

# ERM-INV Model Solutions

## Spring 2019

### 1. Learning Objectives:

4. The candidate will understand the approaches for managing risks and how an entity makes decisions about appropriate techniques.

### Learning Outcomes:

- (4e) Develop an appropriate choice of a risk mitigation strategy for a given situation (e.g., reinsurance, derivatives, financial contracting), which balances benefits with inherent costs, including exposure to credit risk, basis risk, moral hazard and other risks.

### Sources:

ERM-122-14: Captives and the Management of Risk (chapter 1)

### Commentary on Question:

*This question sought to test candidates on various aspects of captive insurance and how it differs from commercial insurance.*

### Solution:

- (a) Explain why this captive would qualify as an Alternative Risk Transfer (ART) program.

### Commentary on Question:

*Most candidates received at least partial credit for this question. Those who defined ART but did not evaluate it in the context of this situation received partial credit.*

ART requires the innovative use of risk financing techniques and utilization of an alternative mechanism to finance risk.

This captive would qualify as an ART because the captive is offshore and is subject to the same regulatory requirements as CYP. Having a captive also means that CYP will be retaining some of the risk, so both criteria are satisfied.

## 1. Continued

- (b) CYP's CFO has proposed creating an offshore Pure Captive with the following characteristics:
- The captive cannot pursue its own external reinsurance business.
  - The offshore location has similar capital requirements but a lower tax rate.
  - Two accountants and an administrative assistant will be hired to run the captive from the offshore location.
- (i) Explain why a Pure Captive might be preferred over other types of captives for CYP.
- (ii) Critique the characteristics of the proposed Pure Captive.
- (iii) Describe two additional key actions that need to be completed with respect to establishing the Pure Captive.

### **Commentary on Question:**

*Candidates generally did well on this part. For subpart (i), candidates received only partial credit if they described a pure captive without saying why it was preferred in this situation. For subparts (ii) and (iii), reasons other than those shown below were eligible for credit if they were supported by a coherent explanation. More credit was given for demonstrating the ability to critique the pure captive of CYP and address additional key actions related to CYP; less credit was given if candidates elaborated on the concept of a pure captive in subpart (i) but did not connect their responses with CYP's circumstances as described in the question.*

- (i) A captive is an insurance company that is wholly owned and controlled by its insureds.

A pure captive is different from other captives because it only insures the business that CYP decides to send it. No outside business from another company will be sent to the captive (e.g., a joint ownership captive), and CYP is not paying fees to send its business to the captive (e.g., "rent-a-captive")

- (ii)
- CYP should set up an actual office and hire or contract people who are insurance professionals and who know the offshore location's rules.
  - CYP should leave open the option for pursuing outside business, as it may be complimentary to CYP's products, or even supplement its earnings.

## 1. Continued

- (iii)
  - Obtain an insurance license for operating in the offshore location.
  - Understand the rules and regulations for operating in the offshore location, especially reporting and tax regulation.
- (c) You are employed by the consulting firm that CYP has hired to perform its actuarial services. CYP's CEO has asked you to offer an opinion on whether CYP should retain this business or cede it to a captive.
  - (i) Evaluate the three key elements that distinguish a captive from a commercial insurer for CYP.
  - (ii) Recommend whether CYP should use a captive. Justify your answer.

### **Commentary on Question:**

*Most candidates received partial credit for this part, but very few demonstrated enough detail to receive full credit. For subpart (i), relevant points other than those shown below were eligible for credit, but most candidates did little more than list some or all of the three elements without providing any evaluation. For subpart (ii), any recommendation with appropriate justification was eligible for credit; most candidates received at least partial credit.*

- (i)
  1. The insurer is putting its own capital at risk
    - CYP is not transferring the risk away; it still retains the risk.
    - The captive would have recourse to CYP's capital if the captive was in default.
    - However, CYP would still have control over the captive because it has a vested interest in the company.
  2. The captive would be working outside the commercially regulated market for insurers
    - There is a risk that CYP's regulators may step in or take a keen interest in the workings of the captive.
    - Using a captive may involve a complicated legal structure, which has risks.
    - Off-shore location has a different regulatory authority which may view taxes and earnings differently than the current regulator.
  3. The captive is used by CYP to achieve its risk financing objectives
    - This gives CYP better control over the risks taken than an alternative such as reinsurance.
    - The captive may have better opportunities to achieve returns because it is not constrained by the Canadian regulatory regime.
    - The captive could insure outside risks to diversify CYP's business.

## **1. Continued**

- (ii) One recommendation is that CYP should not use a captive. Reasons include:
- CYP does not have its own actuarial staff and would have to rely on consultants to set up the captive.
  - CYP does not already have a captive so there is operational risk in setting one up.

## 2. Learning Objectives:

2. The candidate will understand the concepts of risk modeling and be able to evaluate and understand the importance of risk models.
4. The candidate will understand the approaches for managing risks and how an entity makes decisions about appropriate techniques.
5. The candidate will understand the concept of economic capital, risk measures in capital assessment and techniques to allocate the cost of risks within business units.

### Learning Outcomes:

- (2c) Analyze and evaluate risk aggregation techniques, including use of correlation, integrated risk distributions and copulas.
- (4a) Demonstrate and analyze applicability of risk optimization techniques and the impact of an ERM strategy on an organization's value. Analyze the risk and return trade-offs that result from changes in the organization's risk profile.
- (5a) Describe the concepts of measures of value and capital requirements (for example, EVA, embedded value, economic capital, regulatory measures, and accounting measures) and demonstrate their uses in the risk management and corporate decision-making processes.

### Sources:

Risk Appetite: Linkage with Strategic Planning Report

ERM-119-14: Aggregation of Risks and Allocation of Capital

### Commentary on Question:

*The question was intended to assess candidates' ability to calculate RAROC, as well as the usage of a correlation matrix. In addition, the question tests candidates on their understanding of how to use RAROC to make business decisions such as optimizing business risk.*

### Solution:

- (a) You are given the following correlation matrix for the three product lines in 2018.

	Specialty	Homeowners	CAT
Specialty	1.0	0.2	0.2
Homeowners	0.2	1.0	0.2
CAT	0.2	0.2	1.0

## 2. Continued

- (i) Calculate the Risk Adjusted Return on Capital (RAROC) for each product line and the company in total. Show all work.
- (ii) Analyze the profitability of the product line with the lowest RAROC.

### **Commentary on Question:**

*Candidates generally did well on this part. Partial credit was awarded to candidates who solved product line RAROC but were unable to calculate total company RAROC, as well as for candidates who calculated the total company PV of required capital. Most candidates answered the question as described below, but full credit was also awarded for candidates who calculated the total company Required Economic Capital and then prorated to solve for PV of Required Economic Capital.*

- (i)  $\text{RAROC} = \text{PV of After-tax earnings} / \text{PV Required Economic Capital}$

$$\text{RAROC}_{\text{Specialty}} = 1.0 / 5 = 20\%$$

$$\text{RAROC}_{\text{Homeowners}} = 6.0 / 40 = 15\%$$

$$\text{RAROC}_{\text{CAT}} = 27.0 / 450 = 6\%$$

$$\begin{aligned} \text{Required Economic Capital after Diversification} &= [5^2 + 40^2 + 450^2 + (2 * \\ & 0.2 * 5 * 40) + (2 * 0.2 * 5 * 450) + (2 * 0.2 * 40 * 450)]^{1/2} \\ &= [212,305]^{1/2} \\ &= 460.77 \end{aligned}$$

$$\text{RAROC}_{\text{total}} = (1.0 + 6.0 + 27.0) / 460.77 = 7.38\%$$

- (ii) The CAT line has the highest profit margin among all three product lines, being the main contributor to the underwriting profit in terms of amount.

