FETE Complete Illustrative Solutions
Fall 2011

1. Learning Objectives:

2. Corporate Financial Applications

Learning Outcomes:

(2c) Describe the process, methods and uses of insurance securitizations and recommend a structure that is appropriate for a given set of circumstances.

Sources:

FET-148-08

Commentary on Question:

The question tested candidates understanding of general economic reasons giving rise to securitizations, the direct costs associated with implementing securitizations, and the uses of securitizations in the life insurance industry. In particular the question required the candidates to recognize which insurance risks in the Wonka Life case study could be mitigated by securitizations. Candidates with knowledge of the case study and of the study note would be sufficiently prepared to do well on this question.

Many candidates did well on one part of the question but few candidates where able to provide details on all aspects of the question (economic rationale, potential costs, and securitization uses for Wonka Life).

Solution:

(a) Describe the economic rationale and potential costs involved in securitizing financial instruments.

Commentary on Question:

Candidates did relatively well on this section which was generally recall. Regarding the economic rationale, many candidates listed the basics of managing interest rate risk, reducing deadweight, agency and bankruptcy costs, and appealing to investors with varying risk appetites. Many candidates also expressed an understanding of the reduction of informational asymmetries and the improved market efficiency. Less frequently answered was that securitizations reduce transaction costs, improve portfolio efficiency, and provide for the acquisition of specialized investment information.

Many candidates were able to list some or most of the potential costs associated with a securitization. A common mistake was misunderstanding that the question asked for direct costs (or, out of pocket costs).

1. Continued

Economic rationale:
Existence of bankruptcy costs
Manage interest rate risk
Reduction of informational asymmetries
Existence of agency costs (managers pursue their interests over those of the owners)
Reduce deadweight costs from regulation
Appeal to investors with different appetites for risk
Reduce transaction costs and improve portfolio efficiency
Acquisition of specialized investment information

Potential costs:
Legal and admin cost to establish and capitalize special purpose vehicle (SPV)
Costs to model SPV cash flows to provide information to investors
Rating agencies must evaluate and rate
Design, underwrite, and market securities
Compensate swap counterparty
Cost of credit enhancements
Cost of ongoing servicing of assets in trust

(b) Evaluate which risks in Wonka Life's products could be successfully mitigated via securitization.

Commentary on Question:
Candidates did relatively poorly. The best answers were by candidates who organized their answers either by products (and then listed the products' risks) or by risks (and then listed the associated products). Points were earned for listing common risks found in insurance products (and Wonka Life's insurance portfolio is varied), but detailed knowledge of the case study was necessary to do well. This question did not ask for details of the structure of a securitization, which some candidates did provide but did not receive any points for.

Some candidates answered by showing how Wonka Life could hedge its risks using tactics like interest-rate swaps or credit default swaps. While these can be effective in mitigating insurance risks, these are not securitizations per se and did not receive points (if the answer clearly showed that these swaps were within a securitization then some points were given).

Also, the question was focused explicitly on the products' risk, i.e., the liability side of the balance sheet, so references to asset risk, ALM or duration mismatch received only minor credit.

1. Continued

Embedded options
Impose costs on insurers even when OTM
Expose insurers to significant risk

- Minimum interest rate guarantees on whole life, UL and accum annuities
- Maximum loan interest rates
- BV withdrawals on accumulation annuities
- Equity linked GICs – principal and return
- COLA on payout annuities
- GIC puts
- MV guarantees on institutional GICs
- Premium guarantees on Group LTD

Amortization of acquisition costs
Some regulations do not allow DACing of acquisition costs
So writing NB requires a need for cash
Exposed to persistency risk

- High UL acquisition costs

Mortality risk
Exposure to epidemics
Increased exposure to terrorism
- Trad life and non-trad
- GMDB on VAs

Longevity risk
Long term improvements in mortality
Shift from public to private retirement plans
- Payout annuities
- Structured settlements

Foreign exchange risk
- Euro/USD risk on pension GICs

2. Learning Objectives:

2. Corporate Financial Applications

Learning Outcomes:

(2d) Evaluate alternative options for utilizing capital and recommend the most appropriate use in a given situation.

Sources:

Financial Theory and Corporate Policy, 4th ed., Chapter 16

FET-163-08: Financial Markets and Corporate Strategy, 2nd ed., Chapter 19

Commentary on Question:

The purpose of this question is to test the candidate's knowledge of signaling theory as it applies to raising capital and returning value to shareholders. The question is placed in the context of the Case Study. Candidates are further required to perform a simple calculation that illustrates the relevance of shareholder value on decision making.

Solution:

(a) List and explain characteristics of a market environment under which dividend policy is irrelevant.

Commentary on Question:

Candidates did poorly on this section. Many candidates provided a list but did not explain why the characteristic leads to a market environment where dividends are irrelevant. A common mistake was that candidates described an environment where the debt / equity financing decision (not the dividend policy) is irrelevant.

- No taxes or tax on dividends and capital gains are identical): This causes shareholder to be indifferent between receiving value via dividends or capital gains.
- No transaction costs: Method used for delivering value to shareholders (dividends, share repurchase) does not create additional expenses and erode value.
- No information asymmetry: Insiders and outsiders have the same information, so dividends to not have any signaling value.
- Dividends do not affect the investment decision: If dividends and investment are greater than operational cashflows, firm can costlessly seek external financing.

(b) Describe the information conveyed to the market by a company establishing a dividend policy.

2. Continued

Commentary on Question:
Candidates did moderately well. Few candidates provided complete responses but most identified the key items that could be signaled to the market by a dividend.

- Signals that firm expects future cashflows to be sufficiently large to meet future dividend payments (since managers are reluctant to introduce a dividend that may need to be reduced in future), investment needs and debt service without increasing probability of bankruptcy.
- Conveys that company is stronger than other firms, since a dividend is a costly signal that weaker firms will find difficult to mimic.
- Dividends have a tax disadvantage so it must convey information to the market that cannot be conveyed by other means.
- May also convey that firm has exhausted positive net present value investment opportunities.

(c)

(i) Identify two other common favorable signals available to Wonka Life.

Commentary on Question:
Candidates only needed to provide the 2 items below and most got them correct. Credit was not given to "stock split" as this is not a clear and common signal as compared with share repurchase and debt issue.

- Issue new debt
- Repurchase shares

(ii) Compare these types of signals with the dividend signal.

Commentary on Question:
Candidates answered this section relatively well, however few candidates touched on all points.

- Dividends
  - Signals ability to generate stable and sufficient future earnings has improved.
  - Signal does not affect value of firm, rather how value is delivered.
  - Not flexible (because companies reluctant to cut dividends). Good for companies with stable earnings.
  - May signal fewer positive NPV projects.

2. Continued

- Repurchase shares
  - Signals management believe that shares are undervalued.
  - Signal does not affect value of firm, rather how value is delivered.
  - Flexible and good for companies with volatile earnings.
  - Taxed as capital gain, so preferred over dividends by shareholders in high tax brackets.
  - May signal fewer positive NPV projects.

- Issue debt
  - Signals management has confidence in ability to generate stable and sufficient future earnings and management is not concerned about financial distress (bankruptcy) costs.
  - Signal creates value through the creation of a tax shield (compared with other signals which do not create value).
  - Prevents positive NPV projects from being passed up if equity is undervalued.

(iii) Evaluate Wonka Life's ability to use these three signals.

Commentary on Question:
Candidates did poorly on this section. Most candidates identified some relevant facts from the case study. However candidates struggled to identify the financial goals and condition of the company relative to what would be required to execute an effective signal as opposed to just executing the transaction. Very few candidates questioned the credibility of management, which is critical in signaling theory.

Facts from case study:
- Wonka has no long term debt.
- Has desired debt ratio of 30%.
- Has issued dividend in past.
- Has aggressive growth targets for future.
- Past performance has been strong but company has some liquidity and profitability concerns going forward.
- Board of directors are primarily insiders and are all close to end of their terms on board.

Dividend Signal
- Not a good idea given future profit concerns. May not be able to sustain dividend.
- Should not return capital to shareholders as it is needed to fund aggressive growth targets.

2. Continued

- Signal may not be credible given short term outlook of board.

Repurchase Shares
- Should not return capital to shareholders as it is needed to fund aggressive growth targets.
- Signal may not be credible given short term outlook of board.

Issue Debt
- Company has room to issue debt but for it to be an effective signal, needs to be difficult to mimic (i.e. needs to be a substantial debt issue).
- Substantial debt issue could introduce financial distress that would outweigh the tax shield.

(d) Recommend whether Wonka Life should acquire LifeSaver Insurance Company based on shareholder value.

Commentary on Question:
Candidates did well on this section. Candidates that did not provide any quantitative support for a recommendation and gave only qualitative commentary (synergies, execution costs, etc.) received little credit. Many candidates also confused shareholder value (intrinsic value) with share price (market value).

The cost-benefit analysis could be illustrated several ways. Many candidates found alternative ways (from what is shown below) to demonstrate that the intrinsic cost of issuing undervalued shares outweighed the gains from the acquisition. These alternative methods received full credit.

When shares are undervalued relative to their intrinsic value, there is an additional cost to funding projects by issuing new shares. The company should closely evaluate this cost relative to the value of the potential project.

Shareholder value = intrinsic value (long term)
Market value = share price (short term)

Pre-acquisition
- Wonka intrinsic value per share = $1,000M / 10M shares = $100 per share

To acquire
- Cost of project = $300M
- Fund by issuing shares at $60 per share
- Number of shares issued = $300M / $60 = 5M shares
- New shares outstanding = 15M

2. Continued

Post-acquisition

- Wonka intrinsic value = $1,000M + $400M = $1,400M
- Wonka intrinsic value per share = $1,400M / 15M shares = $93 per share

Conclusion: Despite positive value of project, the intrinsic value per share is reduced as a result of the acquisition. Therefore, Wonka should not pursue the acquisition.

3. Learning Objectives:

1. Modern Corporate Financial Theory

Learning Outcomes:

(1b) Calculate the cost of capital for a venture or a firm using the most appropriate method for given circumstances and justify the choice of method.

(1c) Evaluate various profitability measures including IRR, NPV and ROE, etc.

Sources:

FET-175-10: Models, by Derman, E., Financial Analysts Journal, 2009

Financial Theory and Corporate Policy, 4th ed., Chapter 2

Commentary on Question:

This question asked the candidates to calculate the NPV of a project by first determining the appropriate Weighted Average Cost of Capital (WACC). The candidates needed to understand how to assemble the free cashflows and understand that the WACC will change depending on the funding. Ultimately the project generated a negative Net Present Value (NPV), but since the negative NPV was greater than the fixed costs, the company still should have undertaken the project.

Solution:

(a) Describe the aim of financial models.

Commentary on Question:

Candidates did moderately well on this recall question.

To rank securities or options by value
To interpolate or extrapolate from liquid prices to illiquid ones
To transform intuitive linear quantities into nonlinear dollar values
To compare different prices in the present (Financial models do not predict the future)

(b) Calculate the NPV of the project for each of the following:

Commentary on Question:

For both calculations below, candidates did moderately well. In order to do the calculation below, it was important to understand the inputs. Those that did well generally understood the inputs to Free Cashflow and how WACC changes depending on the financing method. Many people adjusted Free Cashflow by the interest on the bond when it wasn't necessary.

