GI ADV Model Solutions
Spring 2019

1. **Learning Objectives:**

5. The candidate will understand methodologies for determining an underwriting profit margin.

Learning Outcomes:

(5d) Allocate an underwriting profit margin (risk load) among different accounts.

Sources:

An Application of Game Theory: Property Catastrophe Risk Load, Mango

Solution:

(a) Calculate the risk load for this account.

The variance is

$1000^2(0.01)(0.99)+3000^2(0.02)(0.98)+5000^2(0.03)(0.97)=913,800.$

The risk load is 913,800(0.0002) = 182.76.

(b) Calculate the risk load for the new account using the marginal variance method.

The variance of the combined accounts is

$1200^2(0.01)(0.99)+3500^2(0.02)(0.98)+5000^2(0.03)(0.97)=981,856.$

The combined risk load is 981,856(0.0002) = 196.37.

The risk load for the new account is 196.37 – 182.76 = 13.61.

(c) Demonstrate that when using the marginal variance method the risk loads for the existing account and the new account are not renewal additive.

The variance of the new account is $200^2(0.01)(0.99)+500^2(0.02)(0.98)=5296.$ The risk load is 5296(0.0002) = 1.06. The marginal risk load for the existing account is 196.37 – 1.06 = 195.31. The sum of the individual risk loads is 195.31 + 13.61 = 208.92 which does not equal 196.37.

(d) Calculate the value of *z* that produces the same risk load for the new account as obtained using the marginal variance method.

1. Continued

The standard deviation of the existing account is $913{,}800^{1/2} = 955.93.$ The standard deviation of the combined accounts is $981{,}856^{1/2} = 990.89.$ The difference is 34.96. Therefore, the multiplier is 13.61/34.96 = 0.3893. The equation to solve is 0.3893 = z(0.15)/1.15 which yields z = 2.985.

2. Learning Objectives:

4. The candidate will understand how to apply the fundamental techniques of reinsurance pricing.

Learning Outcomes:

(4c) Calculate the price for a casualty per occurrence excess treaty.

(4d) Apply an aggregate distribution model to a reinsurance pricing scenario.

Sources:

Basics of Reinsurance Pricing, Clark

Solution:

(a) Calculate the expected losses in the layer using an exposure rating approach.

All calculations are in millions.

Case 1: Policy is 0 to 1, difference is 1.00 – 0 = 1.00. There is no reinsurance exposure. Factor is 0/1.00 = 0.
Case 2: Policy is 0 to 2, difference is 1.16 – 0 = 1.16. Reinsurance coverage is 1 to 2, difference is 1.16 – 1.00 = 0.16. Factor is 0.16/1.16 = 0.138.
Case 3: Policy is 0 to 3, difference is 1.28 – 0 = 1.28. Reinsurance coverage is 1 to 3, difference is 1.28 – 1.00 = 0.28. Factor is 0.28/1.28 = 0.219.
Case 4: Policy is 1 to 3, difference is 1.28 – 1.00 = 0.28. Reinsurance coverage is 2 to 3, difference is 1.28 – 1.16 = 0.12. Factor is 0.12/0.28 = 0.429.
Case 5: Policy is 1 to 4, difference is 1.38 – 1.00 = 0.38. Reinsurance coverage is 2 to 4, difference is 1.38 – 1.16 = 0.22. Factor is 0.22/0.38 = 0.579.

Summing the factors multiplied by the subject premium gives
0(3) + 0.138(4) + 0.219(5) + 0.429(8) + 0.579(9) = 10.29.

The expected losses are 0.6(10.29) = 6.174 or 6,174,000.

2. Continued

(b) Calculate the expected loss ratio on this swing plan.

The average loss cost is 0.10(4%) + 0.75(19%) + 0.15(44%) = 21.25%.
The loaded loss costs are 100/80 times the average loss cost in each range: 5%, 23.75%, and 55%. Applying the minimum and maximum changes them to 12.5%, 23.75%, and 37.5%. The average is 0.10(12.5%) + 0.75(23.75%) + 0.15(37.5%) = 24.69%.

The expected loss ratio is 21.25/24.69 = 86.1%.

(c) Explain the concept of "balance" in the context of swing plans.

In a balanced swing plan, the expected loss ratio is the same as the reciprocal of the loading.

(d) Explain why the provisional rate may be too low.

The provisional rate is well below the expected ultimate swing plan premium rate of 24.69%. This provides a cash flow advantage to the ceding company.

3. Learning Objectives:

2. The candidate will understand the considerations in selecting a risk margin for unpaid claims.

Learning Outcomes:

(2a) Describe a risk margin analysis framework.