However, the profit margin measure does not reflect the risks the company has taken. Its RAROC of 6% is the lowest, i.e., the lowest risk-adjusted return. This is because CAT is known for having low-frequency high-severity losses.

- (b) Orange has developed the following risk appetite statement.

Orange has a long-term aggregate RAROC target of 10%. The company will earn at least 6% RAROC with a probability of 98% in each year.

- (i) Assess Orange's compliance with its risk appetite statement.
- (ii) Analyze Orange's business mix with respect to the company's RAROC.

## 2. Continued

### **Commentary on Question:**

*Candidates generally did well on this section. In order to receive full credit on subpart (i), candidates needed to explain that a single year of sales is insufficient to judge compliance with the 10% long-term target and/or that Orange needs to do additional modeling or assume a probability distribution to assess compliance with the requirement of 6% RAROC with a probability of 98%.*

- (i) Orange's 7.38% RAROC is below the long-term target of 10%, but we cannot tell if they will be in compliance over the long-term because we only know about one year's result. Orange's RAROC this year is above 6%, but we need more information on the probability distribution in order to tell whether it will meet the 6% target 98% of the time.
- (ii) The CAT line produces a much lower RAROC than the other two product lines, but more than half of Orange's business comes from the CAT line. This results in the relatively low overall RAROC of the company. In order to improve on its RAROC, the company should consider increasing its capacity for the other two lines or reducing CAT sales.
- (c) There will be no rate increases in 2019. The 2019 business mix plan needs to address the following requirements.
  - I. Collected premiums for each product line must be at least 90% and at most 150% of those in 2018.
  - II. For product lines with sales increase in 2019, collected premiums must be proportional to the collected premiums for those product lines in 2018.
  - III. The required economic capital for the business written in 2019 needs to be lower than the expected available economic capital of \$150 million.

Assume the following:

- The diversification between product lines results in a 30% reduction in required capital for business written in 2019.
- The ratio of the required capital to collected premiums in any year for each product line is constant.
- (i) Calculate the maximum total premium increase from 2018 to 2019 if Orange aims to maximize overall RAROC, subject to requirements I through III. Show your work.

## 2. Continued

- (ii) Orange has added a requirement that the aggregate RAROC target of 10% needs to be met by the business written in 2019.

Verify that a total premium increase of \$12 million from 2018 to 2019 satisfies this requirement. Show your work.

### Commentary on Question:

*Candidates generally performed poorly on this section. Many candidates omitted the section, and many others misinterpreted requirement II. Requirement II intended to make the premium growth rates of Specialty and Homeowners the same percentage.*

*Full credit was awarded on subpart (i) for candidates who solved for the \$14 million increase and proved that requirements I through III were met.*

*Full credit was awarded on subpart (ii) for candidates who calculated the new premium amounts, proved that requirements I through III were met, and calculated the correct RAROC. The most common answer is shown, but an alternate solution with Specialty at 15, Homeowners at 45, and CAT at 57 was accepted. Candidates who did not follow requirement II but answered the rest of the question correctly were awarded partial credit.*

- (i) Given CAT has the lowest RAROC among all three product lines, a relative shift from CAT to Homeowners and Specialty would increase RAROC. The premium increase for Specialty and Homeowners should be as much as possible subject to requirements I and II, and the decrease for CAT line should be as much as possible subject to requirement I.

Product	Original Premium	Premium Increase %	Calculation	New Premium
Specialty	10	+50%	$10*(1+0.5)$	15
Homeowners	30	+50%	$30*(1+0.5)$	45
CAT	60	-10%	$60*(1-0.1)$	54

Total premium =  $15 + 45 + 54 = 114$   
\$14 million premium increase



## 2. Continued

Required economic capital increases at the same rate as the premium increase.

Product	Original Required Economic Capital	Increase %	Calculation	New Required Economic Capital
Specialty	5	+50%	$5*(1+0.5)$	7.5
Homeowners	40	+50%	$20*(1+0.5)$	30
CAT	450	-10%	$150*(1-0.1)$	135

$$\begin{aligned} \text{Total company Required Economic Capital} &= (1 - 0.3) * (7.5 + 30 + 135) \\ &= 0.7 * 172.5 \\ &= 120.75 \end{aligned}$$

So requirement III is met.

- (ii) We will keep the \$6 million (or 10%) decrease in CAT premium, so the total increase in Specialty + Homeowners is  $12 - (-6) = \$18$  million.

In order to meet requirement II, the sales growth in Specialty and Homeowners must be at the same rate.

$$\begin{aligned} 10 + 30 + 18 &= 10 * (1+x\%) + 30 * (1+x\%) \\ x\% &= 45\% \end{aligned}$$

Product	Original Premium	Premium Increase %	Calculation	New Premium
Specialty	10	+45%	$10*(1+45\%)$	14.5
Homeowners	30	+45%	$30*(1+45\%)$	43.5
CAT	60	-10%	$60*(1-0.1)$	54

PV of after- tax earnings:

$$\begin{aligned} &1.0 * (1 + 0.45) + 6.0 * (1 + 0.45) + 27.0 * (1 - 0.1) \\ &= 1.45 + 8.70 + 24.3 \\ &= 34.45 \end{aligned}$$

Solve for Orange's PV of Required Economic Capital:

$$\begin{aligned} &(1 - 0.3) * [5 * (1 + 0.45) + 40 * (1 + 0.45) + 450 * (1 - 0.1)] \\ &= 0.7 * 470.25 \\ &= 329.18 \end{aligned}$$

$$\begin{aligned} \text{Total Company RAROC} &= 34.45 / 329.18 \\ &= 10.47\% \end{aligned}$$

which is higher than 10%.

### 3. Learning Objectives:

4. The candidate will understand the approaches for managing risks and how an entity makes decisions about appropriate techniques.

#### Learning Outcomes:

- (4a) Demonstrate and analyze applicability of risk optimization techniques and the impact of an ERM strategy on an organization's value. Analyze the risk and return trade-offs that result from changes in the organization's risk profile.
- (4h) Analyze funding and portfolio management strategies to control equity and interest rate risk, including key rate risks. Contrast the various risk measures and be able to apply these risk measures to various entities. Explain the concepts of immunization including modern refinements and practical limitations.
- (4i) Analyze the application of Asset Liability Management and Liability Driven Investment principles to Investment Policy and Asset Allocation.

#### Sources:

ERM-111-12: Key Rate Durations: Measures of Interest Rate Risks

ERM-112-12: Revisiting the Role of Insurance Company ALM within a Risk Management Framework

#### Commentary on Question:

*Candidates were tested on the ability to analyze interest rate risk, knowledge of risk optimization techniques, and the application of asset liability management principles to investment policy and asset allocation.*

#### Solution:

- (a)
  - (i) Compare and contrast the use of key rate durations and effective duration for quantifying interest rate risk.
  - (ii) Verify that the missing key rate duration is 1.44. Show your work.
  - (iii) Verify, using the effective duration, that the key rate durations have been calculated correctly. Show your work.

