QFI ADV Model Solutions
Spring 2019

1. **Learning Objectives:**

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

(2b) Demonstrate an understanding of the basic concepts of credit risk modeling such as probability of default, loss given default, exposure at default, and expected loss.

(2c) Demonstrate an understanding of credit valuation models.

(2k) Demonstrate an understanding of measuring and marking-to-market counterparty credit risk in credit derivatives.

(2l) Understand and apply various approaches for managing credit risk in a portfolio setting.

Sources:

QFIA-104-13: Asset/Liability Management of Financial Institutions, Tilman, Leo M., 2003, Ch. 9, pp 122 – 123

QFIA-104-13: Asset/Liability Management of Financial Institutions, Tilman, Leo M., 2003, Ch. 9, pp 124 – 125

QFIA-104-13: Asset/Liability Management of Financial Institutions, Tilman, Leo M., 2003, Ch. 9, pp 125

QFIA-104-13: Asset/Liability Management of Financial Institutions, Tilman, Leo M., 2003, Ch. 9, pp 132 – 133

Commentary on Question:

This question tested the candidate's knowledge on measuring and modelling counterparty risk with a focus on Potential Future Exposure (PFE) models and credit exposure on derivatives.

1. Continued

Solution:

(a) Compare Potential Future Exposure and Expected Exposure and the methodology to measure them.

Commentary on Question:
The candidates performed as expected on this section. Candidates clearly defined Potential Future Exposure (PFE) and Expected Exposure (EE.) Successful candidates described additional details beyond the definitions.

- The Potential Future Exposure (PFE) is the maximum amount of exposure expected to occur on a future date with a high degree of statistical confidence whereas the Expected Exposure (EE) is the average exposure on a future date.
- Both the PFE and EE can be computed through simulation models.
- The EE(t) curve is used for credit pricing whereas the PFE(t) is typically used to compare credit limits in processing trades

(b) You are designing the simulation engine module that will be used to calculate Potential Future Exposures for your firm.

Your company does a significant amount of foreign exchange rate trading, in both developed market currencies as well as in emerging market currencies. You have been given instructions to set up a model to analyze exchange rate fluctuations using a lognormal diffusion process for all foreign currency trades.

Assess this request.

Commentary on Question:
Candidates performed below average on this section. Successful candidates recognized that Lognormal diffusion models are appropriate for developed markets but not emerging market currencies. Unsuccessful candidates mistakenly identified lognormal diffusion models as inappropriate.

Lognormal diffusion models can be used to simulate developed market currency. However, for emerging market currencies it is less appropriate and it's more suitable to use jump diffusion models since the emerging market currencies are more volatile and likely to have jumps.

(c) Critique your associate's statement and compare advantages and disadvantages of the recommended calibration versus other approaches.

1. Continued

Commentary on Question:
The candidates performed below average on this section. Successful candidates clearly re-stated the associates comments and described why they agreed or disagreed with the statement. Candidates who did poorly did not relate their answers back to the associate's comments or they provided unrelated information.

"calibration is not very important"

- Calibration is an important step in model building
- Future values are impacted by the calibration approach taken

"only consider calibration based on market prices"

- Market based calibrations can be used
  - They are calibrated to market prices from forward curves or option implied volatilities
  - Reflect forward looking views
- Historical data can also be used for calibration
  - Project future values based on historical statistics
  - May be slow to react to changes in market conditions even if more weight given to recent observations

(d) Propose a response to the sales desk at Beta.

Commentary on Question:
The candidates performed above average on this section. Unsuccessful candidates tended to restate facts from the questions with little or no explanation. For example, "Beta is rated BBB and Alpha is AAA which is better." An improved solution is "Beta is rated BBB and Alpha is AAA which means Beta will have a higher default risk than Alpha."

Since Beta is BBB and thus has more credit exposure to our company than Alpha, (an AAA rated firm), we need better compensation to take on the additional credit risk. The terms should be adjusted by either increasing the receive terms (e.g. higher fixed rate received) or having Beta pay an up-front cost.

2. Learning Objectives:

3. Candidate will understand the nature, measurement and management of liquidity risk in financial institutions.

Learning Outcomes:

(3a) Understand the concept of liquidity risk and the threat it represents to financial intermediaries and markets.

(3b) Measure and monitor liquidity risk, using various liquidity measurement tools and ratios.

(3c) Understand the levels of liquidity available with various asset types, and the impact on a company's overall liquidity risk.

(3d) Understand liability termination provisions such as book-value surrender and the impact on a company's overall liquidity risk.

(3g) Understand and apply techniques to manage street liquidity risk.

Sources:

QFIA-105-13 - Sec.3 - AAA Report of the Life Liquidity Working Group

QFIA-106-13 - Matz and New chp 3 ONLY

Commentary on Question:

This question tests the concept of liquidity management, including how to identify, measure, and manage the company specific liquidity risks.

Solution:

(a) Describe how to complete the liquidity monitoring program based on the three levels of liquidity management.

Commentary on Question:

Candidates performed as expected on this section. Most candidates understood the stress liquidity risk part of this section. Unsuccessful candidates did not mention specific actions associated with completing the liquidity monitoring program for each level.

Day to day cash management:

- Keep on top of short-term liquidity need through the combination of cash position and credit line.

Ongoing/intermediate term cash flow management:

- Ongoing liquidity need over 6 – 24 months
- Analyze and monitor the liquidity ratio (cash inflows against outflows).
- Involve management plan or tools to restore liquidity profile if needed.

2. Continued

Stress liquidity risk:

- Ensure company has ability to meet obligation under adverse scenarios.
- Be able to stay in business and remain solvent on a statutory basis.

(b) Evaluate where the liquidity management framework could fail if ABC adopts it from QFI without adjustment.

Commentary on Question:
Candidates performed as expected on this section. Most candidates understood the impact of policy and company size on the liquidity framework. Unsuccessful candidates provided a response that only considered one or two factors.

Policy Size:

- Compared to QFI Life, the average size of ABC's policies is much bigger as it targets high net worth clients (QFI's average size is only (200 billion/10 million) = 20,000.
- Compared to QFI Life, ABC has a lot fewer policies, as it's a start-up company.
  ABC is exposed to the liquidity risk that a large sum of money is under the impact of few contract/policy holders.

Size of company:

- Compared to QFI Insurance, ABC is a much smaller insurance company.
  ABC has less funding sources available compared to QFI Life.

Timing and Predictability:

- Unlike QFI who has a diverse client base, ABC is targeting high net worth clients, who in general are more sophisticated.
- Compared to the average contract/policy holders, sophisticated contract/policy holders normally act more quickly when demanding their money, subject to contract limitations. Therefore, ABC is exposed to higher liquidity risk than QFI from timing perspective.
- ABC and QFI are offering the same product lines in the market.

Other Factors:

- ABC block is younger, policies are earlier in their terms therefore tend to have lower lapses/withdrawals
- ABC may have a higher rate of sales growth due to it being a startup.
- There may be other sources of variation such as product features and demographics that would result in different liquidity needs between the two companies.

2. Continued

(c) Describe possible liquidity strains from Traditional Life Insurance's product features.

Commentary on Question:
Candidates performed above average on this section. Most candidates were able to describe liquidity strains from the product features. Unsuccessful candidates described the liquidity strain in general terms rather than connecting it to ABC specifically.

The product enables policyholders to borrow against their cash surrender value right after the 1st anniversary.

- Potential cash outflows from the demand of policy loans right after the 1st year with uncertainty in time.
- The high net worth clients are likely to leverage the ability to borrow against policies in worse economic conditions, which can add to ABC's liquidity risk under adverse scenario.

The product has no surrender charge after 5^{th} year anniversary.

- Policyholders might choose to surrender their policy after the 5^{th} year when there is no surrender charge, especially in the first month after the 5^{th} year anniversary.
- Given ABC's high net worth client base, this is more likely to happen.
- The uncertainty of policy surrender troubles liquidity management, not only through the demand of cash outflows to cover surrender benefits but also the loss of future premiums.

The product has a fast growing CSV.

- This feature will worsen the two liquidity risks identified above:
- The volume of policy loan that policyholders are eligible to demand can be huge and ABC has no or limited control to the timing of the demand.
- The surrender benefit can be high, which makes fulfilling obligation in a short time more challenging to ABC.

ABC promises to settle benefits in 1 months.

- Given its higher than market average policy size, the settlement amount for one policy caused by a normal course of events can be huge relative to the overall size of the company.
- Under this condition, settling benefits within a month can be a significant source of liquidity risk.

2. Continued

The product offers 7 days for policyholders to cancel policies without penalty.

- If ABC purchases the assets to fund liability right after receiving the premium, the cancelation guarantee could cause pressure to liquidate the assets and fulfill the refund.

(d) You decide to consider the following three options for evaluating liquidity risk of the Traditional Life Insurance's product through cashflow cushions.

Liquidity Risk Evaluation Options				
Option1	7 Days	1 Month	6 Months	1 Year
Option2	7 Days	1 Month	1 Year	5 Years
Option3	3 Days	3 Months	6 Months	1 Year

Recommend one of the above options.

Commentary on Question:
Candidates performed as expected on this section. Most candidates recognized that option 3 did not have time subsets associated with the contractual need of the liability for Traditional Life Insurance. Unsuccessful candidates did not recognize that option 2 contained a time subset of five years that was too long for the purposes of liquidity management.

