LTAM Fall 2018

Model Solutions
Written Answer Questions

Question 1 Model Solution

Learning Outcomes: 2(j), 5(a), 5(c)

Chapter References: AMLCR Chapter 8; Sections 12.5, 12.8; SN LTAM-21-18 Chapter 3

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a) $${}_2p_{60}^{01} = p_{60}^{00}\, p_{61}^{01} + p_{60}^{01}\, p_{61}^{11}$$

$$= (0.9)(0.05) + (0.05)(0.2) = 0.055$$

Comments:

1. *This part was done correctly by almost all candidates.*
2. *For those who did not receive full credit, two common errors were to use an incorrect annual probability of transition and to calculate only one of two terms.*

b)

(i) $${}_2V^{(0)} = 50{,}000\, A_{62:\overline{8}|}^{02} + 150\, \ddot{a}_{62:\overline{8}|}^{00} + 150\, \ddot{a}_{62:\overline{8}|}^{01} - 5{,}000\, \ddot{a}_{62:\overline{8}|}^{00}$$
$$= 50{,}000(0.46667) + 150(4.7328) + 150(0.2533) - 5{,}000(4.7328)$$
$$= 23{,}333.5 + 709.92 + 37.995 - 23{,}664.0 = 417.415$$

(ii) $${}_2V^{(1)} = 50{,}000\, A_{62:\overline{8}|}^{12} + 150\, \ddot{a}_{62:\overline{8}|}^{10} + 150\, \ddot{a}_{62:\overline{8}|}^{11} - 5{,}000\, \ddot{a}_{62:\overline{8}|}^{10}$$
$$= 50{,}000(0.4968) + 150(3.334 + 1.406) - 5{,}000(3.334)$$
$$= 24{,}840 + 711 - 16{,}670 = 8{,}881$$

(iii) $$\left({}_2V^{(0)} + 5{,}000 - 150\right)1.06 = p_{62}^{02}\,(50{,}000) + p_{62}^{00}\,{}_3V^{(0)} + p_{62}^{01}\,{}_3V^{(1)}$$

$${}_3V^{(1)} = \frac{(5583.46 - 3500 - 1573.44)}{0.05} = 10{,}200.40$$

Alternatively

$$\left({}_2V^{(1)} + 0 - 150\right)1.06 = p_{62}^{12}\,(50{,}000) + p_{62}^{10}\,{}_3V^{(0)} + p_{62}^{11}\,{}_3V^{(1)}$$

$${}_3V^{(1)} = \frac{(9254.86 - 6000 - 1215.84)}{0.2} = 10{,}195.10$$

Comments:

1. *Parts (i) and (ii) were done correctly by most candidates.*
2. *For those who did not receive full credit, a common error was to ignore maintenance expenses.*
3. *Although most candidates recognized that a recursive formula was needed to answer part (iii), setting it up correctly proved to be challenging for many candidates.*

c)

(i) $Pr_3^{(0)} = \left({}_2V^{(0)} + P - 60\right)1.057 - p_{62\,3}^{00}V^{(0)} - p_{62\,3}^{01}V^{(1)} - p_{62}^{02}(50{,}000)$

$= (417.415 + 5000 - 60)1.057 - (0.88)(1788) - (0.05)(10{,}200.4) - (0.07)(50{,}000)$

$= 79.328$

$Pr_3^{(1)} = \left({}_2V^{(1)} - 60\right)1.057 - p_{62\,3}^{10}V^{(0)} - p_{62\,3}^{11}V^{(1)} - p_{62}^{12}(50{,}000)$

$= (8881 - 60)1.057 - (0.68)(1788) - (0.2)(10{,}200.4) - (0.12)(50{,}000)$

$= 67.877$

${}_2p_{60}^{01} = 0.055$ from (a)

${}_2p_{60}^{00} = (0.9)(0.89) + (0.05)(0.69) = 0.8355$

$\pi_3 = {}_2p_{60}^{00}\, Pr_3^{(0)} + {}_2p_{60}^{01}\, Pr_3^{(1)}$

$= (0.8355)(\,79.328) + (0.055)(\,67.877) = 70.012$

Alternatively, with ${}_3V^{(1)} = 10{,}195.10$, we get

$Pr_3^{(0)} = 79.593$; $Pr_3^{(1)} = 68.937$; and $\pi_3 = 70.291$

(ii) $NPV_t = NPV_{t-1} + \pi_t\, v^t = \sum_{k=0}^{t} \pi_k\, v^k$; $NPV_0 = \pi_0$; $v = 1/1.08$

t	π_t	NPV_t
0	-200	-200
1	84.74	-121.537
2	80.35	- 52.6499
3	70.012	2.928

Alternatively, with $\pi_3 = 70.291$, we get $NPV_3 = 3.150$

The Discounted Payback Period (DPP) is 3 years.