### 3. Continued

#### Commentary on Question:

Candidates performed fairly well on subpart (i), recognizing the uses of effective and key rate durations but did not perform well on subparts (ii) and (iii). The expectation for (ii) was that candidates would calculate key rate durations using the formula provided in the source and would then calculate effective duration in (iii) to verify. Full credit was given for both parts if effective duration was used in subpart (ii) to back out the 1.44 value and recognize that the solution would be the same for subpart (iii). Most candidates used some form of the effective duration calculation so that the first principles key rate duration calculation was rarely attempted. Common errors included using modified duration instead of effective duration and continuous interest rates instead of annual effective rates as was stated in the question body.

(i)

- Effective duration measures the risk of a parallel shift in the yield curve, whereas key rate durations measure the risk of a shift over a specific portion of the yield curve only
- Effective duration is often inadequate in measuring interest rate risk as the yield curve rarely moves in a parallel fashion. Key rate durations are more effective due to their ability to quantify all types of changes
- Key rate durations are more useful in analyzing complex options and creating replication portfolios compared to effective duration

(ii)

- Calculate the present value of cash flows prior to the shock:  
 $\text{sum}(CF_t/(1+i)^t) = 2834.62$
- The shocked interest rate to use in year 4 is  $0.1/2 = 0.05\%$
- Calculate the present value of cash flows after the shock:  
 $600/(1.005)+200/1.008^2+800/1.013^3+1000/1.0205^4+400/(1.03)^5 = 2830.53$
- Calculate the KRD:  $-(2,830.53 - 2,834.62)/2,834.62/.001 = 1.44$

(iii)

- From subpart (ii) the base PV is 2834.62
- The shocked PV, adding 0.1% to each rate, is 2826.07
- Effective duration:  $-(2,826.07 - 2,834.62)/2,834.62/.001 = 3.02$
- $\text{Sum}(\text{KRD}) = 3.02 \Rightarrow 0.21 + 0.14 + 1.44 + 1.23 = 3.02$

### 3. Continued

- (b) The Chief Investment Officer (CIO) has recommended using surplus volatility as the only risk measure instead because it covers both asset and liabilities.
- (i) Compare and contrast the "bottom-up" and holistic ALM/SAA approaches.
  - (ii) Explain how ABC Life's approach would need to change in order to use surplus volatility instead of asset-only volatility as the risk measure.
  - (iii) Critique the CIO's recommendation.

**Commentary on Question:**

*Candidates performed well on this part of the question. For full credit, candidates needed to recognize the independence of surplus, as this applied to both subparts (i) and (ii). Many candidates recognized that only using one risk measure in subpart (iii) was inadequate and provided several recommendations for others.*

- (i)

The bottom-up approach focuses on assets backing reserves independent of surplus whereas the holistic ALM/SAA considers the entire asset portfolio in aggregate to first optimize risk-adjusted returns within capital constraints and risk tolerance levels while simultaneously determining the most effective constraint for ALM.
- (ii)

When using a surplus volatility risk measure there is an interaction between the liability segments and the surplus segment. Therefore, ABC Life would need to take a more holistic approach and consider the entire portfolio in aggregate rather than the silo approach currently used.
- (iii)
  - The CIO is correct that asset-only volatility ignores half of the balance sheet (i.e. the liabilities). Incorporating a risk measure that accounts for both liabilities and assets such as surplus volatility can provide more information.
  - However, risk measures allow us to assess the effectiveness of different investment strategies by understanding the tradeoff between risk and return. Viewing the solution under a single risk metric as recommended does not provide a complete picture.

### 3. Continued

- (c)
- (i) Match labels A, B, and C to the approaches listed above. Justify your answer.
  - (ii) Recommend which approach to use. Justify your answer.

**Commentary on Question:**

*Candidates performed well on this part of the question. Most were able to deduce which graphs aligned with each description and provide an appropriate recommendation. While there was a clear 'best' recommendation, credit was awarded for other recommendations with reasonable justification.*

- (i)
- Letter A = approach 2 – Based on the first graph, portfolio volatility vs. net excess yield, we can see that A minimizes the portfolio volatility for any given net excess yield. This is consistent with the "Minimize asset-only volatility" portion of approach 2. If we look at the fourth graph, duration gap vs. net excess yield, we can see that A has a minimal duration gap for any net excess yield. This is consistent with the "while constraining the asset-liability duration gap" portion of approach 2.
  - Letter B = approach 3 – Based on the second graph, surplus volatility vs. net excess yield, we can see that C and B are minimizing surplus volatility for a given net excess yield. This implies that they correspond to the two approaches that minimize surplus volatility risk (i.e. approach 1 or 3). If we look at the fourth graph, duration gap vs. net excess yield, we can see that B has a minimal duration gap for any net excess yield. This is consistent with the "while constraining the asset-liability duration gap" portion of approach 3.
  - Letter C = approach 1 – As noted above, the second graph implies B and C are minimizing surplus volatility for a given net excess yield. If we look at the fourth graph, duration gap vs. net excess yield, we can see that C has a larger duration gap for any net excess yield. This is consistent with the "while relaxing the asset-liability duration gap" portion of approach 1.

### 3. Continued

(ii)

- Approach B is recommended.
- Approach A minimizes asset-only risk, which ignores half of the balance sheet and is therefore not recommended.
- Approach C can achieve a lower surplus volatility for a given level of net excess yield as seen in the surplus volatility graph. However, this is achieved by relaxing the asset-liability duration gap as seen in the asset/liability duration graph. The larger asset liability duration gap results in higher capital requirements for approach C as seen in the required capital graph and is therefore not recommended.

#### **4. Learning Objectives:**

1. The candidate will understand the types of risks faced by an entity and be able to identify and analyze these risks.
2. The candidate will understand the concepts of risk modeling and be able to evaluate and understand the importance of risk models.
4. The candidate will understand the approaches for managing risks and how an entity makes decisions about appropriate techniques.
5. The candidate will understand the concept of economic capital, risk measures in capital assessment and techniques to allocate the cost of risks within business units.

#### **Learning Outcomes:**

- (1c) Identify and assess the potential impact of risks faced by an entity, including but not limited to market risk, currency risk, credit risk, counterparty risk, spread risk, liquidity risk, interest rate risk, equity risk, hazard/insurance risk, inflationary risk, environmental risk, pricing risk, product risk, operational risk, project risk and strategic risk.
- (2d) Apply and analyze scenario and stress testing in the risk measurement process.
- (4j) Demonstrate risk management strategies for other key risks (for example, operational, strategic, legal, and insurance risks).
- (4k) Apply best practices in risk measurement, modeling and management of various financial and non-financial risks faced by an entity.
- (5d) Propose techniques for allocating/appropriating the cost of risks/capital/hedge strategy to business units in order to gauge performance (risk adjusted performance measures).

#### **Sources:**

ERM-126-15: ORSA - An International Requirement (Section 3.1 and Section 4.1)

ERM-107-12: Strategic Risk Management Practice, Anderson and Schroder, 2010 Ch. 7  
Strategic Risk Analysis

ERM-117-14: AAA Practice Note: Insurance Enterprise Risk Management Practices  
(pages 4-26)

ERM-120-14: IAA Note on Stress Testing and Scenario Analysis

Risk Appetite: Linkage with Strategic Planning Report

## 4. Continued

ERM-119-14: Aggregation of risks and Allocation of Capital (Sections 4-7)

### **Commentary on Question:**

*This question focuses on strategic risk analysis: risk assessment and measurement, evaluation of its impact on capital, and ultimately decision-making utilizing risk-adjusted performance measures.*

### **Solution:**

- (a) Assess the appropriateness of the CRO's proposal.

