LTAM Spring 2019

Model Solutions
Written Answer Questions

Question 1 Model Solution

Learning Outcomes: 4(a), 4(b), 4(c)

Chapter References: AMLCR Chapters 4, 5 and 6

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a)

(i) Acceptable expressions include:

$$\begin{aligned} L_0^g &= 100{,}000\, v^{K_{50}+1} - 0.9\, G\, \ddot{a}_{\overline{K_{50}+1}|} + 0.4\, G \\ &= 100{,}000\, v^{K_{50}+1} - 0.9\, G\, a_{\overline{K_{50}+1}|} - 0.5\, G \\ &= 100{,}000\, v^{K_{50}+1} - G\, \ddot{a}_{\overline{K_{50}+1}|} + 0.1\, G\, \ddot{a}_{\overline{K_{50}+1}|} + 0.4\, G \\ &= 100{,}000\, v^{K_{50}+1} - 0.9\, G \left(\frac{1-v^{K_{50}+1}}{d}\right) + 0.4\, G \\ &= \left(100{,}000 + \frac{0.9\, G}{d}\right) v^{K_{50}+1} - \frac{0.9}{d} G + 0.4\, G \end{aligned}$$

(ii) Using the first expression in part (i) and solving for $E\left[L_0^g\right] = 0$,

$$G = \frac{100{,}000\, E[v^{K_{50}+1}]}{0.9\, E\left[\ddot{a}_{\overline{K_{50}+1}|}\right] - 0.4} = \frac{100{,}000\, A_{50}}{0.9\, \ddot{a}_{50} - 0.4} = \frac{18931}{0.9(17.0245) - 0.4} = 1268.66$$

Comment: This part was done correctly by almost all candidates.

b)

(i) $$\begin{aligned} L_0^g(G^*) &= 100{,}000 v^{K_{50}+1} - 0.9 G^*\, \ddot{a}_{\overline{K_{50}+1}|} + 0.4 G^* \\ &= \left(100{,}000 + \frac{0.9\, G^*}{d}\right) v^{K_{50}+1} - \frac{0.9}{d} G^* + 0.4\, G^* \end{aligned}$$

The variance of $L_0^g(G^*)$ is:

$$V\left[L_0^g(G^*)\right] = \left(100{,}000 + \frac{0.9\, G^*}{d}\right)^2 V[v^{K_{50}+1}] = \left(100{,}000 + \frac{0.9\, G^*}{d}\right)^2 ({}^2A_{50} - A_{50}^2)$$

Alternatively,

$$V\left[L_0^g(G^*)\right] = 10^{10}\, V[v^{K_{50}+1}] + (0.9\, G^*)^2\, V\left(\ddot{a}_{\overline{K_{50}+1}|}\right) - 2(10^5)(.9G^*)\mathrm{cov}(v^{K_{50}+1}, \ddot{a}_{\overline{K_{50}+1}|})$$

where

$$V[v^{K_{50}+1}] = {}^2A_{50} - A_{50}^2\,, \quad V\left(\ddot{a}_{\overline{K_{50}+1}|}\right) = \tfrac{1}{d^2}({}^2A_{50} - A_{50}^2) \text{ and}$$

$$\mathrm{cov}(v^{K_{50}+1}, \ddot{a}_{\overline{K_{50}+1}|}) = cov\left(v^{K_{50}+1}, \frac{1 - v^{K_{50}+1}}{d}\right) = -\tfrac{1}{d}({}^2A_{50} - A_{50}^2)$$

(ii) $E\left[L_0^g(G^*)\right] = 100{,}000\, A_{50} - 0.9\, G^*\, \ddot{a}_{50} \; + 0.4\, G^* \; = 18{,}931 - 14.92205\, G^*$

$$SD\left[L_0^g(G^*)\right] = \left(100{,}000 \; + \frac{0.9\, G^*}{d}\right)\sqrt{{}^2A_{50} - A_{50}^2}$$

$$= \left(100{,}000 \; + \frac{0.9}{(0.05/1.05)}G^*\right)\sqrt{0.05108 - (0.18931)^2}$$

$$= (100{,}000 \; + 18.9\, G^*)(0.1234574) = 12{,}345.74 \; + 2.3333\, G^*$$

Let $L = \sum_{j=1}^{100} L_{0,j}^g\,(G^*)$ where $L_{0,j}^g(G^*)$ is the loss-at-issue for policy j.