3. Continued

(i) 1500 investment

B = value of bond; S= equity invested

Weighted Avg Cost of Capital (WACC) = bond yield *(1 – tax rate) *(B/(B+S))+Equity return required *(S/(B+S))

B = 500, S = 1000

WACC = .03*(1-.3)*(500/(500+1000)+ .2*(1000/(500+1000) = 14%

Free Cashflow for capital budgeting = EBIT*(1-tax rate) + change in depreciation – change in investment

EBIT = Change in Revenue – change in variable cost – Change in fixed cost – change in depreciation

EBIT(yr 1) = (1575-300-400-900) = -25
EBIT(yr 2) = (1575-300-400-600) = 275

Free Cashflow (yr 1) = -25(1-.3)+900 -0 = 882.5
Free Cashflow (yr 2) = 275(1-.3)+600 -0 = 792.5

NPV = -1500 + 882.5/ (1 +14%) + 792.5/ (1+ 14%)^2 = -116.07

(ii) 3000 investment

B = 500, S = 2500
WACC = .03*(1-.3)*(500/(500+2500)+ .2*(2500/(500+2500) = 17%
Free Cashflow for capital budgeting = EBIT*(1-tax rate) + change in depreciation – change in investment

EBIT = Change in Revenue – change in variable cost – Change in fixed cost – change in depreciation

EBIT(Yr1) = (3000-600-400-1800) = 200
EBIT(Yr2) = (3000-600-400-1200) = 800

Free Cashflow(Yr1) = 200(1-.3)+1800 -0 = 1940
Free Cashflow(Yr2) = 800(1-.3)+1200 -0 = 1760

NPV = -3000 + 1940 / (1+17%) +1736 /(1+17%)^2= -56.20

3. Continued

(c) Recommend and justify a course of action.

Commentary on Question:
Candidates did relatively well. Most candidates made a coherent recommendation, based on an assessment of the relative NPVs. However, very few incorporated the fixed costs into the analysis by not accepting any of the projects which was one of the project options.

In addition to the two options given there is the option to not invest.

NPV (no investment) = Fixed cost * (1- tax rate) = (-400-400) *(1-.3) = -560 because the risk-free rate is 0%

From (b)(i) above, Invest 1500, NPV = -116.07

From (b)(ii) above Invest 3000, NPV = -56.20

All NPVs are negative so none will add shareholder value

However, should invest 3000 as it has the greatest NPV and is greater than the option of doing nothing.

4. Learning Objectives:

2. Corporate Financial Applications

Learning Outcomes:

(2g) Recommend an optimal capital structure and how to implement it for a given business or strategy and be able to justify the recommendation.

Sources:

Doherty, Integrated Risk Management, Chapter 13

Copeland, Weston, Shastri, Financial Theory and Corporate Policy, 4th Ed., Chapter 15

Commentary on Question:

This question tested of how to evaluate two potential Projects. Candidates were asked to identify which Project was more beneficial to the Shareholders and recommend that project. They also had to analyze the impact on bondholders and identify ways to incent the bondholders to accept the Project.

Solution:

(a) Calculate for each of the current situation, Project A, and Project B:

Commentary on Question:

Candidates generally did well this section. Many did the calculations for the two Projects, but not for the current situation. For part (a)(i), a common mistake made was in calculating the value of the firm, some candidates incorrectly subtracted the value of the bonds. For part (a)(iii), many correctly calculated the value to the bondholders, but did not comment on the impact or differentiate the impact between senior and junior bondholders.

(i) The value of the firm

Current Value of firm = (PV current earning scenario 1 * probability current scenario 1) + (PV current earning scenario 2 * probability current scenario 2)

Current Value of firm = (100 * 50%) +(200 * 50%) = 150

Value of firm with project A = Current value + PV earning project A
Value of firm with project A = 150 + 210 = 360

Value of firm with project B
= ((PV current earning scenario 1 + PV earning project B scenario 1)
* (probability current scenario 1 * probability project B scenario 1))
+ ((PV current earning scenario 1 + PV earning project B scenario 2)
* (probability current scenario 1 * probability project B scenario 2))
+ ((PV current earning scenario 2 + PV earning project B scenario 1)

4. Continued

* (probability current scenario 2 * probability project B scenario 1))
+ ((PV current earning scenario 2 + PV earning project B scenario 2)
* (probability current scenario 2 * probability project B scenario 2))

Value of firm with project B = ((100 + 180) * 0.25) + ((100 + 300) * 0.25) + ((200 + 180) * 0.25)) + ((200 + 300) * 0.25)
Value of firm with project B= 390

(ii) The value of equity

Current equity value = current value of firm – current debt (senior).
Current equity value = 150 – 100 = 50

Equity value with project A = Value of firm with project A – Total debt (senior + junior).
Equity value with project A = 360 – 100 – 200 = 60

Equity value with project B = Value of firm with project B – anticipated value of debt

Here are 2 choices to calculate the anticipated value of debt:

1. For all combinations there is only one time that total debt > value of firm.
   This is the case where PV current earnings scenario 1 is 100 and PV project B scenario 1 is 180 for total of 280,
   which is less than total debt of 300 for expected loss of 20.
   This has probability of 25% so expected loss is 20 * 25% = 5.
   Then the expected value of debt is 100 + 200 – 5 = 295.

Or

2. Other way is to calculate as for the value of the firm the expected value for each category of debt. We must recognize that in one scenario the junior bondholders do not recover the full value of the debt.

   Equity value with project B = 390 – 295= 95.

(iii) The impact on bondholders

Project A: no impact on either senior or junior bondholders since value of firm exceeds value of debt.
Under Project A all bondholders will expect to receive full value.

4. Continued

Project B: there is an impact on new junior debt since in one scenario the value of firm is not > value of debt.
Project B, senior debt can recover since value of firm > to senior debt.
Project B, expected loss on junior debt is: $5 as calculated in previous question see ((a)(ii)3).

(b) Recommend the project which maximizes shareholder value.

Commentary on Question:
The students generally did poorly on this section. Students usually made the correct recommendation, but provided too little in terms of an explanation.

Shareholders prefer project B because it offers the better equity value.
Under project B junior debt will not anticipate to receive the full values, only 195 on 200.
Under project B gain to shareholder from current 50 to 95 with project B permit to absorb the loss on junior debt.
So the loss of junior debt of 5 can be absorbed by gain to shareholder and stay in better position than on project A.
If bondholder anticipate project B they can be afraid to enter in the financing thus need incentive.
Leverage ratio with project B is less than with project A giving another reason to accept project B.

(c) Propose incentives to the bondholders so they would accept the project recommended in (b).

Commentary on Question:
Part (c) was generally done well. However many students only listed one incentive. Other candidates provided multiple incentives but did not provide sufficient explanation for the incentive.

Two options to attract bondholders (marks were also given for some other valid incentives):

1. Issue a convertible bond which gives the option to bondholders to exchange the bond for shares.
    - Project less attractive to shareholders.

2. Inclusion of a warrant: gives bondholders the option to purchase firm's shares at a specified price so management can set the value of the new shares.
    - Here the firm pays a lower rate of interest.

5. Learning Objectives:

2. Corporate Financial Applications

Learning Outcomes:

(2f) Describe the process, methods and effects of a potential acquisition or reinsurance of a business including its effect on capital structure, return on equity, price/earnings multiples, and share price.

Sources:

Financial Theory and Corporate Policy, 4th ed., chapter 18

Commentary on Question:

This question tested the candidates understanding of proceeding with an acquisition through an analysis of the impact on earnings per share (EPS). This question required a combination of recall/list, calculations and applied skills to arrive at the solution.

Solution:

(a) Describe earnings dilution from a board of directors' perspective and explain how it is calculated.

Commentary on Question:

Candidates did relatively well on this section. Answers receiving maximum credit included the specific actions/decisions by the board of directors. However few candidates specifically identified the Board's ability to reject unfavorable transactions.

- Earnings dilution will negatively affect earnings per share by spreading earnings over a larger number of shares.
- A board of directors will want to avoid earnings dilution.
- The board of directors may reject a deal if the earnings dilution is judged to be too large.
- Earnings dilution is calculated as the difference in new EPS and existing EPS.
- New EPS = $\frac{\text{net income of the combined company}}{\text{New total shares}}$
- NI of the combined company = NI of acquirer + NI of target
  + after-tax revenue and cost synergies
  – after-tax deal and integration costs
  – after-tax goodwill amortization
  – additional after-tax interest expense

5. Continued

(b)

Commentary on Question:
Candidates did relatively well on this section. Candidates scored well on the current EPS but struggled with calculating a new EPS after a takeover. In addition candidates seem to understand the basic idea of earnings dilution but again struggled at applying the concept.
Candidates receiving full credit were able to perform the calculation error-free. Often overlooked were the nuances of transaction costs in the determination of share issuance, goodwill, and interest calculations.

(i) Calculate the Current EPS for each of K and L.

EPS = NI / outstanding shares

EPS of K = 300 / 65 = 4.62

EPS of L = 50 /40 = 1.25

(ii) Calculate X, the EPS of K for a 100% stock takeover of L.

Calculate stock transaction costs
- Purchase price = 50 of Acquisition Premium
- Purchase price = 750 of Market Value
- Transaction cost rate = 1% of stock issued
- Stock transaction cost = 1% * (50 + 750) = 8

Calculate new number of shares
- Original number of shares = 65
- Additional shares issued = (50 + 750) / 50 = 16
- Total new shares outstanding = 65 + 16 = 81

Calculate the increase in goodwill
- Acquisition Premium = 50
- Goodwill write-off period = 30
- Annual amortization = 50 /30 = 1.67

Calculate combined net income
- Net income of K = 300
- Net income of L = 50
- After Tax Synergies = 25
- Stock Transaction Costs = 8
- Increase in Goodwill Annual Amortization = 1.67
- Combined NI = 300 + 50 + 25 – 8 – 1.67 = 365.33

5. Continued

Calculate X

X = Combined NI / Total new shares outstanding
= 365.33/81
= 4.5102

(iii) Calculate Y, the EPS of L for a 100% cash takeover of K.

Calculate increase in interest

Finance MV = 1200
Acquisition Premium = 50
Interest – After Tax =10%
Increase in interest = 10% * (1200 +50) = 125

Calculate combined net income

Net income of K = 300
Net income of L = 50
After Tax Synergies = 25
Increase in interest = 125
Increase in Goodwill Annual Amortization = 1.67
Combined NI = 300 + 50 + 25 – 125 – 1.67 = 248.33

Calculate Y

Y = Combined NI / Total new shares outstanding
= 248.33/40
= 6.2083

(c) Based on your answer to (b), describe what action the boards of K and L may take in the best interest of their current shareholders.

Commentary on Question:
Candidates did moderately well on this section. Answers receiving maximum credit included the specific actions/decisions by the board of directors on all four options. Credit was given for answers consistent with candidates' part (b) answers regardless of whether or not the part (b) answers were correct.