### **Commentary on Question:**

*The purpose of this question is to test the candidate's understanding of ORSA and how this framework can be used to undertake strategic risk analysis. It is more than simply a compliance framework, as it is often envisioned.*

*Candidates received full credit if they successfully assessed each component separately and demonstrated their understanding of ORSA as a strategic risk analysis framework. Those candidates who simply described the many components of ORSA without relating it to strategic risk analysis did not demonstrate that they understood the concept of ORSA from the desired perspective.*

ORSA is an appropriate framework for strategic risk analysis because:

#### Risk management:

ORSA requires that risk analysis be performed continuously and embedded in normal operations. This approach can be leveraged to perform strategic risk analysis as well.

ORSA takes a holistic, enterprise-wide point of view, which is also essential for evaluating strategic options.

#### Capital management:

Strategic options must be evaluated based on their impact on the overall capital of XYZ, both actual, projected and allocated.

This evaluation must be done prospectively and be aligned with strategy.



## 4. Continued

### Business strategy:

XYZ must integrate the results of the two previous steps into its strategic decision-making.

This process must be integrated and used by management and the Board to decide whether it should pursue an international expansion using risk-adjusted performance measures.

- (b) The CRO proposes that you use the PESTEL method to perform a preliminary strategic risk identification analysis.
- (i) Describe advantages and disadvantages of the PESTEL method for XYZ.
  - (ii) Identify, using the PESTEL method, the strategic risks that XYZ may face.
  - (iii) Propose an alternative risk identification method that may improve XYZ's assessment. Justify your response.

### **Commentary on Question:**

*The purpose of this question is to test the candidate's understanding and application of a particular qualitative method in assessing strategic risk.*

*For the PESTEL method, students had to state advantages and disadvantages, and then apply the method by classifying external threats and opportunities provided. The purpose was not to elaborate on more threats as some candidates did.*

*Additionally, students had to propose and describe an alternative risk identification method that could enhance the XYZ's risk evaluation. Some students wrote about an alternative risk management method, which was not the purpose of the question.*

*Globally, this question was answered well by most candidates.*

(i)

Some of the advantages of the PESTEL method are:

- The method is conceptually simple and easy to implement.
- It focuses on identifying general environmental risks, creating a foundation for assessing industry- and company-specific risks.

## 4. Continued

- It encourages XYZ to establish a regular environmental scanning of potential developments and external factors to capture a broad amalgam of risks.

Some of the disadvantages are:

- It is based solely on qualitative assessment. There is no quantification per se, therefore it is harder to translate risks into impact on XYZ in terms of ranking or prioritization of risks.
- It might be too narrow in its approach: risk categories might be too narrow/non-inclusive, some important risks might be ignored.
- It is less formalized than developing scenarios, thus unable to anticipate drastic or abrupt changes that may occur over time.

(ii)

Although presented in this manner, these factors intersect many categories, both causes and effects.

### Political:

- Changes in social programs affecting customers in terms of their long-term sustainability.

### Economic:

- Low interest rates by historical standards.
- Lingering impact of the 2008 financial crisis.
- Unknown economic impact of climate change. *Linked with Environmental.*

### Social:

- New challenges: longevity and changing family patterns.
- Demographic changes and migration patterns.

### Technology:

- Online-insurance intense competition.
- Insurance Tech and artificial intelligence entail an unknown future.
- Big data and predictive modelling opportunities.

### Environmental:

- Climate changes and its unknown impact on investments. *Linked with Economic.*

## 4. Continued

### Legal:

- New international capital regulation. *Linked with Economic.*
- Privacy regulations resulting from technological advances. *Linked with Technology.*

(iii)

*The following are a sample of acceptable responses. Other accepted responses include, but are not limited to, Risk Influence Matrix/Causal Loop, Case Study, and Group Risk Identification.*

SWOT analysis. The advantage of this method is that it also considers both strengths and weaknesses of XYZ, the ability of the organization to be resilient to threats, and the capacity/willingness to take advantage of opportunities.

A Risk Map can be created to analyze the frequency and severity of strategic risk. However, capital is not estimated explicitly. A Risk Timing Map can be implemented, which is a modified version of a static risk map, measuring the evolution of risk over time.

The Delphi method can be implemented where experts, in rounds, determine and then refine their findings on risks and opportunities. It focuses on a wider range of issues than PESTEL and can be used for more complex topics.

- (c) Describe four shortcomings of the assessment methodology used for this analysis.

### **Commentary on Question:**

*The purpose of this question was to test the knowledge of candidates concerning the scenario methodology and how scenarios can be used in the context of strategic risk analysis.*

*Some candidates simply commented on the results as shown in the table without referring to the underlying theory and best practices of scenarios as described in the source material. This was not the purpose of the question.*

## 4. Continued

Some of the shortcomings that can be inferred from the table and the proposed theory are:

- Some scenarios are limited in their assessment to potential short-term developments only while others focused solely on long-term ones. Some scenarios focused on threats only, while others focused on opportunities. Prospective, synthetic and option-like scenarios should be done consistently between all potential expansion options.
- The advantage of scenarios, compared to other more limited qualitative methods, is their capacity to define and describe complex situations. They can be expanded and refined as desired. In this case, it appears that the full use of their potential was not realized as they are purely descriptive, their terminology is vague, and content varies with each option.
- There is no account of potential interaction/interdependence between the different threats and opportunities, an aspect that scenarios should consider.
- There is no evaluation of their potential financial and operational impact of each described scenario if XYZ were to pursue one of these options.

(c)

- (i) Compare and contrast the following performance measures in terms of their use in strategic decision making. Justify your analysis by performing appropriate calculations.
  - ROE
  - RAROC
  - RARORAC
- (ii) Recommend to the Board which strategic expansion should be pursued based on your financial analysis. Justify your recommendation.

### **Commentary on Question:**

*The purpose of this question was to test the candidate's understanding of risk-adjusted performance measures that are used to inform strategic decisions.*

*Most candidates performed well on this question, calculating the measures correctly and providing sufficient comparison between them. In some cases, the RAROC and RARORAC could be the same depending on the Study Note referenced. Either was considered valid as long as the calculation was done properly.*

## 4. Continued

*However, some candidates performed the calculations by integrating all three options simultaneously. This was not appropriate as it was clearly stated in the stem of the question that XYZ was planning to pursue a single option.*

*Some candidates confounded ROE and RAROC. Importantly, ROE does not account for expected losses while RAROC does.*

(i)

### ROE:

ROE = Net Income (Revenue – Costs) / XYZ Equity

European ROE = 5% = 10 / 200

Asian ROE = 25% = 50 / 200

African ROE = -5% = -10 / 200

- ROE – like ROA – is not a risk-adjusted metric. It is based solely on accounting values.

### RAROC:

RAROC = Risk-adjusted return (Net Income – Expected loss) / Available Capital

European RAROC = (10 - 1) / 100 = 9%

Asian RAROC = (50 - 5) / 120 = 33%

African RAROC = (-10 - 5) / 75 = -20%

- RAROC is used by management usually within business units to gauge performance but can also be relevant to shareholders as well.

### RARORAC:

RARORAC = Risk-adjusted return (Net Income – Expected losses) / Required Capital

European RARORAC = (10 - 1) / 54 = 17%

Asian RARORAC = (50 - 5) / 127 = 35%

African RARORAC = (-10 - 5) / 9 = -167%

- RARORAC is more complete than the previous two as it integrates both expected and unexpected losses.