Sources:

A Framework for Assessing Risk Margins, Marshall, et al.

Solution:

(a) State four concerns expressed regarding this methodology.

Commentary on Question:

Any four of the five items listed below are sufficient for full credit. Other valid concerns could also receive credit.

- The approaches used to determine the coefficients of variation vary significantly.
- These approaches often ignore the individual characteristics of the valuation portfolio.
- Estimation of correlations relies heavily on judgement.
- It is uncommon to test the adopted distribution against past experience or to adjust it for future expectations.
- Separate analyses are conducted to estimate the central estimate of insurance liabilities and the risk margin.

(b) Explain how each concern stated in part (a) is addressed by the framework discussed in the paper.

Commentary on Question:

Each statement must align with a concern expressed in part (a) to receive credit.

- A single approach is recommended that can be consistently applied.
- A step-by-step analysis of the portfolio is conducted to ensure that all aspects of risk are captured.
- A consistent mix of qualitative and quantitative approaches is used.
- Hindsight analysis is employed.
- The two aspects are developed simultaneously and in a coherent manner.

4. Learning Objectives:

1. The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:

(1c) Identify alternative models that should be considered depending on the results of the tests.

(1d) Estimate the standard deviation of a chain ladder estimator of unpaid claims.

Sources:

Measuring the Variability of Chain Ladder Reserve Estimates, Mack

Testing the Assumptions of Age-to-Age Factors, Venter

Solution:

(a) Calculate the natural starting values for $f(1)$, $f(2)$, $f(3)$ and $f(4)$ implied from the standard chain ladder method. These values should sum to 1.

First calculate the age-to-age factors (claim amounts are in thousands):
1 to 2: 45/30 = 1.5; 2 to 3: 39/30 = 1.3; 3 to 4: 23.1/21 = 1.1.

Next, calculate the age-to-ultimate factors:
1-ult: 1.5x1.3x1.1 = 2.145; 2-ult: 1.3x1.1 = 1.43; 3-ult: 1.1.

The natural starting values are:
$f(1) = 1/2.145 = 0.46620$;
$f(2) = (1.5 – 1)/2.145 = 0.23310$;
$f(3) = (1.3 – 1)/1.43 = 0.20979$;
$f(4) = (1.1 – 1)/1.1 = 0.09091$.

(b) Calculate starting values for $h(1)$, $h(2)$, $h(3)$ and $h(4)$ using the $f(d)$ values from part (a).

$$h(1) = \frac{0.46620(8,000) + 0.23310(7,000) + 0.20979(6,000) + 0.09091(2,100)}{0.46620^2 + 0.23310^2 + 0.20979^2 + 0.09091^2} = 21,024$$

$$h(2) = \frac{0.46620(10,000) + 0.23310(5,000) + 0.20979(3.000)}{0.46620^2 + 0.23310^2 + 0.20979^2} = 20,453$$

$$h(3) = \frac{0.46620(12,000) + 0.23310(3,000)}{0.46620^2 + 0.23310^2} = 23,166$$

$$h(4) = \frac{0.46620(14,000)}{0.46620^2} = 30,030$$

4. **Continued**

(c) Calculate the values of $f(1)$, $f(2)$, $f(3)$ and $f(4)$ that minimize the sum of the squared residuals.

$$f(1) = \frac{21,209(8,000) + 20,633(10,000) + 23,626(12,000) + 30,435(14,000)}{21,209^2 + 20,633^2 + 23,626^2 + 30,435^2} = 0.46000$$

$$f(2) = \frac{21,209(7,000) + 20,633(5,000) + 23,626(3,000)}{21,209^2 + 20,633^2 + 23,626^2} = 0.22494$$

$$f(3) = \frac{21,209(6,000) + 20,633(3,000)}{21,209^2 + 20,633^2} = 0.21604$$

$$f(4) = \frac{21,209(2,100)}{21,209^2} = 0.09901$$

(d) Estimate the unpaid claims for each of accident years 2, 3 and 4 using the parameterized BF model.

AY2: 20,633(0.09901) = 2,043
AY3: 23,626(0.21604 + 0.09901) = 7,443
AY4: 30,435(0.22494 + 0.21604 + 0.09901) = 16,435

(e) Estimate the unpaid claims for each of accident years 2, 3 and 4 using the standard chain ladder method.