Comments:

1. *For part (i), many candidates did not recognize that the expected emerging profit depends on the state at the beginning of the period.*
2. *Most candidates did well on part (ii). Some candidates discounted the profit signature values at a rate other than the hurdle rate.*

Question 2 Model Solution

Learning Outcomes: 1(c), 2(b), 2(k), 3(a), 3(c)

Chapter References: AMLCR Chapters 3, 4 and 5

Comment: Most candidates did very well on this question.

a) The benefit being valued is that of a 5-year deferred whole life annuity of 1000 per year payable continuously, issued to (x).

Comments:

1. *Most candidates did very well on this part, many receiving full credit.*
2. *For those who did not receive full credit, a common error was to omit some details in the description of the benefit, for example, the amount or the continuous nature of the payments, for which a small deduction was applied.*

b) Acceptable expressions include:

$$\begin{aligned} E[Y] &= 1000\, v^5\, {}_5p_x\, \overline{a}_{x+5} \\ &= 1000\, {}_5E_x\, \overline{a}_{x+5} \\ &= 1000\, {}_{5|}\overline{a}_x \\ &= 1000\, (\overline{a}_x - \overline{a}_{x:\overline{5}|}) \end{aligned}$$

Comment: This part was answered very well, most candidates receiving full credit.

c)

(i) $\text{Var}[\text{Y}|\text{I}=1] = V\left[1000\, v^5\, \overline{a}_{\overline{T_x-5}|} \,\middle|\, T_x > 5\right] = 10^6\, v^{10}\, V\left[\overline{a}_{\overline{T_{x+5}}|}\right]$

$$= 10^6\, v^{10}\, V\left[\frac{1-v^{T_{x+5}}}{\delta}\right] = 10^6\, v^{10}\, \frac{\left({}^2\overline{A}_{x+5} - \overline{A}^2_{x+5}\right)}{\delta^2}$$

(ii) $E[\text{Y}|\text{I}=0] = 0$ $\qquad$ $\text{Var}[\text{Y}|\text{I}=0] = 0$

$E[\text{Y}|\text{I}=1] = 10^3 v^5\, \overline{a}_{x+5}$ $\qquad$ $\text{Var}[\text{Y}|\text{I}=1] = 10^6 v^{10}\, \frac{\left({}^2\overline{A}_{x+5} - \overline{A}^2_{x+5}\right)}{\delta^2}$

$$Var(Y) = \text{E}\left[\text{Var}[\text{Y}|\text{I}]\right] + \text{Var}\left[\text{E}[\text{Y}|\text{I}]\right]$$

$$Var(Y) = 10^6 \left\{ {}_5p_x\, v^{10}\, \frac{\left({}^2\overline{A}_{x+5} - \overline{A}^2_{x+5}\right)}{\delta^2} + {}_5q_x\, (0) \right\} + 10^6\, (v^5\, \overline{a}_{x+5})^2\, {}_5p_x\, {}_5q_x$$

Comments:

1. *Most candidates did well on part (i).*
2. *For part (ii), only well-prepared candidates showed all key steps of the proof needed to get full credit.*

d)

$$\text{EPV} = 1000\ {}_5E_{65}[\alpha(\infty)\ \ddot{a}_{70} - \beta(\infty)]$$
$$= 1000(0.75455)[1.0002(12.0083) - 0.50823] = 8679.19$$

Alternatively

$$\overline{A}_{70} = \frac{i}{\delta} A_{70} = \frac{0.05}{\ln(1.05)}(0.42818) = 0.4387975$$

$$\overline{a}_{70} = \frac{1 - \bar{A}_{70}}{\delta} = \frac{1 - 0.4387975}{\ln(1.05)} = 11.50237$$

$$\text{EPV} = 1000\ {}_5E_{65}\ \overline{a}_{70} = 1000(0.75455)(11.50237) = 8679.11$$

