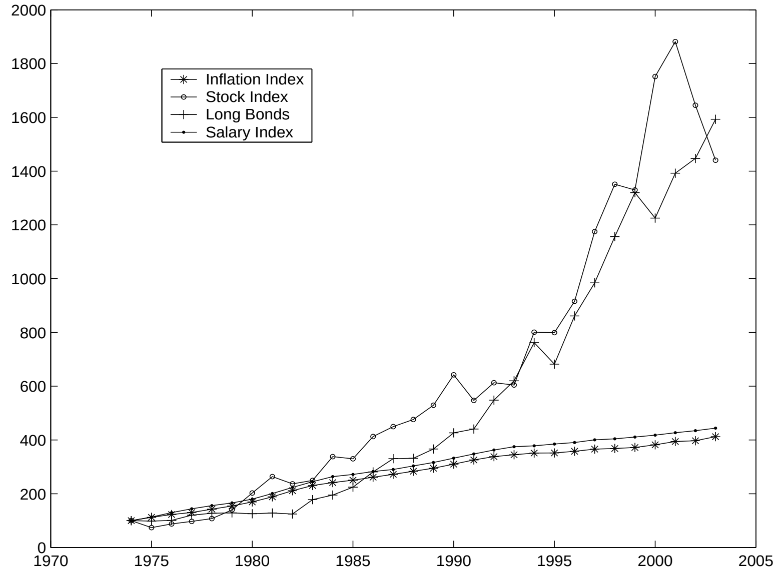

Figure 3.3: The accumulated values of four indexes in the latest 30 years

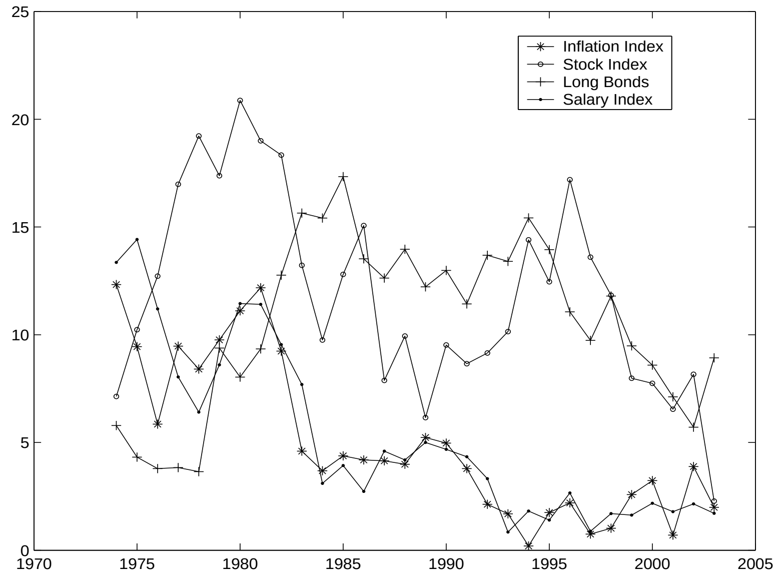

Figure 3.4: The annual increase rates of four indexes in the latest 30 years

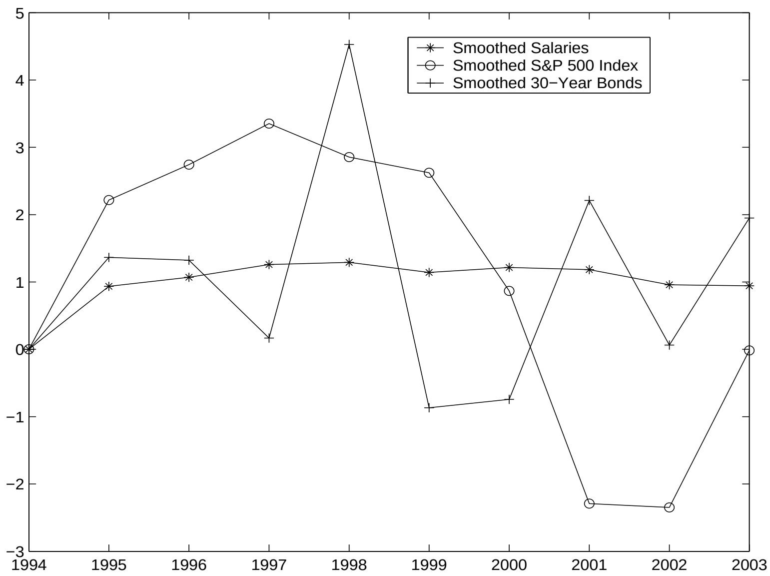

Figure 3.5: The annual increase rates of three smoothed indexes

From figure 3.5 and figure 3.3, we find stock indexes increased much faster in the 1990's and decreased after 2000. We will compare these two special situations in later chapters.