$E[L] = 100E\left[L_0^g(G^*)\right] = 1{,}893{,}100 - 1492.205G^*$

$V[L] = 100V\left[L_0^g(G^*)\right]$ (*independent lives*)

$SD[L] = 10SD\left[L_0^g(G^*)\right] = 123{,}457.4 + 23.333G^*$

Solving for G^* such that $P[L \le 0] = 0.95$

$$P\left[\frac{L - E[L]}{SD[L]} \le \frac{0 - (1{,}893{,}100 - 1492.205\, G^*)}{123{,}457.4 \; + 23.333G^*}\right] = 0.95$$

$$\Rightarrow \frac{(-1{,}893{,}100 + 1492.205\, G^*)}{123{,}457.4 \; + 23.333\, G^*} = 1.645$$

$$G^* = \frac{123{,}457.4(1.645) + 1{,}893{,}100}{1492.205 - 23.333(1.645)} = 1441.85$$

Comments:

1. *Part (i) was done correctly by most candidates. A common mistake made by those who used the alternative approach was to ignore the covariance term.*
2. *Many candidates received partial credit for part (ii); only a few received full credit.*

c) As the size of the portfolio increases, the percentile premium, G^*, decreases for a given probability of a loss. As the size of the portfolio tends to infinity, G^* tends to the equivalence premium, $G \; = 1268.66$. If ABC sells 1000 policies instead of 100, each with a premium of 1442, it will receive more premiums than needed to meet its 5% target of a positive loss.

The probability of a positive loss on the portfolio would decrease.

Comment: Most candidates correctly determined that the probability would decrease, earning partial credit. Relatively few candidates provided a sufficiently thorough justification for full credit.

Question 2 Model Solution

Learning Outcomes: 1(a), 3(a), 3(b), 5(a), 5(b)

Chapter References: AMLCR Chapters 1, 8; SN LTAM-21-18 Revised Chapters 1, 5

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a) 1) To better replicate the loss to the injured party (*IP*):
- lost wages, medical and other expenses due to injury, offset inflation (if increasing annuity).

2) To relieve the *IP* from the investment/interest risk, or from the burden of managing funds.

3) To reduce the dissipation risk:
- risk of running out of funds from squandering/over spending.

b) 1) Tax :

The structured settlement annuity might not be taxed, or taxed at a lower rate than salary. Less than 100% is needed to replace the net pre-injury earnings.

2) Incentive to return to work:

The *IP* may choose to return to work if that would increase his/her (net) income.

3) *IP* at fault:

The payments may be reduced if the *IP* is (partially) at fault.

Comments:

1. *Performance on parts (a) and (b) was mixed. Some candidates did not answer those two parts, others provided only one relevant reason for each part and/or repeated the same reasons for both parts.*
2. *A number of candidates discussed pricing or underwriting issues, in one or both parts, for which no credit was awarded.*
3. *Another common reason given for which no credit was awarded was to argue that an annuity is preferred because the insurance company would earn interest.*

c) The benefits are 90,000 when in States 0 or 2 plus 20,000 when in State 0 for up to 2 years. So,

$$EPV = 90{,}000[\,\bar{a}_x^{00} + \bar{a}_x^{02}] + 20{,}000[\,\bar{a}_{x:\overline{2|}}^{00}\,]$$