Timing is a key element for liquidity risk - over a long enough period of time, liquidity risks go away as most assets either mature or can be sold without losses to fulfill policyholder's claim. Therefore, the time subset for liquidity management should be based on the contractual demand of their liability and should include and focus on short term and intermediate term. (From (b), ongoing cashflow management looks at ongoing cashflow need over next 6 – 24 months).

Choose option 1 over option 3:
Option 1 has time subsets associated with the contractual need of the liability for Traditional Life Insurance.
7 days: uncertainty of cash outflows when clients choose to cancel the policies
1 months: uncertainty of cash outflows that are required to settle benefits

Choose option 1 over option 2:
Option 1 and option 2 both have time subset associated with the contractual need of the liability for Traditional Life Insurance. However, option 2 include time subset of 5 years, which is too long for the purpose of liquidity risk management as lots of assets can be liquidated over 5 years to cover liability needs. Option 1, on the other hand, has time subset of 6 months, an intermediate term where the risk of not being able to cover a liability need is higher.

2. Continued

(e) Recommend two liquidation strategies that bring the one-year cashflow cushion back to the allowable limit, while minimizing the costs of liquidation.

(i) one excluding management actions

(ii) one including management actions

Commentary on Question:
Candidates performed as expected on this section. Most candidates were able to calculate the cash inflow required from asset liquidation. Unsuccessful candidates did not apply the liquidation strategy correctly.

Cashflow Cushion = Forecasted cash inflows for the period/ Forecasted cash outflows for the period
Based on the cashflow cushion of 0.95:1 and cash outflows of 10 million, the forecasted cash inflow in the period of 1 year is 9.5 million (9.5 = 10 * 0.95).
To get to cashflow cushion of 1.1:1, the required cash inflow is 11 million (11 = 10 * 1.1)
Required cash inflow from asset liquidation is 1.5 million. (1.5 = 11 – 9.5)

Liquidation Strategy: To minimize the cost of liquidation, we should first start from the asset with the least haircut and use up the liquidation capacity of the asset within the required period before moving on to the asset with the next smallest haircut. Note that the asset classes are arranged in credit quality order, not liquidation order so the candidate must identify the correct classes in the correct order, not just go left to right.

First calculation – Before management actions – without security lending program:

The total of 1.5M needed within a year while minimizing costs would be supplied as following:

5 Year AA Corporate Bond :
- Volume that can be liquidated in year : 0.5 million
- Volume that need to be liquidated : 0.5 million
- Cashflow from selling the asset : 0.5 * (1- 0%) = 0.5 million
- Cashflow still needed : 1.5 - 0.5 = 1 million

10 Year A Corporate Bond:
- Volume that can be liquidated in year : 0.4 million
- Volume needed to be liquidated : (0.4) * (1-10%) = 0.36 million
- Cashflow still needed : 1 - 0.36 = 0.64 million

2. Continued

10 Year BBB Corporate Bond:
- Volume that can be liquidated in year : 1.2 million
- Maximum cashflow that can be obtained: 1.2 * (1-20%) = 0.96
- Sell (0.64/0.96)*1.2 = 0.8 million to obtain cashflow of 0.64

Second Calculation – After management actions (with security lending program):

The first year additional cash inflow from security lending program is 0.7 million = 0.2 million 5Y AA bond + 0.2 million 10Y A bond + 0.2 million 20Y BBB bond + 0.1 million AA Private Fixed Income

The rest of cash needed within a year is 1.5 – 0.7 = 0.8 million which could be supplied as following:

5 Year AA Corporate Bond :
- Volume that can be liquidated in year : 0.5 million
- Volume that need to be liquidated : 0.5 million
- Cashflow from selling the asset : 0.5 * (1- 0%) = 0.5 million

10 Year A Corporate Bond:
- Volume that can be liquidated in year : 0.4 million
- Cash inflow needed: 0.8 – 0.5 = 0.3 million
- Maximum cashflow that can be obtained = 0.4 * (1-10%) = 0.36
- Sell (0.3/0.36)*0.4 = .33 million to obtain cash flow of 0.3 million

3. Learning Objectives:

1. The candidate will understand the standard yield curve models, including:
   - One and two-factor short rate models
   - LIBOR market models

   The candidate will understand approaches to volatility modeling.

Learning Outcomes:

(1a) Identify and differentiate the features of the classic short rate models including the Vasicek and the Cox-Ingersoll-Ross (CIR) models.

(1b) Understand and explain the terms Time Homogeneous Models, Affine Term Structure Models and Affine Coefficient models and explain their significance in the context of short rate interest models.

(1c) Explain the dynamics of and motivation for the Hull-White extension of the Vasicek model.

Sources:

Brigo, D and Mecurio F, Interest Rate Models – Theory and Practice, 2nd Edition, Section 3.2.1 2.

Brigo, D and Mecurio F, Interest Rate Models – Theory and Practice, 2nd Edition, Section 3.3-3.4 3.

Brigo, D and Mecurio F, Interest Rate Models – Theory and Practice, 2nd Edition, Section 3.8 – 3.9

Commentary on Question:

This question tests the concept of affine term structure and its significance in bond pricing.

Solution:

(a) Describe the characteristics of the Vasicek model.

Commentary on Question:

Candidates did brilliantly on this questions. Almost all candidates listed a sufficient number of characteristics of the Vasicek model to obtain full credit.

3. Continued

The Vasicek Model

- Vasicek model: $dr = a(b-r)dt + \sigma\, dz$, $(a > 0)$
- r is the short-term interest rate
- b is the long-run interest rate
- Contain mean reversion (b – r)
- a is the mean reversion speed adjustment rate
- σdz is the random noise term
- r is normally distributed
- Analytically tractable for bond prices
- Affine term structure model
- Constatnt theta/time homogeneous
- One factor model
- Rates can go negative
- Endogeneous/Can't be calibrated to market rates

(b) Vasicek model
CIR model
Hull-White Extended Vasicek model

Compare and contrast the Vasicek model with the other two models above.

Commentary on Question:
Candidates performed above average on this question. Candidates did well in comparing the three models. Unsuccessful candidates typically listed characteristics of the other two models without attempting to directly compare them to the Vasicek model.

Similarity between the CIR Model and the Vasicek Model

- Both models are one-factor models
- both models exhibit mean reversion (b – r)
- both models are equilibrium models
- both models are affine term structure models
- both models can analytically price bonds and bond options
- both models produce an endogenous term structure of interest rates, i.e. the initial term structure does not match observed term structure in the market.

Differences between the CIR and the Vasicek Model

- The main difference of the CIR model is to guarantee a nonnegative short rate since σ is related to r in $\sigma\sqrt{r}\, dz$ (use a square-root process for the short rate)
- Vasicek is a normal model and CIR is a $NC\chi^2$ model

3. Continued

Similarity between the HW and the Vasicek Model

- Both models are one factor normal models
- both models are mean reverting
- both models are normally distributed
- both models allow negative interest rates
- both models are possible of pricing analytically bonds and bond options

Differences between HW and the Vasicek Model

- HW is arbitrage-free, but Vasicek is an Equilibrium model
- HW has time-varying θ, but Vasicek has a constant one
- Vasicek is endogenous, but HW is not

(c) Explain which model is more accurate in pricing interest rate derivatives, such as options on bonds.

Commentary on Question:
Candidates did as expected on this question. Most candidates were able to identify that the Hull White Extended Vasicek model is most accurate, highlighting the ability to reproduce the initial yield curve as critical to match the initial pricing of interest rate options. Unsuccessful candidates failed to point out the shortcomings of the other models in this respect when answering this question.

- HWEV model is more accurate
- CIR and Vasicek model are equilibrium models
- They do not reproduce the initial term structure well.
- They lack the ability to model the short term and long term end of the term structure due to time-homogeneity.
- HWEV is an arbitrage free model, with time varying component
- HWEV can capture the current term structure

(d)

(i) Identify the type of process followed by $\boldsymbol{J}_t$.

(ii) Compare and contrast this model to CIR model.

Commentary on Question:
Candidates performed below average on this question. Most candidates were not able to identify the process/distribution of the model; however, candidates in general were reasonably proficient in identifying the model as a JCIR model, and comparing the JCIR and CIR processes.

3. Continued

(i) J_t is a compound Poisson process.

(ii) This model consists of a CIR model and a jump component *J*.
This model preserves the features of the basic CIR model, e.g. affine term structure, guaranteeing nonnegative short rates, etc.
In the CIR model, the short rate is a continuous function of time, while in this model, the short rate can have positive jumps.

4. **Learning Objectives:**

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

(2a) Demonstrate an understanding of events and causes of the 2008 global credit crisis.

(2l) Understand and apply various approaches for managing credit risk in a portfolio setting.

(2m) Understand the rationale, markets and risks of structured finance.

Sources:

QFIA-101-13: Managing Credit Risk: The Great Challenge for Global Financial Markets, Caouette, John B., et. al., 2nd Edition, 2008, Ch. 24: Structured Finance

Commentary on Question:

This question tests the understanding of structured finance concepts, by applying analysis to a hypothetical situation covering material in the referenced reading.

Solution:

(a) You work for a small investment bank that is thinking of originating structured finance business. Your colleague recently made the following comments:

"Over the years investment firms have benefited from innovation in terms of securitization and credit availability. Our firm should seriously consider issuing structured finance products.

