SOCIETY OF ACTUARIES

Quantitative Finance and Investment Advanced Exam

Exam QFIADV

MORNING SESSION

Date: Thursday, May 2, 2019
Time: 8:30 a.m. – 11:45 a.m.

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has a total of 100 points. It consists of a morning session (worth 60 points) and an afternoon session (worth 40 points).

   a) The morning session consists of 9 questions numbered 1 through 9.

   b) The afternoon session consists of 6 questions numbered 10 through 15.

   The points for each question are indicated at the beginning of the question.

2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.

3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.

2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.

3. The answer should be confined to the question as set.

4. When you are asked to calculate, show all your work including any applicable formulas. When you are asked to recommend, provide proper justification supporting your recommendation.

5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate morning or afternoon session for Exam QFIADV.

6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Recognized by the Canadian Institute of Actuaries.

Tournez le cahier d'examen pour la version française.

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Exam QFIADV

****BEGINNING OF EXAMINATION***
Morning Session

1. (*5 points*) You are part of a team tasked with the measuring and managing counterparty risk.

(a) (*1 point*) Compare Potential Future Exposure and Expected Exposure and the methodology to measure them.

You are designing the simulation engine module that will be used to calculate Potential Future Exposures for your firm.

Your company does a significant amount of foreign exchange rate trading, in both developed market currencies as well as in emerging market currencies. You have been given instructions to set up a model to analyze exchange rate fluctuations using a lognormal diffusion process for all foreign currency trades.

(b) (*1 point*) Assess this request.

Your team is now working on what approach to take regarding the calibration of parameters for use in the simulation models. An associate on your team states that calibration is not very important and that the team should only consider calibration based on market prices as it is the only calibration approach that makes sense.

(c) (*1.5 points*) Critique your associate's statement and compare advantages and disadvantages of the recommended calibration versus other approaches.

You have been moved to your firm's derivative trading desk. You are involved in getting rate quotes for a five year interest rate swap in which you want to pay the floating rate leg and receive the fixed rate leg. You are negotiating with two firms, Alpha, a "AAA" rated firm and Beta, a "BBB" rated firm. You have negotiated the following terms with Alpha.

Pay terms	Receive terms	Up front cost
3 month Libor	2.90%	-0-

You believe the terms with Alpha are fair.

The sales desk at Beta also wishes to execute on these terms.

(d) (*1.5 points*) Propose a response to the sales desk at Beta.

2. (*9 points*) ABC Life Insurance is a recent start-up company marketing insurance and investment products toward high net-worth clients. ABC's primary strategy is to offer customized policies constructed by bundling products from its three lines of business: Traditional Life Insurance, Segregated Fund with GMAB Option, and Fixed Annuity.

The Chief Risk Officer (CRO) of ABC is aware of the importance of liquidity management and intends to apply the three levels of liquidity management:

- Day to day cash management
- Intermediate term cash flow management
- Stress liquidity risk

(a) (*1 point*) Describe how to complete the liquidity monitoring program based on the three levels of liquidity management.

Although ABC is still small, it has had significant growth since it started. At the end of 2017, ABC's cumulative sales are as follows.

Product Line	**Cumulative Sales**	**Average Policy/Contract Size**
Traditional Life Insurance	30 million	0.4 million
Segregate Fund with GMAB	200 million	3.0 million
Fixed Annuity	150 million	1.5 million

QFI Life is one of the largest life insurance companies in North America and is known for its successful liquidity risk management. QFI Life has a diverse client base and offers the same three product lines as ABC. At the end of 2017, QFI Life has accumulated 200 billion of sales with 10 million policies in-force.

Before leaving to build ABC, the CRO was previously employed by QFI and was on its Risk Management Committee. He understands the key contributors to failure for stress liquidity risks but due to limited resources at ABC, is considering leveraging the liquidity management framework already developed by QFI.

(b) (*2 points*) Evaluate where the liquidity management framework could fail if ABC adopts it from QFI without adjustment.

2. Continued

To adapt QFI's liquidity management framework for ABC, the CRO asks you to analyze ABC's liquidity profile, starting with Traditional Life Insurance.

#	**Traditional Life Insurance – Product Features**
1	20 Pay premiums
2	Benefit paid at the earlier of death or age 100
3	Fast growing cash surrender value (CSV)
4	Cancellation of policy allowed without penalty within 7 days
5	Benefit settlement within 1 month
6	Policy loan allowed after 1st anniversary
7	Surrender charge waived after 5th anniversary

(c) (*1.5 points*) Describe possible liquidity strains from Traditional Life Insurance's product features.

You decide to consider the following three options for evaluating liquidity risk of the Traditional Life Insurance's product through cashflow cushions.