3.2 Selection of Hedging Assets

In current approaches used by pension fund managers to hedge the pension plan liabilities, almost every kind of financial assets can be used to construct the hedging portfolio. For example, all of the three assets(the inflation-linked bonds, the stock index, the long bonds) can be used to hedge. The selection of the hedging assets should be determined by the nature of benefits at retirement.

If the amount of the retirement benefit is fixed, it is easy to hedge the liabilities. Regular long-term government bonds can be used to hedge the fixed pension benefits. In July 2001, the Boots Company moved their pension plan fund to a 100% bond portfolio. Since the default risk of bonds is low, the value of assets at retirement is almost certain. The fund managers can match the values of assets and fixed liabilities by using the government bonds. If the value of future benefits is not variable, bonds with fixed return will hedge the liabilities perfectly. The risky assets or the combination of the risky assets and the risk-free assets may provide a closer match.

The pension plan benefits considered in this paper are based on final salaries. Since the final salary plays a significant role in the pension benefits, the increase rate of the hedging portfolio should be highly associated with the salary growth. From the data analysis, it is easy to find that the best hedging asset would be inflation-linked bonds which have high correlation with the salary growth.

However, inflation-linked bonds are not overwhelmingly used in practice. The main problems is due to the availability. There may also be more subjective reasons, to do with the performance measurement of fund managers, behind the apparent unattractiveness of the inflation.

The stock index and the long bond index represent two popular financial assets in pension fund management. Since the stock index has the higher correlation with the salary growth than the long bonds, the stock index should provide a better hedging job of pension plan liabilities. The S&P 500 index is the weighted-average of 500 major stocks in the financial market. It should reflect the main direction of the US economy(and therefore the global economy). The better the economy is, the greater salaries grow, and the greater the increase rate of the S&P 500 index is. By observation, the association appears stronger in a strong economy, such as the 1990's. During 1990's, the correlation coefficient between the salary growth and the increase rate of the S&P 500 index is around 70%.

Comparing bonds and stocks as a salary hedge, we note the advantages and the disadvantages of stock indexes.

Advantages:

1. *Higher correlation with the salary increase.*

   It is the most important reason that we use stocks to replace bonds. Since the stock indexes have a higher correlation with salary growth, the values of hedging assets and pension plan liabilities, the future value of assets and liabilities should be a closer match.

2. *The availability.*

   Pension fund managers can invest the funds into the stock index at every business day. And they can hold the assets as a long time as they want. But bonds have their own issue date and maturity date. Sometimes the companies can not hold bonds until the retirement date since the bonds mature early.

Disadvantage:

1. *High volatility.*

   It is obvious that stock indexes are more volatile than salaries. The high volatility of the stock indexes causes the uncertain value in the future. Hence, it is impossible to find the perfect hedging portfolio.

Chapter 4

Hedging using a bivariate lognormal model

This chapter introduces our proposed method. Before describing that, we discuss the assumptions and the hedging problem again.

4.1 Assumption

In this section, the accrual rate, the projected effective interest rate after retirement and the mortality rate are assumed fixed. Salaries are assumed to increase annually, by some random amount. The fund managers can find the appropriate annuity to pay the benefits to employees. So the value of future benefits at retirement is defined as equation 2.2.

Also, the contribution rate is fixed and pre-determined. To make the problem simpler, we assume in this section the contribution rate c is equal to the product of the accrual rate α and the annuity factor $\ddot{a}_{65}^{(12)}$. So the contribution rate $c = \alpha \times \ddot{a}_{65}^{(12)}$ can be cancelled. In practice, traditional actuarial techniques might give a contribution rate rather lower than this, in general.

4.2 Hedging Problem

To simplify the hedging problem, this paper considers an individual pension plan with only one active member. The pension plan has the inception age 35 and the normal retirement age is 65. The salary in the t^{th} year is denoted S_t. Hence the contribution of the employee at the start of the year t is $c \times S_t$, where c is the contribution rate. The contributions will be invested to meet the liability at the normal retirement age. We simulate the process of the salary growth and the increase rate of the stock index to examine the difference between the future values of assets and liabilities at retirement, and try to minimize the investment risk to make sure that the future values of assets and liabilities are matched.