The efficient use of capital is the only advantage for investors investing in this product. I like the fact that the burden of repayment is not on the originator but on a pool of assets generating cash flows and on the entity providing credit support.

Since only commercial mortgages generating future cash flows can be securitized, I say we consider issuing these products."

Critique your colleague's comments.

Commentary on Question:

The candidates performed brilliantly on this section. Well structured answers were explicit on analyzing each of the comments and making clear statements of agreement or not.

4. Continued

Yes, investment firms have benefitted from innovations but there also still risk (as evidenced by the 2007 financial crisis.)

There are many advantages for investors including the following:

- Liquidity
- Reduced Borrowing Costs
- Tax Management
- Efficient Use of Capital (not the only advantage)
- Regulatory Capital Arbitrage
- Reach additional Investors

The originator often still needs to have a stake in the entity that provides credit support and must be careful that underwriting standards are not relaxed. This may be an important consideration given the size of our firm.

Other assets besides commercial mortgages can be securitized such as:

- Residential Mortgages (RMBS)
- Auto Loans
- Credit Card Receivables
- Collateralized Bond Obligations (CBOs)
- Collateralized Loan Obligations (CLOs)

(b) Assess which tranche (if any) that each of the above investors would be more inclined to invest taking into account any additional risks that need to be considered.

Commentary on Question:
The candidates performed above average on this section. Most candidates made explicit recommendations with enough explanation to justify their choice. Unsuccessful candidates failed to recognize the hedging possibilities and provide a recommendation for the equity tranche for the company seeking highly rated assets.

Farming Company:
Equity Tranches as for hedge against not having rainy weather. This investment would offset farming losses that occurred during very sunny weather.

Institutional investor:
Tax tranche to offset gains that may be realized when assets need to be sold in a couple of years. (Same time that tax credits would be needed)

4. Continued

Pension Investor:
Either the equity or the main tranche should be appropriate depending on the asset needs of the pension. Separately, this investment should help satisfy the environmental mandate in place, no matter which tranche is selected.
Insurance Company:
The Main tranche gives the opportunity for adding a highly rated asset with some additional yield. The risk would be whether the supports are sufficient.

Traditional Power Generation Company:
The Equity tranche as a hedge against changing business environments to help with diversification of investments.

(c) Discuss the implications of these two developments on each of the tranches of this securitized asset, both individually and in relation to each other.

Commentary on Question:
The candidates performed below average on this section. Most candidates did not adequately address all parts of this question. The analysis of the developments was expected to be done on each individually as well as on both at the same time. Unsuccessful candidates failed to do a full analysis of each criteria as the question asked..

Better battery storage:
Favorable development for all three tranches as this makes the likelihood that there will be buyers for all of the electricity produced by the project. Any electricity produced when demand is low can be now be stored and sold when there is demand.

Lower Electricity Prices:
No impact to the tax credit tranche. This tranche was not dependent on the price of the electricity being produced.
For the Main Tranche, this change in price likely will some minor adverse effects as the floor likely will be more in play. If the new prices are well below the floor, then if the supports fail, impacts would be much more impactful.
The equity tranche will be adversely affected by the lower prices. This tranche could additionally need to provide support to the main tranche if the prices drop below the stated floors for the main tranche.

Interaction between the two items:
The tax credit tranche is likely not to be affected much if at all by either of these developments. Some political risk will always exist and may be slightly higher as solar power becomes more established.

4. Continued

The main tranche should most likely be slightly favorable given that the two items are working against each other but the supports in the main tranche should protect the tranche from the lower electricity prices.
It is difficult to ascertain the impacts to the equity tranche. The lower electricity prices will definitely hurt the tranche but that should be offset to some extent by the improvement in battery technology. Without additional information, the risk to the tranche cannot be fully evaluated.

5. Learning Objectives:

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

(2b) Demonstrate an understanding of the basic concepts of credit risk modeling such as probability of default, loss given default, exposure at default, and expected loss.

(2d) Demonstrate an understanding of Merton asset value models in the context of credit risk.

Sources:

Bluhm 2nd Ed, Ch 3

Commentary on Question:

This question tests candidates understanding of basic concepts of credit risk modeling and the application of Merton asset valuation model in the context of credit risk.

Solution:

(a) Calculate an upper bound for the asset value of EcL.

Commentary on Question:

The candidates performed above average on this section. Some candidates received partial credit for computing equity value correctly but not the debt value.

Total asset value A = E + B
Where:
E = EcL's equity value = 1 million shares * $40/share = 40 million
B = EcL's debt value. Since B is unknown, we solve for the upper bound of B assuming EcL has no default risk

Upper bound of debt vale = B_{max} = 350 * exp(-2%*9) = 292.3 million
Therefore: A = E + B ≤ E + B_{max} = 40 + 292.3 = 332.3 million
The upper bound of asset value = A_{max} = 332.3 million

(b) Show that your analyst's estimate of EcL's asset value volatility is not correct.

Commentary on Question:

The candidates performed as expected on this section. There are many methods to answer this section and all methods were given full credit as long as the candidates correctly demonstrated their understanding of the relationship that holds for σ_A . Partial credit was given for understanding the relationship; however, not providing commentary on their analysis.

5. Continued

Under Merton's model, the asset volatility, σ_A , satisfies

$$\sigma_A = \frac{\sigma_E E}{A\, N(d_1)}$$

Where:
σ_E = EcL's equity volatility = 45.5%
E = 40 million from (a)
$N(d_1) =$ Probability from a normal distribution ≤ 1
A = EcL's asset value $\leq A_{max}$ from (a)

Thus,

$$\sigma_A \geq \frac{\sigma_E E}{A_{max} * 1} = \frac{45.5\% * 40}{332.3 * 1} = 5.62\%$$

This indicates that the asset volatility σ_A cannot be lower than 5.62%, which implies the analyst's estimate of "$\sigma_A = 5\%$" is incorrect.

(c) Determine if your analyst either overstated or understated the asset value volatility by testing the impact of a +1% shock on the asset value volatility.

Commentary on Question:
The candidates performed as expected on this section. There are many methods to answer this section and all methods were given full credit as long as the candidates correctly demonstrated their understanding of the constraints in their analysis.

Based on part (b), the asset volatility σ_A cannot be lower than 5.62%. Therefore, the analyst's estimate of "$\sigma_A = 5\%$" understated the asset volatility.

6. Learning Objectives:

4. The candidate will understand important quantitative techniques relating to financial time series, performance measurement, performance attribution and stochastic modeling.

Learning Outcomes:

(4i) Demonstrate an understanding of the general uses and techniques of stochastics modeling.

Sources:

QFIA-124-16 IAA, Stochastic Modeling, Theory and Reality from and Actuarial Perspective, sections I-1 to I-29 and II-1 to II-24

Commentary on Question:

This question tests the candidate's recognition of foreign exchange rate dynamics in relation to market volatilities. Volatility reduction techniques are also addressed as a secondary objective.

Solution:

(a) Describe the historical and market volatility approaches and recommend the approach for your company.

Commentary on Question:

Candidates performed as expected on this section, often defining both approaches and making a recommendation while not going into great detail.

The market approach solves for the implied volatility in current market prices. It is market consistent and has a term structure but is only appropriate for short-term projections.

Historical volatility uses the standard deviation measured as a constant over the same time period as the expected life of the business modeled. This is the preferred approach given the company's long-term view, a margin may be added as needed.

(b) Project the CAD to USD exchange rate one year hence based on the five scenarios above.

Commentary on Question:

Candidates performed above average on this section, often achieving full credit and at a minimum able to define and set up the problem.

6. Continued

Given 10% volatility, RFD as 1.2%, RFF as 1.5%, an initial X of 0.8, and a time delta of 1, the five random numbers correspond to future values of 0.739, 0.841, 0.868, 0.781, and 0.714 based on the formula

$X(t+\Delta t) = X(t) * \exp((RFF-RFD - vol^2/2)* \Delta t + sqrt(\Delta t) *vol * Z)$

The average of these values is 0.789.

(c) Explain the reaction of the FX return you expect to observe.

Commentary on Question:
Candidates performed as expected on this question. Unsuccessful candidates often did not recognize the correct direction of the FX return with the RFD shock. Successful candidates provided details on their reasoning.

The FX return increases with the RFD shock. Since foreign bonds don't have a duration with respect to RFD the returns measured in the domestic currency for both foreign bonds and foreign equity will show an increase the same direction as the RFD shock because of the FX movement.

(d) Describe four variance reduction techniques that can be applied in stochastic modeling.

Commentary on Question:
Candidates performed brilliantly on this question, typically achieving full credit by detailing each of the four approaches.

- Antithetic Variables: Creating two trials, one using randomly generated u1, second using -u1, then the estimator of f(u) is the average of f(u1) and f(-u1).
- Control Variates: Instead of using a given function f(u), the modeler chooses another function, g(u) for the run and converts results back to f(u).
- Stratified Sampling: Stratified sampling divides the distribution of a market variable into homogeneous subgroups or "stratum".
- Importance Sampling: Importance sampling generates samples under the heavy loads and adjusts the results for the difference.

7. Learning Objectives:

1. The candidate will understand the standard yield curve models, including:
   - One and two-factor short rate models
   - LIBOR market models

   The candidate will understand approaches to volatility modeling.

Learning Outcomes:

(1a) Identify and differentiate the features of the classic short rate models including the Vasicek and the Cox-Ingersoll-Ross (CIR) models.

(1l) Define and explain the concept of volatility smile and some arguments for its existence.