Since returning to State 0 after leaving it is impossible, we have

$$\bar{a}_{x:\overline{2|}}^{00} = \bar{a}_x^{00} - v^2\ {}_2p_x^{00}\ \bar{a}_{x+2}^{00} = 0.559\ - (1.04)^{-2}\ (0.0301)(0.559) = 0.5434435$$

and

$$EPV = 90{,}000[\,0.559 + 7.161] + 20{,}000[\,0.5434435\,] = \ 705{,}668.87$$

Comment: Most candidates did very well on this part.

d)

(i) $\quad {}_2V^{(0)} = 90{,}000\left[\bar{a}^{00}_{x+2} + \bar{a}^{02}_{x+2}\right] = 90{,}000\,[0.559 + 7.104] = 689{,}670$

(ii) $\quad {}_2V^{(2)} = 90{,}000\,\bar{a}^{22}_{x+2} = 90{,}000\,(10.623) = 956{,}070$

(iii) Let $E_0[{}_2V]$ be the EPV at t=0 of the reserve at t=2. Note that ${}_2V^{(1)} = {}_2V^{(3)} = 0$.

$$E_0[{}_2V] = v^2\,{}_2p^{00}_x\,{}_2V^{(0)} + v^2\,{}_2p^{02}_x\,{}_2V^{(2)}$$
$$= (1.04)^{-2}\,[(0.0301)(689{,}670) + (0.6164)(956{,}070)] = 564{,}053.82$$

(iv) EPV[payments in first 2 years] = EPV[all payments] – EPV[reserve at t=2]

$$= EPV - E_0[{}_2V]$$
$$= 705{,}668.87 - 564{,}053.82 = 141{,}615.05$$

Alternatively,

$$EPV = 20{,}000\,\bar{a}^{00}_{45:\overline{2|}} + 90{,}000\left(\bar{a}^{00}_{45:\overline{2|}} + \bar{a}^{02}_{45:\overline{2|}}\right)$$
$$= 110{,}000(0.5434435) + 90{,}000(0.9092918) = 141{,}615.05$$

where

$$\bar{a}^{02}_{45:\overline{2|}} = \bar{a}^{02}_{45} - v^2\,{}_2p^{00}_{45}\,\bar{a}^{02}_{47} - v^2\,{}_2p^{02}_{45}\,\bar{a}^{22}_{47} = 0.9092918$$

Comments:

1. *Most candidates did well on this part.*
2. *A common error made by those using the alternative solution in part (iv) was to ignore the last term, $v^2\,{}_2p^{02}_{45}\,\bar{a}^{22}_{47}$, when calculating $\bar{a}^{02}_{45:\overline{2|}}$.*

e)

(i) The reserve at time 2 given State 2, ${}_2V^{(2)}$, will stay the same.
This reserve is conditional on *IP* being in State 2 and transitions out of State 2 are not affected.

(ii) Here, ${}_tp^{00}_x = exp\left\{-\int_0^t \mu^{01}_{x+s} + \mu^{02}_{x+s} + \mu^{03}_{x+s}ds\right\}$ and ${}_tp^{0i}_x = \int_0^t {}_sp^{00}_x\mu^{0i}_{x+s}\,ds$

So, increasing μ^{01}_{x+t} for all t will: increase ${}_tp^{01}_x$ for all t

decrease ${}_tp^{00}_x$ for all t; => decrease $\bar{a}^{00}_x$ and ${}_2V^{(0)}$

decrease ${}_tp^{02}_x$ for all t (with same ${}_2V^{(2)}$ from (i))

Therefore, $E_0[{}_2V] = v^2\,{}_2p^{00}_x\,{}_2V^{(0)} + v^2\,{}_2p^{02}_x\,{}_2V^{(2)}$ will decrease.

Comments:

1. *Most candidates determined that the reserve would stay the same in part (i) but a number of candidates provided an incorrect or incomplete justification, for example, simply stating that there are no transitions from State 1 to State 0.*
2. *Part (ii) proved to be challenging. Only well-prepared candidates correctly determined the change in the EPV at time 0 of the reserve at time 2 with a complete justification.*

Question 3 Model Solution

Learning Outcomes: 6(d), 6(e), 6(f)

Chapter References: AMLCR Chapter 10; SN LTAM-21-18 Revised Chapter 6.