4.3 Description of Model

From the definition of the future benefits, we divide the future benefits into n parts, where the amount of each part is equal to the final salary(S_n) times the other factor($\alpha \times \ddot{a}_{65}^{(12)}$) and n is the years of service. On the other hand, the employee should contribute the amount of $c \times S_t$ each year of service to guarantee his own benefits at retirement. In our model, all of the contributions will invest into the stock index and hold to the retirement date. The future value of assets at retirement also has n parts. The amount of each part is equal the accumulated amount of the contribution in every year. Let ST_t be an index of stocks at start of year t and no fund at time 0. Therefore, the future value of assets at retirement (A_n) can be expressed by the sum of n parts:

$$A_n = ST_n \times \sum_{t=1}^{n} Share_t \tag{4.1}$$

where:

$$Share_t{}^1 = \frac{the\ amount\ of\ contribution\ at\ year\ t}{the\ price\ of\ the\ stock\ index\ at\ year\ t}$$

$$ST_n : The\ projected\ price\ of\ the\ stock\ index\ at\ retirement$$

[1] Assume the share of the stock index can be split.

$$n : The\ years\ of\ service\ to\ normal\ retirement\ age\ 65$$

We assume the projected price increases of the salary(S_t) and the stock index(ST_t) follow a 2-dimension lognormal distribution:

$$s_t = S_t/S_{t-1}; \qquad r_t = ST_t/ST_{t-1};$$

$$\begin{pmatrix} s_t \\ r_t \end{pmatrix} \sim Lognormal\left(\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}\right)$$

The future value of assets can be expressed as:

$$A_n = ST_0 \times \prod_{t=1}^{n} r_t \times \sum_{i=1}^{n} \frac{c \times S_0 \times \prod_{t=1}^{i} s_t}{ST_0 \times \prod_{t=1}^{i} r_t} \tag{4.2}$$

where:

ST_0 is the price of the stock index at beginning.
r_t is the projected price increase of the stock index at year t.

Therefore, there are two series of random variables in this hedging model: the projected salary increase rate(s_t) and the projected increase rate of the stock index(r_t). They determine the future values of benefits and assets. We use the stochastic model to generate these two random variables. These two series of random variables follow the 2-dimension lognormal distribution with the mean vector $\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}$ and the variance matrix $\begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}$. The parameters could be determined by the historical data observed from the website.

4.4 Problem of the Model

Using this method, there is substantial basis risk, because the salary increase rate and the increase rate of the stock index both follow different stochastic processes.

To assess this method, we consider the probability that the value of assets is greater than the value of liabilities at retirement.

From equation 2.2 and equation 4.2, we are interested in the ratio of the future value of liabilities and the future value of assets. Equivalently, the probability of this ratio less than 1 is equal to the probability that the value of assets is greater than the value of liabilities. We consider these two measures to assess the hedging job.

We call this ratio '*Hedging Ratio*' in this paper.

where $Hedging\ Ratio = \frac{The\ value\ of\ liabilities\ at\ retirement}{The\ value\ of\ assets\ at\ retirement}$

4.5 Estimation of Parameters

In the proposed model, there are 6 parameters need to be pre-determined. The five of six parameters are the parameters in the stochastic model to simulate the increase rate of the stock index and the salary increase rate. The means, variances, and correlation coefficient are determined using the American historical monthly data from 1994 to 2003. The choice of these five parameters influences the final results significantly, especially the correlation coefficient. This is why we choose the financial assets which have the high correlation with the salary growth. The mean and the variance of the increase rate of the stock index are also important. In the later of this chapter, we will show the effect of different choices by the different values of parameters.

Another parameter is the contribution rate c, we assume this rate is equal to the accrual rate times the annuity factor($c = \alpha \times \ddot{a}_{65}^{(12)}$). This makes sense. In practice, the accrual rate is around 1.5% and the annuity factor is between 9 and 10. Hence the contribution rate is between 12% and 15% of the annual salary. The future benefit can be expressed by the projected final salary(S) times years of service(n) times the contribution rate($c = \alpha \times \ddot{a}_{65}^{(12)}$). The contribution rate appears both on the expression of the future liabilities and the future assets. They will be cancelled when we consider the *Hedging Ratio*. The assumption of the accrual rate and the mortality can be neglected. And the assumption that the share of the stock index can be split can affect the final result slightly. We use this assumption to make the calculation easier.

We use two different parameter sets:

1. Using the annual data in Canada from 1974 to 2003
2. Using the monthly data in US from 1994 to 2003

We also apply a sensitivity test in a later section.