Board of directors for K will use the following information to make decisions.
Original EPS = 4.62
100% Cash Takeover EPS = 4.51
100% Stock Takeover EPS = 4.5102

The board of directors for K should reject a cash purchase of L due to slight EPS dilution and reject a stock purchase of L due to slight EPS dilution.

5. Continued

Board of directors for L will use the following information to make decisions.
Original EPS = 1.25
100% Cash Takeover EPS = 6.2083
100% Stock Takeover EPS = 5.91

The board of directors for L should approve a cash purchase of K for an EPS increase and also a stock purchase of K for an EPS increase.

The cash takeover should be preferred to the stock takeover due the larger increase in EPS.

6. Learning Objectives:

3. Derivatives and Pricing

Learning Outcomes:

(3e) Derive the Black Scholes Merton pricing formula.

(3f) Demonstrate understanding of option pricing techniques and theory for equity and interest rate derivatives.

(3g) Identify limitations of each option pricing technique.

(3h) Describe and evaluate equity and interest rate models.

Sources:

Hull Chapter 28

FET-167-09: How to use the holes in Black-Scholes

Commentary on Question:

This question tested interest rate derivatives and call option pricing using Black's model. Parts (a), (d) and (e) were recall questions. Parts (b) and (c) were comprehension/calculation questions.

For parts (a), (d) and (e), candidates had to simply list out all the points to get maximum marks. For parts (b) and (c), candidates had to provide the correct formula, as well as calculate the correct final answer to get full marks.

See below for specific comments on each section which indicate areas where candidates did well and where candidates had trouble.

Solution:

(a) Describe three reasons why interest rate derivatives may be more difficult to value than equity or foreign exchange derivatives.

Commentary on Question:

Candidates answered this section well.

1. For the valuation of many products it is necessary to develop a model describing the behavior of the entire zero-coupon yield curve.

2. The volatilities of different points on the yield curve are different.

3. Interest rates are used for discounting the security as well as defining its payoff.

(b) Calculate the eleven month forward price of this bond.

6. Continued

Commentary on Question
Most candidates were able to provide the formula for the bond forward price. However, many candidates incorrectly calculated the present value of coupon payments, I, by failing to include 2 coupon payments. As the method of interest rate compounding was not specified in the question, we gave full marks if the candidate calculated the answer correctly using either continuous compounding or annual compounding. Notice how much of a difference this minor assumption makes.

(Formula 28.3) The bond forward price is $F_B = \dfrac{B_0 - I}{P(0,T)}$

B_0 is the bond price at time 0 = \$975

K is the strike price = 1000

T=11/12

Continuous Compounding:

$$P(0,T) = e^{-0.12x(11/12)} = .8958$$

I is the present value of the coupon payments
Each coupon payment is \$60

$$I = 60e^{-0.25x.08} + 60e^{-0.75x.09} = 114.90$$

F_B is the forward bond price

$$F_B = \frac{B_0 - I}{P(0,T)} = \frac{(975 - 114.90)}{.896} = 960.15$$

Annual Compounding:

$$P(0,T) = \left(\frac{1}{1.12}\right)^{11/12} = 0.9013$$

I is the present value of the coupon payments
Each coupon payment is \$60

$$I = 60\left(\frac{1}{1.08}\right)^{3/12} + 60\left(\frac{1}{1.09}\right)^{9/12} = 115.10$$

F_B is the forward bond price

$$F_B = \frac{B_0 - I}{P(0,T)} = \frac{(975 - 115.10)}{0.9013} = 954.03$$

6. Continued

(c) Calculate the price of the call option using Black's model.

Commentary on Question
Many candidates were able to provide the formula for the call option price using Black's model, as well as the formulas for d_1 and d_2. Credit was given for the correct approach, even though many made arithmetic errors or carried over the wrong answer from (b). The correct solutions (both continuous and annual compounding) are shown below.

(formula 28.1) The price of the call option is

$$c = P(0,T)[F_B N(d_1) - KN(d_2)]$$

$$d_1 = \frac{\ln(F_B / K) + \sigma_B^2 T / 2}{\sigma_B \sqrt{T}} \qquad d_2 = d_1 - \sigma_B \sqrt{T}$$

σ_B=.10

Continuous Compounding:

$$d_1 = \frac{\ln(960.15/1000) + ((0.10^2\, x(11/12))/2}{0.10\sqrt{11/12}} = -0.377$$

$$N(d_1) = 0.3530$$

$$d_2 = -0.377 - .10\sqrt{11/12} = -0.473$$

$$N(d_2) = 0.3181$$

$$c = P(0,T)\left[F_B N(d_1) - KN(d_2)\right] = 0.8958\left[960.15 \times N(d_1) - 1000 \times N(d_2)\right] = 18.64$$

Annual Compounding:

$$d_1 = \frac{\ln(954.03/1000) + ((0.10^2\, x(11/12))/2}{0.10\sqrt{11/12}} = -0.4436$$

$$N(d_1) = 0.3287$$

$$d_2 = -0.4436 - .10\sqrt{11/12} = -0.5394$$

$$N(d_2) = 0.2948$$

$$c = P(0,T)\left[F_B N(d_1) - KN(d_2)\right] = 0.9013\left[954.03 \times N(d_1) - 1000 \times N(d_2)\right] = 16.88$$

6. Continued

(d) Describe two measures of this uncertainty.

Commentary on Question
Most candidates answered this section poorly. Common wrong answers included Vega, volatility smile and volatility surface, which do not measure the uncertainty of volatility. Vega is wrong as it measures the change in option value with respect to the change in volatility of the underlying asset. The volatility smile is a plot of the implied volatility of an option as a function of its strike price. The volatility surface is a graph or table showing the variation of implied volatilities with strike price and time to maturity.

One measure of the uncertainty of volatility is the "volatility of the volatility." A formula that takes account of changes in volatility will include both current and expected future levels of volatility. Another measure is the relation between the future price and its volatility.

(e) Recommend a strategy other than buying just the call option that should perform well.

Commentary on Question
Most candidates answered this section poorly. Common wrong answers included writing a volatility swap, and creating synthetic call options on the underlying bond. A volatility swap is incorrect as the purchaser is generally unsure of which direction volatility will move, while the question stated that the volatility of the bond is expected to rise. Creating a synthetic call option is incorrect as it is too similar to simply purchasing a call option.

Some possible strategies other than buying just the call option are buying straddles, buying strangles, or buying both calls and puts in a ratio that makes the trade delta neutral.

7. Learning Objectives:

2. Corporate Financial Applications

3. Derivatives and Pricing

Learning Outcomes:

(2a) Describe the steps necessary to obtain funds for a given project or firm from any specified source, and be able to recommend a specific approach to raising capital in a given situation.

(2e) Apply real options analysis to recommend and evaluate firm decisions on capital utilization.

(3a) Define the cash flow characteristics of complex derivatives including exotic options, interest rate derivatives, swaps, and other non-traditional derivatives.

(3o) Use numerical methods to effectively model complex assets or liabilities.

Sources:

Hull

- Chapter 13, page 295
- Chapter 17, page 369
- Chapter 19, pages 407-414
- Chapter 24, pages 562-564

Commentary on Question:

This question asks the candidates to demonstrate their understanding of warrants and ability to price them using a numerical approach.

While most candidates knew the definition and formula of theta, a significant portion did not apply the calculation correctly.
A few candidates used Black-Scholes to evaluate delta which was not necessary for this problem.

Solution:

(a) Advantages and Disadvantages of the stock and warrant as compared to just issuing stock.

Commentary on Question

Candidates did well on this section which was a mainly recall question.

<u>Advantages</u>

Lower price for stock due to option to convert
Grow business quicker with money raised from shareholders
No dilution until exercise of warrant
It is like a call option, may attract another set of investors

7. Continued

Disadvantages
May dilute stock price if exercised
Too complicated to price
Rating agency view may be unfavorable
May be harder to raise additional funds at a future date

(b) Explain why the company would structure the warrant to have the exercise price change with the exercise date.

Commentary on Question
Candidates did moderately well on this section. While many candidates got one of the reasons below, full credit was only given to those who gave multiple reasons.

The candidate should have explained various reasons including but not limited to the following reasons:

1. Firm expects price to grow in the future and thus wants to limit the loss of potential funds raised.
2. It will help to reduce the impact of dilution of shares for current shareholders.
3. Brings different risk appetites / time horizons into better alignment between the firm's management and the shareholders.
4. Help encourage earlier exercise of warrant allowing for better alignment of cash flow needs for a rapidly growing company.

(c) Calculate the value of the warrant using the binomial tree.

Commentary on Question
Candidates did very well on this section. However, some candidates divided the warrant value in half which was incorrect as they were not asked what the dilution impact to the current shareholders would be.

Identify the inputs

Time step $\Delta t = 1$
Asset yield $q = 0$
Risk-free rate $r = 0.04$
Up step size $u = 12.21/10 = 1.221$,
Down step size $d = 8.19/10 = 0.819$
Strike price $K_1 = 9$ at time 1
Strike price $K_2 = 10$ at time 2

Calculate the probability of up move

Growth factor $a = e^{(r-q)\,\Delta t} = e^{0.04} = 1.0408$
Probability of up move $p = (a - d)/(u - d) = 0.5515$

7. Continued

Determine backward the warrant value fi,j at each node (i, j)

$f_{2,j} = \max(0, S_{2,j} - K_2)$ for $0 \le j \le 2$

$f_{2,0} = \max(0, 6.7 - 10) = 0$

$f_{2,1} = \max(0, 10 - 10) = 0$

$f_{2,2} = \max(0, 14.9 - 10) = 4.92$

$f_{1,j} = \max(e^{-r\Delta t}[p\, f_{2,j+1} + (1-p) f_{2,j}],\ S_{1,j} - K_1)$ for $0 \le j \le 1$

$f_{1,0} = \max(e^{-r\Delta t}[p\, f_{2,1} + (1-p) f_{2,0}],\ S_{1,0} - K_1)$

$= \max(0, 8.19 - 9)$

$= 0$

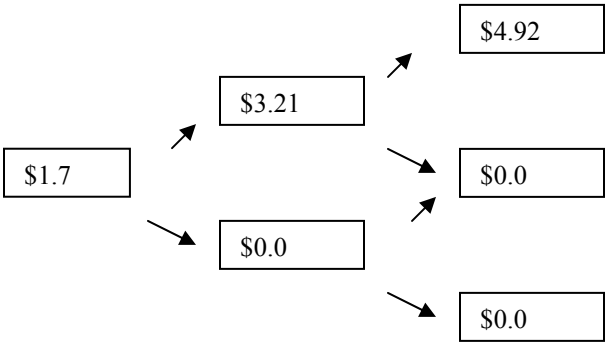
$f_{1,1} = \max(e^{-r\Delta t}[p\, f_{2,2} + (1-p) f_{2,1}],\ S_{1,1} - K_1)$

$= \max(2.60, 12.21 - 9)$

$= 3.21$

$f_{0,0} = (e^{-r\Delta t}[p\, f_{1,1} + (1-p) f_{1,0}])$

$= 1/1.0408*(.5518*3.21+(1-0.5518)*0) = 1.7$



(d) Estimate the delta and theta of the warrant at time 0.

