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**SOCIETY OF ACTUARIES**  
**Exam FETE**  
**Financial Economic Theory and Engineering Exam (Finance/ERM/Investment)**

**Exam FETE**  
**AFTERNOON SESSION**

**Date:** Thursday, November 4, 2010

**Time:** 1:30 p.m. – 4:45 p.m.

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**INSTRUCTIONS TO CANDIDATES**

**General Instructions**

1. This afternoon session consists of 9 questions numbered 10 through 18 for a total of 60 points. The points for each question are indicated at the beginning of the question.
2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.
3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

**Written-Answer Instructions**

1. Write your candidate number at the top of each sheet. Your name must not appear.

2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.
3. The answer should be confined to the question as set.
4. When you are asked to calculate, show all your work including any applicable formulas.
5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets since they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate morning or afternoon session for Exam FETE.
6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d'examen pour la version française.





**\*\*BEGINNING OF EXAMINATION\*\***  
**AFTERNOON SESSION**  
*Beginning with Question 10*

**The following material applies to questions 10 and 11.**

Carmenére Life Insurance Company has a significant block of in-force term life policies that have been very profitable. The Company has launched a highly successful new Universal Life product and projects that the large increase in sales and the front-loaded commissions on the product will cause significant surplus strain.

You are given the following financial information:

First Year Statutory Income Statement for the Universal Life Product (\$'000)	
Gross Premium	4,000
Investment Income	80
Commissions	2,400
Expenses	1,200
Death Benefits	320
Change in Reserves	2,400
Tax Rate	30%

Liabilities risk distribution:

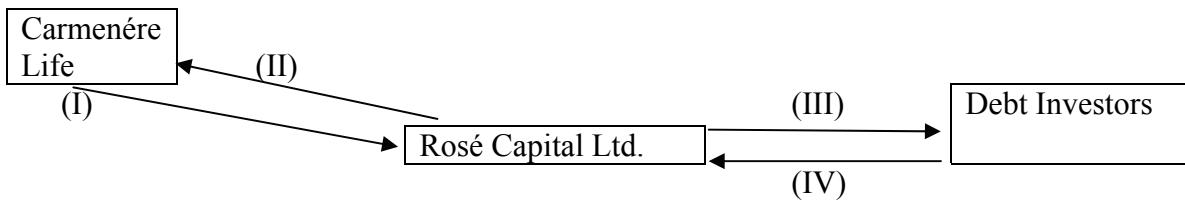
Universal Life Block		Term Life Block	
Loss (\$000)	Probability of Loss	Loss (\$000)	Probability of Loss
0	44.2%	0	44.2%
250	38.2%	0	38.2%
500	10.1%	0	10.1%
1,000	4.5%	2,000	4.5%
1,200	1.3%	4,000	1.3%
2,600	0.9%	5,000	0.9%
3,000	0.8%	10,000	0.8%

Assume the amount of capital Carmenére Life requires is based on 137.5% of CTE (97) and all risks are independent. The total capital available for Carmenére Life is \$13,000,000. Carmenére Life allocates required capital using the proportional allocation method. Free capital is allocated using the percentile layers method.

The Chief Actuary is reviewing different surplus enhancement solutions to address the surplus strain problem of the Universal Life block.

Solution 1: Obtain reinsurance for the Universal Life block. Carmenére Life will cede 30% of the Universal Life business on a modified coinsurance basis to Gewürztraminer Re (GRe). In return, GRe will provide a first year ceding allowance of 15% of ceded premium. First year surplus strain is defined as the negative net earnings after tax but before the cost of capital.

Solution 2: Carmenére Life is considering securitizing future cash flows from the term life block as a means of raising capital. The following mortality index bond structure is suggested.



If the cumulative adverse mortality exceeds 125% of the actual number of deaths in the mortality index pool of 2010, Carmenére Life would be permitted to withdraw proceeds from Rosé Capital Limited. The full amount of proceeds would flow to Carmenére Life if the cumulative adverse mortality reached 150% or more of the actual deaths of the mortality indexed pool in 2010, with proportionate payment from Rosé Capital for adverse mortality falling in between.

- I - \$6,700,000
- II - \$7,000,000

- 10.** (6 points) Calculate the total amount of free capital for Carmenére Life and allocate the free capital to each of the two products by the allocation-by-layers method.
  
  
  
  
  
  
- 11.** (9 points)
  - (a) (3 points) Determine the reduction in first year surplus strain in Solution 1.
  - (b) (1 point) Briefly describe the economic rationale for securitization.
  - (c) (3 points) Specific to Carmenére Life's Solution 2,
    - (i) Explain how the securitization of the Term Life in-force block would address Carmenére Life's Universal Life surplus strain problem.
    - (ii) Describe the cash flows (I, II, III, IV) and role each party plays.
    - (iii) Derive a function for the option payoff in terms of CM (Cumulative Mortality) and D (Actual deaths in the indexed pool of 2010).
  - (d) (2 points) Outline 2 or 3 advantages for both solutions and recommend a solution for Carmenére Life.

**12.** (*4 points*)

- (a) (*2 points*) Describe how investment risk is assessed by the capital adequacy requirements of the Canadian and U.S. regulatory capital regimes. Address:
- (i) Valuation methodologies
  - (ii) Capital formulas
  - (iii) Diversification benefits
- (b) (*2 points*)
- (i) Compare Solvency II investment risk regulatory requirements in Canada and the U.S.
  - (ii) Describe the impact of Solvency II on Canadian and U.S. insurer decisions on portfolio allocation.