4.6 Case 1: Using the annual data in Canada from 1974 to 2003

In this case, the parameters are determined by the annual data in Canada. We use a 30-year data set from 1974 to 2003. The annual salary increase rate and the increase rate of the stock index follow the lognormal distribution with the mean vector $\begin{pmatrix} 0.051191223 \\ 0.098522832 \end{pmatrix}$ and the variance matrix

$$\begin{pmatrix} 0.031731869^2 & 0.3662 \times 0.031731869 \times 0.153363695 \\ 0.3662 \times 0.031731869 \times 0.153363695 & 0.153363695^2 \end{pmatrix}.$$

The valuation date is assumed to occur at the start of each year. The active member is assumed to pay the contribution, and receives a salary increase at the valuation date. The accrual rate ($\alpha = 1.5\%$) and the annuity factor ($\ddot{a}_{65}^{(12)} = 9.43$) are both assumed.

Therefore, the single active member will pay the contribution which is about 14.15% of his annual salary to the accumulated fund each year. And he will be paid a pension of 45% of his final annual salary after the retirement.

The result in this case is fine. The mean of the *Hedging Ratio* is 0.5172. Figure 4.1 shows the distribution of the *Hedging Ratio*.

From figure 4.1, the *Hedging Ratio* has high probability of it less than 1. This means that the probabilities that assets exceed liabilities at retirement is 0.9481 in this case.

We also consider a strategy of 100% of investment in long bonds. Using the long bonds index implicitly assumes the bonds are not held to maturity, but are sold

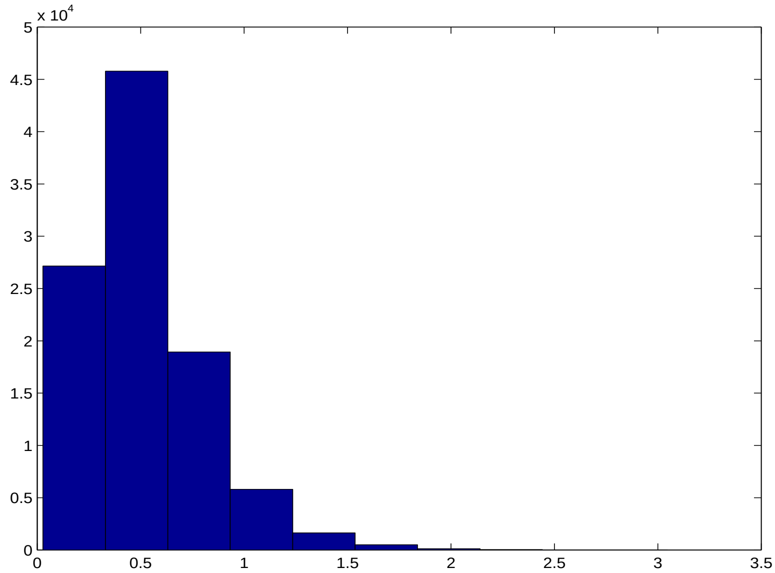

Figure 4.1: The Distribution of the *Hedging Ratio*

and replaced with new long bonds at each year end. We assume the increase rate of long bonds have zero correlation with the increase rate of salaries here, although the increase rate of long bonds always have negative correlation with salaries growth. The increase rates of longs bonds and salaries are still assumed to follow lognormal distribution. From the 30-year annual data set, the mean vector is the mean vector $\begin{pmatrix} 0.051191223 \\ 0.069305968 \end{pmatrix}$ and the variance matrix is

$$\begin{pmatrix} 0.031731869^2 & 0 \\ 0 & 0.093464481^2 \end{pmatrix}.$$

Using the same method, we get the distribution of the *Hedging Ratio* for investing all contributions in long bonds. See figure 4.2.

This is a worse hedging job than investing in stocks. The mean of the *Hedging Ratio* is 0.8054 in this case, and the probability of the *Hedging Ratio* less than 1 is only 0.7886.

If we consider the negative correlation coefficient between the increase rates of long bonds and salaries which is -0.4435, the mean of the *Hedging Ratio* is 0.8536 and the probability of the *Hedging Ratio* less than 1 is 0.7023.