## 4. Continued

- (ii) The Asian expansion should be pursued if the decision is made using RARORAC as it exhibits the largest value. Even when required capital is not accounted for, the expansion still represents the largest contributor to return. ROE, on a forward basis, will increase which will benefit shareholders. The only concern is the fact that available capital is below required capital. This should be addressed before pursuing this expansion option.

## 5. Learning Objectives:

2. The candidate will understand the concepts of risk modeling and be able to evaluate and understand the importance of risk models.
3. The candidate will understand how the risks faced by an entity can be quantified and the use of metrics to measure risk.

### Learning Outcomes:

- (2a) Demonstrate how each of the financial and non-financial risks faced by an entity can be amenable to quantitative analysis including an explanation of the advantages and disadvantages of various techniques such as Value at Risk (VaR), stochastic analysis, and scenario analysis.
- (2c) Analyze and evaluate risk aggregation techniques, including use of correlation, integrated risk distributions and copulas.
- (2g) Analyze and evaluate model and parameter risk.
- (2h) Construct approaches to modeling various risks and evaluate how an entity makes decisions about techniques to model, measure and aggregate risks including but not limited to stochastic processes.
- (3b) Analyze and evaluate the properties of risk measures (e.g., Delta, volatility, duration, VaR, TVaR, etc.) and their limitations.

### Sources:

ERM-101-12: Measurement and Modeling of Dependencies in Economic Capital (Ch 3-5)

ERM-103-12: Basel Committee - Developments in Modelling Risk Aggregation, pages 72 – 89

Value-at- Risk, Third Edition, The New Benchmark for Managing Financial Risk, Jorion Ch. 5 Computing VaR

Value-at- Risk, Third Edition, The New Benchmark for Managing Financial Risk, Jorion Ch. 12 Monte Carlo Methods

### Commentary on Question:

*Commentary is included below under each part.*

## 5. Continued

### Solution:

- (a) VaR is calculated independently for NorthPole and SouthPole. It is your task to aggregate VaR at the enterprise level.
- (i) Determine the 96% daily absolute VaR of aggregate claims for each city. Show your work.
  - (ii) Determine the theoretical range of the aggregate 96% daily absolute VaR based on the VarCovar method. Show your work.
  - (iii) Assume claim amounts in NorthPole and SouthPole have a correlation of  $\rho = 0.8$ .  
  
Determine the 96% daily VaR of aggregate claims for both cities combined using the VarCovar method.
  - (iv) Discuss the appropriateness of the VarCovar method for calculating enterprise capital.

### Commentary on Question:

*Many candidates struggled with the calculation of VaR in (i), though were generally able to aggregate the calculated VaRs correctly in (iii). Additionally, while most candidates realized that assuming perfect dependence ( $\rho = 1.0$ ) would result in the highest aggregate VaR, many assumed that the lowest aggregate VaR would result from independence ( $\rho = 0.0$ ) rather than using  $\rho = -1.0$ .*

- (i) 96% is greater than the 95% of the time claims are based on normal weather.

Thus, we are in the extreme weather category, which is 5%. If we divide the range into 5 parts (the “highest” 5 percentiles), the “breakpoint” of the first part is the point at which 4% of claims will exceed that amount, which is 1/5 of the way through the higher range of both cities:

- \$3,400 for NorthPole
- \$4,600 for SouthPole



## 5. Continued

(ii) Overall VaR using VarCovar method  
 $= (\text{VaR}_{\text{NP}}^2 + \text{VaR}_{\text{SP}}^2 + 2\rho * \text{VaR}_{\text{NP}} * \text{VaR}_{\text{SP}})^{0.5}$

Using  $\rho$  at the extreme values of -1 and 1, we get the following:

$$(3400^2 + 4600^2 + 2 * (-1) * (3400) * (4600))^{0.5} = \underline{1200}$$

$$(3400^2 + 4600^2 + 2 * (1) * (3400) * (4600))^{0.5} = \underline{8000}$$

(iii)  $(3400^2 + 4600^2 + 2 * (.8) * (3400) * (4600))^{0.5} = \underline{7599}$

- (iv) VarCovar only uses a single correlation coefficient calculated over the whole distribution. With VaR, we are more interested in the behavior of the tails of the loss distributions, so the aggregate correlation coefficient may understate tail risk.

However, the VarCovar method is easy to compute and could be easier to explain and understand than some alternative methods, e.g. copula.

- (b) (5 points) Your manager asks you to perform a simulation exercise to estimate warranty claims for NorthPole and SouthPole. He suggests using Monte Carlo simulation with correlated uniform random variables for this task.
- (i) Explain how you would apply Cholesky factorization to address your manager's request.
- (ii) Calculate the Cholesky factors using the correlation coefficient from part (a). Show your work.

You have partially populated the following table using the Cholesky factors from part (b)(ii). Z1 and Z2 represent random draws from a N(0,1) distribution. Your manager said to initially induce correlation on these variables, producing Z3 and Z4, before translating them to U(0,1) variables.

*Claim Amount Simulation from 1,000 replications  
Ordered from lowest aggregate claims to highest aggregate claims*

## 5. Continued

Simulation Number	Z1	Z2	Z3	Z4	U1 = F(Z3) is U(0,1)	U2 = F(Z4) is U(0,1)	Aggregate NorthPole Claims from U1	Aggregate SouthPole Claims from U2	Total Aggregate Claims
959	1.08	1.99	1.08	2.06	0.86	0.98	905	5,800	6,705
960	1.52	1.39							
961	0.13	3.53	0.13	2.22	0.55	0.99	579	6,400	6,979

- (iii) Determine the absolute VaR using your manager's suggested method by completing the above table. Show your work.

While the simulated random variables have a correlation of  $\rho = 0.8$ , the simulated claim amounts are correlated with  $\rho = 0.7$ . Your manager suggests the correlation discrepancy may be due to the choice of dependency measure and recommends using Spearman's rho.

- (iv) Explain your manager's feedback.

### Commentary on Question:

*In subpart (i), most candidates failed to explain the key steps of implementing Cholesky factorization or may not have approached the question by providing a step-by-step outline as desired. Not every step shown below was required for full credit, but full points were awarded if it was clear the candidate understood how the method could be applied to the stated problem.*

*Calculations in subparts (ii) and (iii) were generally done well. Answers resulting from minor rounding discrepancies in subpart (iii) were awarded full points.*

- (i) Take the correlation matrix and decompose it (using Cholesky factorization) into an upper and a lower triangular matrix.

Since there are two variables, the lower triangular matrix  
 $L = \begin{bmatrix} 1 & 0 \\ \rho & (1-\rho^2)^{0.5} \end{bmatrix}$

If  $N$  is a vector of two independent normal random variables with unit variance, then the matrix product  $LN$  gives two variables of unit variance that have the same correlation as the original correlation matrix.

For each replication, generate two independent normal random variables of unit variance.

## 5. Continued

Multiply the matrix product LN.

Determine  $D = F(x)$ , the value of the distribution function at  $x$ , for each of the correlated random variables.  $D$  is a uniform random variable from 0 to 1; and the two  $D$ s will have a correlation of 0.8

Determine  $C = F^{-1}D$ , the value of the NorthPole and SouthPole claims that gives the same point in the distribution as the correlated random variable (i.e. the inverse distribution method of simulation)

Determine total claims for the replication by summing the simulated claims for NorthPole and SouthPole.

After all replications have been completed, rank from lowest aggregate claim cost to highest and find the 96th percentile. This is the simulated VaR.