General comment: Many candidates omitted this question entirely or only answered one part.

a) $_0V = AL_0 = (TPE)_{63} \cdot \alpha \cdot \left(\frac{r_{63}}{l_{63}} v^{0.5} \ddot{a}^{(12)}_{63.5} + \frac{r_{64}}{l_{63}} v^{1.5} \ddot{a}^{(12)}_{64.5} + \frac{r_{65}}{l_{63}} v^2 \ddot{a}^{(12)}_{65} \right)$

$$AL_0 = (2{,}500{,}000)(0.02)\left(\frac{4515.2}{47{,}579.3}(1.05)^{-0.5}(13.514) + \frac{4061.0}{47{,}579.3}(1.05)^{-1.5}(13.231) + \frac{38{,}488.3}{47{,}579.3}(1.05)^{-2}(13.086)\right)$$

$$= (50{,}000)(1.251550 + 1.049600 + 9.601498) = 595{,}132.40$$

Comments:

1. *Performance on this part was mixed. Candidates either did very well, earning full credit, or not at all well, earning little or no credit.*
2. *The most common error was to incorrectly value the liability for mid-year exits.*

b) $AL_0 + NC = EPV(\textit{benefits for mid-year exits}) + v\, p^{00}_{63}\, AL_1$

where

$$EPV(\textit{benefits for mid-year exits}) = ((TPE)_{63} + 0.5\, s_{63})\, \alpha\, \frac{r_{63}}{l_{63}} v^{0.5} \ddot{a}^{(12)}_{63.5}$$

$$= \big(2{,}500{,}000 + (0.5)(160{,}000)\big)(0.02)(1.25155) = 64{,}579.98$$

$$AL_1 = ((TPE)_{63} + s_{63}) \cdot \alpha \cdot \left(\frac{r_{64}}{l_{64}} v^{0.5} \ddot{a}^{(12)}_{64.5} + \frac{r_{65}}{l_{64}} v^1 \ddot{a}^{(12)}_{65}\right)$$

$$v\, p^{00}_{63}\, AL_1 = ((TPE)_{63} + s_{63}) \cdot \alpha \cdot \left(\frac{r_{64}}{l_{63}} v^{1.5} \ddot{a}^{(12)}_{64.5} + \frac{r_{65}}{l_{63}} v^2 \ddot{a}^{(12)}_{65}\right)$$

$$= (2{,}500{,}000 + 160{,}000)(0.02)(1.0496 + 9.601498) = 566{,}638.41$$

$$NC = 64{,}579.98 + 566{,}638.41 - 595{,}132.40 = 36{,}085.99$$

Alternatively,

$$NC = s_{63} \cdot \alpha \cdot \left[(0.5)\ \frac{r_{63}}{l_{63}} v^{0.5} \ddot{a}^{(12)}_{63.5} + \frac{r_{64}}{l_{63}} v^{1.5} \ddot{a}^{(12)}_{64.5} + \frac{r_{65}}{l_{63}} v^2 \ddot{a}^{(12)}_{65}\right]$$

$$= (160{,}000)(0.02)[(0.5)(1.25155) + 1.0496 + 9.601498] = 36{,}085.99$$

Comments:

1. *Only the most well-prepared candidates achieved full or nearly full credit.*
2. *Partial credit was awarded to candidates who showed some understanding of the question by writing down some relevant formulas before evaluating them numerically.*

c) EPV(Health benefits) $= B(63,0.5)\frac{r_{63}}{l_{63}}\, v^{0.5}\, \ddot{a}_B(63.5, 0.5) + B(64,1.5)\frac{r_{64}}{l_{63}}\, v^{1.5}\, \ddot{a}_B(64.5, 1.5)$

$$+B(65,2)\frac{r_{65}}{l_{63}}\, v^2\, \ddot{a}_B(65, 2)$$

where
$\ddot{a}_B(x,t) = \ddot{a}_B(x,0) \quad \forall t > 0,$ since inflation is constant;