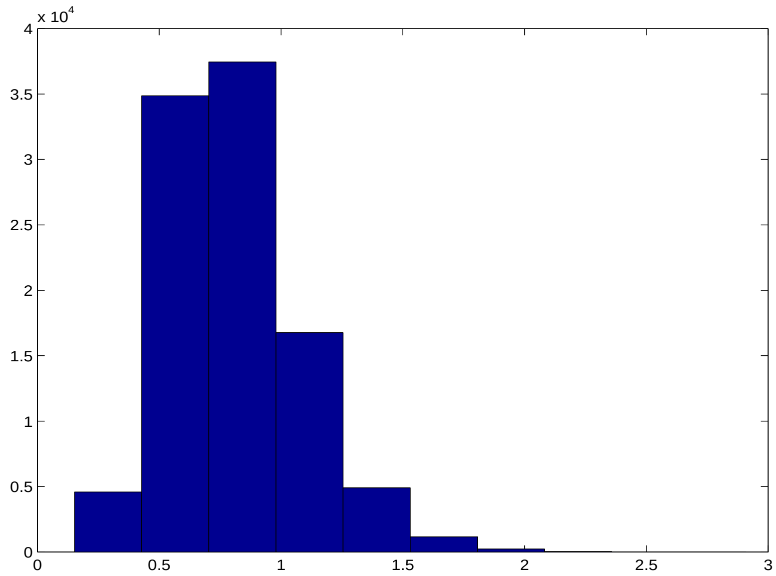

Figure 4.2: The Distribution of the *Hedging Ratio*

Comparing the results from investing all the contributions in stock indexes with a 100% long bonds strategy, we find that stock indexes can hedge future pension benefits better than long bonds.

4.7 Case 2: Using the data from 1994 to 2003

In this section, we use the 10-year US data set to determine the parameters. This is to simulate the actions of actuaries who tend to place very high credibility on the more recent economic history.

In the 1990's, the economy was in a good situation. The increase rate of stocks was much greater than salaries. The price of the S&P 500 index kept increasing from 1994 to 2000. In this period, we find a fairy high correlation between the salary increase rate and the increase rate of the S&P 500 index. During this period, the correlation coefficient is 0.7124.

After 2000, the stock market turned, but salaries still kept increasing. The correlation between increase rates of salaries and the S&P 500 index decreased at this time. If we consider the period from 1994 to 2003, the correlation coefficient changed significantly, to around 23%. The lower correlation coefficient means the salary increase and the index increase have little effect on each other.

From 1994 to 2000, the mean vector is $\begin{pmatrix} 0.032779323 \\ 0.144066351 \end{pmatrix}$ and the variance matrix is

$$\begin{pmatrix} 0.005385414^2 & 0.7124 \times 0.005385414 \times 0.157709696 \\ 0.7124 \times 0.005385414 \times 0.157709696 & 0.157709696^2 \end{pmatrix}.$$

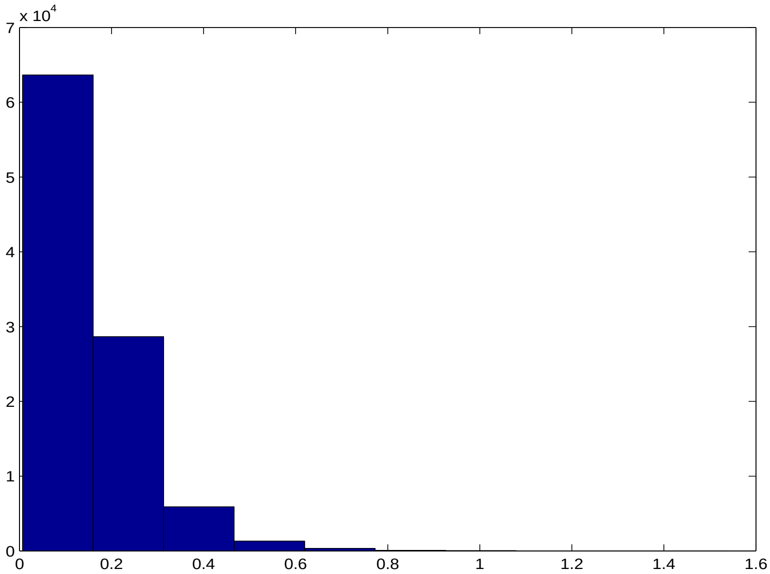

Figure 4.3: The Distribution of the *Hedging Ratio*

From 1994 to 2003, the mean vector and the variance matrix are both changed. The mean vector is $\begin{pmatrix} 0.031252868 \\ 0.083670887 \end{pmatrix}$ and the variance matrix is

$$\begin{pmatrix} 0.005425116^2 & 0.2311 \times 0.005425116 \times 0.202671668 \\ 0.2311 \times 0.005425116 \times 0.202671668 & 0.202671668^2 \end{pmatrix}.$$

The mean of the increase rate of the S&P 500 index decreases and the variance of it increases. After the simulation, we find the means of the *Hedging Ratio* are 0.1542 and 0.5115, respectively. And the probabilities of the *Hedging Ratio* less than 1 are 0.9997 and 0.8979 in these two cases.

Figure 4.3 and figure 4.4 show the different results of the distribution. Pension fund managers can hedge their benefits better when the stock market has good performance.