(1m) Calculate the hedge ratio for a call option given the dependency of the Black-Scholes volatility on the underlying.

(1q) Describe and contrast several approaches for modeling smiles, including: Stochastic Volatility, local-volatility, jump-diffusions, variance-gamma and mixture models.

Sources:

Brigo, D and Mecurio F, Interest Rate Models – Theory and Practice, 2nd Edition, Ch. 3

Rebonato, Volatility Correlation – The Perfect Hedger and the Fox, 2nd Edition, Ch. 6, 7, 8, 9

Commentary on Question:

This question tests a variety of implications for Black-Scholes modeling in a foreign exchange environment.

Solution:

(a) Describe the observed behavior(s) of FX option implied volatility between JPY/AUD.

Commentary on Question:

Candidates performed above average on this question, typically identifying the markets as mature and more volatile resulting in them getting full credit.

The shape of the volatility smile is highly variable in mature markets, sometimes forming a smirk or flattening.

(b) Explain why the synthetic put option might not hedge the GMDB exposure in the presence of a volatility smile.

7. Continued

Commentary on Question:
Candidates performed above average on this question, comparing the Black-Schole's deficiencies in regard to hedging.

The put option is not at-the-money so u sing the Black-Schole's constant volatility assumption will produce the wrong hedge ratio due to an incorrect delta.

(c) Describe other models that will help the company better capture the volatility smile.

Commentary on Question:
Candidates performed as expected on this question. Unsuccessful candidates just identified one of the models that is appropriate, where as successful candidates identified all three.

Full stochastic models such as the Heston model identifies several sources of variance, Local volatility models that have a deterministic sub-model for the volatility are also appropriate. Jump diffusion models that add a discontinuity on top of a Brownian motion model could also be used.

(d) Identify the process of the instantaneous variance v_t.

Commentary on Question:
Candidates performed brilliantly on this section, almost always achieving full credit.

This is a CIR (Cox-Ingersoll-Ross) model.

(e) Explain how this model produces a volatility smile.

Commentary on Question:
Candidates performed poorly on this section. Unsuccessful candidates often failed to recognize the smile in the model.

This is a stochastic process that is thought of as an integral across all Black-Schole's sub call options. The formula is almost exactly linear when at-the-money but becomes convex away from that point as show in Jenkin's inequality, this causes the volatility smile.

7. Continued

(f) Explain what does $\text{CTE}_{1-\alpha}(\chi) = 1$ implies, with respect to the hedging efficiency.

Commentary on Question:
Candidates performed at a fair level on this question, recognizing what CTE is but not always its implications, achieving about half credit.

The CTE is the additional amount of money at offset to keep the insurer's portfolio from ever going negative. Value of 1 means that over the worst scenarios one additional unit must be held at issue for every 100 units of premium.

(g) Comment on why setting up a Vega hedge portfolio within the Black-Scholes model can lead to potential "over-hedging".

Commentary on Question:
Candidates performed above average on this question, recognizing the constant nature of the Black-Shole's model. Unsuccessful candidates did not recognize the implications for Vega hedging.

Since volatility is constant in the Black-Scholes the changes in Vega would be overestimated, leading to over-hedging.

8. Learning Objectives:

7. The candidate will understand various investment related considerations with regard to liability manufacturing and management.

Learning Outcomes:

(7a) Identify and evaluate the impact of embedded options in liabilities, specifically variable annuities guaranteed riders (GMAB, GMDB, GMWB and GMIB).

(7g) Demonstrate an understanding of liability driven investing (LDI) for pension plans.

Sources:

ERM-407-14: Equity Index Annuities – Downside protection, but at what cost?

QFIA-116-13: The Impact of Stochastic Volatility on Pricing, Hedging and Hedging Efficiency

Commentary on Question:

This question tests the understanding of the Equity Indexed Annuity (EIA) product and the use of derivatives for hedging purposes.

Solution:

(a) Critique the product features and investment strategy, recommending potential changes.

Commentary on Question:

The candidates performed below average on this section. Nearly all candidates correctly noted that surrenders should not be free of charge. Many unsuccessful candidates were unable to identify the need to reduce the bond portfolio duration or the need to adjust allocations based on market conditions.

- Surrender should not be free of charge. There are large upfront costs when issuing a policy and the investments are illiquid. There should be a high surrender charge in the early years, which then grades down to 0 over time after the upfront costs have been recouped.
- The duration of the bond portfolio is too long since the EIA liability has a maturity of 12 years, so its duration is less than 12 years. The duration of the bond portfolio should be reduced (i.e. invested in shorter-term bonds) to achieve a better asset-liability match and reduce possible liquidation costs.
- The fixed allocation of the investment portfolio doesn't adapt to changes in market conditions. The allocation to bonds and options should be adjusted to reflect their costs in the market. Additionally, the company should be able to change the participation, cap and floor rates in response to changes in market conditions.

8. Continued

(b) Outline an appropriate hedging strategy for the product. Justify your answer.

Commentary on Question:
The candidates performed as expected on this section. Many candidates were able to correctly identify the appropriate hedging strategy. Some candidates did not provide a justification or payoff diagram of the hedging strategy and received partial credit. Some candidates recommended longing ATM puts & shorting OTM calls instead of longing ATM calls & shorting OTM calls and received partial credit.

The EIA product can be hedged by longing ATM call options and shorting an OTM call options with a strike 5% above the current index level. This strategy provides downside protection and 100% upside participation in the index returns ranging between 0-5%. For any index returns in excess of 5%, the long and short positions offset each other, and the net payoff is 5%.

(c) Calculate the total hedge cost for your proposed strategy in (b). State all assumptions made.

Commentary on Question:
The candidates performed as expected on this section. Some candidates were able to correctly calculate the number of long ATM calls needed. Many candidates missed the 10 multiplier in the calculation of the number of long contracts needed and received a minor reduction for doing so. Many candidates correctly recognized the need to short an equal number of OTM calls as the number of long ATM calls. Most candidates were able to correctly use linear interpolation to obtain the price of the short call. Many candidates understood the concept of hedge cost but missed the 10 multiplier in the calculation and received a minor reduction for doing so. Most candidates did not state the assumption that the bid/ask spread was 0 in the calculation of the hedge cost.

# of long ATM calls needed = 4,000,000/(2,775*10) = 144

Given the 5% cap, we will also need to short 144 call options with strike of 2,775*1.05 = 2,914

Using linear interpolation, we obtain the price of the short call to be:
(2,925-2,914)*122/(2,925-2,900) + (2,914-2,900)*111/(2,925-2,900) = 116

Hedge cost = 10*#contracts*(price long ATM call - price short OTM call) = 10*144*(183-116) = 96,480

The hedge cost calculation assumes that the bid/ask spread is 0 (i.e. no transaction costs)

8. Continued

(d) Propose a modification to the hedging strategy based on the modified credited rate specification.

Commentary on Question:
The candidates performed poorly on this section. Some candidates correctly identified the appropriate modification to the hedging strategy. Some candidates did not provide a justification or payoff diagram of the hedging strategy and received partial credit. Many candidates omitted this part of the question.

In addition to the options used in (b) - long ATM calls & short OTM calls, the hedger should short ATM puts. This strategy leads to the same payoff if the index goes up, but it results in losses if the index falls.

(e) Calculate y% such that the hedge cost equals 2% of the fund value under the modified hedging strategy, assuming the same $4 million of projected sales and starting index value of 2775.

Commentary on Question:
The candidates performed poorly on this section. Many candidates were able to obtain the hedge budget. Some candidates were able to come up with the amount of the shortfall that needed to be funded. Most candidates struggled with the calculations of the number of ATM puts to short and y%. Many candidates omitted this part of the question.

Hedge budget = 4,000,000*2% = 80,000

Given the cost of the original hedge, we need to fund a shortfall of 96,480-80,000 = 16,480

We short ATM put options with a price of $142. The number of ATM puts to short is: 16,480/142/10 = 11.6

y% = 11.6*10*2775/4,000,000 = 8.0%

9. **Learning Objectives:**

6. The candidate will understand and be able to describe the variety and assess the role of alternative assets in investment portfolios. The candidate will demonstrate an understanding of the distinguishing investment characteristics and potential contributions to investment portfolios of the following major alternative asset groups:
   - Real Estate
   - Private Equity
   - Commodities
   - Hedge Funds
   - Managed Futures
   - Distressed Securities
   - Infrastructure

Learning Outcomes:

(6a) Demonstrate an understanding of the types of investments available in each market, and their most important differences for an investor.

(6b) Demonstrate an understanding of the benchmarks available to evaluate the performance of alternative investment managers and the limitations of the benchmarks.

(6c) Demonstrate an understanding of the investment strategies and portfolio roles that are characteristic of each alternative investment.

Sources:

QFIA-111-13: Maginn & Tuttle, Managing Investment Portfolios, 3rd Ed. 2007, Ch. 8

QFIA-113-13: Secular and Cyclic Determinants of Capitalization Rates: The Role of Property Fundamentals, Macroeconomic Factors and "Structural Changes"

Commentary on Question:

This question tests the understanding of real estate investments with regard to the impact of macro-economic factors on the capitalization rate and asset value, appraisal measures of performance, and benchmark and assessment of risks for investors.

Solution:

(a) Discuss the advantages and risks associated with real estate as they relate to the Education fund.