$B(63,0.5) = 2000\,(1.035)^3\,(1.03)^{0.5} = 2250.45$
$B(64,1.5) = 2000\,(1.035)^4\,(1.03)^{1.5} = 2399.09$
$B(65,2) = 2000\,(1.035)^5\,(1.03)^2 = 2520.03$

$$\text{EPV(Health benefits)} = 2250.45\,\frac{r_{63}}{l_{63}}\, v^{0.5}\,(30.420) \;+\; 2399.09\,\frac{r_{64}}{l_{63}}\, v^{1.5}\,(29.124)$$
$$+\, 2520.03\,\frac{r_{65}}{l_{63}}\, v^2\,(28.487)$$
$$= 6340.05 + 5542.79 + 52672.63 = 64{,}555.47$$

Comments:

1. *Many candidates omitted this part entirely.*
2. *Many of the candidates who answered this part found it challenging. A common error was incorrectly calculating the benefits at exit times, B(x,t).*

Question 4 Model Solution

Learning Outcomes: 1(a), 1(c), 2(h), 3(a), 3(b), 4(a), 4(b), 4(c), 5(a), 5(b)

Chapter References: AMLCR Chapter 9

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a) $100{,}000\, A_{\overline{40:40}} = P\, \ddot{a}_{\overline{40:40}:\overline{20}|}$

where

$A_{\overline{40:40}} = 2A_{40} - A_{40:40} = 2(0.12106) - 0.16055 = 0.08157$

$\ddot{a}_{\overline{40:40}:\overline{20}|} = 2\, \ddot{a}_{40:\overline{20}|} - \ddot{a}_{40:40:\overline{20}|} = 2\,(12.9935) - 12.9028 = 13.0842$

$$P = \frac{8157}{13.0842} = 623.42$$

Comments:

1. *Candidates did very well on this part, most of them achieving full credit.*
2. *For those who did not receive full credit, common errors included ignoring the 20-year premium paying period and making the premiums payable until the first death only,* $P\, \ddot{a}_{40:40:\overline{20}|}$.

b) - **Common shock**: both lives may die at the same time, e.g. as a result of an accident.

- **Common lifestyle**: couples tend to share a common lifestyle, e.g. healthy (exercising) or unhealthy (smoking) habits.
- **Broken heart syndrome**: mortality may increase temporarily after the death of one's partner.

Comments:

1. *Almost all candidates stated at least one valid reason.*
2. *Most candidates received full credit for this part.*

c)

(i) $A_{40:40} = 1 - d\, \ddot{a}_{40:40}$

where

$\ddot{a}_{40:40} = \ddot{a}_{40:40:\overline{20}|} + {}_{20}E_{40:40}\, \ddot{a}^{SULT}_{60:60}$

$= 12.9254 + (0.35912)(13.2497) = 17.683632$

$$A_{40:40} = 1 - \frac{0.05}{1.05}(17.683632) = 0.157922$$

Alternatively,

$A_{40:40} = A_{40:40:\overline{20|}} - {}_{20}E_{40:40}\,(1 - A^{SULT}_{60:60})$

where

$A_{40:40:\overline{20|}} = 1 - d\,\ddot{a}_{40:40:\overline{20|}} = 1 - (0.05/1.05)\,(12.9254) = 0.384505$

$A_{40:40} = 0.384505\ - (0.35912)(1 - 0.36906) = 0.157922$

(ii) ${}_{20}E_{40:40} = {}_{10}E_{40:40} \cdot {}_{10}E_{50:50}$ => ${}_{10}E_{40:40} = 0.35912/0.59290 = 0.605701$