Comments:

1. *Most candidates did well on this part.*
2. *Partial credit was given to the candidates who did not use or incorrectly used UDD to calculate the EPV.*

e)

(i) $\Pr[Y > E[Y]] = \Pr[T_{65} > t + 5]$

where t is such that $1000\ v^5\ \bar{a}_{\overline{t}|} = E[Y] = 8679.19$

$$\Rightarrow\ v^5\ \left(\frac{1 - v^t}{\delta}\right) = 8.6792 \qquad \text{and} \qquad t = \ln\left(1 - \frac{8.6792(0.04879)}{0.783526}\right) / \ln\left(\frac{1}{1.05}\right) = 15.9359$$

$$\Pr[T_{65} > 20.9359] = {}_{20.9359}p_{65}$$
$$= \frac{(0.9359)l_{86} + (0.0641)l_{85}}{l_{65}}$$
$$= \frac{(0.9359)(57656.7) + (0.0641)(61184.9)}{94579.7} = 0.612$$

Alternatively

$$\Pr[Y > E[Y]] = \text{E}[\Pr[\ Y > E[Y]\ |I]$$
$$= {}_5q_{65} \cdot 0 + {}_5p_{65} \cdot \Pr[\ Y > E[Y]\ |\ T_{65} > 5]$$
$$= {}_5p_{65} \cdot \Pr[1000\ v^5\ \bar{a}_{\overline{T_{70}|}} > 8679.2]$$
$$= {}_5p_{65} \cdot \Pr[T_{70} > 15.9359]$$
$$= {}_{20.9359}p_{65} = 0.612$$

(ii) The Normal approximation can be used when the distribution is close to a Normal distribution. From the Central Limit Theorem, this is true for sums or means of large numbers of independent random variables. Here we have a single random variable with a skewed distribution that is not close to a Normal distribution.

Alternatively

The Normal distribution is symmetric which would give $\Pr[Y > E[Y]] \approx 0.5$. This approximation is not appropriate for skewed distributions like to one here.

Comments:

1. *Only well-prepared candidates received full credit for part (i).*
2. *A common error for part (i) was to ignore or incorrectly reflect the deferral period.*
3. *Another common error for part (i) was to calculate the survival probability by rounding the number of years to 21.*
4. *A few candidates used the normal approximation which resulted in no credit for part (i).*
5. *Candidates who answered part (ii) did very well.*

Question 3 Model Solution

Learning Outcomes: 2(b), 2(g), 2(k)

Chapter References: SN LTAM-21-18 Chapter 4 Section 4

General comment: Many candidates omitted this question entirely or only answered one or two parts.

a)

$$m_x = \frac{\int_0^1 {}_rp_x\,\mu_{x+r}dr}{\int_0^1 {}_rp_x\,dr} = \frac{\int_0^1 e^{-\int_0^r \mu_{x+s}ds}\,\mu_{x+r}dr}{\int_0^1 {}_rp_x\,dr} = \frac{q_x}{\int_0^1 e^{-\int_0^r \mu_{x+s}ds}\,dr}$$

Comments:

1. *Performance on this part was mixed. Candidates either received full credit or they received little or no credit for omitting this part or answering it incorrectly.*
2. *The most common incorrect answer was to provide a definition specific to an assumption about mortality between integer ages, such as a constant force of mortality or UDD.*

b)

$$\log m(80,2018) = -2.4 + (0.05)(-4 - 0.2 + 1.5\,Z_{2018}) = -2.61 + 0.075\,Z_{2018}$$

(i) $E[m(80,2018)] = \exp\left(-2.61 + (0.5)(0.075^2)\right)$
$= \exp(-2.6071875) = 0.073741651$

(ii) $SD[m(80,2018)] = \sqrt{0.073741651^2(\exp(0.075^2) - 1)} = 0.005538$

Alternatively

$E[m^2(80,2018)] = \exp(-2.61 \times 2 + (0.5)(0.075^2) \times 2^2) = 0.005468505$
$SD[m(80,2018)] = \sqrt{0.005468505 - 0.073741651^2} = 0.005538$

(iii) $Q_{10\%}\big(m(80,2018)\big) = \exp\big(Q_{10\%}(\log m(80,2018))\big)$
$= \exp\big(-2.61 + (0.075)(-1.282)\big) = 0.066793$