Commentary on Question

Candidates did relatively well on this section. While most candidates knew the definition and formula of theta, the most common mistake was not applying the calculation correctly. Additionally, a few candidates used Black-Scholes to evaluate delta which was not necessary for this problem.

Delta

$\Delta = (CB-CC)/(SB - SC)$ (formula 19.8)

$\Delta = (3.21 - 0.00)/(12.21 - 8.19) = 0.798$

Theta

$\Theta = (f2,1 - f0,0)/(2\,\Delta t)$ or $(CE - CA)/(2\,\Delta t)$ (formula 19.10)

$\Theta = (0.00-1.7)/(2) = -0.85$

7. Continued

(e) Define gamma and explain how you could estimate the gamma of the warrant at time 0.

Commentary on Question
Candidates did relatively well on this section. To get full credit it was necessary to explain the approach to calculating gamma, however it was not necessary to actually calculate it. Additionally, some candidates also made the observation that theta could be used to help estimate gamma and those that did received additional credit.

Gamma is the rate of change of the portfolio delta with respect to the price of the underlying asset S.
It is the second partial derivative of the portfolio with respect to the asset price.

At time $2\Delta t$ we have two estimates of Δ:

One based on the second and third node, and the other based on the first and second node.

Gamma is the change in delta divided by the difference between the two values of S.

8. Learning Objectives:

3. Derivatives and Pricing

Learning Outcomes:

(3f) Demonstrate understanding of option pricing techniques and theory for equity and interest rate derivatives.

Sources:

Hull, Chapter 17 – The Greek Letters

Hardy, Chapter 8 – Dynamic Hedging for Separate Account Guarantees

Commentary on Question:

The question tested hedging using the Greeks with several calculation questions. This is exactly what variable annuity hedge analysts work on every day; candidates aspiring to work in that field must master this material.

Candidates who scored the highest in this question knew how to calculate the Greeks using a finite difference approach, and were able to set up the hedges properly.

Solution:

(a) Calculate the number of S&P futures contracts you need to buy long or sell short to neutralize delta.

Commentary on Question:

Candidates did relatively well on this section. The most common mistake was to rely on the Black-Scholes formulas to calculate the Greeks, which doesn't work because the question does not assume a Black-Scholes market.

The question did not specify the current volatility. The following calculations are based on a current volatility of 25%. Full credit was given to candidates who used a different current volatility; candidates are reminded to do the best they can with the information given – if nothing is written the graders can give no credit.

An increase in the liability value is a loss to MLIC. When the S&P decreases, the liability increases, causing a loss. Hence, we need to short the S&P futures so that the gain in the futures position will offset the loss due to the increase in the liability. The delta of the liability is calculated as

$$\Delta\text{Liability} = \frac{-71.8 - (-73.9)}{1100 \times 1.01 - 1100 \times 0.99} = 0.1$$

where we have used central differencing with a shock size of 1%, and we use negative liability values to indicate that this presents a loss to MLIC. The delta of the futures is given by

$$\Delta\text{Futures} = e^{[(r-q)\tau]} = e^{[(0.04-0.02)\times 0.25]} = 1.005$$

8. Continued

The short position in futures is given by $\frac{\Delta \text{Liability}}{\Delta \text{Futures}} = 0.1$. Since each futures contract is on 250 times the S&P index, the delta of futures per contract is then $\Delta \text{Futures} \times 250 = 251.25$

The number of futures contract that needs to be shorted is

$$\text{Number of contracts} = \text{Round}\left(\frac{0.1 * 1000000}{251.25}\right) = 398$$

(b) Determine the hedging positions required to neutralize both delta and Vega, and calculate the initial cost to set up these positions.

Commentary on Question:
Candidates did moderately well. A common mistake made was to fail to notice that the future contracts had nothing to do with the cost of the hedge.

The put option is used to hedge the Vega risk whereas the futures contract is used to hedge delta risk. The liability value increases (a loss to company) when S&P volatility rises. The put option value increases when S&P volatility rises, so we need to go long put options so the gain from the put position will offset the loss in liability.

$$\vartheta \text{Liability} = \frac{-65.7 - (-80.2)}{-1 - 1} = -7.25$$

The number of put options to long is $\frac{7.25}{0.75} \approx 10$, and this position gives a delta of $10 \times -0.11 = -1.1$. The current unhedged delta is $0.1 - 1.1 = -1$. Hence, we need to add delta (long futures contract) in order to remain delta neutral. The long position in futures needed is $\frac{1}{1.005} = 0.995$, which means the total number of futures contract needed is given by

$$\text{Number of contracts} = \text{Round}\left(\frac{0.995 * 1000000}{251.25}\right) = 3980$$

There is no initial cost to enter a futures contract, and there are no margin requirements. Hence, the cost of this hedge only comes from the put options, which is given by $10 \times 0.18 = 1.80$.

8. Continued

(c) Estimate the gain or loss on the put position using the greeks values provided.

Commentary on Question:
The scores distribution was bimodal - either the candidates knew it or they didn't.

The option gain from delta is

$$-2\% \times 1100 \times -0.11 \times 10 = 24.2$$

The option gain from Vega is

$$1 \times 0.75 \times 10 \approx 7.25$$

On the basis of a first order approximation, we obtain a total gain of
$24.2 + 7.25 = 31.95$.

(d) Calculate the overall net gain or loss due to the market movements.

Commentary on Question:
Candidates did very well on this section. Most candidates recognized the 3 relevant components of the gain/loss.

The liability increases from 72.8 to 83.6 which is a loss of $83.6 - 72.8 = 10.8$. The futures contract gain $0.995 \times 1100 \times 2\% = 21.89$. Together with the gain from part (c), the total gain is $-10.8 - 21.89 + 31.95 = -0.74$ million.

9. Learning Objectives:

3. Derivatives and Pricing

Learning Outcomes:

(3f) Demonstrate understanding of option pricing techniques and theory for equity and interest rate derivatives.

Sources:

Hull, 7th ed. Pages 653 or 8th ed. Page 653

Commentary on Question:

This question required the candidate to derive the put-call parity relationship for interest rate options and swaps and to spot an arbitrage opportunity from an option misprice. The explanations are largely derived directly from the syllabus material and required recall, comprehension and a little analysis.

Solution:

(a) Describe the payoff of each instrument at time 12 months.

Commentary on Question:

Candidates did well on this section. The most common mistake was to provide the pricing formula where the question asked for the payoff.

The following two solutions would allow a candidate to achieve full marks.

I. $R_L > R_K$:

Cap: $L*(R_L - R_K) *\delta_K$
Floor: 0
Swap: $L*(R_K - R_L) *\delta_K$

II. $R_K > R_L$
Cap: 0
Floor: $L*(R_K - R_L) *\delta_K$
Swap: $L*(R_K - R_L) *\delta_K$

Alternative Solution:
Cap: $L*\max(R_L - R_K, 0) *\delta_K$
Floor: $L*\max(R_K - R_L, 0) *\delta_K$
Swap: $L*(R_K - R_L) *\delta_K$

9. Continued

(b) Indicate the put-call parity relationship for caps and floors.

Commentary on Question:
Candidates did relatively poorly. The most common mistake was putting the put-call parity for equities instead of interest rate derivatives.

$$\text{cap} = \text{floor} + \text{swap}$$

Since the swap holder pays LIBOR the put call parity in this case can be written as:

$$\text{cap} = \text{floor} - \text{swap}$$

(c) Identify the arbitrage opportunity and recommend a trade to take advantage of it.

Commentary on Question:
Candidates did moderately well. Some candidates did not explicitly identify the arbitrage opportunity but provided a method to take advantage of it. However, for the candidate to receive full marks at a minimum the following was expected.

$$2\ (\text{cost of cap}) > 3(\text{cost of floorlet}) - 2(\text{cost of swap}) = 1$$

The cap is overpriced, therefore one can achieve arbitrage profit by selling the cap and buying the (floor – swap) portfolio, equivalently: sell cap, buy floor and sell the swap.

10. Learning Objectives:

1. Modern Corporate Financial Theory

Learning Outcomes:

(1d) Define and compare risk metrics used to quantify economic capital and describe their limitations.

Sources:

FET-115-08: Specialty Guide on Economic Capital

Commentary on Question:

This question tested the candidates' overall understanding of Economic Capital (EC) as presented in the source material. More specifically, it tested the ability of candidates to calculate EC components and to apply this knowledge in a practical exercise, the use of EC in pricing.

Overall, candidates did poorly on this question. As with all calculation type questions, it is important for candidates to show their work (formulae, data used in the formulae, numerical answer), as this allows them to demonstrate their knowledge and understanding of the source material. It is usual for greater emphasis to be placed on the development of the numerical answer rather than on the correct numerical answer itself. The latter will receive credit, but it is not possible to receive full credit for the question by simply providing a correct numeric answer. One must demonstrate mastery by showing intermediate steps in calculations as well.

If part of a question relied on a previous answer which had not been calculated, it was often possible to receive full credit for the later part by assuming a previous numeric answer for purposes of working through that part, as long as candidates were able to demonstrate their knowledge and understanding of the source material being tested in that later part.

Candidates generally did better on part (a) than part (b). Part (a) was a recall and calculation type question, based on an example found in the source material.

Part (b) was an application and knowledge utilization type question, based on formulas found in the source material as well as the descriptions of indirect and direct approaches.

Solution:

(a)

(i) Calculate the unadjusted EC before covariance.

Commentary on Question:

Overall, the candidates did relatively poorly on this section.

Some candidates tended to write down numbers in a formula without showing what those numbers represented, or else mentally calculated intermediate results and wrote those results. Both methods missed out on grading points because it was not clearly demonstrated that candidates understood the calculation.

10. Continued

Other candidates showed values in a labeled table or provided a written formula which clearly indicated what the intermediate results represented, both methods earning full credit where data and answers were correct.

Another common error was to calculate only the Sum (mean losses) or to calculate only the Sum (98^{th} percentile losses).

Sum (98^{th} percentile losses) - Sum (mean losses), or
Sum over each factor (98^{th} percentile loss – mean loss)

(300,000 + 40,000+55,000) – (200,000+12,000+40,000)

= 143,000

(ii) Calculate the unadjusted EC after covariance.

Commentary on Question:
Overall, the candidates did relatively poorly on this section.

A common approach was to introduce covariance matrices and attempt to develop a calculation of covariance from first principles. This did not earn credit, as the question clearly asked for a traditional approximation, which was defined in the source material. Although the data provided in the question directed the candidates to use the traditional approximation, one candidate successfully did it from first principles.

For the traditional approximation for a covariance adjustment, total unadjusted EC after covariance:

= Square root [(EC for interest rate risk) ^{2} + (EC for mortality risk) ^{2} + (EC for lapse risk) ^{2}]

= Square root [(300,000 – 200,000) ^{2} + (40,000 - 12,000) ^{2} + (55,000 - 40,000) ^{2}]

= 104,924

(iii) Calculate the adjusted EC after covariance.

Commentary on Question:
Overall, the candidates did relatively poorly on this section.

The most common error was to subtract the total actuarial liability from the total balance sheet liabilities resulting in $25,000.