Alternatively, ${}_{10}E_{40:40} = v^{10}\,{}_{10}p_{40:40} = 0.605687$

(iii) $\ddot{a}_{40:40:\overline{20|}} = \ddot{a}_{40:40:\overline{10|}} + {}_{10}E_{40:40} \cdot \ddot{a}_{50:50:\overline{10|}}$

$$\ddot{a}_{50:50:\overline{10|}} = \frac{\ddot{a}_{40:40:\overline{20|}} - \ddot{a}_{40:40:\overline{10|}}}{{}_{10}E_{40:40}} = \frac{12.9254 - 8.0703}{0.605701} = 8.016$$

Comments:

1. *Performance on this part was mixed.*
2. *Many candidates failed to correctly reflect the fact that future lifetimes are independent after 20 years when calculating the required EPVs.*
3. *Identifying appropriate relationships between EPVs that were useful for this model proved to be challenging for many candidates.*

d) $100{,}000\,A_{\overline{40:40}} = P\,\ddot{a}_{\overline{40:40}:\overline{20|}}$

where

$A_{\overline{40:40}} = 2\,A_{40} - A_{40:40} = 2(0.12106) - 0.157922 = 0.084198$

$\ddot{a}_{\overline{40:40}:\overline{20|}} = 2\,\ddot{a}_{40:\overline{20|}} - \ddot{a}_{40:40:\overline{20|}} = 2\,(12.9935) - 12.9254 = 13.0616$

$$P = \frac{8419.80}{13.0616} = 644.62$$

Comments:

1. *Performance on this part was mixed.*
2. *Candidates who did well on part (c) did very well on this part.*

e)

(i) $E[{}_{10}L \mid only\ Pat\ alive] = 100{,}000\, A_{50} - P\, \ddot{a}_{50:\overline{10}|}$

$$= 18{,}931 - (644.62)(8.055) = 13{,}738.59$$

(ii) $E[{}_{10}L \mid both\ alive] = 100{,}000\, A_{\overline{50:50}} - P\, \ddot{a}_{\overline{50:50}:\overline{10}|}$

$$= 100{,}000\, A_{\overline{50:50}} - P\,(2\, \ddot{a}_{50:\overline{10}|} - \ddot{a}_{50:50:\overline{10}|})$$

$$= 13{,}441 - (644.62)(\,2(8.055) - 8.0157) = 8223.25$$

(iii) $E[{}_{10}L \mid at\ least\ one\ alive] =$

$$= \frac{E[{}_{10}L \mid exactly\ one\ alive] \cdot P(exactly\ one\ alive) + E[{}_{10}L \mid both\ alive] \cdot P(both\ alive)}{P(exactly\ one\ alive) + P(both\ alive)}$$

$P(at\ least\ one\ alive) = {}_{10}p_{\overline{40:40}} = P(exactly\ one\ alive) + P(both\ alive)$

$P(both\ alive) = {}_{10}p_{40:40} = 0.9866$

⇨ $P(exactly\ one\ alive) = {}_{10}p_{\overline{40:40}} - {}_{10}p_{40:40} = 0.998 - 0.9866 = 0.0114$

$$E[{}_{10}L \mid at\ least\ one\ alive] = \frac{(13{,}738.59)(0.0114) + (8223.25)(0.9866)}{0.998} = 8286.25$$

Comments:

1. *Most candidates found this part challenging.*
2. *Many candidates omitted this part entirely or only received partial credit for parts (i) and (ii).*
3. *Only well-prepared candidates attempted and received credit for part (iii).*

f) With the reserve being defined as the EPV of future benefits less future premiums, the reserve calculated in e) (iii), i.e. given at least one alive, is the most suitable value if the insurance company has no knowledge of the number of survivors. For a large portfolio, the total reserve is expected to be sufficient in this case.
The reserve calculated in e) (i) is too conservative. For most policies, both insureds would still be alive at time 10 resulting in lower EPV of benefits and larger EPV of premiums than the values used to calculate the reserve in e) (i). The reserve calculated in e) (ii) is insufficient. For some policies, only one insured would still be alive at time 10. Assuming that both are alive to calculate the reserve would underestimate the funds required at time 10.