Comments:

1. *Many candidates did well on parts (i) and (ii).*
2. *Part (iii) proved to be challenging for most candidates.*
3. *A common error in part (iii) was to use a normal distribution for m_x.*
4. *Another common error was to calculate the 90% quantile instead of the 10% quantile.*

c)

(i) Under UDD, ${}_rq_x = r \cdot q_x$, $0 \le r \le 1$.

$$So, \qquad m_x = \frac{q_x}{\int_0^1 {}_rp_x\, dr} = \frac{q_x}{\int_0^1 (1 - r \cdot q_x)\, dr} = \frac{q_x}{1 - 0.5\, q_x}$$

Then,

$$\frac{1 - 0.5\, m_x}{1 + 0.5\, m_x} = \frac{1 - \frac{0.5\, q_x}{1 - 0.5\, q_x}}{1 + \frac{0.5\, q_x}{1 - 0.5\, q_x}} = \frac{1 - q_x}{1} = p_x$$

(ii) Since p_x is a decreasing function of m_x, the median of p_x corresponds to the median of m_x.

$$Q_{50\%}\big(p(80, 2018)\big) = \frac{1 - 0.5\, Q_{50\%}\big(m(80, 2018)\big)}{1 + 0.5\, Q_{50\%}\big(m(80, 2018)\big)}$$

$$= \frac{1 - 0.5 \exp(-2.61)}{1 + 0.5 \exp(-2.61)} = \frac{1 - 0.5 \times 0.073535}{1 + 0.5 \times 0.073535} = 0.929073$$

Comments:

1. *Performance on this part was mixed.*
2. *Few candidates provided a proof starting with the UDD assumption and with enough details to achieve full credit on part (i).*
3. *Most candidates found part (ii) challenging. A number of candidates who attempted it confused the median and the mean of p_x.*

d)

1. A cohort effect is observed in population data but not present in the Lee Carter model.
2. The Lee Carter model assumes improvements at different ages are perfectly correlated. This is not the case in population data.

Comments:

1. *Many candidates omitted this part.*
2. *For those who attempted it, performance was mixed. Some achieving full credit, others little or no credit.*
3. *Little or no credit was given to incorrect or incomplete statements about features of population mortality not captured in the Lee-Carter model.*
4. *Some candidates discussed features of other mortality models, such as the Cairns-Blake-Dowd model, instead of the Lee-Carter one which resulted in no credit for this part.*

Question 4 Model Solution

Learning Outcomes: 1(a), 4(b), 5(a)

Chapter References: AMLCR Chapters 1, 5, 6 and 7

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a) To avoid the problem that older lives may be less able to pay the premiums due to less disposable income. To avoid lapses due to health or financial conditions.

Comments:

1. *Few candidates provided a coherent rationale for a maximum age for paying premiums.*
2. *Candidates who simply stated that it would make the product more attractive to policyholders without any explanation received no credit.*
3. *A number of candidates provided an incorrect explanation. For example, arguing that a maximum age for paying premiums would make the product cheaper.*

b)

(i) $G\ \ddot{a}_{60:\overline{20}|} = 100{,}000\ A_{60} + 0.3G + (0.1G)\ddot{a}_{60:\overline{20}|} + 450 + 50\ \ddot{a}_{60}$

$$G = \frac{100{,}000\ A_{60} + 450 + 50\ \ddot{a}_{60}}{0.9\ \ddot{a}_{60:\overline{20}|} - 0.3} = \frac{29{,}028 + 450 + 50(14.9041)}{(0.9)(12.3816) - 0.3} = 2787.234$$

(ii) ${}_2V = 100{,}000\ A_{62} + 50\ \ddot{a}_{62} - (0.9)G\ddot{a}_{62:\overline{18}|}$

$= 31{,}495 + 50(14.3861) - (0.9)(2787.234)(11.58493) = 3153.39$

where

$$\ddot{a}_{62:\overline{18}|} = \frac{\ddot{a}_{60:\overline{20}|} - 1 - v\ p_{60}}{v^2\ {}_2p_{60}} = \frac{12.3816 - 1 - (1 - 0.003398)/1.05}{(1 - 0.003398)(1 - 0.003792)/1.05^2} = 11.58493$$