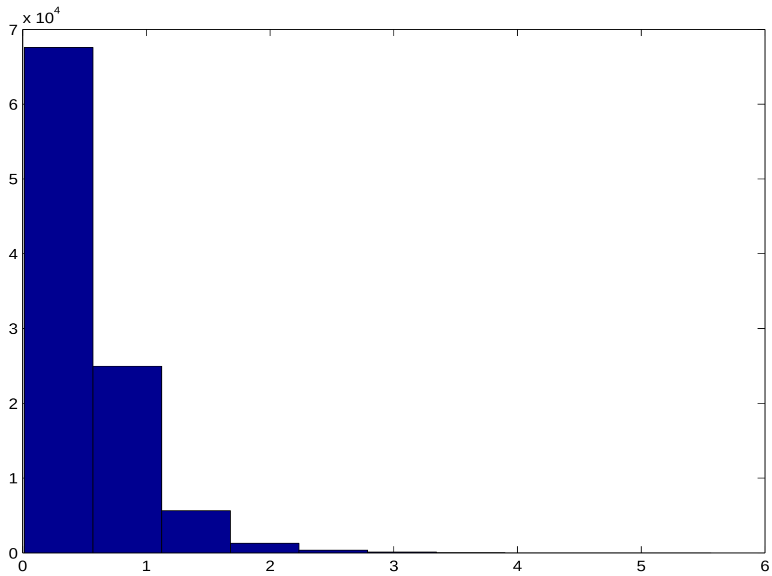

Figure 4.4: The Distribution of the *Hedging Ratio*

In conclusion, the different values of parameters affect the final hedging result. If pension fund managers overestimate parameters to invest, they maybe mismatch benefits when the stock market goes down.

4.8 Sensitivity Test

In this case, we discuss the effect on the mean *Hedging Ratio*, and the probability that the *Hedging Ratio* is less than 1 of using different values of the mean and the variance of the increase rate of the S&P 500 index. We fix the correlation coefficient is 0.3758. And we assume the mean of the salary increase rate is 5% per year and the standard variance of the salary increase rate is 0.03.

We test the mean of the increase rate of stock indexes from 0.08 to 0.14, the standard deviation from 0.09 to 0.14. In Table 4.1, we show the mean of the *Hedging Ratio*, and in Table 4.2 the ruin probability(i.e. Probability that the *Hedging Ratio* is less than 1).

Mean-Stdev	0.08	0.09	0.10	0.11	0.12	0.13	0.14
0.09	0.6560	0.5526	0.4636	0.3846	0.3189	0.2617	0.2141
0.10	0.6611	0.5579	0.4676	0.3877	0.3210	0.2640	0.2162
0.11	0.6663	0.5604	0.4698	0.3920	0.3249	0.2673	0.2189
0.12	0.6688	0.5655	0.4745	0.3953	0.3279	0.2699	0.2215
0.13	0.6747	0.5708	0.4788	0.4003	0.3322	0.2736	0.2250
0.14	0.6811	0.5758	0.4863	0.4054	0.3364	0.2769	0.2278

Table 4.1: The Mean of the *Hedging Ratio*

Mean-Stdev	0.08	0.09	0.10	0.11	0.12	0.13	0.14
0.09	0.05213	0.01522	0.00321	0.00034	0.00003	0	0
0.10	0.07265	0.02404	0.00670	0.00136	0.00013	0	0
0.11	0.09205	0.03647	0.01088	0.00307	0.00067	0.00015	0
0.12	0.11012	0.04995	0.01866	0.00600	0.00182	0.00035	0.00004
0.13	0.12788	0.06270	0.02717	0.00997	0.00312	0.00088	0.00017
0.14	0.14394	0.07758	0.03847	0.01568	0.00603	0.00193	0.00037

Table 4.2: The Probability of the *Hedging Ratio* less than 1

Chapter 5

Sampling Method

In this chapter, the sampling method will be used to simulate the salary growth and the increase rate of indexes and long bonds.

We substitute sampling method for the lognormal distribution to simulate the future increase rates of salaries, indexes and bonds. We assume increase rates of these three assets have same distributions as the historical data in next 30 years.