(ii)  $L = [[1,0], [\rho, (1-\rho^2)^{0.5}]]$

Where  $\rho = 0.8$ ,

$$L = [[1, 0], [0.8, 0.6]]$$

(iii) Correlated Normal random variables: 1.52 and  $0.8 * 1.52 + 0.6 * 1.39 = 2.05$

$$P(Z \leq 1.52) = 0.94$$

$$P(Z \leq 2.05) = 0.98$$

$$\text{Point on the NorthPole claim distribution} = .94/.95 * 1000 = 989$$

$$\text{Point on the SouthPole claim distribution} = (.98 - .95)/.05 * 3000 + 4000 = 5800$$

$$\text{Total claims} = 989 + 5800 = 6789$$

The 96 percentile is the  $0.96 * 1000 = 960$ th observation. The 960th observation is 6789.

## 5. Continued

- (iv) The Pearson's correlation coefficient of 0.8 was calculated based on empirical loss data. The simulation exercise does not impose this correlation on the losses themselves, but rather on the loss percentile for a given distribution. Instead of Pearson's correlation coefficient, we can calculate a rank correlation metric such as Spearman's rho or Kendall's tau, and use this value in the simulation based on Cholesky factorization.

Spearman's rho has the property that it is invariant under monotonic scaling of the losses because it depends only on the relative rank of the observations within a data set rather than the actual values of the observations. Therefore, rank correlation does not depend on marginal distributions of both variables.

- (c) After rerunning the analysis in part (b) using Spearman's rho, your manager explains that this method is equivalent to simulation using a Gaussian copula. You are evaluating the following options:

- Monte Carlo Simulation using a Gaussian Copula
- Monte Carlo Simulation using an Archimedean Copula
- Copula methods should not be used

- (i) Describe the pros and cons of the two copulas listed.
- (ii) Identify three key considerations for determining the most appropriate option.

### **Commentary on Question:**

*Most candidates were able to describe the key relevant attributes for the two copulas in (i). In general, discussions regarding additional considerations were focused on copula choice (often by reiterating items from (i)) and seemed to ignore the fact that "not using copulas" was also an option.*

- (i) Gaussian copulas are easy to simulate, but do not have distributions that can be described in closed form and do not exhibit tail dependence.

Archimedean copulas required advanced techniques for simulation, have a distribution that can be described in a closed form, and are capable of modeling tail dependence.

## 5. Continued

- (ii) Examples of additional considerations:

Does empirical data suggest that correlation between NorthPole and SouthPole losses is not constant over time? We may need a copula if empirical data suggest correlation changes under extreme weather circumstances.

Is there enough data available to accurately calibrate a copula model?

Is the potential aggregate loss significant enough to warrant addition complexity in the modelling approach?

- (d) Describe one key advantage and one key disadvantage for each of the following methods as they pertain to Pleasant Air's analysis:

I. VarCovar

II. Simulation using Cholesky factorization / Gaussian Copula

III. Simulation using Archimedean copula

**Commentary on Question:**

*Many candidates did not focus their responses on the pros and cons of each method given the situation described in the stem, i.e. adding additional cities to the company's portfolio. Therefore, answers tended to be an overly generalized comparison of each method.*

I. VarCovar –

Simple, which is important as the number of cities expands, but not necessarily accurate, as it does not consider variations in correlation along the distribution (i.e. ignores tail correlation), which may be important for cities that are close to each other.

II. Simulation Using Cholesky Factorization/Gaussian Copula –

Easy to simulate, which is important as the number of cities expands (otherwise you could have models that quickly become unmanageable); but cannot exhibit tail dependence, which may be important for cities that are close to each other.

## 5. Continued

### III. Simulation Using Archimedean Copula –

Can be calibrated to model tail correlation, which may be important for cities that are close to each other new locations are added; however, it requires advanced techniques to simulate, which could be difficult to understand and evaluate.

## 6. Learning Objectives:

5. The candidate will understand the concept of economic capital, risk measures in capital assessment and techniques to allocate the cost of risks within business units.

### Learning Outcomes:

- (5a) Describe the concepts of measures of value and capital requirements (for example, EVA, embedded value, economic capital, regulatory measures, and accounting measures) and demonstrate their uses in the risk management and corporate decision-making processes.

### Sources:

ERM-123-14: S&P Enterprise Risk Management Criteria

ERM-501-12: Risk Based Capital-General Overview

### Commentary on Question:

*This question tests the candidate's ability to knowledge of regulatory capital and rating agency requirements, particularly extensions of requirements into corporate strategy.*

*This question also tests the candidate's ability to make decisions and recommendations, including providing support or explanation for how a determination was made.*

*Candidates generally did not demonstrate the depth of knowledge required for this question.*

### Solution:

- (a) The CEO has asked you to evaluate the three blocks. PQR has \$5 million of seed money available for the acquisition and capital support.

- (i) Describe the risks associated with each RBC component identified above.

- (ii) PQR evaluates each block separately. For example, when evaluating block A, blocks B and C are ignored.

Populate the missing RBC Ratio values in the table. Show all work.

- (iii) Evaluate the pros and cons of each block for PQR.

### Commentary on Question:

*Subpart (i) required candidates to describe the risks associated with the RBC components given. Many candidates simply listed the risks and did not describe them. Descriptions of each risk were required for full credit.*

## 6. Continued

*Subpart (ii) required candidates to calculate RBC ratios and show work supporting the calculations for each of the potential acquisition blocks. Very few candidates received full credit. Nearly all candidates failed to recognize that when a block is acquired that PQR would pay the purchase price, leaving the seed money less the purchase price as PQR's TAC. This was required for full credit. Many candidates made assumptions for TAC, with resulted in partial credit. However, some candidates recalculated the ACL and used this for TAC, resulting in RBC ratios very close or equal to 100%. Candidates that did so did not demonstrate appropriate depth of knowledge and subsequently received only partial or no credit.*

*Subpart (iii) required candidates to evaluate the pros and cons of each block for PQR. Candidates generally were able to evaluate pros and cons of each block, however very few candidates related the blocks to PQR. Full credit required either one pro or con statement for each block and that the statement related to PQR. Candidates were not required to have the correct responses from subparts (i) and/or (ii) to receive full credit on (iii).*

(i) C0: Asset Risk – Affiliates: This is the risk of default of assets for affiliated investments. The parent company is required to hold an equivalent amount of risk-based capital to protect against financial downturns of affiliates.

C1: Asset Risk – Other: This is the risk of default of principal and interest or fluctuation in fair value of assets.

C2: Insurance Risk: The life insurance risk factors calculate the surplus needed to provide for excess claims, both from random fluctuations and from inaccurate pricing for future level of claims.

C3: Interest Rate Risk: The interest rate risk encompasses the risk of losses due to changes in interest rate levels. The factors in this calculation represent the surplus necessary to provide for a lack of synchronization of asset and liability cash flows.

C4: Business Risk: Business risk for life insurers is based on premium income, annuity considerations, and separate account liabilities. Also, included in business risk exposures is litigation.



## 6. Continued

- (ii) Step 1: Calculate  $TAC = 5,000,000 - \text{Price}$  for each block  
Step 2:  $RBC = TAC / ACL$

	Block A	Block B	Block C
ACL	820,000	1,360,000	710,000
Price	3,500,000	1,500,000	3,200,000
TAC	1,500,000	3,500,000	1,800,000
RBC	183%	257%	254%

- (iii) Block A – Block A is not appropriate to purchase. Its RBC is at the company action level (150-200%) while the other blocks are much higher (both above 250%).