9. Continued

Commentary on Question:
Candidates performed as expected and identified attributes of liquidity and cost associated with Real Estate (RE). Unsuccessful candidates were not able to distinguish Direct and REIT's investments and the advantages and risks associated with each one, especially with the risk of liquidity. Many candidates assumed direct property without mentioning it. In addition unsuccessful candidates were not able to discuss advantages associated with the potential enhancement for the Educational fund to enhance the return and lower the volatility.

As advantages:

Real estate (RE) which include Direct property and Real estate investment trusts (REIT's) are both types of investments known as a potential return enhancement in a portfolio of stocks and bonds;
RE provide lower volatility in return and is less affected by short-term economic conditions;
Direct investment in commercial real estate properties with rental income component increases the stability of its returns;
Direct real estate is not highly correlated with the performance of other assets (stock or bonds) and lower correlated than REIT's;
REIT's are publicly traded equities that securitize illiquid real estate assets
RE permit high risk-adjusted returns for investors who can obtain cost-efficient and high-quality information;
REIT's permit small investors with as small outlay to gain real estate exposure with also an exposure to a professionally managed portfolio.

As risks:

Direct real estate has a lack of liquidity and not easy to divide the property to sell a portion;
REITs avoid risk of concentration in direct ownership property and idiosyncratic;
Direct ownership has a higher cost of acquiring information, brokers charge, high commissions, substantial operating and maintenance costs which are not incurred by REITs investors;
Lack of knowledge or experience in direct ownership by the Education fund management justify additional cost for direct investment.

(b) Discuss limitations that this performance measure may have when used to measure the performance of an asset class such as real estate.

9. Continued

Commentary on Question:
Candidates performed below average on this question. Successful candidates were able to mention almost all limitations of the Sharpe ratio. Unsuccessful candidates only listed one or two limitations, not enough to align with the point value of the question. Nearly all candidates were able to describe the impact of low volatility of Real Estate and its impact to increase the Sharpe ratio.

This measure is the Sharpe ratio as implied by the 0.65 value.
The main limitations for this measure are:

- The Sharpe ratio is time dependent;
- Illiquid investment bias Sharpe ratio upward;
- Sharpe ratios overestimated when investment returns are serially correlated which causes a lower standard deviation;
- Sharpe ratio is primarily a risk-adjusted performance measure for stand-alone investments and does not take into consideration the correlations with other assets in a portfolio;
- Sharpe ratios have no predictive ability;
- Sharpe ratios can be gamed; that is, the reported Sharpe ratio can be increased without the investment really delivering higher risk-adjusted returns.
- Sharpe ratio of individual asset will be different than its impact on the fund's Sharpe. Therefore, adding an asset with lower Sharpe but also lower correlation to fund may raise Sharpe.

(c) Explain reasons supporting the addition of these two components including their interaction with $RRI_{j,t}$ and RTB_t, and impact on asset values and capitalization rates.

Commentary on Question:
Candidates performed as expected. Almost all candidates were able to explain the impact on the capitalization rate for the new components. For the asset side, unsuccessful candidates were not able to recognize the negative impact of the spread to lower the asset values but were able to justify the impact for the debt flow. Most candidates did not recognize the absence of change and correlation with RRI and RTB.

The new variables SPREAD AND DEBTFLOW will not change sign and significance of Real rent index (RRI) and Real T-bond (RTB) since they are orthogonal to RRI and RTB factors;

The SPREAD reflects compensation for higher risk. This result in a higher discount rate of net operating earnings which reduce the asset price or asset value. As the SPREAD increase, the asset value decreases and the capitalization rates increase since capitalization rates = earnings / price or asset value.

9. Continued

The DEBTFLOW reflect flood of global capital which increased the availability of debt in the market. This increase in volume of debt availability permit it is easier to trade real estate and push up the asset prices or asset values.
This positive correlation on assets or prices with DEBTFLOW translates into a lower capitalization rate.

Then, inclusion of the SPREAD and DEBTFLOW are significant and have strong effects on real estate asset values and capitalization rates model.

10. Learning Objectives:

3. Candidate will understand the nature, measurement and management of liquidity risk in financial institutions.

6. The candidate will understand and be able to describe the variety and assess the role of alternative assets in investment portfolios. The candidate will demonstrate an understanding of the distinguishing investment characteristics and potential contributions to investment portfolios of the following major alternative asset groups:
   - Real Estate
   - Private Equity
   - Commodities
   - Hedge Funds
   - Managed Futures
   - Distressed Securities
   - Infrastructure

Learning Outcomes:

(3a) Understand the concept of liquidity risk and the threat it represents to financial intermediaries and markets.

(3b) Measure and monitor liquidity risk, using various liquidity measurement tools and ratios.

(6c) Demonstrate an understanding of the investment strategies and portfolio roles that are characteristic of each alternative investment.

(6e) Demonstrate an understanding of infrastructure investments.

Sources:

Quantitative Credit Portfolio Management, Ben-Dor, et. al., 2012, p. 133-135

Quantitative Credit Portfolio Management, Ben-Dor, et. al., 2012, p.133-134

Quantitative Credit Portfolio Management, Ben-Dor, et. al., 2012, p.138-139

Commercial Real Estate Analysis & Investment, Chapter 12, p. 264-266, p.271, p.274, p.282-285

Commentary on Question:

This question tests a candidate's understanding of the components of OAS. It also tests the ability to apply their knowledge of the commercial real estate market. Finally, it tests their knowledge of the investment characteristics of infrastructure in a portfolio management scenario.

10. Continued

Solution:

(a) Describe the relationship between a bond's bid-ask spread and its liquidity risk.

Commentary on Question:
Candidates performed as expected in this question. Most candidates were able to describe appropriately the positive relationship between liquidity risk and a bond's bid-ask spread; however, unsuccessful candidates failed to give an account of how liquidity risk affects the pricing and trading of the bond to the investor.

- The wider the spread, the higher the liquidity cost and the higher the bond's liquidity risk
- A wider spread means a bond investor at the time of selling will want to be compensated for the greater risk in the form of a risk premium.
- A narrower spread means the bond is more readily and easily traded (requiring a lower risk premium).
- The bid-ask spread is only one indicator that affects a bond's liquidity risk.

(b)

(i) Explain to your manager why spread decomposition is useful.

Commentary on Question:
Candidates performed as expected in answering this question. Successful candidates were able to identify the importance of spread decomposition for strategic portfolio management. Unsuccessful candidates were unable to use the basics of the OAS regression model in order to explain spread decomposition. Partial credit was awarded if candidates demonstrated an understanding of the model without describing its importance.

- The portfolio strategy will depend on the reason why the credit spread moved. For example, whether the wide spreads are due to large expected default losses, high liquidity cost or a high-risk premium.
- The portfolio can ride out periods of high liquidity cost and risk aversion if a buy-and-hold strategy is used.
- If the wide spread reflects an increase in expected default losses, you may need to reposition or hedge its portfolio.
- Can use the decomposition to monitor separately the liquidity and credit risk.
- Credit bond spreads are generally much larger than is justified by their subsequent default and recovery experience. A portion of the credit spread may reflect an expected liquidity cost. Another portion of the credit spread may reflect a risk premium demanded by risk-adverse investors because of uncertainty of cashflow and liquidity costs.

10. Continued

(b)

(ii) Explain some of the issues with the use of CDS spreads in the context described above.

Commentary on Question:
Overall, candidates performed poorly. Successful candidates were able to analyze the features of the market-quoted CDS in order to conclude its drawbacks in measuring INT's long-duration investment portfolio.

- Using CDSs data is not appropriate for every bond.
- CDS markets are not necessarily liquid and therefore cannot always be considered as a pure default proxy.
- Five-year CDSs are usually the most liquid. The liability has long duration, so bonds in the portfolio tend to have long durations and the CDS market will not be liquid for the long bonds.
- CDSs may not be available for all the bonds in the portfolio. (Bonds in this portfolio may not be part of the CDS universe.)

(c) Evaluate the validity of each of these four reasons.

Commentary on Question:
Candidates performed below average on this section. Unsuccessful candidates failed to demonstrate an understanding of the role of commercial real estate investments in portfolio management.

1. This is not entirely a valid reason. INT should perform a due diligence before entering the market. Also, the decision to enter the commercial estate market should not be the only factor in determining changes in INT's asset allocation.
2. This reason is invalid as the risk lies in the asset and not the investor
3. It is true that the commercial real estate market exhibit lower market inefficiency compared to REIT.
4. Since the market has limited data, any new information can be significant and viewed as relevant. The potential for new information may cause INT to be overconfident

(d) Finally, your manager is considering expansion to one additional alternative asset class not yet considered above. The team has been given the following goals to be achieved for any asset class to be added:

Goal 1: Help the portfolio outperform in times of financial crises
Goal 2: Opportunity to take advantage of an inefficient market
Goal 3: Support duration matching hedging activities

Recommend an asset class that could meet all of these goals.

10. Continued

Commentary on Question:
Candidates performed poorly on this section of the question. Successful candidates were able to identify infrastructure investment and provide reasonable explanation for each goal supporting its selection given INT's long-duration asset portfolio. Unsuccessful candidates recommended commodities and hedge funds. While these were valid asset classes, they did not support all of INT's goals. Maximum points were awarded if candidates provided at least comprehensive explanations in support of each infrastructure asset

Goal 1:

- Attractive returns;
- Low sensitivity to swings in the economy and markets;
- Low default rates;
- Good inflation hedge
- Infrastructure outperformed during the financial crisis.
- Infrastructure provides diversification benefits which could help the portfolio during financial crises (even though evidence is not conclusive).