10. Continued

Another common error was to add the adjustment amount to the unadjusted EC after covariance. This in effect double counts the amount of capital already in the balance sheet liabilities, whereas the intention is to take credit for the amount already in the balance sheet liabilities by subtracting the adjustment amount.

Adjustment for amount of capital included in balance sheet liability = actuarial liability on balance sheet – sum of mean losses

270,000 – (200,000+40,000_12,000) = 18,000.

Adjusted EC after covariance = Total unadjusted EC after covariance – adjustment for amount of capital included in balance sheet liability

104,924 – 18,000 = 86,924.

(b) Calculate the PV of annual distributable earnings under:

(i) The indirect approach

(ii) The direct approach

Commentary on Question:
Overall the candidates did poorly on this section.

The question describes a relationship between inforce EC components (80% interest rate risk and 20% mortality risk). All reasonable interpretations as to what this relationship meant were accepted as solutions. For example, while a more simple interpretation might be that the amounts for each component were in the ratio 80% to 20%, another equally sound interpretation was that the unknown factors in the linear formula given in the source material were in the ratio 80% to 20%.

Candidates need take care that the use of either set of factors must be consistent. For example, "a" and "c" factors in the linear formulas are used differently (one is applied to NAR and ${}_tq_x$, one is only applied to NAR), whereas "b" factors are the same.

A correct solution required candidates to recognize what data to use and how to use it. Data from the inforce VUL block is used to determine the factors to be used to calculate projected EC for the VUL pricing model. Data from the VUL pricing model is used to project EC and ultimately calculate the PV of annual distributable earnings.

10. Continued

A common error was to include inforce amounts in the projections (i.e. not treating these blocks separately). For example, in calculating the change in EC for year 1, the amount of inforce EC at time 0 is irrelevant.

It was assumed that successful candidates knew the definition of distributable earnings. An incorrect definition and use of incorrect amounts did not earn credit. A common error was to use Change in Reserves in the definition of distributable earnings instead of Change in EC. Candidates at this level of exam are expected to recognize that Change in Reserves is already removed in the calculation of After Tax Book Profits. To remove it again when calculating distributable earnings is incorrect.

Another common calculation error was found in the discounting of projected distributable earnings. All distributions occur at end of year, whereas some candidates incorrectly assumed distributions occurred at beginning of year.

Both indirect and direct methods require the calculation of projected EC.

Linear formula approximation should be either:

NAAR x ${}_tq_x(d)$ x (Mortality risk factor "c") + ${}_tV_x$ x (Interest rate risk factor "b"),

or NAAR x (Mortality risk factor "a") + ${}_tV_x$ x (Interest rate risk factor "b")

110,000 x a + 252,000 x b = 70,000

and 110,000 x a = 20% and 252,000 x b = 80%

so a = 0.2 * 70,000/110,000, and b = 0.8 * 70,000/252,000

→ a = 12.73% → b = 22.22%

EC(k) = (NAAR(k) x a) + (tVx (k) x b)

EC(1) = 30,000 x 12.73% + 105,000 x 22.22% = 27,150
EC(2) = 20,000 x 12.73% + 110,000 x 22.22% = 26,988
EC(3) = 15,000 x 12.73% + 115,000 x 22.22% = 27,463

10. Continued

Chg in EC(1) = EC(1) – 0 = 27,150
Chg in EC(2) = EC(2) – EC(1) = 26,988 – 27,150 = -162
Chg in EC(3) = EC(3) – EC(2) = 27,463 – 26,988 = 475

(i) Under the indirect method, the model recognizes establishment and later release of Economic capital as an annual product flow making up distributable earnings.

AT Inc on EC (k) = (1 – tax) x (Inv on EC(k)) x EC(k)

AT Inc on EC (1) = (1 – 35%) x 8% x 27,150 = 1,412
AT Inc on EC (2) = (1 – 35%) x 8% x 26,988 = 1,403
AT Inc on EC (3) = (1 – 35%) x 8% x 27,463 = 1,428

Calculate Distributable Earnings (DE)

DE (k) = ATBP(k) – Chg in EC(k) + AT Inc on EC (k)

DE (1) = -5,000 – 27,150 + 1,412 = -30,738
DE (2) = 12,000 – (-162) + 1,403 = 13,565
DE (3) = 10,000 – 475 + 1,428 = 10,954

Calculate Present Value Distributable Earnings (PVDE)

PVDE = DE(1) x v + DE(2) x v^2 + DE(3) x v^3 @ 15%
PVDE = -30,738/1.15 + 13,565/1.15^2 + 10,954/1.15^3 = -9,269

(ii) Under the direct method, must calculate the Net Cost of EC
= Hurdle rate – (1 – tax) x (Inv on EC)
= 15% - (1 – 35%) x 8%
= 9.80%

This factor is applied to the factor-based calculation of projected EC to arrive at an annual cost of capital for the EC:

Charge for EC (k) = (Net Cost of EC) x EC(k)

Charge for EC (1) = 9.80% x 27,150 = 2,661
Charge for EC (2) = 9.80% x 26,988 = 2,645
Charge for EC (3) = 9.80% x 27,463 = 2,691

DE (k) = ATBP(k) – Charge for EC(k)

10. Continued

DE (1) = -5,000 – 2,661 = -7,661
DE (2) = 12,000 – 2,645 = 9,355
DE (3) = 10,000 – 2691 = 7,309

PVDE = DE(1) x v + DE(2) x v^2 + DE(3) x v^3 @ 15%
PVDE = -7,661/1.15 + 9,355/1.15^2 + 7,309/1.15^3 = 5,218

11. Learning Objectives:

2. Corporate Financial Applications

3. Derivatives and Pricing

Learning Outcomes:

(2c) Describe the process, methods and uses of insurance securitizations and recommend a structure that is appropriate for a given set of circumstances.

(3c) Identify embedded options in assets and liabilities.

Sources:

Securitization of Life Insurance Assets and Liabilities, Pages 201-202, 221-222

Hull, Options Futures & Derivatives, Chapter 13, The Black-Scholes-Merton Model.

Commentary on Question:

The question asked the candidates to do the following:

- Identify and describe the embedded options in the reverse mortgage product;
- Analyzes the risks of adding reverse mortgage to the current GIC portfolio;
- Identify parties involved and their roles in a securitization structure that is appropriate for longevity risk.

The main considerations were:

- To fully understand the implications from a cashflows/ALM perspective of adding reverse mortgages product with their GIC portfolio products;
- To realize that the securitization structure required the bank to move away from the government agency and turn to the market to finance any shortfalls when the longevity index is triggered;
- To recognize that the bank wanted to securitize its longevity risk only e.g. bank would pay a premium that would represent the expected longevity cash flows in order to receive actual payments from the SPV. Such payments would have been payable only when the longevity index is being triggered.

Solution:

(a)

Commentary on Question:

This question tested the ability to recognize the type of options embedded in a financial product and identify its components: underlying, strike price and volatility.

11. Continued

Candidates did well on this section. A common mistake was identifying the option as a call or GMDB in part (i). Additionally, although most of the candidates correctly identified the put option few described the option completely and thus did not get full credit.

(i) Describe the option that the bank is currently writing.

By guaranteeing that the loan will be reimbursed at the time of death even if the value of the property is lower than the loan amount, Sturgeon Bay Bank is writing a put option to the reverse mortgage customer on the property value against the accumulated value of the loan. The customer contracting a reverse mortgage does in effect buy the right to sell his/her property at time of death for a minimum price equal to the accumulated value of the loan (put option).

(ii) Describe the underlying, strike price, and volatility implicit in the context of this particular option.

- The underlying of the put option written by Sturgeon Bay Bank is the value of the residential property.
- The strike price is the accumulated value of the loan.
- The volatility implicit of the put option is the volatility of the property price.

(b) Analyze the risks associated with this line of business as currently managed.

Commentary on Question:
The question asked the candidates to recognize the risks created by selling a reverse mortgage in conjunction with the GICs, not the risks embedded in the reverse mortgage. It was important to understand whether a reverse mortgage was a good complementary product to the GICs currently sold from a cash flow and ALM perspective.

The candidates did relatively poorly on this question. A common mistake was analyzing the risks of the reverse mortgage as opposed to analyzing the risks of adding the new product to the line of business. Very few candidates answered the question correctly.

11. Continued

The risks associated to the line of business are:

- Duration mismatch risk between reverse mortgage and GIC.
- Spread risk: Risk that the bank will not be able to earn enough interest from the reverse mortgage to pay out the fixed interest rate on the GICs. The bank earns a variable interest rate on the reverse mortgage that can drop below the fixed credited rate on the GIC.
- Cash flow mismatch risk: The reverse mortgage does not seem to be a good fit to match the GIC cash flows. Because of their relatively short maturities and the possibility of the GIC holder withdrawing their account before maturity, the reverse mortgage product does not seem to be a good fit. The reverse mortgage cash flows are uncertain in amount and time, as opposed to the GIC cash flows which are for 5 years.
- Liquidity risk: Because the GIC holders have the option to withdraw their balance before maturity it might be necessary but difficult to sell the reverse mortgage in order to pay out the balance of the GIC holder.
- Longevity risk: The fact that the homeowner might live longer than expected; increasing the value of the homeowner's put option embedded in the reverse mortgage. It also increases duration mismatch risk between the GIC and the reverse mortgages.

(c) Recommend a securitization structure,

Commentary on Question:
This part tested understanding of the securitization process for longevity risk, the parties involved and their respective roles.

Candidates did well on part (i) and moderately well on parts (ii) and (iii). Candidates excelled in identifying the parties involved in the securitization structure. They missed on accurately describing the role of each party. In general candidates were able to draw the cash flow diagram.

Common mistakes were not recognizing the following:

- The bank wanted to securitize its longevity risk only and not finance reverse mortgages through investors or GIC investors.
- To securitize its longevity risk the bank would have to move away from the government agency and go to the capital markets.
- The bank needed to pay a premium in order to receive actual payments from the SPV and such payments were conditional to the longevity index being triggered.
- Investors incurred the risk of losing coupons when the longevity index is triggered.

11. Continued

- The GIC investors were different from the investors buying securities bearing the longevity risk in the securitization.

(i) Identifying the parties to the transaction,

The securitization structure should involve the following parties:

i. Borrowers - Homeowners
ii. Lender/Originator - Sturgeon Bay Bank
iii. Special purpose vehicle (SPV)
iv. Capital market investors

(ii) The role of each party, and

i. Homeowners:
   - Purchase the reverse mortgages from SBB and receive a fixed loan.
   - Upon the homeowners' death, the loan is repaid using the proceeds from the sale of the property.
   - Homeowners are not responsible for the shortfall if the sale proceeds exceed the accumulated value of the loan.