Block B – Block B has a high concentration of both asset risk and investment / market risk. This aligns nicely with the hedge fund expertise of the parent company / investment advisor.

Block C – Block C has most of its capital tied up in insurance risk. PQR has hired several actuaries with strong mortality background, so this block aligns nicely with that expertise.

(b)

- (i) You anticipate that PQR's Emerging Risk Management and Strategic Risk Management subfactors will be scored as Negative by S&P.

Assess each of the remaining three ERM score subfactors for PQR. Justify your answer.

- (ii) Determine PQR's expected overall ERM score. Justify your answer.
- (iii) Recommend two improvements to PQR's ERM program to increase the overall ERM score to the next level. Justify your recommendation.

### **Commentary on Question:**

*Subpart (i) required candidates to assess each of the three remaining subfactors for PQR. Most candidates did very well on this subpart. Full credit required correct identification of each of the three remaining subfactors, correctly scoring the subfactor, and providing supporting statements for the subfactors.*

*Subpart (ii) required candidates to determine PQR's overall ERM score. Most candidates struggled with this subpart. Many candidates continued to use the positive, neutral, negative rating scheme as was used for the subfactors. Candidates were not required to have correctly scored the ERM subfactors in subpart (i) to receive full credit.*

## 6. Continued

*Subpart (iii) required candidates to recommend improvements to PQR's ERM program that would increase the overall ERM score to the next level. Full credit required two improvements PQR should make, with each requiring a justification. Candidates generally did well, with many receiving full credit.*

- (i) Risk Management Culture – Neutral
- The PQR Life Board of Directors participation in the ERM process is infrequent.
  - PQR Life manages risks within the business units.

Risk Controls – Neutral

- PQR Life has identified and monitors its main sources of material risks.
- PQR Life has a formal limit enforcement policy.

Risk Models – Positive

- PQR Life model limitations are documented and understood within the organization.
- PQR Life has implemented a strict model governance process.

- (ii) PQR would be scored as “Adequate.”

Overall, PQR does not satisfy the requirement for “Adequate with Strong Risk Control” because:

- Risk control subfactor is not scored positive
- Strategic risk management is not scored at least neutral
- At least one subfactor was scored negative

Furthermore, the risk controls and risk management culture subfactors are scored at least neutral. A “Weak” score would be assessed if one or both of these were scored negative.

- (iii) To increase PQR's overall ERM score to the next level:

1. Risk control subfactor needs to be scored positive. To be scored positive, the insurer has identified all material risks from all sources and frequently monitors its risk exposures with multiple metrics.

## **6. Continued**

2. Strategic risk management cannot be scored negative. To be scored as neutral, the insurer's capital allocation is risk-based but mainly reflects the views of external constituents.

“Adequate with Strong Risk Control” requires:

- Risk control subfactor is scored positive
- Strategic risk management is scored at least neutral
- No subfactor is scored negative

## 7. Learning Objectives:

3. The candidate will understand how the risks faced by an entity can be quantified and the use of metrics to measure risk.
4. The candidate will understand the approaches for managing risks and how an entity makes decisions about appropriate techniques.

### Learning Outcomes:

- (3a) Apply and construct risk metrics to quantify major types of risk exposure such as market risk, credit risk, liquidity risk, regulatory risk, etc., and tolerances in the context of an integrated risk management process.
- (3b) Analyze and evaluate the properties of risk measures (e.g., Delta, volatility, duration, VaR, TVaR, etc.) and their limitations.
- (4d) Demonstrate how derivatives, synthetic securities, and financial contracting may be used to reduce risk or to assign it to the party most able to bear it.
- (4e) Develop an appropriate choice of a risk mitigation strategy for a given situation (e.g., reinsurance, derivatives, financial contracting), which balances benefits with inherent costs, including exposure to credit risk, basis risk, moral hazard and other risks.
- (4f) Analyze the practicalities of market risk hedging, including dynamic hedging.

### Sources:

Value at Risk, Jorion, Chapter 8, Multivariate Models

Options, Futures, and Other Derivatives, Chapter 18, The Greek Letters

### Commentary on Question:

*Commentary listed underneath question component.*

### Solution:

- (a)
  - (i) Calculate the portfolio volatility. Show your work.
  - (ii) Calculate the portfolio VaR at 97.5%. Show your work.

## 7. Continued

### **Commentary on Question:**

*Candidates performed well on this part of the question. Most candidates were able to correctly recall and apply the formulas for portfolio volatility under the beta model and portfolio VAR. Answers expressed as either percentages or dollar amounts were both acceptable for full credit.*

Under the beta model,

$$\sigma_{Port}^2 = \beta^2 * \sigma_{Mkt}^2$$

Where  $\sigma_{Port}$ ,  $\sigma_{Mkt}$ , and  $\beta$  are portfolio volatility, market volatility, and portfolio beta respectively.

Using the given quantities,

$$\sigma_{Port}^2 = 1.25^2 * 0.24^2 = 0.09$$

$$\text{Yielding } \sigma_{Port} = \sqrt{0.09} = 30\%.$$

To obtain the 97.5% VAR, use the formula

$$VAR_{Port} = \alpha * \sigma_{Port} * W$$

Substituting the appropriate quantities,

$$VAR(97.5\%) = 1.96 * 0.30 * 40,000,000 = 23,520,000$$

- (b) Assess whether the cost of implementing the hedge complies with the Board's mandate.

### **Commentary on Question:**

*Candidates performed poorly on this part, with few receiving full credit. Many candidates were able to correctly determine the appropriate number of put options required for the hedge using the formula from the source reading; however only a few were able to determine the correct strike price by applying the Capital Asset Pricing Model. Most candidates arrived at a strike price using a crude simplification based on the \$36 million portfolio floor, the current \$40 million portfolio value, and the current S&P index value of \$1,000. Such candidates received partial credit if they were able determine a hedging cost and arrive at a logical conclusion based on the cost calculated.*

The number of put options underlying the S&P 500 market index required for the hedging strategy is determined by the formula

$$(\text{Portfolio beta}) * \frac{(\text{Portfolio value})}{(\text{Index value}) * 1,000} = 1.25 * \frac{40,000,000}{1,000 * 1,000} = 500$$

The appropriate strike price for the put option contracts will be the S&P 500 index level that corresponds to a \$4 million (i.e. 10%) decrease in portfolio value.

## 7. Continued

Let X represent the S&P 500 index return that corresponds to a 10% decrease in portfolio value.

Apply the Capital Asset Pricing Model:

$$(\text{Excess portfolio return}) = (\text{Portfolio beta}) * (\text{Excess index return})$$

Rearranging:

$$\text{Portfolio return} = (\text{Portfolio beta}) * (\text{Excess index return}) + (\text{Risk free rate})$$

Substituting:

$$-10\% = 1.25 * (X - 2.9\%) + 2.9\%$$

Yielding  $X = -7.5\%$ .

Thus the appropriate strike price for the put options is

$$1,000 * (1 - 0.075) = 925.$$

Referencing the table provided in the question stem, the price of each put option contract is 4,888.

The total hedging cost is

$$4,888 * 500 = 2,443,960 > 5\% * 40,000,000.$$

Therefore the hedging cost exceeds the limit specified by the Board of Directors.

- (c)
- (i) Evaluate each of the alternatives in terms of both cost and hedge effectiveness.
  - (ii) Recommend one of the alternatives. Justify your answer.