AY2: 18,000(1.1 – 1) = 1,800
AY3: 15,000(1.43 – 1) = 6,450
AY4: 14,000(2.145 – 1) = 16,030

(f) Estimate the proportionality constants α_1^2, α_2^2, and α_3^2.

$$\alpha_1^2 = \frac{1}{2}\left[8,000\left(\frac{15,000}{8,000} - 1.5\right)^2 + 10,000\left(\frac{15,000}{10,000} - 1.5\right)^2 + 12,000\left(\frac{15,000}{12,000} - 1.5\right)^2\right] = 937.5$$

$$\alpha_2^2 = \frac{1}{1}\left[15,000\left(\frac{21,000}{15,000} - 1.3\right)^2 + 15,000\left(\frac{18,000}{15,000} - 1.3\right)^2\right] = 300$$

$$\alpha_3^2 = \frac{300^2}{937.5} = 96$$

5. Learning Objectives:

1. The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:

(1a) Identify the assumptions underlying the chain ladder estimation method.

(1e) Apply a parametric model of loss development.

Sources:

LDF Curve Fitting and Stochastic Reserving: A Maximum Likelihood Approach, Clark
Measuring the Variability of Chain Ladder Reserve Estimates, Mack

Solution:

(a) Provide two advantages that are relevant for this triangle.

Commentary on Question:
One of the general advantages of Clark's model is that it can handle triangles without evenly spaced evaluation dates. However, this triangle does not have that issue. Candidates did not receive credit for stating this advantage.

- There are fewer parameters to estimate.
- Random fluctuations are smoothed.

(b) Explain why the Cape Cod method is generally preferred to the LDF method.

The Cape Cod method has significantly fewer parameters to estimate compared to the LDF method.

(c) Calculate ULT_{2017}.

$$G(18) = 1 - (21.897 / 39.897)^{4.3335} = 0.92572$$
$$ULT_{2017} = 10,000 / 0.92572 = 10,802$$

(d) Estimate the expected payments in 2020 for accident year 2018.

$$G(30) = 1 - (21.897 / 51.897)^{4.3335} = 0.97623$$
$$G(6) = 1 - (21.897 / 27.897)^{4.3335} = 0.64987$$
Expected payments = $8,000(0.97623 - 0.92572) / 0.64987 = 622$

5. Continued

(e) Identify which of these assumptions are not made by Mack. Justify your responses.

(i) is not made by Mack. Mack assumes that within an accident year, incremental losses depend on previous losses and hence are not independent of them.
(ii) is made by Mack. The same model for development is used for each accident year.
(iii) is not made by Mack. Mack uses squared error to obtain variance estimates.

6. **Learning Objectives:**

3. The candidate will understand excess of loss coverages and retrospective rating.

Learning Outcomes:

(3c) Explain and calculate the effect of economic and social inflationary trends on first dollar and excess of loss coverages.

Sources:

The Mathematics of Excess of Loss Coverages and Retrospective Rating – A Graphical Approach, Lee

Solution:

Consider the following six possibilities:

(i) $t_1 < t_2 < t_3$

(ii) $t_1 < t_3 < t_2$

(iii) $t_2 < t_1 < t_3$

(iv) $t_2 < t_3 < t_1$

(v) $t_3 < t_1 < t_2$

(vi) $t_3 < t_2 < t_1$

For each of (i) through (vi), either

- Identify a probability distribution with positive probability at exactly two discrete points (loss sizes) for which the inequality is satisfied; or
- Explain why there is no probability distribution for which the inequality is satisfied.

Commentary on Question:

For (i)-(iii) there are many examples that satisfy the inequality. Any example that does so received full credit.

(i) Place probability 0.5 at each of 150 and 250.
For Layer 1 the initial expected payment is 0.5(100) + 0.5(100) = 100. With 10% trend it is 0.5(100) + 0.5(100) = 100. The trend factor is 0%.
For Layer 2 the initial expected payment is 0.5(50) + 0.5(100) = 75. With 10% trend it is 0.5(65) + 0.5(100) = 82.5. The trend factor is 10%.
For Layer 3 the initial expected payment is 0.5(0) + 0.5(50) = 25. With 10% trend it is 0.5(0) + 0.5(75) = 37.5. The trend factor is 50%.

6. **Continued**

(ii) Place probability 0.9 at 100 and 0.1 at 250.
For Layer 1 the initial expected payment is 0.9(100) + 0.1(100) = 100. With 10% trend it is 0.9(100) + 0.1(100) = 100. The trend factor is 0%.
For Layer 2 the initial expected payment is 0.9(0) + 0.1(100) = 10. With 10% trend it is 0.9(10) + 0.1(100) = 19. The trend factor is 90%.
For Layer 3 the initial expected payment is 0.9(0) + 0.1(50) = 5. With 10% trend it is 0.9(0) + 0.1(75) = 7.5. The trend factor is 50%.