Liquidity Risk Evaluation Options				
Option1	7 Days	1 Month	6 Months	1 Year
Option2	7 Days	1 Month	1 Year	5 Years
Option3	3 Days	3 Months	6 Months	1 Year

(d) (*1.5 points*) Recommend one of the above options.

Question 2 continued on the next page.

2. Continued

According to your analysis of Traditional Life Insurance under a baseline scenario, total forecast cash outflow in one year is 10 million, bringing the cashflow cushion down to 0.95:1, below an allowable limit of 1.10:1. You are given the profile of stand-by liquidity below:

Time (Years)	Volume to Liquidate (in millions)			
	5 Year AA Corporate Bond	10 Year AA Private Fixed Income	10 Year A Corporate Bond	20 Year BBB Corporate Bond
0.25	0.2			
0.5	0.15		0.2	
0.75	0.1		0.1	0.8
1	0.05	6	0.1	0.4
1.25		3	0.1	0.3
1.5		2	0.05	0.2
1.75		1		0.2
2		1		0.1
Haircut	0%	30%	10%	20%

In addition to forecasted cash flows, the management is also considering entering a security lending program soon, using a separate portfolio of bonds. The following table summarizes the expected cash inflows from this program:

Time (Years)	Additional cash inflow from Security Lending Program (in millions)			
	5 Year AA Corporate Bond	10 Year AA Private Fixed Income	10 Year A Corporate Bond	20 Year BBB Corporate Bond
0.25	0.1			
0.5		0.1	0.1	
0.75	0.1			0.2
1			0.1	
1.25	0.1			
1.5		0.1	0.1	
1.75	0.1			0.2
2			0.1	
Haircut	0%	0%	0%	0%

2. Continued

(e) (*3 points*) Recommend two liquidation strategies that bring the one-year cashflow cushion back to the allowable limit, while minimizing the costs of liquidation.

(i) one excluding management actions

(ii) one including management actions

3. (*5 points*) You are responsible for building the interest rate economic scenario generators (ESGs) for your company to value interest rate derivatives. You have been given the following three models to consider:

- Vasicek model
- CIR model
- Hull-White Extended Vasicek model

(a) (*1 point*) Describe the characteristics of the Vasicek model.

(b) (*1.5 points*) Compare and contrast the Vasicek model with the other two models above.

(c) (*1.5 points*) Explain which model is more accurate in pricing interest rate derivatives, such as options on bonds.

Consider the following model:

$$dr_t = k(\theta - r_t)\,dt + \sigma\sqrt{r_t}\,dW_t + d\boldsymbol{J}_t$$

with

$$\boldsymbol{J}_t = \sum_{i=1}^{M_t} Y_i$$

where

M is a time-homogenous Poisson process with intensity λ

and

Y_i are i.i.d. exponential random variables with parameter μ.

(d) (*1 point*)

(i) Identify the type of process followed by $\boldsymbol{J}_t$.

(ii) Compare and contrast this model to CIR model.

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4. (*6 points*) You work for a small investment bank that is thinking of originating structured finance business. Your colleague recently made the following comments:

> *"Over the years investment firms have benefited from innovation in terms of securitization and credit availability. Our firm should seriously consider issuing structured finance products.*
>
> *The efficient use of capital is the only advantage for investors investing in this product. I like the fact that the burden of repayment is not on the originator but on a pool of assets generating cash flows and on the entity providing credit support.*
>
> *Since only commercial mortgages generating future cash flows can be securitized, I say we consider issuing these products."*

(a) (*1 point*) Critique your colleague's comments.

A selection of Solar Projects across the country are being securitized. The various projects are geographically diverse and are expected to produce similar amounts of electricity. These projects result in tax credits that are available to be used for the next ten years.

- Recent incentives have allowed for tax credits to be transferred to corporations with larger tax burdens.
- The construction on these projects has ended today.

These projects are being securitized in an innovative manner (with the introduction of a tax credit tranche) and are divided into 3 tranches:

1. Main Tranche: Highly rated tranche based on income from expected electricity generation and conservative prices being paid for that electricity, subject to a cap and floor on electricity prices.
2. Equity Tranche: Receives excess income or is responsible for shortfall, if any, from electricity generation or prices.
3. Tax Credit Tranche: Provides the right to use tax credits that become available two years after the completion of construction.

4. Continued

The following investors are interested in investing in this Solar Project:

1. Farming company with diverse geographical representation but dependent on rainy weather each year.
2. Institutional investor holding a large realized gain in their existing portfolio, and a large cash outflow starting in two to three years.
3. Pension investor with an environmental protection mandate.
4. An insurance company seeking highly rated assets with some additional yields compared to treasuries.
5. Traditional Power Generation Company looking to diversify their investment portfolio.

(b) (*3 points*) Assess which tranche (if any) that each of the above investors would be more inclined to invest taking into account any additional risks that need to be considered.