### **Commentary on Question:**

*Candidate performance was relatively strong on this part of the question, with several candidates receiving full credit. In evaluating the cost and hedge effectiveness of the three alternatives, candidates were expected to provide supporting reasoning to obtain full credit. Responses simply stating that cost and/or hedge effectiveness would be favorable or unfavorable without providing further justification received minimal credit. Candidates were required to recommend Alternative III to receive full credit, as it is the only option that satisfies the objective of placing a \$36 million floor on the portfolio value, as explicitly stated by Hawke in the question stem.*

## 7. Continued

Alternative I: This would lower the cost of the put options purchased, given that option prices decrease as they become more out-of-the-money. However, the key disadvantage of this approach is that it lowers the protection level of the hedging strategy.

Alternative II: The premiums received upfront from the written puts would reduce the net hedging cost. The disadvantage is that the company's protection is reduced in the event of a severe drop in the index value that results in the written put options becoming in-the-money at maturity.

Alternative III: The premiums received upfront from the written calls would reduce the net hedging cost. The disadvantage is that the company is obligated to make a payout in the event of the index value increasing beyond the strike price of the written calls at maturity.

Recommend Alternative III. Alternatives I and II do not accomplish the desired outcome of placing a \$4 million floor on losses when market returns are poor. Furthermore, when market returns are high, the loss from the call options written would be offset by gains in the portfolio market value.

## 8. Learning Objectives:

2. The candidate will understand the concepts of risk modeling and be able to evaluate and understand the importance of risk models.
3. The candidate will understand how the risks faced by an entity can be quantified and the use of metrics to measure risk.

### Learning Outcomes:

- (2a) Demonstrate how each of the financial and non-financial risks faced by an entity can be amenable to quantitative analysis including an explanation of the advantages and disadvantages of various techniques such as Value at Risk (VaR), stochastic analysis, and scenario analysis.
- (2b) Evaluate how risks are correlated, and give examples of risks that are positively correlated and risks that are negatively correlated.
- (2c) Analyze and evaluate risk aggregation techniques, including use of correlation, integrated risk distributions and copulas.
- (2f) Analyze the importance of tails of distributions, tail correlations, and low frequency/high severity events.
- (3a) Apply and construct risk metrics to quantify major types of risk exposure such as market risk, credit risk, liquidity risk, regulatory risk, etc., and tolerances in the context of an integrated risk management process.
- (3b) Analyze and evaluate the properties of risk measures (e.g., Delta, volatility, duration, VaR, TVaR, etc.) and their limitations.
- (3c) Analyze quantitative financial data and insurance data (including asset prices, credit spreads and defaults, interest rates, incidence, causes and losses) using modern statistical methods. Construct measures from the data and contrast the methods with respect to scope, coverage and application.

### Sources:

Value at Risk, Jorion, Chapter 17, VAR and Risk Budgeting in Investment Management (excluding 17.3 and 17.4)

The devil is in the tails: actuarial mathematics and the subprime mortgage crisis;  
Catherine Donnelly and Paul Embrechts

Value at Risk, Jorion, Chapter 8, Multivariate Models

### Commentary on Question:

*Commentary listed underneath question component.*



## 8. Continued

### Solution:

- (a) Estimate the one-year VaR (95%) of the tracking error based on the 5 year data sample, assuming the tracking error is normally distributed.

#### Commentary on Question:

*Few candidates achieved full credit for this part. Candidates were required to correctly calculate the tracking error for each year, the mean and sample standard deviation of the tracking errors, as well as the VAR to earn full credit. Many candidates calculated the population standard deviation instead of sample standard deviation and thus received partial credit. Credit was given to candidates who calculated VAR as a percentage instead of a dollar amount, since the question did not specify which form is required.*

$$\text{Year 1 tracking error} = 8.5\% - 6.0\% = 2.5\%$$

$$\text{Year 2 tracking error} = -4.1\% - (-2.2\%) = -1.9\%$$

$$\text{Year 3 tracking error} = 5.3\% - 1.4\% = 3.9\%$$

$$\text{Year 4 tracking error} = 12.0\% - 9.4\% = 2.6\%$$

$$\text{Year 5 tracking error} = 1.8\% - (-3.1\%) = 4.9\%$$

$$\text{Mean tracking error} = (2.5\% - 1.9\% + 3.9\% + 2.6\% + 4.9\%) / 5 = 2.4\%$$

$$\text{Std. dev. of tracking error} = \sqrt{(2.5\% - 2.4\%)^2 + (-1.9\% - 2.4\%)^2 + (3.9\% - 2.4\%)^2 + (2.6\% - 2.4\%)^2 + (4.9\% - 2.4\%)^2} / (5 - 1) = 2.6\%$$

$$\text{VAR}(95\%) = 1.645 * 50,000,000 * 2.6\% = 2,145,000$$

- (b) Assess each of Hawke's statements. Justify your answer.

#### Commentary on Question:

*Overall, candidates did not perform well on this part. Many candidates did not understand that policy mix risk remains unchanged for funds using the same benchmark, and that the major component of variation in portfolio performance is the choice of asset mix (policy mix risk) instead of the choice of managers (active management risk). Candidates who correctly stated whether the statements are true or false without justifying their answers did not receive any credit.*

Statement I is false:

Policy-mix risk is the risk of loss due to the policy mix selected by the fund. Jones Capital and Flamingo Funds both use the S&P 500 index as their benchmark, which is the same as the existing benchmark that ABC Group is using. Therefore policy-mix risk will remain unchanged.

## 8. Continued

Statement II is false:

Empirical studies have shown that the majority of variation in portfolio performance can be attributed to the choice of asset classes (captured in policy-mix risk) rather than the choice of manager (captured in active-management risk).

Statement III is true:

Correlation interactions between managers may result in a cross-product term that is another component of total fund VAR. For example, a negative correlation between policy-mix VAR and active-management VAR could result in the sum of the two terms being less than total fund VAR.

- (c) Calculate the standard deviation for the new portfolio in percentage terms. Show your work.

**Commentary on Question:**

*Candidates performed very well on this part. Most candidates were able to recall the formula for portfolio variance and standard deviation and correctly identify the correlation matrix and portfolio weights.*

Portfolio standard deviation =  $\sqrt{(w'\Sigma w)} = 44\%$

Where  $w = \begin{bmatrix} 0.75 \\ 0.25 \end{bmatrix}$

$$\Sigma = \begin{bmatrix} 0.25 & 0.12 \\ 0.12 & 0.16 \end{bmatrix}$$

- (d) Identify the main shortcoming of using a Gaussian copula framework for modeling credit risk in extreme market scenarios.

**Commentary on Question:**

*Most candidates received full credit for this part by pointing out that the Gaussian copula does not model default clustering in the tail scenarios since the coefficient of upper tail dependence is 0 when  $\rho < 1$ .*

The Gaussian copula has a coefficient of upper tail dependence of zero, provided  $\rho < 1$ .

Thus, under the Gaussian copula model, corporate defaults are asymptotically independent as the size of default increases. Default clustering is not modeled. In reality, corporate defaults occur in clusters during times of economic stress.

## 8. Continued

- (e) Recommend an alternative copula model that overcomes the shortcoming identified in (d). Justify your answer.

**Commentary on Question:**

*Most candidates received full credit for this part. Any copula that exhibits upper tail dependence (Joe copula, Gumbel copula,  $t$  copula, etc.) was acceptable for full credit.*

I recommend the Gumbel copula, as it shows upper tail dependence and thus is capable of modeling default clustering in extreme scenarios.