Comments:

1. *Only well-prepared candidates correctly identified the most suitable value for the reserve for each in-force policy.*
2. *Even for those who correctly identified the most suitable reserve, providing a good explanation was a challenge.*

Question 5 Model Solution

Learning Outcomes: 5(a), 5(b), 5(c)

Chapter References: AMLCR Chapters 8, 12 (Section 12.8)

General comment:
Overall performance on this question was mixed. Many candidates achieved maximum marks, while many others omitted this question entirely.

a) The emerging profit in year 3, conditional on being in State *j* at time 2 is

$$Pr_3^{(j)} = \left({}_2V^{(j)} + P_3^{(j)} - E_3^{(j)}\right)(1 + i_t) - \sum_k E\ B_3^{jk} - \sum_k E_3 V^{jk}$$

where $i_t = 0.06$.

For State 0,
${}_2V^{(0)} = 12{,}000$
$P_3^{(0)} = 30{,}000$
$E_3^{(0)} = (0.05)P_3^{(0)} = 1500$

$E\ B_3^{01} = p_{x+2}^{01}\ 600{,}000 = (0.014)(600{,}000) = 8400$
$E\ B_3^{02} = p_{x+2}^{02}\ 1{,}000{,}000 = (0.014)(1{,}000{,}000) = 14{,}000$
$E\ B_3^{03} = p_{x+2}^{03}\ 1{,}000{,}000 = (0.004)(1{,}000{,}000) = 4000$

$E_3V^{00} = p_{x+2}^{00}\ {}_3V^{(0)} = (1 - 0.014 - 0.014 - 0.004)(9000) = 8712$
$E_3V^{01} = p_{x+2}^{01}\ {}_3V^{(1)} = (0.014)(210{,}000) = 2940$

So,
$Pr_3^{(0)} = (12{,}000 + 30{,}000 - 1500)(1.06) - 8400 - 14{,}000 - 4000 - 8712 - 2940$
$= 4878$

Comments: For those who did not achieve full marks, common errors included :

1. *Omitting the value of the benefit payable in case of death after CI,*
2. *Incorrectly including a value for the 500,000 benefit payable in case of transition from State 1 (at the beginning of the year) to State 2,*
3. *Using wrong expenses for year 3.*

b) For State 1, we have

$_2V^{(1)} = 280{,}000$

$P_3^{(1)} \ = \ 0$

$E_3^{(1)} = 100$

$E\ B_3^{12} = p_{x+2}^{12}\ \ 500{,}000\ =\ (0.25)(500{,}000) = 125{,}000$

$E_3V^{11} = p_{x+2}^{11}\ _3V^{(1)} =\ (1-0.25)(210{,}000)\ =\ 157{,}500$

$Pr_3^{(1)} = (280{,}000 - 100)\ \ (1.06)\quad - 125{,}000 - 157{,}500 =\ 14{,}194$

Comment:

Common errors included ignoring or using wrong expenses, and adding a premium when none is paid while in State 1.

c) $\Pi_3\ = {}_2p_x^{00}\ \ Pr_3^{(0)} + {}_2p_x^{01}\ \ Pr_3^{(1)}$

$$_2p_x^{00} =\ p_x^{00}\ \ p_{x+1}^{00} = (1 - p_x^{01} - p_x^{02} - p_x^{03})(\ 1 - p_{x+1}^{01} - p_{x+1}^{02} - p_{x+1}^{03})$$
$$=\ (1 - 0.01 - 0.008 - 0.004)(1 - 0.012 - 0.011 - 0.004) = 0.951594$$

$$_2p_x^{01} =\ p_x^{00}\ \ p_{x+1}^{01} +\ p_x^{01}\ \ p_{x+1}^{11} =\ (0.978)(0.012) + (0.01)((0.75) = 0.019236$$

$$\Pi_3\ = (0.951594)(4878) + (0.019236)(14{,}194) = 4914.91$$

Comment:

Some candidates used incorrect probabilities for the two emerging profits, or calculated the 2-year transition probabilities incorrectly.

d) $NPV(3) = \sum_{k=0}^{3} \Pi_k\ \ v_{10\%}^{k}\ =\ -500 - \frac{770}{1.1} + \frac{3536}{1.1^2} + \frac{4914.91}{1.1^3} = 5414.96$

Comment:

Candidates who attempted this part did well.