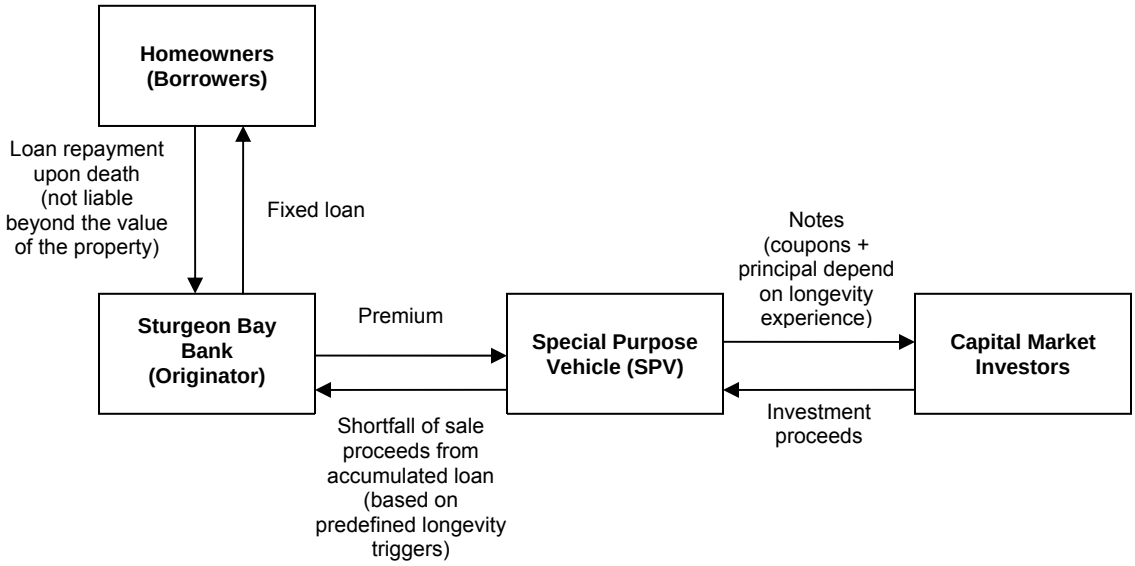
ii. Sturgeon Bay Bank:
   - Sells the reverse mortgage to homeowners and provides a fixed loan.
   - Exposed to the risk that sale proceeds are less than the accumulated value of the loan upon the homeowners' death.

iii. Special purpose vehicle (SPV):
   - Collects a premium from SBB and securitizes the longevity risk by repackaging the reverse mortgage cashflows into debt securities and selling them to capital market investors.
   - Debt securities have predefined longevity triggers to determine payout.
   - Pays a benefit to SBB if SBB experiences a loss.
   - The premium and capital from selling the bond are assumed to have been invested at a risk free rate.

iv. Capital market investors:
   - Purchases the debt securities and assumes the longevity risk.
   - Provides investment to the SPV in exchange for coupon payments and principal.
   - Actual payout depends on the actual longevity experience relative to the predefined longevity triggers.
   - Receives a reduced payout if actual experience is worse than expected.

11. Continued

(iii) Diagramming all cash flows.



12. Learning Objectives:

2. Corporate Financial Applications

Learning Outcomes:

(2h) Describe how behavioral characteristics and biases of users and providers of capital affect the capital structure.

Sources:

Grinblatt & Titman, Financial Markets and Corporate Strategy, 2nd Edition, Chapter 19, Pages 676, 678-679

Copeland, Weston, Shastri, Financial Theory and Corporation Policy, Chapter 15, Page 624

Commentary on Question:

This question asked about different alternatives to finance a project using capital or debt and the resulting impact on shareholder value.

Solution:

(a) Describe the advantages and disadvantages of each financing alternative.

Commentary on Question:

This section requires the candidates to list the main advantages and disadvantages under current shareholder's and manager's point of view using three ways to finance a new project.

Candidates did moderately well on this section which was primarily recall. Most of the candidates received points from explaining debt and preferred stock financing through a discussion of dilution, leverage and bankruptcy. A smaller number discussed impact of the current stock price which is higher than book value for equity financing. A common mistake was to view the investment from the perspective of an investor instead of MiMo Management, as the question asked.

(i) Issue new equity:

Advantages:
Lowers the overall leverage of the company
There is an incentive to use it is highest when current stock price is overvalued by the market (as is the case for MiMo)
New issue with overpriced equity increase intrinsic value of existing shares
Provides an inexpensive financing because relatively low expected return

12. Continued

Disadvantages:
Issuing equity when stock price is low is not suitable
Likely reluctant to finance with underpriced shares
Dilutive to earnings of current shareholders

(ii) Issue new senior debt:

Advantages:
Reduce risk with positive NPV
Even with positive NPV, if high financial distress and adverse selection cost the firm must pass up the project
Taxable earning makes debt financing attractive since can deduct interest payment from profit
No dilution of earning and control as with issue of new equity
Manager will favor that option if they have private information making them highly confident that the project will be a financial success

Disadvantages:
Increases the leverage of MiMo
Can create financial distress for Mimo if obliged to pay interests and capital
Could ultimately force bankruptcy of the company
Debts have priority over shareholder in case of liquidation

(iii) Issue new cumulative preferred stock

Advantages:
No problem with financial distress
Cannot force bankruptcy as bond
No dilution of earning
Offer a fixed claim, so dilution costs of underpriced shares is not a problem
A good security with temporary financial difficulties

Disadvantages:
May have a voting right (for example, in New York)
Cumulative dividends
All past dividends must be paid before common stock dividends

(b) Calculate the impact on shareholder value for each financing alternative.

12. Continued

Commentary on Question:
The candidates did well on this section. However, many candidates deduct the initial expense from NPV twice, thus misunderstanding the real value of NPV. In this case we did not give credit for that part of the NPV calculation in the formula. Some candidates incorrectly used the market value ($50) to calculate the intrinsic value of a share instead of the current present value stated in the question.

(i) Calculation shareholder value with new equity:

Share **value before project**= (Current Business Value) / (Number shares);

Share value before new project = 40,000,000 / 1,000,000 = 40.

Share value **with new project**= (Current Business Value + PV of new project) / (Number of original shares + Number of new shares);
Number of new shares = cost of investment / current share price

15,000,000 / 50 = 300,000 of new shares.
Share value with project = ((Current business value) + (NPV of the new project)) / (Total number of shares)

Share value with project = (40,000,000 + 20,000,000) / (1,000,000 + 300,000)

= 46.15

(ii) Calculation shareholder value with senior debt:

Share value with project= (Current Present Value + NPV of new project after additional reduction) / (Number of original shares).

New debt **reduces NPV** by 30%, so NPV reduced to:
20,000,000 * (100%– 30%) = 14,000,000
Share value with project = (40,000,000 + 14,000,000) / (1,000,000 of original shares)
= 54

(iii) Calculation shareholder value with preferred stock:

Share value with project= (Current Present Value + NPV of new project after additional reduction) / (Number of original shares).

Issuing preferred shares **reduces NPV** by 20% so NPV reduced to:
20,000,000 * (100% –20%) = 16,000,000

12. Continued

Share value with project = (40,000,000 + 16,000,000) / (1,000,000 of original shares)
= 56

(c) Recommend and justify the best funding alternative for MiMo.

Commentary on Question:
Candidates did poorly on this section. While many candidates performed the calculations correctly, very few provided reasons to select the best one. Based on the share values with the project, the obvious choice is the highest value (preferred stock). However, most of the grading points were awarded for the candidate discussing any implications and justifying the answer.

To justify we need to compare the share values under the various options, and consider any implications for each funding alternative.

Summary of share values:

	(b)(i)	(b)(ii)	(b)(iii)
Share value before	40.00	40.00	40.00
Share value with project	46.15	54.00	56.00

The current price of 50 may appear overpriced relative to actual value at 40. This may provide a good opportunity to issue shares. However, buyers may not purchase these shares since intrinsic value remain less than current value.
So the increase in share value most likely does **not justify issuing new common stock** instead of debt or preferred stock (especially given that other funding options give higher values).

The issue of new debt provides a larger increase in value than issuing shares.
This debt may lead to drop in the firm's credit rating. This increases the risk to MiMo of financial distress.
In the case of liquidation the value of the business will cover the senior debt but shareholders will suffer a loss.
Then debt financing is not the best alternative.

Finally, issuing **preferred stock** provides a better increase in the shareholder value.
Also the preferred stockholders cannot force bankruptcy if dividends are not paid as payment of dividend may be skipped.
So there is no dilution and MiMo does not have to deal with the impact on common stock price in case of overpriced share or underpriced share.
Then, as a source of capital to finance the investment, the best choice for MiMo is to issue preferred stock.

13. Learning Objectives:

1. Modern Corporate Financial Theory

3. Derivatives and Pricing

Learning Outcomes:

(1d) Define and compare risk metrics used to quantify economic capital and describe their limitations.

(3i) Contrast commonly used equity and interest rate models.

Sources:

Hardy Chapter 2: Modeling Long-Term Stock Returns

Commentary on Question:

This question related to Modeling Stock Returns for GMMB liabilities. The candidates needed to be able to recall the features of both deterministic and stochastic models and the suitability of such models with a GMMB liability. Lastly the candidates needed to form an opinion on what data source would be most appropriate and be able to explain their reasoning.

Solution:

(a) Describe the problems with using a deterministic method to model GMMB liabilities.

Commentary on Question:

Candidates did moderately well on this section. A common mistake was to describe the GMMB feature but this did not address the question.

No single path intended to model extreme behavior is likely to be credible.
It is difficult to interpret the results, having enough to pay on one tested path does not guarantee that all potential scenarios are covered.
A single path may not capture the risk appropriately for all contracts particularly if the guarantee may be ratcheted upward.
GMMB benefit payout is path dependent. For example, ratchets make a rise followed by a fall more costly than a fall followed by a rise.

(b) Describe the features of the above models.

Commentary on Question:

Because of the similarity in what was being asked in part (b) and part (c), explanations appearing in either part were accepted.

13. Continued

Candidates did relatively well on parts (b) and (c). The most common mistake was to not relate their recommendation to economic capital purposes. A less common mistake was to explain the advantages and disadvantages of each model but not make a clear recommendation and thus not receive full credit.

Lognormal (LN) Model
Assumes equity return follows a lognormal distribution.
Is simple and tractable.
Provides a reasonable approximation over short time intervals, but is less appealing for long time intervals.
Empirical studies indicate that it fails to capture more extreme price movements.
It does not allow for autocorrelations in the data.
It fails to capture volatility bunching.

Regime Switching Lognormal Model (RSLN)
The process switches randomly at each time step between two or more LN models (regimes) based on a Markov process.
Maintains the tractability of the LN model.
More accurately captures extreme observed behavior than does the LN model, i.e. RSLN has fatter tails.
Introduces stochastic volatility by allowing the volatility to switch randomly between the volatility values corresponding to each regime.
Has been shown to provide a very good fit to the stock index data relevant to equity linked insurance.

The Empirical Model
Historical returns are used as the sample space for future returns.
Each return is equally likely and sampling occurs with replacement.
Makes modeling of multivariate distributions easy by sampling the pair of values from the same date.
Is simple and quick to use.
Analytical development is not possible.
No autocorrelation due to assumption of independence.
Suffers from the same weaknesses as the LN model, which it closely resembles in distribution (i.e. tails not fat enough, no volatility clustering).

(c) Assess the suitability of each model for modeling your company's GMMB liability for economic capital purposes.