We combine the yield rate of 30-year zero coupon bonds, the monthly increase rates of salaries, common stocks and Canada long bonds to a column vector. Using 50-year annual data from 1954 to 2003, we have 50 column vectors as our sampling data. The future monthly increase rates of these three assets will be randomly picked up from the sampling data. The valuation date is still once per year, although we consider the monthly increase rates. At the valuation date, the contribution will be separated into two part, $p\%$ of the contribution will be invested into the common stock index and $1-p\%$ of the contribution will be invested into long bonds. We assume the contribution rate(c) is 0.1 and the accrual rate(α) is 0.015. But the annuity factor($\ddot{a}_{65}^{(12)}$) is determined by the yield rate of long bonds at retirement.

$$\ddot{a}_{65}^{(12)} \approx \ddot{a}_{\overline{13}|}^{(12)} = \frac{1-(1+i)^{-13}}{d^{(12)}}$$

where

i is the yield rate of 30-year zero coupon bonds at retirement

At the retirement, the results are similar to the one using the lognormal distribution. Figure 5.1 shows the mean of the *Hedging Ratio* and figure 5.2 shows the probability that the *Hedging Ratio* is less than 1.

Both of two figures shows that investing all of the contributions into stocks to hedge the salary-related benefits is not the best strategy. The best point is around 95% of contributions in stocks and 5% in long bonds. However, it is a good choice using stocks to hedge the salary-related DB pension benefits.

To test the sensitivity of the contribution rate, we change the contribution rate to 7.5%. We also examine the mean of the *Hedging Ratio* and the probability that the *Hedging Ratio* is less than 1. See figure 5.3 and figure 5.4. The results are quite similar. The curves parallel shift by changing the contribution rate.

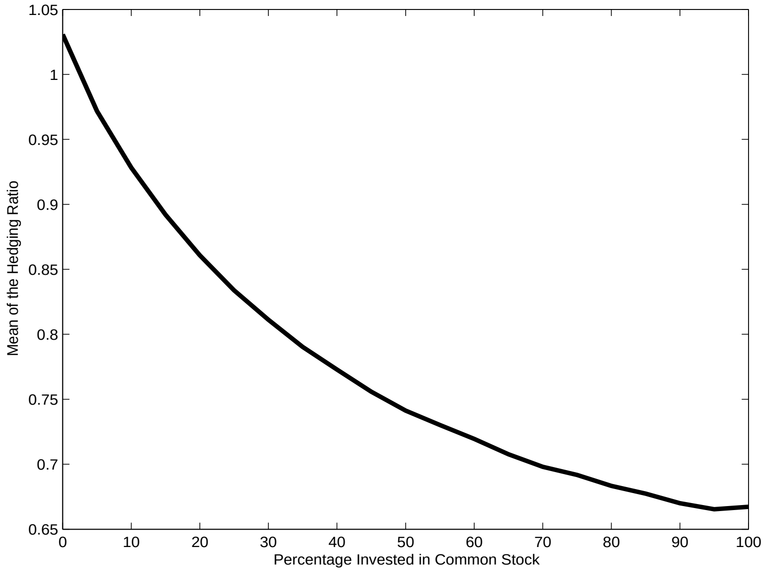

Figure 5.1: The Mean of *Hedging Ratio* and the Asset Allocation

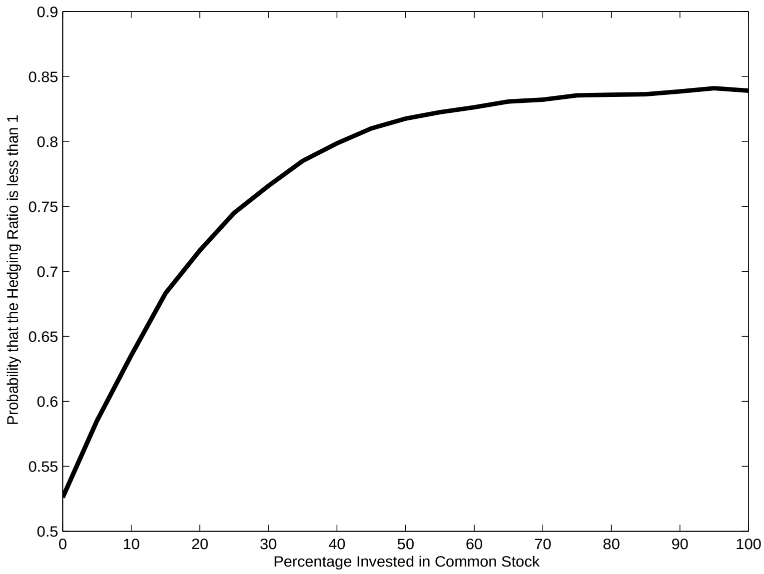

Figure 5.2: The Probability of *Hedging Ratio* less than 1 and the Asset Allocation