Question 6 Model Solution

Learning Outcomes: 2(a), 2(e)

Chapter References: SN LTAM-22-18 Chapter 12

General Comment: A number of candidates omitted this question entirely.

a)

$$\hat{S}(t)=\begin{cases}1 & t<t_1\\ \prod_{i=1}^{j}\left(1-\frac{s_i}{r_i}\right) & t_j\le t<t_{j+1},\ \ j=1,2,3,4\\ \prod_{i=1}^{5}\left(1-\frac{s_i}{r_i}\right) & t_5\le t<t_{max}=9\end{cases}$$

For this study, we have

i	t_i	s_i	r_i
1	*2*	*2*	*12-1 = 11*
2	*4*	*1*	*11-2-1 = 8*
3	*5*	*1*	*8-1-1 = 6*
4	*7*	*1*	*6-1-1 = 4*
5	*8*	*1*	*4-1 = 3*

So,

$$\hat{S}(t)=\begin{cases}1 & 0<t<2\\ 1-\frac{2}{11}=0.81818 & 2\le t<4\\ (0.81818)\left(1-\frac{1}{8}\right)=0.71591 & 4\le t<5\\ (0.71591)\left(1-\frac{1}{6}\right)=0.596591 & 5\le t<7\\ (0.59659)\left(1-\frac{1}{4}\right)=0.44744 & 7\le t<8\\ (0.44744)\left(1-\frac{1}{3}\right)=0.29830 & 8\le t<9\end{cases}$$

$\hat{S}(6)=\ 0.596591$

Comments:

1. *Most candidates who attempted this question did well on this part.*
2. *Some candidates received partial credit for estimating S(t) at t=6 only. Others failed to estimate S(t) for all values of t, for which a small deduction was applied.*

b) Using Greenwood's formula provided in the Table,

$$V[\hat{S}(6)] \approx \left(\hat{S}(6)\right)^2 \left[\frac{2}{(11)(9)} + \frac{1}{(8)(7)} + \frac{1}{(6)(5)}\right] = (0.3559207)(0.071392496) \ = \ 0.0254101$$

$$=> \ \ SD(\hat{S}(6)) \approx \ 0.159405$$

Comment: Candidates did very well on this part.

c)

(i) An approximate 95% linear CI is $\hat{S}(6) \pm \ 1.96 \ \cdot \ SD(\hat{S}(6))$

$$0.596591 \pm \ (1.96)(0.159405)$$

CI: (0.284157, 0.909025)

(ii) An approximate 95% log-transformed CI is $\left(\hat{S}(6)^{1/U}, \hat{S}(6)^{U}\right)$
where

$$U = exp\left(\frac{1.96 \ \cdot \ SD(\hat{S}(6))}{\hat{S}(6) \ \ ln(\hat{S}(6))}\right)$$

$$= \ exp\left(\frac{1.96 \ (0.159405)}{(0.596591) \ ln(0.596591)}\right) = exp(-1.0138909) = 0.362805$$

95% log-transformed CI: $(0.596591^{1/0.362805}, 0.596591^{0.362805})$

$$(0.240823, 0.829114)$$

Comments:

1. *Most candidates did very well on part (i).*
2. *Candidates either did very well or very poorly on part (ii).*

d) The log-transformed CI has the advantage of always producing an interval inside the (0-1) range.

e) The thirteenth observation will have no effect on the estimate of $S(6)$.
It does not change the deaths (number and timing) or the risk sets (exposures) at the observed times of death in this study.

Comment: Well-prepared candidates were able to correctly determine the effect and provide a complete and clear explanation.