Economic capital is a tail risk metric. Among the three models above, I would recommend RSLN because RSLN is easy to use, gives fatter tails as needed in the EC measurement. It's also shown in Hardy's book that RSLN is a very good fit to the stock index data so RSLN is suitable for the GMMB liability.

13. Continued

(d) Describe the advantages and disadvantages of each data source and make a recommendation of which source to use. Support your choice.

Commentary on Question:
Below shows one set of explanations which would have received full credit. Although the solution below recommends historical returns since 1956, full credit was possible for a recommendation of either of the other two data sources provided it was justified.

The candidates did relatively poorly on this section, especially on the discussion on current market statistics.

Historical Returns since 1926

Advantage:
It maximizes the history used to project future returns, which is valuable given the long term projections needed for GMMB liabilities.

Disadvantages:
1. Some have argued that there was a shift in stock return data at the end of the great depression.
2. Returns may also have been distorted by fiscal restraints during World War II. Thus it is attractive to only consider data from 1956 onward.

Historical Returns since 1956

Advantages:
1. It provides an estimate of the distribution of long term returns, which is appropriate for modeling GMMB liabilities.
2. It excludes data from time periods that may not be a good fit to future conditions.

Disadvantage:
GMMB contract may require stock prices to be projected 40 years ahead, and using only 55 years of history to fit it may not be enough. However, due to policy lapses, mortality and discounting, cash flows beyond 20 years may not be very substantial.

13. Continued

Current Market Statistics

Advantage:
Volatility parameters can be derived from market prices and the price volatility relationship.

Disadvantage:
Current implied market volatility usually varies substantially from long term volatility. But for GMMB liabilities a long term view is needed, so Current Market Statistics may not be suitable for modeling the GMMB liability.

Bases on the discussion above, I would recommend using Historical Returns since 1956 to model GMMB liability.

14. Learning Objectives:

3. Derivatives and Pricing

Learning Outcomes:

(3f) Demonstrate understanding of option pricing techniques and theory for equity and interest rate derivatives.

(3j) Define and apply the concepts of martingale, market price of risk and measures in single and multiple state variable contexts.

Sources:

Hull v7: Pages 290, 623-625 and 628 or Hull v8: pages 312, 631-623 and 630

Commentary on Question:

This was a straightforward question on the basics of the martingale in risk-neutral valuation.

Solution:

(a) Modify the above processes to be risk-neutral. Define any variables you introduce.

Commentary on Question:

Most of the candidates did well recognizing a simple substitution was all that was needed and a few gave good explanations even though it was not required. A common mistake was to use Ito's Lemma to derive the drift term to be $\mu - (\sigma^2)/2$. Another common mistake was to use the risk-free rate minus the dividend yield.

Replace μ with r, the risk free rate.

(b) Write down an expression for the market price of risk, λ.

Commentary on Question:

Most candidates wrote down the expression correctly.

$\lambda = (\mu_1 - r)/\sigma_1 = (\mu_2 - r)/\sigma_2$

λ is **not** dependent on the nature of the derivative.
λ must be the same for all derivatives that are dependent on μ and t for the no arbitrage condition.
σ_j is the volatility of F_j.
μ_j is the expected growth rate of F_j.
r is the risk free rate.

14. Continued

(c) Express μ_2 in terms of σ_2 and the market price of risk.

Commentary on Question:
Most candidates wrote down the expression correctly.

This answer required a rearranging of the terms in (b).
$\mu_2 = r + \lambda \sigma_2$

(d) Explain in words the concept of a martingale.

Commentary on Question:
Most candidates answered this correctly.

Full credit was given for any of the answers below:

A random variable follows a martingale if the expected value of the variable over the next period is equal to the last known value.

The expected change of the variable over the next period is zero. The trend line is flat

It's a zero drift stochastic process.

(e) Identify, with reasons, which of the following processes are martingales:

Commentary on Question:
Most candidates answered this section correctly and had a good explanation. A few answered correctly with insufficient explanations and received fewer points. Even fewer got the answers backwards or answered both as martingales.

(i) The asset value at each time step in a risk-neutral process

The asset value is expected to grow at the risk-free rate, which results in a different value than it is today, so it is **not** a martingale.

(ii) The discounted asset value in a risk-neutral process

The asset is expected to grow at the risk-free rate and the discounted value at risk-free rate will stay the same. The discounted asset value is a martingale.

Other explanations were also acceptable.

15. Learning Objectives:

3. Derivatives and Pricing

4. Efficient and Inefficient Markets, Complete and Incomplete Markets, Information Theory & Market Misbehavior

Learning Outcomes:

(3e) Derive the Black Scholes Merton pricing formula.

(3h) Describe and evaluate equity and interest rate models.

(3i) Contrast commonly used equity and interest rate models.

(4c) Describe empirical evidence and results regarding market efficiency.

Sources:

Hull, Options Futures & Other Derivatives, Chapter 13 The Black-Scholes-Merton Model

- Ch. 17 The Greek Letters

Copeland, Chapter 12 Information Asymmetry and Agency Theory

Hardy, Investment Guarantees, Ch. 2, Modeling Long-Term Stock Returns

Commentary on Question:

The question asked candidates to demonstrate their understanding of the signaling theory described in Copeland's book and to use several methods from Hull to solve option pricing questions.

Solution:

(a) Outline the key costly and costless signaling announcements.

Commentary on Question:

The candidates answered this section well as this is a recall question from Copeland's book.

Costly signaling announcements

- Amount of firm's equity that is retained by an entrepreneur
- Amount of debt issued by the firm
- Size of dividends declared
- Type of financing used for an investment
- Decisions to split the stock

Costless signaling announcements

- Amount of equity issued/repurchased
- Type of debt issued

15. Continued

(b) Calculate the price and delta of the 2-month call option on the stock with strike price equal to $30 using the binomial tree.

Commentary on Question:
The candidates answered this section well.

U = 25 * u = 25 *1.3 = 32.5, D = 25* d = 25 * 0.7 = 17.5
f(1,1) = max(25 * 1.3 -30, 0) = 2.5
f(1,0) = max(25 * 0.7 - 30, 0) = 0
P = (exp(rd) – d)/(u-d)
P = (exp(4%*2/12) – 0.7)/(1.3-0.7) = 0.511 Or 0.447 (if d = 1/1.3)
Price = exp(-rt) * (p *f(1,1) + (1-p) *f(1,0))
= 1.26938 Or 1.111 (if d = 1/1.3)

Delta = (f(1,1) - f(1,0))/ S(0) * (u - d)
Delta = (2.5 - 0) / (25 * 0.6) = 0.16666 Or 0.188 (if d = 1/1.3)

(c) Solve for the Black-Scholes-Merton model's annualized implied volatility using the price from (b).

Commentary on Question:
The candidates answered this section poorly. Very few candidates knew how to use an iterative procedure to determine the answer. While most candidates had the price correct from (b), points were still possible even if part (b) was done incorrectly.

C = S(0) * N(D1) – K * exp(-rT) * N(D2),
D1 = (ln(S(0)/K) + (r + σ^2/2)*T)/(σ*T^.5)
D2 = D1 – σ*T^.5

Use an iterative search procedure to find the implied volatility.

Use u = exp (σ)(T)^.5 to determine an initial estimate of σ => σ = 64.27%
You can use this estimate as one bound. 64.27% will give a value of C < 1.269.
Because C is an increasing function of σ, a higher value is required.
You can then try 74.27% (10% higher); this will gives a value too high.
σ is around 69.78.

(d) Calculate the Black-Scholes-Merton model's delta using the implied volatility from (c).

15. Continued

Commentary on Question:
Most candidates were able to apply the formula below correctly. Even though most candidates did not have the correct volatility from (c), most points were still earned.

Delta = $N(d_1)$
$d_1 = (\ln(S(0)/K) + (r + \delta^2/2)*T)/(\delta*T^{.5})$
$d_1 = (\ln(25/30) + (4\% + 69.777\%^2/2)*2/12)/(69.777\%*(2/12)^{.5})$
$d_1 = -0.4742$
Delta = $N(d_1) = 0.3177$

(e) Evaluate the appropriateness of delta hedging changes in the stock price, versus using an option hedge strategy.

Commentary on Question:
Less than half the candidates did well on this section. Few candidates knew the assumptions behind the two hedging strategies and/or realized the announcement is expected to cause a stock price jump (not a continuous stock price movement).

- Delta hedging not appropriate as the announcement is expected to cause a stock price jump.
- Dynamic delta hedging assumes continuous price movements.
- A discontinuous price movement does not provide time to rebalance the delta hedge.

16. Learning Objectives:

4. Efficient and Inefficient Markets, Complete and Incomplete Markets, Information Theory & Market Misbehavior

Learning Outcomes:

(4a) Define capital market efficiency and the value of information.

Sources:

Financial Theory and Corporate Policy, Copeland, Weston, Shastri, 4th Edition, Chapter 6

Commentary on Question:

This question tested the candidate's understanding of the assumptions behind CAPM and APT. Part (b) tested APT via applying the formula for the expected return. Part (c) asked the candidate to apply this to develop an arbitrage strategy.

Solution:

(a) Describe the assumptions of:

(i) Capital Asset Pricing Model (CAPM)

(ii) Arbitrage Pricing Theory (APT)

Commentary on Question:

Candidates did well overall on this part. A common mistake what to describe the CAPM and APT formula rather than the underlying assumptions.

Capital Asset Pricing Model (CAPM)

- Investors are Risk Averse and maximize the expected utility of their Wealth.
- All can borrow and lend at the risk free rate.
- There are no restrictions on short sales and short sale proceeds.
- Markets are competitive and frictionless.
- Investors have homogeneous expectations about asset returns.
- All assets are marketable and divisible.

Arbitrage Pricing Theory (APT)

- Markets are competitive and frictionless.
- Individuals assume the market is governed by the same number of k variables. Also known as the "k-factor" model.
- Number of assets available is greater than the number of factors.
- The noise term or error term is independent of all factors.

16. Continued

(b) Derive the Arbitrage Pricing Line for this set of portfolios, using the table above.

Commentary on Question:
Candidate responses varied widely. Some candidates obtained full marks on this question. Common mistakes included: ignoring the risk free rate and putting the expected return on the wrong side of the equation.

The general form of the arbitrage pricing model is:

$E(R_i) - R_f = (\delta_1 - R_f)\, b_{i1} + (\delta_2 - R_f)\, b_{i2}$
Let $\lambda_i = \delta_i - R_f$

Portfolio 1:
$0.20 = 0.05 + 0.08\, \lambda_1 + 0.05\, \lambda_2$

Portfolio 2:
$0.15 = 0.05 + 0.04\, \lambda_1 + 0.06\, \lambda_2$
$\lambda_1 = 1.4286$
$\lambda_2 = .7143$

The arbitrage pricing line is:
$E(R_i) = .05 + 1.4286\, b_{i1} + .7143\, b_{i2}$

(c) Identify the reason an arbitrage opportunity exists here and explain how Mr. Jones can take advantage of it.