Goal 2:

- High barriers to entry
- Economies of scale (*e.g.* high fixed, low variable costs);
- Inelastic demand for services (giving pricing power);
- Low operating cost and high target operating margins; and
- Low correlation of returns with other asset classes;
- INT is in the U.S. and most of the historical data is from Australia and Canada, so the U.S. market is not yet fully established.
- Most carriers invest less than 10% of their portfolio in infrastructure, so the market is largely untapped.
- There is limited historical data, and no evidence indicating the market for infrastructure is mature.

Goal 3:

- Duration mismatch has been a concern for investors because the lifetime of the underlying assets and the lifetime of the vehicle into which they are packaged often differ and investors have to deal with reinvestment risks.
- Recently new funds are created with longer lifetime and flexible exist strategies and of open-ended funds. INT Life should invest in these funds when duration mismatch is a concern.
- Long duration (e.g. concessions of 25 years, leases of 99 years).
- Long term, stable and predictable cash flows;
- Natural fit with long-lasting, often inflation-linked pension liabilities;

11. Learning Objectives:

4. The candidate will understand important quantitative techniques relating to financial time series, performance measurement, performance attribution and stochastic modeling.

5. The candidate will understand the behavior characteristics of individuals and firms and be able to identify and apply concepts of behavioral finance.

Learning Outcomes:

(4b) Apply various techniques for analyzing factor models including Principal Component Analysis (PCA) and Statistical Factor Analysis.

(5b) Describe how behavioral finance explains the existence of some market anomalies.

(5c) Identify and apply the concepts of behavioral finance with respect to individual investors, institutional investors, portfolio managers, fiduciaries and corporate managers.

Sources:

QFIA-109-13: A Survey of Behavioral Finance, by Barberis & Thaler

QFIA-125-16: Market Models: A Guide for Financial Data Analysis, Ch. 6, Principal Component Analysis

Commentary on Question:

This question tests the understanding of behavioral finance and the application of principle component analysis (PCA).

Solution:

(a) Explain the strategy and rationale an arbitrageur could use to take advantage of the CIO's expectation.

Commentary on Question:

The candidates performed above average on this section. Most candidates understood that Stock A price was likely to go up due to inclusion in the index. Unsuccessful candidates tended to take a long position in stock A without taking a mitigating short position in A, which is something an arbitrageur would do.

An arbitrageur could short Stock C and purchase Stock A in anticipation of Stock A being included in the index. This assumes the following:

- Since Stock C and Stock A are both financial institutions they would be a good substitute for each other
- Shorting Stock C would fund the purchase of Stock A, allowing the arbitrage strategy to be costless

11. Continued

Studies have shown that when a stock is added to an index, the price will jump, much of which will be permanent. Once Stock A is included in the index we would expect to see a permanent increase in value, resulting in a profit on the arbitrage strategy.

(b) Describe factors that may cause this strategy to be unprofitable.

Commentary on Question:
Candidates performed brilliantly on this section. Most candidates understood how the opportunity to profit on Stock A joining the index may not materialize. Unsuccessful candidates did not provide an answer that addressed the question.

Fundamental Risk – The arbitrage strategy assumes that Stock A and Stock C can be used as a substitute for each other. In reality substitute securities are rarely perfect which means that this arbitrage strategy does not remove all fundamental risk

Implementation Costs – Other costs such as transaction costs, commissions, bid-ask spreads could impact the profitability of this strategy, potentially making the strategy not attractive to execute.

(c) Describe Ambiguity Aversion and how it would impact the decision making of a dividend-focused investor deciding between investing in either Stock A or Stock B.

Commentary on Question:
The candidates performed brilliantly on this section. Successful candidates understood ambiguity aversion and how to apply it. Unsuccessful candidates only answered part of the question.

Ambiguity Aversion – People dislike ambiguity or when the probability distribution of the outcome is unknown.

Impact on Investor – This investor is focused on dividends and therefore when deciding between two investments, he will pick the investment that has a guaranteed dividend. Stock A pays a regular dividend every quarter while Stock B has not paid a dividend but is considering it for the first time. Therefore, the investor will select Stock A since the dividend is guaranteed.

(d) Simulate the standardized daily return for stock B over the two years preceding the past year.

11. Continued

Commentary on Question:
The candidates performed as expected on this section. Most candidates understood how to determine the number of principal components that would account for 75% of the total variation. Many candidates could determine the proper weights to use with the principal components. Unsuccessful candidates failed to calculate the principal components themselves.

The number of principal components that explains at least 75% of the variation of the system of 5 stocks is 3:
2.46/5 = 0.492
(2.46+0.72)/5 = 0.636
(2.46+0.72+0.71)/5 = 0.778 > 0.75

$B_r = (0.45)*W_1 – (0.19)*W_2 – (0.63)*W_3$

$W_1 = (0.54)*(-0.13) + (0.52)*(0.91) + (0.46)*(0.01) + (0.48)*(2.12) = 1.42$
$W_2 = (-0.22)*(-0.13) + (0.04)*(0.91) + (0.80)*(0.01) + (-0.56)*(2.12) = -1.12$
$W_3 = (-0.30)*(-0.13) + (-0.61)*(0.91) + (0.39)*(0.01) + (0.62)*(2.12) = 0.80$

$B_r = (0.45)*W_1 – (0.19)*W_2 – (0.63)*W_3$
$B_r = (0.45)*(1.42) – (0.19)(-1.12) – (0.63)(0.80) = 0.348$

(e) Explain an effective way to convert a PCA simulated standardized daily return to prices.

Commentary on Question:
Candidates performed poorly on this section. Successful candidates recognized that the data in the PCA had to be standardized first before doing the PCA. Unsuccessful candidates left this section blank or suggested methods such as regression which were unnecessary.

Multiply the simulated return by the standard deviation for stock B in the pre-sample (the last year of data)

Add this result to the mean of all the returns in the pre-sample, not just the mean of stock B to produce the adjusted daily return.

Divide the next day's price by (1+the adjusted daily return) to find the prior day's price.

12. Learning Objectives:

1. The candidate will understand the standard yield curve models, including:
   - One and two-factor short rate models
   - LIBOR market models

   The candidate will understand approaches to volatility modeling.

Learning Outcomes:

(1f) Explain how deterministic shifts can be used to fit any given interest rate term structure.

(1i) Explain the set up and motivation of the Lognormal Forward LIBOR Model (LFM).

(1j) Describe the calibration of the LFM to Cap and Floor prices.

Sources:

Brigo, D and Mecurio F, Interest Rate Models – Theory and Practice, 2nd Edition, Section 3.8, p.68.

Brigo, D and Mecurio F, Interest Rate Models – Theory and Practice, 2nd Edition, Section 6.1-6.2, p.195-199.

Brigo, D and Mecurio F, Interest Rate Models – Theory and Practice, 2nd Edition, Section 6.3, p.220-222.

Brigo, D and Mecurio F, Interest Rate Models – Theory and Practice, 2nd Edition, Section 6.3, p.210-211.

Commentary on Question:

This question tests the knowledge of market models and its application to caplet prices.

Solution:

(a) Describe the caplet that achieves the CRO's recommendation.

Commentary on Question:

Candidates performed above average on this section. Many candidates correctly identified all terms of the caplet and received full points. Some candidates did not recognize the caplet was forward starting and received partial credit. Some candidates did not identify the appropriate strike level and received no credit.

The CRO should take a long position on a caplet to floor the interest spread for a single period while retaining upside potential. The loan is funded in 6 months and so the caplet should be forward starting. To achieve a minimum 1.5% spread, the company should purchase a one-year caplet with cap rate of 6% - 1.5% = 4.5% starting in 6 months.

12. Continued

(b) Compare and contrast the properties of the CIR++ and the CIR short-rate models.

Commentary on Question:
Candidates performed above average on this section. Most candidates were able to provide at least one similarity and difference between the two models. Some candidates mistook the CIR++ to be a two-factor market model. Some candidates did not identify the difference between the two models and so only received partial credit.

Both models are mean-reverting models that have analytical formulas for bond prices, bond-option prices, swaptions and cap prices.
Both models have their distribution of the instantaneous spot rate with tails that are fatter than in the Gaussian case.
Through restriction on the parameters, it is always possible to guarantee positive rates for both models without worsening the volatility calibration in most situations.
Both models are affine one-factor models in the short rate.
Unlike the CIR, the CIR++ provides an exact fit of any observed term structure and reproduces market prices.

(c) Describe the advantages of using a market model over a short rate model when pricing interest rate derivatives.

Commentary on Question:
Candidates performed as expected on this section. Most candidates correctly noted that market models better reflected market prices. Some candidates did not identify that market models are consistent with the market standard Black formulas and so only received partial credit.

Unlike short rate models, market models use interest-rate dynamics that are compatible with market standard Black formulas for the two main markets for interest rate derivatives, caps and swaptions markets.
Market models provide a rigorous derivation of Black's formulas rather than using simplifying and inexact assumptions on interest rate distributions in short rate models.
Market models use observable market rates whereas short rate models use unobservable instantaneous interest rates.
Market models are easy to calibrate and are widely used in practice today.

12. Continued

(d) Calculate the price of the caplet the CRO recommends using Black's formula.