Alternatively

$${}_1V = \frac{[(0 + (1 - 0.4)G - 500)(1.05) - 100{,}000\ q_{60}]}{p_{60}} = \frac{1230.9574 - 339.8}{0.996602} = 894.1959$$

$${}_2V = \frac{[(894.1959 + (1 - 0.1)G - 50)(1.05) - 100{,}000\ q_{61}]}{p_{61}} = \frac{3141.142}{0.996208} = 3153.10$$

Comments:

1. *Candidates did very well on this part.*
2. *For those who did not receive full credit, a common error was to assume that per policy expenses stopped after the premium paying period.*

c)

(i) Let P^{Mod} be the renewal modified net premium.

$$P^{Mod}(\ \ddot{a}_{60:\overline{20}|} - 0.5) = 100{,}000\ A_{60}$$

$$P^{Mod} = \frac{29{,}028}{12.3816 - 0.5} = 2443.105$$

The first year modified premium is $(0.5)P^{Mod} = 1221.553$

(ii) $${}_2V^{Mod} = 100{,}000\ A_{62} - P^{Mod}\ddot{a}_{62:\overline{18}|}$$
$$= 31{,}495 - (2443.105)(11.58493) = 3191.80$$

Alternatively

$${}_1V^{Mod} = \frac{[(0 + (0.5)P^{Mod})(1.05) - 100{,}000\ q_{60}]}{p_{60}} = 946.045$$

$${}_2V^{Mod} = \frac{[(946.045 + P^{Mod})(1.05) - 100{,}000\ q_{61}]}{p_{61}} = 3191.51$$

Comments:

1. *Most candidates did well on this part.*
2. *For those who did not receive full credit, two common errors were to include expenses in parts (i) and (ii), and to calculate the FPT reserve instead of using the modified reserve basis given in this question.*

d)

(i) Under FPT, the first year modified premium is
100,000 $v\ q_{60}$ = 339.8/1.05 = 323.62

(ii) By definition, the EPV of premiums is the same under both methods.

EPV of FPT premiums = EPV Mod premiums.

Since the first premium under FPT is lower, renewal premiums under FPT must be higher.

$$P^{FPT} > P^{Mod}$$

So, ${}_2V^{FPT} = 100{,}000\ A_{62} - P^{FPT}\ddot{a}_{62:\overline{18}|} <\ 100{,}000\ A_{62} - P^{Mod}\ddot{a}_{62:\overline{18}|} = {}_2V^{Mod}$.

The FPT reserve at the end of year 2, ${}_2V^{FPT}$, will be lower than the modified reserve, ${}_2V^{Mod}$.

Comments:

1. *Performance on this part was mixed.*
2. *Most candidates did well on part (i).*
3. *For part (ii), only a few candidates provided a coherent explanation for why the FPT reserve would be lower than the modified reserve at time 2.*

Question 5 Model Solution

Learning Outcomes: 1(c), 3(a), 4(b)

Chapter References: AMLCR Chapters 4, 5 and 6

General comment:
Overall performance on this question was poor. Most candidates omitted this question entirely or only answered part (a).

a)

(i) $X = 250{,}000/a_{\overline{20}|} = \frac{250{,}000}{(1-1.05^{-20})/.05} = \frac{250{,}000}{12.46221} = 20{,}060.65$

(ii) Before the payment at time 5, there are 16 payments remaining with an EPV of

$$OLB = X\ \ddot{a}_{\overline{16}|}$$
$$= 20{,}060.65\ \frac{(1-1.05^{-16})}{(0.05/1.05)} = 20{,}060.65\ (11.37966) = 228{,}283.38$$

(iii) OLB at *K+1* is

$$OLB_{K+1} = \begin{cases} X\ \ddot{a}_{\overline{20-K}|} = \dfrac{X}{d}\ (1-v^{20-K}) & K = 0, 1, 2, \ldots, 19 \\ 0 & K = 20, 21, \ldots \end{cases}$$

Alternatively

$$OLB_{K+1} = \begin{cases} 250{,}000(1+i)^{K+1} - X\ \ddot{s}_{\overline{K}|} & K = 0, 1, 2, \ldots, 19 \\ 0 & K = 20, 21, \ldots \end{cases}$$

Comments:

1. *Most candidates did very well on part (i).*
2. *Most candidates did well on part (ii).*
3. *Only well-prepared candidates received some credit for part (iii).*

b)

