Fall 2018 LTAM

Multiple Choice Solutions

Comments on candidates' performance on the multiple choice questions.

Questions 9, 11, 14, 15 and 19 attracted the lowest number of correct answers.

Questions 2, 4, 7 and 13 attracted the highest number of correct answers.

Overall the average number of correct answers was lower than in recent MLC exams.

Question 1: E

Question 2: B

$${}_5p_{30} - {}_{21}p_{30} = e^{-0.3} - e^{-(0.6+0.33)} = 0.35$$

Question 3: C

$$q_{50}^{(1)} = {}_{0.75}q_{50}^{(1)} + {}_{0.75}p_{50}^{(\tau)}\ {}_{0.25}q_{50.75}^{(1)}$$

$${}_{0.75}q_{50}^{(1)} = {}_{0.75}\ q_{50}'^{(1)} = 0.75(0.02) = 0.015$$

$$0.02 = {}_{0.75}\ q_{50}'^{(1)} + {}_{0.75}\ p_{50}'^{(1)}\ {}_{0.25}q_{50.75}'^{(1)} \Rightarrow {}_{0.25}\ q_{50.75}'^{(1)} = {}_{0.25}\ q_{50.75}^{(1)} = \frac{0.02 - 0.015}{{}_{0.75}p_{50}'^{(1)}}$$

$${}_{0.75}p_{50}^{(\tau)} = 0.9\,{}_{0.75}p_{50}'^{(1)}$$

$$\Rightarrow q_{50}^{(1)} = 0.015 + 0.9\,{}_{0.75}p_{50}'^{(1)}\left(\frac{0.02 - 0.015}{{}_{0.75}p_{50}'^{(1)}}\right) = 0.015 + 0.9(0.005) = 0.0195$$

Question 4: D

Paths	Probability
$1 \to 1 \to 1 \to 1$	0.2^3
$1 \to 1 \to 3 \to 1$	$(0.2)(0.5)(0.9)$
$1 \to 3 \to 1 \to 1$	$(0.2)(0.5)(0.9)$
$1 \to 3 \to 1 \to 1$	0

$\Rightarrow$Total probability 0.1880

Question 5: C

$$\hat{S}(6) = \left(\frac{18}{20}\right)\left(\frac{15}{18}\right)\left(\frac{9}{14}\right) = 0.482$$

Question 6: B

$$\text{Prob} = 0.76(1-0.962\times 0.25)(1-0.964\times 0.965\times 0.26) = 0.4376$$

Question 7: B

$$\hat{S}(15) = e^{-\left(\frac{2}{200}+\frac{2}{192}+\frac{3}{182}+\frac{17}{170}\right)} = 0.8721$$

Question 8: C

Question 9: A

$$\begin{aligned}\bar{A}^{02}_{60:\overline{10}|} &= \bar{A}^{02}_{60} - {}_{10}p^{00}_{60}\, v^{10}\, \bar{A}^{02}_{70} - {}_{10}p^{01}_{60}\, v^{10}\, \bar{A}^{12}_{70} \\ &= 0.164\end{aligned}$$

Question 10: E

$$\begin{aligned}EPV &= 9000\mathrm{v}^{10}\left({}_{10}p_{50}\,{}_{10}q_{60}\,\ddot{a}_{60} + {}_{10}p_{50}\,{}_{10}q_{60}\,\ddot{a}_{60}\right) \\ &= 9000\left({}_{10}E_{50}\,{}_{10}q_{60}\,\ddot{a}_{60} + {}_{10}E_{60}\,{}_{10}q_{50}\,\ddot{a}_{70}\right) \\ &= 5870\end{aligned}$$