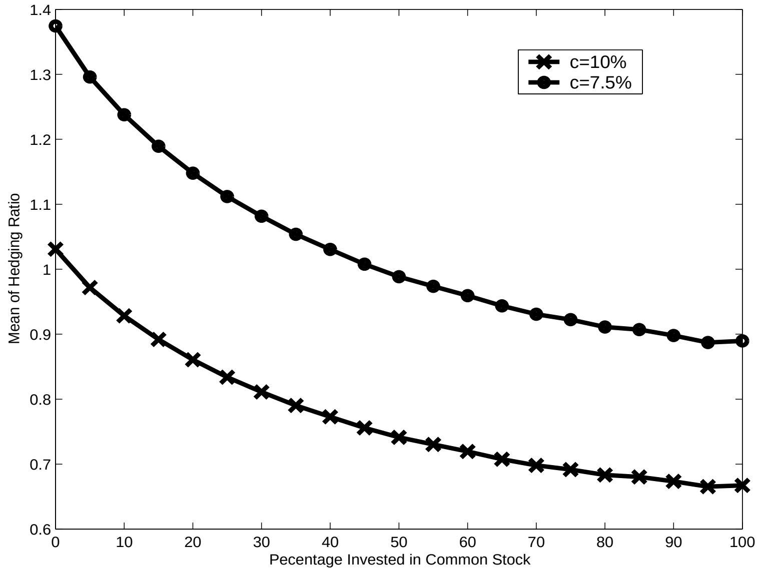

Figure 5.3: The Mean of *Hedging Ratio* and the Asset Allocation

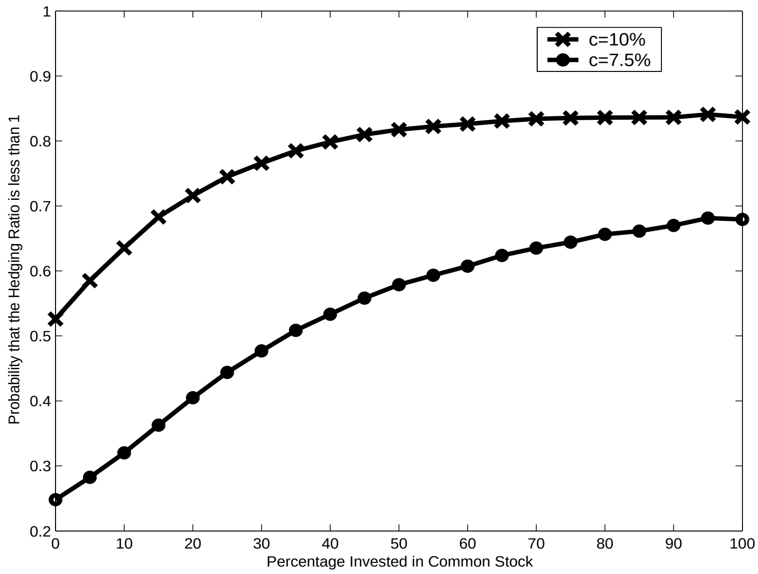

Figure 5.4: The Probability of *Hedging Ratio* less than 1 and the Asset Allocation

Chapter 6

Conclusion

From the discussion, association between salaries, bonds, stocks and inflation is complex. It is hard to determine which one is the best. Using a static hedge, there is a place for a significant stock holding in hedging salary-related pension benefits than long bonds. The reason is that stocks have higher return and higher association with salaries than long bonds. Historically, there has been little use of inflation-linked bonds. Since we usually don't use inflation-linked bonds to hedge pension liabilities, stocks are still a better choice.

In chapter 4, we assume the contribution rate(c) is equal to the product of the accrual rate(α) and the annuity factor($\ddot{a}_{65}^{(12)}$). This is not a good assumption. Under this assumption, we can cancel the contribution rate on both sides of assets and liabilities. This makes our model easier, since we don't need to concern about the choice of different contribution rates. When we determine the contribution rate arbitrarily, our model still works and results are similar. Hence, pension fund managers can change the definition of the contribution rate to setup their own objective of the *Hedging Ratio*. The lower contribution rate is, the higher mean of the *Hedging Ratio*.

Another problem in this paper is the volatility of the *Hedging Ratio*. Since the change in stock markets affects our *Hedging Ratio*, we have to set our objective to a low *Hedging Ratio* in order to promise that the probability of the *Hedging Ratio* less than 1 is high. One solution of this problem is that we consider a large number of active members. When many employees retire at the same time, we can focus our objective on the mean of the *Hedging Ratio*.

We still need to explore different dynamic strategies. For examples, we can invest all contributions to stocks and move into bonds near retirement. And the future work will focus on the combination of stocks and long bonds or some other assets and explore bonds holding strategy in place of sell/buy strategy. Also, we will concern to hedge the hybrid pension benefits in the future work.

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