Commentary on Question:
Many candidates understood how to come up with the expected return of 17.5%. A common mistake was to identify the strategy as sell Portfolios 1 and 2 and buy Portfolio 3. The expected return of Portfolio 3 lies <u>below</u> the Arbitrage Pricing Line, which means Portfolio 3 should be sold, <u>not</u> bought.
Other candidates who did not do part (b) correctly were able to identify that the beta of Portfolio 3 was equal to the average of the betas of Portfolio 1 and 2 and the expected return of 16% was less than the average of the expected returns of portfolios 1 and 2, which allowed them to correctly identify the arbitrage opportunity.

$E(R_i) = .05 + 1.4286\ (.06) + .7143\ (.055) = .175$
The expected return (16%) of portfolio 3 lies below the Arbitrage Pricing Line (17.5%).

He should sell Portfolio 3 and buy a combination of Portfolio 1 and 2, which lies on the Arbitrage Pricing Line, with $\beta_1 = 0.06$ and $\beta_2 = 0.055$, to make a riskless profit.

17. Learning Objectives:

4. Efficient and Inefficient Markets, Complete and Incomplete Markets, Information Theory & Market Misbehavior

Learning Outcomes:

(4c) Describe empirical evidence and results regarding market efficiency.

Sources:

Copeland, Weston, and Shastri, Chapter 10

Commentary on Question:

This question tested the candidate's understanding of, and ability to apply their knowledge to, an aspect of market efficiency: the value of information and how it may (or may not) be used by market participants.

This question required candidates to first recall the four key hypotheses regarding what information is used by investors and then apply this knowledge to interpret the behavior of four analysts as evidenced by samples of their pricing behavior against given, "known" data.

The text followed the introduction of these hypotheses by describing experiments conducted by Forsythe, Palfrey, and Plott that repudiated or supported these hypotheses. Although the question specifically referenced this material by use of the key word "experiment", most candidates did not seem to link that portion of the text with the hypotheses.

Finally, application of the concept of the value of information was tested through a calculation nearly identical to that appearing in the text. To properly answer this portion of the question the candidate was required to apply their understanding of the behavior of the analysts with the inputs to the calculation. For the candidate able to successfully do so, the calculation would have been relatively straightforward.

Candidates tended to do very poorly on parts (b) and (c) though these simply covered the information in the text that followed the part (a) material, testing it in more depth.

Solution:

(a)

(i) Identify the analyst conforming most closely to the "Naïve Hypothesis" valuation approach. Justify your answer.

Commentary on Question:

Some candidates did well on this question.

The successful candidate likely approached the solution to (a) (both parts (i) and (ii)) by one of two paths: (1) Identifying the relationship between the analysts' quotes and the given information, then naming the hypothesis which fit that interpretation or (2) Listing the hypotheses and then supplying the descriptions or reasons for each hypothesis and which analyst fit that description.

17. Continued

Candidates are also reminded that using a term to define itself is not a logically valid method of answering a question. Maximum points were available to those candidates correctly supplying reasons for identifying particular analysts with each hypothesis, not simply reusing words from the question itself.

Analyst 2 most closely conforms to someone using the Naïve Hypothesis. His price quotes appear arbitrary and unrelated to either future payoffs or their probabilities; the progression of price quote appears to be nonsensical.

(ii) Identify and explain each of the other analysts' hypotheses on asset prices.

Analyst 1 appears to follow the Intrinsic Value Hypothesis. His price quotes are always identical to the present value of future cash flows which indicates that he considers information related only to payouts and disregards the expectations of others.

Analyst 4 appears to follow the Speculative Equilibrium Hypothesis. His price quotes are always identical to those identified as the resale value. His pricing most closely conforms to an analyst who anticipates the behavior of other individuals in pricing without regard to the actual payoffs themselves.

Analyst 3 appears to follow the Rational Expectations Hypothesis. His price quotes are always the maximum of the two bases for expected values. His willingness to pay the maximum of the payoffs and the value to others is indicative of an efficient market where all information is reflected.

(b) Describe experiments which have been conducted to provide empirical evidence which supports or repudiates each hypothesis.

Commentary on Question:
Candidates did poorly on this section. Candidates often described certain studies which had been performed on economic data and not the experiments which were conducted. This was not the information requested by the question. Candidates who conceptually described the experiments below received some credit.

One experiment that has been conducted involved an oral double auction with two periods over several trials. Participants submitted purchase and sales prices for an asset in each period. Each participant knew the value of payoffs to themselves but not to other participants. The experiment then sought to identify the market price of the asset.

17. Continued

Over two time periods, the auction results initially supported the intrinsic value hypothesis as participants submitted prices reflecting the asset value to themselves. However, as the trials proceeded, the additional information gained by the participants caused the market price to quickly converge on the maximum value to all participants. The convergence value itself and speed at which the trials converged were both indicative of a market price expected by the rational expectations hypothesis. The participants climbing the learning curve rapidly repudiates the naïve hypothesis (the pricing was not arbitrary). The rapid learning also repudiated the speculative equilibrium hypothesis since the trend in market pricing was not reflective of the value the asset had to the other participants.

The auction experiment was followed up by one performed in a futures market environment, with a similar information structure but pricing in the spot market concurrent with the futures market. Here, the convergence toward a price supporting the rational expectations hypothesis was even more rapid than the previous experiment.

(c) Calculate the value that Wauwatosa Investments LLC believes is the equilibrium price today, under a stable mixed strategy.

Commentary on Question:
Candidates did poorly on this section. Most candidates missed two important concepts needed to effectively answer this question. First, candidates tended to include all of the analyst prices rather than identifying those which would be used under a stable mixed strategy. Second, candidates often discounted valuations at the risk-free rate although the valuation occurs "today" using pricing from "today". Correctly identifying the analysts to use and matching their hypothesis with the strategies required under the stable mixed strategy were required to correctly compute the answer.

A stable mixed strategy identifies an equilibrium price from prices gained from "analysts" and "random selectors." Here, we know that analyst 2 is the naïve hypothesis analyst and therefore the "random selector" while analyst 3 does appear to perform analysis and he is the "analyst." Since Wauwatosa is not aware of the price quotes of analysts 1 and 4, ignore those quotes.

First, determine the "competitive advantage" of the analyst strategy:

$d = c_2/c_1$ = cost of analyst strategy / cost for no analysis = 10%/2% = 5

Second, calculate the weight to apply to the analyst price under equilibrium using the normal rate of return of 5%:

17. Continued

$p = (rd-c_2)/(rd-r) = (5\%*5 - 10\%) / (5\%*5 - 5\%) = 15\%/20\% = .75$

Finally, the equilibrium price uses the analyst strategy with weight p and the random strategy with weight (1-p). Thus, today's price under equilibrium is:

75%*(Analyst 3 price today) + 25%*(Analyst 2 price today) = 75%*70+25%*80 = 72.5

18. Learning Objectives:

4. Efficient and Inefficient Markets, Complete and Incomplete Markets, Information Theory & Market Misbehavior

Learning Outcomes:

(4e) Define principal-agency theory and explain how it affects capital structure, portfolio management and risk management.

Sources:

Copeland, Weston, and Shastri, Chapter 12

Commentary on Question:

This question tests principal-agency theory and asks the candidate to show his/her understanding through a numerical question.

Solution:

(a) Describe the major sources of conflicts between

Commentary on Question:

Many candidates did well on this part.

(i) The company's managers and its stockholders

Choice of effort by managers: Managers may not give their best effort depending upon their ownership of the firm and their compensation structure.

Managers may choose suboptimal level of investment: Managers may make investment decisions that help diversify the firm but may not be in the best interest of shareholders.

Differential time horizons between managers and stockholders: Managers may put less weight on future cash flows occurring after their tenure with the company.

(ii) The company's stockholders and its bondholders

Dividend Payout:
If a firm unexpectedly changes its dividend payout and finances this increase by reducing the asset base or by reducing the planned investment, it may result in transfer of wealth from bondholders to shareholders.

Claim dilution:
If a firm chooses to issue new debt with equal or higher priority than existing debt, the claim of the existing debt holders is diluted.

18. Continued

Asset substitution:
The firm may pursue risky projects for a potentially higher payoff accruing to shareholders. This action reduces the value of its debt by making the debt payoff less certain / more risky.

Underinvestment:
The firm may pass up (under-invest in) a potentially good investment opportunity when the investment benefit accrues to bondholders, causing bondholders an opportunity loss.

(b) Construct Eau Claire Life's Balance Sheet (as in Table 1 above) as of 12/31/2011 if the company had paid a $20 million cash dividend to its shareholders from its $100 million cash holdings on 12/31/2011. Assume the discount rate of the zero coupon bond is unaffected by the dividend action.

Commentary on Question:
This is an application of "dividend conflict" between stockholders and bondholders as in (a)(ii).

Candidates did relatively poorly. Many candidates did not realize that dividend payout may impact the market value of the zero-coupon bond as of 12/31/2011 - they either showed a balance sheet with no change in zero-coupon bond value (which makes this question trivial) or tried to construct the balance sheet as of the zero-coupon bond maturity date, despite the question specifically asking for new balance sheet as of 12/31/2011 assuming the $20 million dividend had been paid.

Cash amount = 100 – 20 = 80
Invested assets = 500, no change due to dividend payout
Policy reserve = 450, no change due to dividend payout

Dividend payout may affect the market value of the 0-coupon bond due to reduced asset base. Need to derive the new market value of the 0-coupon bond.

Determine the bond payoff at maturity if Invested Asset is 570:
Available asset to pay off bond = 570 + 100 – 480 = 190 before dividend payout and 190 – 20 = 170 after dividend payout, so the bond face value of 140 will be paid in full in this scenario.

Determine the bond payoff at maturity if invested asset is 490:
Available asset to pay off bond = 490 + 100 – 480 = 110 before dividend payout and 110 – 20 = 90 after dividend payout. In this scenario, the bond will not be paid off in full even before dividend payout.

18. Continued

The expected payoff of the bond:
= 140*0.9 + 110 * 0.1 = 137 before dividend payout
= 140*0.9 + 90 * 0.1 = 135 after dividend payout

Since the market value of the bond before dividend payout = 120, this implies a present value discount factor = 120/137

Since the discount rate is unaffected by the dividend payout, this implies the new market value of the bond = 135 * 120/137 =118.25

Finally, the new Equity value = 600 – 20 – 450 – 118.25 = 11.75

The new balance sheet is shown below:

Assets (in $millions)		Liabilities and Equity ($millions)	
Cash	80	Policy Reserve	450
Invested Assets	500	0-coupon bond	118.25
Total	580	Equity	11.75

(c) Calculate the wealth transfer between bondholders and shareholders assuming the $20 million cash dividend payout had occurred.

Commentary on Question:
This is a follow-up from part (b). Candidates did relatively poorly.

Since many candidates didn't realize the zero-coupon bond value would change if dividend were paid, they tried to compute the expected wealth transfer at the zero-coupon bond maturity date instead of the current balance sheet date. The concept is: if there is an expected wealth transfer in the future, the present market value will and should reflect this expectation, which is the essence of part (b). Nevertheless, credit was given if the candidate demonstrated knowledge of the wealth transfer concept by correctly computing the expected wealth transfer using the incorrect zero-coupon bond maturity date.

There is a wealth transfer effect if the company paid dividend:
Shareholder gain = 11.75 – (30 – 20) = 1.75
Bondholders loss = 120 –118.25= 1.75