Commentary on Question:
Candidates performed poorly on this section. Some candidates correctly stated the caplet formula and performed most or all of the required calculations. Many candidates did not come up with the correct formula while others incorrectly attempted to use the Black-Scholes call option formula instead of the Black formula.

A one year caplet with cap rate K = 4.5% at t = 0.5 is needed to be priced at t = 0

Caplet Price = $L * P(0,T) * \tau * [F * N(d1) - K * N(d2)]$

Where:

$F = F(0,0.5,1.5) = 4\%$
t_k = 6 months = 0.5 years
t_{k+1} = 1.5 years
$\tau = t_{k+1} - t_k$ = 1 (year)
Volatility of all forward rates = $\sigma = 30\%$
Notional of the loan = \$200,000 = L

$d1 = [\ln(F/K) + v^2/2] / v$
$d2 = [\ln(F/K) - v^2/2] / v$ where, $v^2 = \tau \times \sigma^2$ since we are assuming constant forward volatilities.

P(0,0.5) = (1/(1 + spot rate))^0.5= (1/1.04)^0.5
P(0.5,1.5) = (1/(1+F(0,0.5,1.5)))^(1.5-0.5) = (1/1.04)
P(0,1.5) = P(0,0.5)*P(0.5,1.5) = (1/1.04)^1.5

$d_1 = \{\ln(4\%/4.5\%) + 1 * (30\%)^2/2\} / \{(1)^{(0.5)} * 30\%\} = -0.242610$
$d_2 = \{\ln(4\%/4.5\%) - 1 * (30\%)^2/2\} / \{(1)^{(0.5)} * 30\%\} = -0.542610$
$N(d_1) = N(-0.24) = 0.4052$
$N(d_2) = N(-0.54) = 0.2946$

Caplet Price = \$200,000 * (1/1.04)^1.5) * 1 * [4% * 0.4052 - 4.5% * 0.2946] = \$556

(e) Calculate the minimum profit that ABC Trust could earn on the loan after implementing the CRO's hedge recommendation.

12. Continued

Commentary on Question:
Candidates performed below average on this section. Many candidates correctly determined the minimum future payoff (excluding the hedge) of $3,000. Most candidates did not take into account the time value of money by either calculating the present value of the future spread or accumulated value of the hedge premium.

After the caplet, the minimum profit for ABC will occur if rates rise above 4.5% and will be equal to the present value of the 1.5% minimum spread less the premium paid for the hedge.

The minimum net payoff (after the hedge) at t=1.5 will be equal to $200,000 * 1.5% = $3,000.

PV of minimum payoff = $3,000 x exp(-4% x 1.5) = $2,825

Add caplet premium to determine the minimum PV of profit earned by ABC company = $2,825 - $556 = $ 2,269

(f) Suggest two alternative assumptions to simplify her specification of the volatility surface.

Commentary on Question:
Candidates performed above average on this section. Most candidates received full credit by correctly suggesting two alternate specifications for the volatility surface. Some candidates did not provide any suggestion at all and so received zero points.

The CRO's volatility surface specifies piecewise-constant instantaneous volatilities for the forward rates with many parameters.

Suggestion 1: Assume instantaneous volatility of $F_k(t)$ depends only on time to maturity of a forward rate rather than on time t and maturity T_k separately:

$$\sigma_k(t) = \sigma_{k,\beta(t)} =: \eta_{k-(\beta(t)-1)}$$

For example, this leads to the following table:

Instant. Vols	Time: $t \in (0, T_0]$	$(T_0, T_1]$	$(T_1, T_2]$	...	$(T_{M-2}, T_{M-1}]$
Fwd Rate: $F_1(t)$	η_1	Dead	Dead	...	Dead
$F_2(t)$	η_2	η_1	Dead	...	Dead
$\vdots$	...	...	...	...	...
$F_M(t)$	η_M	η_{M-1}	η_{M-2}	...	η_1

12. Continued

Suggestion 2: Assume instantaneous volatility of $F_k(t)$ has a constant instantaneous volatility regardless of t and only depends on maturity T_i of the considered forward rate $F_k(t)$:

$\sigma_k(t) = \sigma_{k,\beta(t)} := s_k$ where s_k is the constant instantaneous volatility regardless of t.

For example, this leads to the following table:

Instant. Vols	Time: $t \in (0, T_0]$	$(T_0, T_1]$	$(T_1, T_2]$	...	$(T_{M-2}, T_{M-1}]$
Fwd Rate: $F_1(t)$	s_1	Dead	Dead	...	Dead
$F_2(t)$	s_2	s_2	Dead	...	Dead
$\vdots$	...	...	...	...	...
$F_M(t)$	s_M	s_M	s_M	...	s_M

13. Learning Objectives:

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

4. The candidate will understand important quantitative techniques relating to financial time series, performance measurement, performance attribution and stochastic modeling.

Learning Outcomes:

(2b) Demonstrate an understanding of the basic concepts of credit risk modeling such as probability of default, loss given default, exposure at default, and expected loss.

(2d) Demonstrate an understanding of Merton asset value models in the context of credit risk.

(2e) Demonstrate an understanding of the term structure of default probability.

(4i) Demonstrate an understanding of the general uses and techniques of stochastics modeling.

Sources:

QFIA-124-16

Introduction To Credit Risk Modeling, Bluhm, Christian, 2nd Edition, pages 4 to 21 and 235 to 238.

Commentary on Question:

This question tests candidates' understanding of basic concepts of credit risk modeling and the application of Merton's model in the context of credit risk. It also emphasized candidates' awareness of when to use risk-neutral model parameters and when to use real-world parameters. Parts c and d tests candidates' general understanding of stochastic modeling in the context of a credit risk model.

Solution:

(a) Determine a numerical upper bound for the probability of default over a 1-year time horizon, if market participants are risk-adverse.

Commentary on Question:

Candidates performed above average on this section. Successful candidates recognized that the upper bound is the risk-neutral probability under the Merton model.

13. Continued

Under the Merton model, the risk-neutral probability of default is $N(-d_2)$ or $1 - N(d_2)$

$d_2 = \frac{\ln 1200/1000 + 0.03 - 0.5 \times 0.1^2}{0.1} = 2.0732$
$N(-d_2) = 0.01908$

Because market participants are risk-adverse, the actual probability of default must be lower than 0.01908.

(b) Calculate the annual effective credit spread of the one-year zero coupon bond.

Commentary on Question:
Candidates performed below average on this section. Successful candidates recognized that the Merton model can be used to calculate today's value of the bond, which can be used to calculate the yield and credit spread; however, most candidates that recognized this calculated a continuously compounded credit spread instead of a risk-free credit spread. Some unsuccessful candidates incorrectly used the risk-neutral probability and risk-free discount rate to try and calculate a loss given default ratio and credit spread.

E = today's fair value of the firm's equity = $AN(d_1) - Fe^{rT}N(d_2)$

$d_1 = \frac{\ln 1200/1000 + 0.03 + 0.5 \times 0.1^2}{0.1} = 2.1732$
$d_2 = 2.1732 - 0.1 = 2.0732$

$N(d_1) = 0.9851$
$N(d_2) = 0.98092$

E = today's fair value of the firm's equity
$E = 1200 \times 0.9851 - 1000 \times e^{-0.03} \times 0.98092 = 230.208$

D = today's fair value of the bond = $A - E$
$D = 1200 - 230.208 = 969.792$

Bond yield = $1000/969.792 - 1 = 3.115\%$
Risk-free rate = $e^{0.03} - 1 = 3.04\%$
Credit spread = bond yield − risk-free rate = $3.115\% - 3.04\% = 0.07\%$

(c) Describe four problems with the migration matrix M.

Commentary on Question:
Candidates performed above average on this section. Most candidates were able to identify and describe 3 or 4 problems.

13. Continued

1. The Default state is not absorbing / the probability of migrating out of the Default state is non-zero.
2. Lower-risk state A shows a higher default probability than higher-risk state B.
3. Row monotony towards the diagonal: State A should be more likely to migrate to closer state B than more distant state C. Alternatively, column monotony towards the diagonal: the chance of migration into state C should be higher for closer state B than more distant state A.
4. The transition probabilities of state C do not add to 1.

(d) Your colleague decides to use Monte-Carlo simulation to simulate 10,000 scenarios of the state of XYZ Corp after one year. He describes his methodology in this way:

"For each scenario, I generate two random numbers uniformly selected between 0 and 1. Then, for each 0.5 year period, given the starting state S, the ending state E is the one such that

- the probability of being in state S or better is greater than the random number R
- the probability of being in state S or worse is smaller than R"

Critique your colleague's methodology.

Commentary on Question:
Candidates performed below average on this section. Most candidates were not able to identify that the conditions outlined by the colleague overlap.

The colleague's method is not accurately described. One potential correction could be as follows:

"…given the starting state, the ending state E is the one such that

- the probability of being in state S or better is greater than the random number R
- the probability of being in a state strictly better than S is smaller than R"

(e) Calculate the loss given default ratio, using the estimated probability of default.

Commentary on Question:
Candidates performed below average on this section. Some unsuccessful candidates incorrectly used results from the risk-neutral framework of the Merton model instead of real-world results. Some unsuccessful candidates calculated the 'recovery ratio' instead of the 'loss given default ratio'.

13. Continued

In one year, the bond will pay either 1000 with probability 0.95, or x with probability 0.05.