Question 11: E

EPV Premiums: $P(1+0.99v) = 1.9519P$

EPV Benefits:

$$1000\, v^2\,{}_2p_{40}\left(1+1.04v\,p_{42}+1.04^2v^2\,{}_2p_{42}+\ldots\right) = 1000v^2\,{}_2p_{40}(1+e_{42})$$

$$e_{40} = 11.06 = p_{40} + {}_2p_{40} + {}_3p_{40} + \ldots = p_{40} + {}_2p_{40}(1+e_{42})$$

$$\Rightarrow (1+e_{42}) = 10.379$$

$$\Rightarrow P = \frac{9310}{1.9519} = 4770$$

Question 12: E

$$P = 100\frac{(1-d\ddot{a}^{ns}_x)}{\ddot{a}^{ns}_x} = 1.6896$$

EPV Premiums $= 1.6896(12.40) = 20.95$

EPV Benefits $= 100(1-d\ddot{a}^{s}_x) = 40.95$

EPV Loss at issue $= 40.95 - 20.95 = 20$

Question 13: D

$$\text{EPV Annuity } = 50\,000\,{}_{10}E_{55}\,\ddot{a}_{65} = 402\,040$$
$$\text{EPV Survival Benefit } = 200\,000\,{}_{25}E_{55} = 45\,667$$
$$\text{EPV Premiums minus expenses } = P(0.95\,\ddot{a}_{55:\overline{10}|} - 0.15) = 7.4682P$$
$$\Rightarrow P = 59\,950$$

Question 14: D

$$P\ddot{a}_{55:65:\overline{10}|} = 55,000\,{}_{10}E_{55}\;\ddot{a}_{65} + 55,000\,{}_{10}E_{65}\;\ddot{a}_{75} - 10,000\,{}_{10}E_{55:65}\;\ddot{a}_{65:75}$$
$${}_{10}E_{55:65} = (1+i)^{10}\,{}_{10}E_{55}\,{}_{10}E_{65} = 0.53459$$
$$\Rightarrow P = \frac{705,352}{7.7596} = 90,900.6$$

Question 15: A

$${}_{10}V = 200000\bar{A}^{1}_{75:\overline{10}|}v^{0.5} - 12P\ddot{a}^{(12)}_{75:\overline{10}|}$$
$$\ddot{a}^{(12)}_{75:\overline{10}|} = 7.3203 - \frac{11}{24}(1 - 0.44085) = 7.0640$$
$$\bar{a}_{75:\overline{10}|} = 7.3203 - \frac{1}{2}(1 - 0.44085) = 7.0407$$
$$\bar{A}^{1}_{75:\overline{10}|} = 1 - \delta\bar{a}_{75:\overline{10}|} - {}_{10}E_{75} = 0.21563$$
$$\Rightarrow {}_{10}V = 18,860$$

Question 16: C

$$\left({}_{5}V^{(0)} + P\right)(1+i) = p^{00}_{55}\,{}_{6}V^{(0)} + p^{01}_{55}(200,000) + p^{02}_{55}(100,000)$$
$$\Rightarrow {}_{6}V^{(0)} = 2998.3$$

Question 17: A

$$P^{*} = \frac{200,000A_{51}}{\ddot{a}_{51:\overline{14}|}} = \frac{39,560}{\ddot{a}_{51} - \frac{94579.7}{98457.2}v^{14}\ddot{a}_{65}} = 3851.23$$
$${}_{5}V^{FPT} = 200,000A_{55} - P^{*}\ddot{a}_{55:\overline{10}|} = 16,164$$

Question 18: B

$$\Pi_{10} = {}_{9}p_{35}Pr_{10}$$
$$\text{Pr}_{10} = \left({}_{9}V + P - E\right)(1.06) - q_{44}(500,000)(1.06)^{(0.5)} - p_{44}\,{}_{10}V = 4070.5$$
$$\Rightarrow \Pi_{10} = 3989$$

Question 19: E

EPV at age 62 is:

$$5000(1.0268)^{32}\left\{1+(1.0226)(1.0268)vp_{62}+\left((1.0226)(1.0268)v\right)^2{}_2p_{62}\right\}$$

$$=5000(1.0268)^{32}\left\{1+p_{62}+{}_2p_{62}\right\}=34812$$

$\Rightarrow$ EPV at age 30 is: $34{,}812\,{}_{32}p_{30}\,v_{5\%}^{32}=7028$

Question 20: A

Projected total salary: $S_{35}\left(1+1.04+1.04^2+...+1.04^{29}\right)=S_{35}\dfrac{1.04^{30}-1}{0.04}=56.08S_{35}$

Projected final salary: $S_{35}(1.04)^{29}=3.12S_{35}$

Projected replacement rate: $\dfrac{0.02(56.08)}{3.12}=35.95\%$