(i) $$Z = \begin{cases} \frac{X}{d}\ (1-v^{20-K})v^{K+1} & K = 0, 1, \ldots, 19 \\ 0 & K = 20, 21, \ldots \end{cases}$$

Alternatively

$$Z = \begin{cases} (250{,}000(1+i)^{K+1} - X\ddot{s}_{\overline{K}|})v^{K+1} & K = 0, 1, \ldots, 19 \\ 0 & K = 20, 21, \ldots \end{cases}$$

(ii) Noting that $\frac{X}{d}\,(1-v^{20-K})v^{K+1}=\frac{X}{d}\,(v^{K+1}-v^{21})$,

alternatively

$(250{,}000(1+i)^{K+1}-X\ddot{s}_{\overline{K}|})v^{K+1}=250{,}000-Xa_{\overline{K}|}=X(a_{\overline{20}|}-a_{\overline{K}|})=\frac{X}{d}\,(v^{K+1}-v^{21})$,

we have

$$Z=\begin{cases}\frac{X}{d}\,(v^{K+1}-v^{21}) & K=0,1,\dots,19\\ 0 & K=20,21,\dots\end{cases}$$

and

$$E[Z]=\frac{X}{d}\,(E[v^{K+1}\cdot I(K<20)]-E[v^{21}\cdot I(K<20)])=\frac{X}{d}\left(A_{\,\frac{1}{35:\overline{20}|}}-v^{21}\;{}_{20}q_{35}\right)$$

Alternatively

$Z=\frac{X}{d}[Z_1-Z_2]$

where $Z_1=\begin{cases}v^{K+1} & K=0,1,\dots,19\\ 0 & K=20,21,\dots\end{cases}$ and $Z_2=\begin{cases}v^{21} & K=0,1,\dots,19\\ 0 & K=20,21,\dots\end{cases}$.

So, $E[Z]=\frac{X}{d}[E(Z_1)-E(Z_2)]=\frac{X}{d}\left(A_{\,\frac{1}{35:\overline{20}|}}-v^{21}\;{}_{20}q_{35}\right)$

Comments:

1. *Few candidates attempted this part.*
2. *A number of those who attempted it found the proof in part (ii) challenging.*

c)

$E[Z]=P\;\ddot{a}_{35:\overline{20}|}$.

$$P=\frac{X}{d\;\ddot{a}_{35:\overline{20}|}}\left(A_{\,\frac{1}{35:\overline{20}|}}-v^{21}\;{}_{20}q_{35}\right)=\frac{X}{d\;\ddot{a}_{35:\overline{20}|}}\left(A_{35:\overline{20}|}-{}_{20}E_{35}-v^{21}\;{}_{20}q_{35}\right)$$

$$P=\frac{20{,}060.65}{(0.05/1.05)(13.024)}\left(0.37981-0.37041-1.05^{-21}\left(1-\frac{97{,}846.2}{99{,}556.7}\right)\right)=104.573$$

Comments:

1. *Few candidates attempted this part.*
2. *Most of those who attempted it did well by simply using the information given in part b).*

d)

The level annual premiums are very likely to eventually exceed the decreasing expected death benefit of such mortgage insurances.

For example, $X\ v\ q_{54} = 34$ which is less than $P=104$. Policyholders may choose to lapse and not buy insurance for the relatively small death benefit, OLB $=X=20{,}060$; or buy a new policy for less than 104. Healthier policyholders would tend to lapse earlier in the contract. The risk of adverse selection for ABC Life would be high.

Comments:

1. *Many candidates did not attempt this part.*
2. *Only well-prepared candidates were able to provide a satisfactory rationale as to why the lapse rate would tend to be high at later durations.*

Question 6 Model Solution

Learning Outcomes: 5(a), 5(b), 5(c), 5(d), 5(e), 5(f)

Chapter References: AMLCR Chapter 10

General Comment: Many candidates omitted this question.

a)

$$S_{64} = 40{,}000\left(1+\frac{0.036}{12}\right)^{(64-30)(12)} s^{(12)}_{\overline{1}|i=1.003^{12}-1} = 138{,}044.47$$