Or, $920 = \frac{0.95(1000)+0.05x}{1.05}$

$x = 320$

Therefore the loss given default ratio is $1 - \frac{x}{1000}$
$= 68\%$

14. **Learning Objectives:**

3. Candidate will understand the nature, measurement and management of liquidity risk in financial institutions.

5. The candidate will understand the behavior characteristics of individuals and firms and be able to identify and apply concepts of behavioral finance.

6. The candidate will understand and be able to describe the variety and assess the role of alternative assets in investment portfolios. The candidate will demonstrate an understanding of the distinguishing investment characteristics and potential contributions to investment portfolios of the following major alternative asset groups:
   - Real Estate
   - Private Equity
   - Commodities
   - Hedge Funds
   - Managed Futures
   - Distressed Securities
   - Infrastructure

Learning Outcomes:

(3f) Apply liquidity scenario analysis with various time horizons.

(5c) Identify and apply the concepts of behavioral finance with respect to individual investors, institutional investors, portfolio managers, fiduciaries and corporate managers.

(6a) Demonstrate an understanding of the types of investments available in each market, and their most important differences for an investor.

(6b) Demonstrate an understanding of the benchmarks available to evaluate the performance of alternative investment managers and the limitations of the benchmarks.

Sources:

QFIA-111-13: Maginn & Tuttle, Managing Investment Portfolios, 3rd Ed. 2007, Ch. 8;

QFIA-109-13: A Survey of Behavioral Finance, by Barberis & Thaler

QFIA 106-13 Chapter 3 of Liquidity Risk Management

Commentary on Question:

This question tests candidates' understanding of real estate alternative investments, the behavioral characteristics of individuals, and liquidity scenario analysis.

14. Continued

Solution:

(a) List the attributes the analyst should look for in the appropriate index to represent direct real estate.

Commentary on Question:
The candidates performed poorly on this section. Some candidates were able to identify at least one of the attributes. Many candidates provided responses that were not reflective of what the question asked.

Index based on appraisal-based US property values
Index is diversified geographically and by property type
Index is investable and unambiguous
Index is unlevered

Since typically appraisals are infrequent, an unadjusted index tends to underestimate both the volatility in market value and the correlation with other asset classes. The unsmoothing corrects for this bias.

(b) Determine an asset allocation that would maximize the expected cash flows, while ensuring the required distribution is met under both scenarios.

Commentary on Question:
The candidates performed as expected on this section. Many candidates provided answers that received full credit. Several different proposed asset allocations met the conditions of the question, and those answers received full credit. Some candidates provided answers that did not meet both criteria set out in the question.

Expected Cash Flows per 1% of portfolio allocation:

Bonds	600
Equities	620
Direct Real Estate	640
REIT	520

Should maximize Direct Real Estate
If Direct Real Estate is more than 5%, (with bonds at 75%) then requirement not met in recession

14. Continued

The intended allocation is:

Bonds	75
Equities	20
Direct Real Estate	5

Two other asset allocations that met the requirements are

85% bonds and 15% direct real estate

80% bonds, 10% equities, and 10% direct real estate

(c) Compare the two proposals:

(i) from the liquidity perspective;

(ii) from the diversification perspective and;

(iii) from the acquisition and operating costs perspective.

Commentary on Question:
Candidates performed below average on this section. Unsuccessful candidates did well in discussing daily trading of publicly listed REITs and the diversification benefits of direct real estate; contrasting themselves from successful candidates who also discussed additional liquidity, acquisition and operating cost perspectives for the proposals. Unsuccessful candidates were those that that simply stated less liquid/diversifying/costs without providing any rationale.

Liquidity perspective:
Direct investment not liquid/long-term commitment
Publicly listed is liquid exchange-traded on a daily basis
Large transaction size. Large transaction size when buying/selling an office building is in contrast to the flexibility of trading small amounts on public exchanges.

Portfolio diversification perspective
Direct investment not diversified – risk of neighborhood deterioration
Publicly listed usually well diversified geographically and by property type
Direct investment historically provided more diversification benefit to the total portfolio

Acquisition and operating costs perspective
Higher transaction cost. Direct investments involve broker commissions and financing costs of buying physical assets. Publicly listed securities have low transaction costs.

14. Continued

Higher cost of acquiring information. Direct investments' lack of availability and timeliness of information result in extensive valuation and due diligence issues. Information about publicly listed securities is readily available and accessible.
Direct investment involves substantial operating costs

(d) Describe behavioral biases exhibited by Analyst B and Analyst C.

Commentary on Question:
The candidates performed below average on this section. While some candidates did receive full credit. Unsuccessful candidates received no credit for answers that were inconsistent with the definitions of the respective behavioral biases. The paragraph introducing Analyst C erroneously stated that she recommended direct real estate while then providing a strong statement in favor of indirect real estate. Candidates received credit for answers reflective of this contradiction.

Analyst B demonstrates Availability Bias. This is a bias whereby people judge the probability of an event based on their memories of relevant information.

Analyst B recommends a direct investment just because he has recently read some blogs and seen some advertisements regarding benefits of direct investments. This is a judgment based on his memories.

Analyst C demonstrates Belief Perseverance.

Analyst C says always better despite information to contrary.

15. Learning Objectives:

4. The candidate will understand important quantitative techniques relating to financial time series, performance measurement, performance attribution and stochastic modeling.

Learning Outcomes:

(4c) Describe and assess performance measurement methodologies for assets portfolios.

(4f) Calculate and interpret performance attribution metrics for a given asset, portfolio.

(4g) Explain the limitations of attribution techniques

Sources:

Ch 69: Handbook of Fixed Income Securities, Fabozzi, F.J., 8th Edition, 2012

Commentary on Question:

This question tested candidates' understanding of the performance attribution model. It tests candidates ability to identify the characteristics of a flexible attribution model and the limitations of the attribution model and how to remediate.

Solution:

(a) Discuss the characteristics of a flexible performance attribution model.

Commentary on Question:

Candidates did poorly on this question. Most candidates answered characteristics of a "good" attribution model instead of "flexible" attribution model. Successful candidates listed the characteristics of the flexible attribution model as below.

A flexible attribution model must satisfy the following requirements:

- Measure the risk factors corresponding to the portfolio managers' decision
- Independently measure the contributions of each factor in the portfolio management process
- Closely follow the published returns of the fund, and its benchmark
- Be available in a timely manner
- Have sufficient adaptability to explain both short and long time periods

15. Continued

(b) Based on the firm's current portfolio, you designed a performance attribution process and prepared the following attribution report for a certain time period.

Sector	W_p	W_b	R_p	R_b	Outperformance	
					Asset Allocation	Security Selection
Large Value	30%	35%	8%	6%	-0.30%	A
Large Core	25%	30%	7%	8%	B	-0.25%
Mid Value	C	20%	5%	3%	0.15%	0.50%
Small Core	20%	15%	D	9%	0.45%	0.20%

Calculate A, B, C and D from the table above.

Commentary on Question:
Candidates performed brilliantly on this question. Most candidates were able to identify the correct formula and get the correct numerical answer.

A: Security selection = $W_SP * (R_SP - R_SB)$ (69-5)
$A = 30\% * (8\% - 6\%) = 0.6\%$
B: Asset Allocation = $(W_SP - W_SB) * R_SB$ (69-4)
$B = (25\% - 30\%) * 8\% = -0.4\%$
$C = 1 - 30\% - 25\% - 20\% = 25\%$
D: $0.20\% = 20\% * (D - 9\%)$
$D = 10\%$

(c) Critique the CFO's comment including a recommended improvement to the performance attribution process.

Commentary on Question:
Candidates performed as expected on this question. Most candidates were able to identify the need for the hurdle rate and relative return in the attribution model. Some were able to identify that the current model would assign a positive asset allocation to any over weighted sector. Candidates who received full marks were those who commented why this is a bad decision.

Formulae used in the report above assign a positive contribution to asset allocation outperformance to any overweighted sector with positive benchmark sector return. Thus, if multiple sectors have positive benchmark returns, overweighting the sector with worst return would still be considered a good decision. One possible improvement to using absolute sector benchmark return, is to use the sector return relative to the benchmark return for Asset Allocation = $(W_SP - W_SB) * (R_SB - RB)$

15. Continued

(d) Recalculate the contribution from asset allocation for each sector using your improved performance attribution process from part (c) above.

Commentary on Question:
Candidates performed as expected on this question. Many candidates were able to identify the correct formula and calculate the correct numerical value. Some candidates identified the formula but failed to solve the correct numerical value.

Formula to be used: Asset Allocation = (WSP – WSB) * (RSB – RB)

RB = $\sum_S W_S B * R_S B$ = 35% * 6% + 30% * 8% + 20% * 3% + 15% * 9% = 6.45%

[2 points] Large Value: Asset Allocation = $(W_S P - W_S B) * (R_S B - RB)$ = (30% - 35%) * (6% - 6.45%) = 0.02.25%

[2 points] Large Core: Asset Allocation = $(W_S P - W_S B) * (R_S B - RB)$ = (25% - 30%) * (8% - 6.45%) = -0.07.75%

[2 points] Mid Value: Asset Allocation = $(W_S P - W_S B) * (R_S B - RB)$ = (25% - 20%) * (3% - 6.45%) = -0.17.25%

[2 points] Small Core: Asset Allocation = $(W_S P - W_S B) * (R_S B - RB)$ = (20% - 15%) * (9% - 6.45%) = 0.12.75%