(iii) Place probability 0.8 at 25 and 0.2 at 250.
For Layer 1 the initial expected payment is 0.8(25) + 0.2(100) = 40. With 10% trend it is 0.8(27.5) + 0.2(100) = 42. The trend factor is 5%.
For Layer 2 the initial expected payment is 0.8(0) + 0.2(100) = 20. With 10% trend it is 0.8(0) + 0.2(100) = 20. The trend factor is 0%.
For Layer 3 the initial expected payment is 0.8(0) + 0.2(50) = 10. With 10% trend it is 0.8(0) + 0.2(75) = 15. The trend factor is 50%.

(iv)-(vi) are not possible. The maximum trend factor for Layer 1 is 10% (when there is only a limit, the trend factor is never larger than the underlying trend) while the trend factor for Layer 3 is bounded below by 10% (when there is only a retention, the trend factor is always greater than the underlying trend). Hence, $t_1 < t_3$. This contradicts the last three cases.

7. Learning Objectives:

5. The candidate will understand methodologies for determining an underwriting profit margin.

Learning Outcomes:

(5a) Calculate an underwriting profit margin using the target total rate of return model.

(5b) Calculate an underwriting profit margin using the capital asset pricing model.

(5c) Calculate an underwriting profit margin using the risk adjusted discount technique.

Sources:

Ratemaking: A Financial Economics Approach, D'Arcy and Dyer

Solution:

(a) Calculate the target total rate of return using the Capital Asset Pricing Model (CAPM).

Target total rate of return =
$\text{risk-free rate} + \beta \times \text{market risk premium} = 2 + 1.5(6) = 11\%$

(b) Calculate the underwriting profit margin.

$$UPM = \frac{S}{P}\left(TRR - \frac{IA}{S}\times IR\right) = \frac{500,000}{850,000}\left(11 - \frac{1,200,000}{500,000}\times 7\right) = -3.41\%$$

(c) Explain how the existence of catastrophe risk makes the use of CAPM problematic for insurers.

CAPM provides a risk premium only for risks that are systematic with market returns. Catastrophe risk is not such a risk and hence is not considered by CAPM.

(d) Explain the relationship between the internal rate of return and net present value when employing discounted cash flow analysis.

The internal rate of return is the discount rate that gives a net present value of zero. If cash flows change sign more than once, there may be multiple solutions.

8. Learning Objectives:

4. The candidate will understand how to apply the fundamental techniques of reinsurance pricing.

Learning Outcomes:

(4e) Describe considerations involved in pricing property catastrophe covers.

Sources:

Basics of Reinsurance Pricing, Clark

Solution:

(a) Calculate the net profit or loss for Specialist assuming:

(i) No losses

(ii) One or more losses

All calculations are in millions. In both cases the premium is 50 and the margin is 10%, or 5.

(i) If there are no losses, the loss is 0 and the profit is 45 – 0 = 45. The profit commission is 0.95(45) = 42.75. Then loss + margin – premium = 0 + 5 – 50 = –45. Additional premium is 0. Net profit = premium – loss – profit commission + additional premium = 50 – 0 – 42.75 + 0 = 7.25.

(ii) If there are one or more losses, the loss is 200 and the profit is 45 – 200 = –155. The profit commission is 0. Then loss + margin – premium = 200 + 5 – 50 = 155. Additional premium is 0.6(155) = 93. Net profit = premium – loss – profit commission + additional premium = 50 – 200 – 0 + 93 = –57.

(b) Calculate the rate on line for an equivalent traditional risk cover.

Commentary on Question:

The solution depends on the answers to part (a). Candidates who had incorrect answers to that part but used them correctly here (*and in part* (*c*)) *could receive full credit.*

With no loss the net profit is 7.25 while with a loss it is –57. The balancing item is the difference, 64.25. The rate on line is 7.25/64.25 = 11.3%.

8. **Continued**

(c) Calculate the Additional Premium percentage required to match the rate on line of 15%.

The no loss net profit is 7.25. For a rate on line of 15%, the balancing item is 7.25/0.15 = 48.333. This implies that with a loss the net profit is 7.25 – 48.333 = –41.083. Hence, –41.083 = 50 – 200 – 0 + additional premium and so the additional premium is 108.917. The additional premium percentage is 108.917/155 = 70.3%.

(d) State the two conditions that a finite reinsurance arrangement must fulfill for a ceding company to consider it insurance.

- The reinsurer must assume significant insurance risk.
- It must be reasonably possible that the reinsurer will realize a significant loss.