Alternatively

$$S_{64} = 40{,}000\left(1+\frac{0.036}{12}\right)^{(64-30)(12)}\left(\frac{1}{12}\right)s_{\overline{12}|.036/12}$$

$$= 40{,}000\,(3.394538)\frac{(1.003^{12}-1)}{0.003}\left(\frac{1}{12}\right) = 135{,}781.52\left(\frac{12.199993}{12}\right) = 138{,}044.47$$

The monthly pension, B, is:
$B = (0.02)(35)S_{64}\,/12 = 8052.594$

Comments:

1. *Many candidates did well on this part, especially those who drew a timeline to show the increasing monthly earnings.*
2. *For those who did not receive full credit, the most common error was to calculate the final year salary, S_{64}, incorrectly, for which a small deduction was applied.*

b)

$$EPV = (0.02)(35)S_{64}\,\ddot{a}^{(12)}_{65:\overline{10}|} = (0.02)(35)(138{,}044.47)(13.38208) = 1{,}293{,}125.93$$

where

$$\ddot{a}^{(12)}_{65:\overline{10}|} = \ddot{a}^{(12)}_{\overline{10}|} + {}_{10}E_{65}\,\ddot{a}^{(12)}_{75} = \ddot{a}^{(12)}_{\overline{10}|} + {}_{10}E_{65}\,(\ddot{a}_{75} - 11/24)$$

$$= \left(\frac{1-1.05^{-10}}{12(1-1.05^{-1/12})}\right) + (0.55305)(10.3178 - 11/24) = 13.38208$$

Comments:

1. *Performance on this part was mixed.*
2. *Common errors included calculating the EPV at the wrong date, using annual payments, incorrectly calculating the EPV of the 10-year guaranteed annuity and using UDD instead of the Woolhouse formula.*

c)

The accumulated value of contributions, AV , is

$$AV = (0.06)\left(\frac{40{,}000}{12}\right) \text{ x } [1.008^{419} + (1.003)(1.008^{418}) + \cdots + (1.003^{419})(1.008^{0})]$$

$$= 200\left[(1.008^{419})\left(\frac{1-\left(\frac{1.003}{1.008}\right)^{420}}{1-\left(\frac{1.003}{1.008}\right)}\right)\right] = (200)(28.181425)(176.627679) = 995{,}523.94$$

Alternatively

$$AV = (0.06)\left(\frac{40{,}000}{12}\right)\left(\frac{1.008^{420}-1.003^{420}}{0.008-0.003}\right) = 995{,}523.94$$

Comments:

1. *Most candidates did poorly on this part.*
2. *The candidates who used a timeline to show the contribution amounts found it helpful in answering this part.*

d) Since the lump sum of 1,293,126 is larger than the AV of contributions at 9.6% (995,524), the IRR that Riley has earned by taking the lump sum is more than 9.6% convertible monthly.

Comments:

1. *Most candidates did poorly on this part.*
2. *Many candidates tried to answer the question in general terms instead of comparing Riley's options using the results from parts b) and c).*
3. *Another common error was answering the question by comparing the assumed interest rate on contributions to the rate of salary increase.*

e) The revised monthly benefit, B^*, is such that

$$EPV = 1{,}293{,}125.93 = (12\ B^*)\ \ddot{a}_{65}^{(12)} = (12\ B^*)(\ddot{a}_{65} - 11/24)$$

$$B^* = \frac{1{,}293{,}125.93}{12\left(13.5498 - \frac{11}{24}\right)} = 8{,}231.35$$

Comments:

1. *The candidates who attempted this part did well.*
2. *Some candidates used the AV of contributions instead of the EPV of benefits, for which no credit was given.*

f)

(i) Adverse selection in insurance refers to a situation where a policyholder makes a decision based on asymmetric information about the risk, i.e. information known to the policyholder but unknown to the insurer. Policyholders who represent higher risk tend to buy more insurance.

Here, adverse selection would occur whenever an employee would choose the most advantageous option at the time of retirement based on his/her health and other risk factors unknown to the pension plan sponsor.

(ii) Adverse selection is likely to <u>increase the cost</u> to the plan. The healthiest employees mostly choosing the life annuity and those in poor health choosing the lump sum.

Comments:

1. *Most candidates either did poorly or received full credit on part (i).*
2. *Only well-prepared candidates provided a coherent rationale as to why the costs to the plan would increase due to adverse selection.*
3. *The candidates who discussed the pricing issues rather than the impact on the plan's costs received no credit for part (ii).*