It is now two years later and the following two things have occurred in the electricity generation market:

1. Solar electricity storage and subsequent delivery has progressed considerably allowing for the ability to provide electricity at any time.
2. A complete change in the pricing structure of solar electricity resulting in costs being equal at all times. These prices are 20% less than the average prices two years earlier.

(c) (*2 points*) Discuss the implications of these two developments on each of the tranches of this securitized asset, both individually and in relation to each other.

5. (*6 points*) EcL is a publicly traded company that does not pay dividends. EcL issued a zero-coupon bond and used the proceeds to fund its large corporate project.

You are given:

- The remaining time-to-maturity of EcL's zero coupon bond = 9 years.
- The face value of EcL's zero-coupon bond = \$350 million
- EcL has no other liabilities or debts except the above zero-coupon bond.
- EcL's total number of outstanding shares = 1 million
- EcL's stock price = \$40.00 per share
- EcL's stock price volatility = 45.5% (the volatility given for the equity is the level of the equity volatility given the current values of the firm's assets/equity)
- The continuously compounded risk-free interest rate = 2% for all maturities
- The relationship between σ_A and $\sigma_A \mathrm{N}(d_1)$ is monotone

(a) (*1 point*) Calculate an upper bound for the asset value of EcL.

Based on the above information and some additional work, your analyst, making use of the Merton model estimated that EcL's asset value volatility = 5%.

(b) (*3 points*) Show that your analyst's estimate of EcL's asset value volatility is not correct.

(c) (*2 points*) Determine if your analyst either overstated or understated the asset value volatility by testing the impact of a +1% shock on the asset value volatility.

6. (*8 points*) Your company is Canadian-based with large operations in the U.S., and which takes a long-term view of its business and is less concerned with short-term market fluctuations.

You are asked to use Monte Carlo simulation with a lognormal FX rate model to project the CAD to USD exchange rate one year from now.

You are given the following information:

- Time-zero exchange rate: CAD 1.00 = USD 0.80
- Canadian risk-free rate = 1.2%
- U.S. risk-free rate = 1.5%
- Annual FX rate volatility based on historical volatility approach = 0.1
- Annual FX rate volatility based on market volatility approach = 0.15

(a) (*2.5 points*) Describe the historical and market volatility approaches and recommend the approach for your company.

You are given the following:

Scenario	Random number (standard normal distribution)
1	- 0.767
2	0.525
3	0.840
4	- 0.221
5	- 1.120

(b) (*2.5 points*) Project the CAD to USD exchange rate one year hence based on the five scenarios above.

You have decided to apply an upward shock to the Canadian risk-free rate and re-run your simulation.

(c) (*1 point*) Explain the reaction of the FX return you expect to observe.

(d) (*2 points*) Describe four variance reduction techniques that can be applied in stochastic modeling.

7. (*8 points*) A large Japanese insurance company is launching a new single premium Variable Annuity (Segregated Fund) product. Due to the low interest rate environment in Japan, the company decides to invest completely in a different mature market, the Australian market. The premium is collected in Japanese Yen (JPY). Then the premium is converted into Australian dollars (AUD) and is invested in the Australian index (ASX 200). The benefits are paid in JPY when claims are made.

This VA product comes with a GMDB which pays the maximum of:

1) the current fund value converted into JPY, and
2) the premium accumulated at 1.5% per annum.

The company decides to cover the JPY/AUD exposure using an FX option.

(a) (*0.5 points*) Describe the observed behavior(s) of FX option implied volatility between JPY/AUD.

The company models the Australian index using the Black-Scholes model and decides to synthetically replicate the required put options to hedge the GMDB exposure.

(b) (*1 point*) Explain why the synthetic put option might not hedge the GMDB exposure in the presence of a volatility smile.

(c) (*2 points*) Describe other models that will help the company better capture the volatility smile.

The company eventually chose the Heston model as their equity model. The Heston model is presented as the following:

$$dS_t = \mu_t S_t dt + \sqrt{v_t} S_t dW_t^s$$
$$dv_t = \alpha(\theta - v_t)\, dt + \varepsilon \sqrt{v_t} S_t dW_t^v$$

Where W_t^s, W_t^v are Wiener processes with correlation ρ.

(d) (*0.5 points*) Identify the process of the instantaneous variance v_t.

(e) (*2 points*) Explain how this model produces a volatility smile.

7. Continued

With respect to the hedging strategy, the company measures $\text{CTE}_{1-\alpha}(\chi)$, where χ is defined as the minimum of the distributed values of the insurer's profit/loss at all policy calculation dates, as a percentage of the sum of the premiums paid to the insurer at time 0.

(f) (*1 point*) Explain what does $\text{CTE}_{1-\alpha}(\chi) = 1$ implies, with respect to the hedging efficiency.

(g) (*1 point*) Comment on why setting up a Vega hedge portfolio within the Black-Scholes model can lead to potential "over-hedging".

8. (*8 points*) You are the Pricing Actuary at Company ABC. Your company is developing a new Equity Indexed Annuity (EIA) product with the following features:

- Yearly credited rate = 100% x S&P 500 Index return, subject to 5% cap and 0% floor
- Product matures after 12 years
- No surrender charges

ABC plans to use the following investment strategy for the product:

- 97% of the fund value invested in bonds with a duration of 12 years
- 3% of the fund value used to purchase index options

(a) (*2 points*) Critique the product features and investment strategy, recommending potential changes.

You are given the following additional information:

- Initial S&P index value is 2775 points
- Projected sales in the first month are $4 million.

Traders from your in-house asset management company plan to use 1-year over-the-counter options to hedge the product, since they believe exchange-traded options matching the required strike prices are not available.

The following mid-market price quote table was provided (with each options contract having a 10 multiplier or $10 per index point). You are told prices for strikes between those given will be linearly interpolated.

Strike	Calls	Puts
2700	226	111
2725	211	121
2750	197	131
2775	183	142
2800	169	153
2825	157	165
2850	144	177
2875	133	190
2900	122	204
2925	111	218

(b) (*1 point*) Outline an appropriate hedging strategy for the product. Justify your answer.

8. Continued

(c) (*2 points*) Calculate the total hedge cost for your proposed strategy in (b). State all assumptions made.

To significantly reduce hedge costs, ABC is considering modifying the credited rate guarantee such that a portion y% of any losses on the index are passed on to the policyholder.

(d) (*1 point*) Propose a modification to the hedging strategy based on the modified credited rate specification.

(e) (*2 points*) Calculate y% such that the hedge cost equals 2% of the fund value under the modified hedging strategy, assuming the same $4 million of projected sales and starting index value of 2775.

9. (*5 points*) The Board of a small non-taxable Education fund has asked you to analyze an opportunity to invest in real estate within the context of improving yield in the current low interest rate environment while minimizing both the risks of liquidity and increased investment costs. The Education fund currently invests exclusively in bonds and common stocks of US companies.

(a) (*2 points*) Discuss the advantages and risks associated with real estate as they relate to the Education fund.

The current management team specializes in US bonds and stocks but is responsible for providing monthly valuations for the entire fund and have given you the following performance statistics.

Annualized performance for 2002-2017	**Average return**	**Standard deviation**
Education fund	5.0%	6.0%
US Treasury 30-day bill	1.1%	1.5%

You are to select one of the following two classes of real estate assets: Direct ownership of a major commercial center in the metropolitan area of the fund or, equity real estate investment trusts (REIT) listed on the New York Stock Exchange.

Annualized performance for 2002-2017	**Average return**	**Standard deviation**
Commercial center	7.1%	5.0%
REIT equity	8.9%	7.8%

The Board requests that you consider only their existing performance measure, a quotient relating excess returns and risk, in making your recommendation and indicates that the fund's current value for this measure is 0.65. They suggest that the larger the ratio is, the better it is.

(b) (*1.5 points*) Discuss limitations that this performance measure may have when used to measure the performance of an asset class such as real estate.

9. Continued

In the real estate world, the capitalization rate is a critical component in asset price determination. One possible model uses only rental and interest rate fundamentals variables to predict future real estate capitalization rates using the following predictive model:

$$\log(C_{j,t})=a_0+a_1\log(C_{j,t-1})+a_2\log(C_{j,t-4})+a_3\log(RRI_{j,t})+a_4RTB_t+a_7Q2_t+a_8Q3_t+a_9Q4_t+a_{10}D_j$$

Variable	Description
$RRI_{j,t}$	Real rent index calculated as the ratio of real rent data for a given Metropolitan Statistical Area (MSA) in a given quarter to the historical average of real rent for this MSA.
RTB_t	Real T-bond yield calculated as nominal yield minus inflation rate.
$Q2_t$, $Q3_t$, $Q4_t$	Seasonal dummy variables
D_j	Fixed market-level effect associated with each MSA
a_k	Constants

A recent study suggests adding 2 macroeconomic financial factors as the risk premium and the debt on the market identified by: $SPREAD_t$ and $DEBTFLOW_t$

Variable	Description
$SPREAD_t$	Risk premium calculated as the spread between Moody's AAA Corporate Bond Index and 10-year T-bond yield.
$DEBTFLOW_t$	Debt flow calculated as a ratio of Total Net Borrowing and Lending, from the Federal Reserve's Flow of Funds Database to that quarter's nominal GDP level.

(c) (*1.5 points*) Explain reasons supporting the addition of these two components including their interaction with $RRI_{j,t}$ and RTB_t, and impact on asset values and capitalization rates.

****END OF EXAMINATION****
Morning Session

USE THIS PAGE FOR YOUR SCRATCH WORK