**13.** (7 points) You are analyzing the risks under the guaranteed minimum maturity benefit (GMMB) rider for a block of variable annuity contracts. The GMMB was always thought of as a free rider and no hedging program was ever established. The terms of the GMMB contracts are as follows:

- 10-Year term
- Management charge:  $m = 0.15\%$  of the policyholder's fund per month, deducted at the beginning of the month
- The initial premium is \$250
- The maturity benefit guarantee is 80% of the initial premium
- There is no guaranteed minimum death benefit.

You assume that the stock accumulation each month follows an independent lognormal process with parameters  $\mu = 0.005$  and  $\sigma = 0.06$ , per month. Ignore mortality and lapse decrements. The risk-free rate of interest is 5% per year compounded continuously.

- (a) (2 points) Calculate the probability that the GMMB matures in the money.
- (b) (3 points) Calculate the 95% Value at Risk (VaR) for the present value of the GMMB liability at inception, showing that it falls within the range \$42 to \$48.

You have separately estimated CTE(95) of the present value at inception of the GMMB liability to be \$60. You have also calculated the value of the GMMB as an option at inception, using the assumptions above, at \$11.6.

The Chief Financial Officer of the firm says “The expected management charge income is almost exactly equal to the liability VaR for the contract, and since we have tens of thousands of different policies, we do not need any additional economic capital to support this risk.”

- (c) (2 points) Outline the points you would make in response to this comment.

- 14.** (7 points) Petit Verdot Inc., an all-equity financed viticulture company, expects to earn \$10 million before interest and tax (EBIT) per year in perpetuity. You are given the following:

Petit Verdot's current systematic risk	2
Risk free rate, annual	4%
Market risk premium	3%
Corporate income tax rate	35%

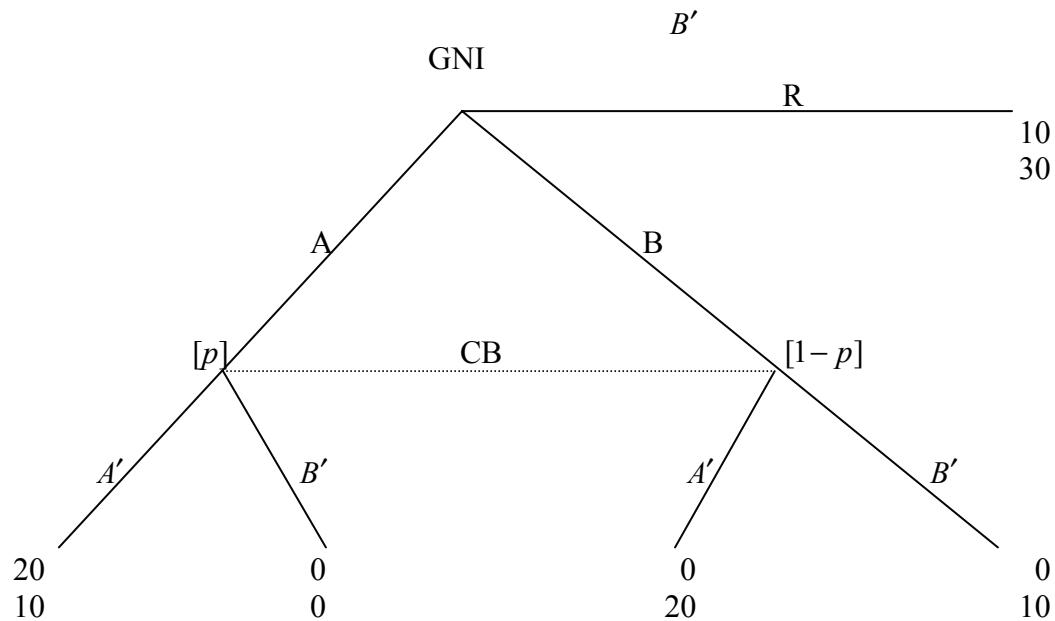
The Chief Financial Officer wants to explore the effects on the capital structure under various levels of debt financing, with business disruption costs and lost interest tax shields taken into account. The following financial information for the various debt financing scenarios is provided:

Value of debt	Cost of debt	Present value of \$1 contingent on future business disruption	Business Disruption Cost
\$6,500,000	6.00%	0.20	\$275,000
\$8,500,000	6.25%	0.25	\$300,000
\$10,500,000	6.50%	0.30	\$325,000
\$12,500,000	6.75%	0.35	\$400,000

- (a) (1 point) Determine the value of Petit Verdot Inc.
- (b) (3 points) Calculate under the four different debt scenarios:
  - (i) Expected value of the tax benefit from the deductibility of interest payments
  - (ii) Expected present value of the business disruption costs
  - (iii) Petit Verdot's weighted average cost of capital.
- (c) (3 points) Evaluate the benefits of the four debt scenarios in terms of optimal capital structure and recommend the best alternative for Petit Verdot.

- 15.** (6 points) Gamay Noir Inc. (GNI) is considering whether to enter negotiations with one of its key unions, Chenin Blanc (CB). GNI may unilaterally decide to enter or reject the contract negotiations but must decide on strategy A or B once negotiations are entered. CB will know whether GNI decides to enter or reject negotiations but will not know GNI's negotiation strategy before deciding on its own strategy  $A'$  or  $B'$ . CB believes that GNI has the probability  $p$  of selecting strategy A and the probability  $[1-p]$  of selecting B.

The following figure illustrates the situation:



- (a) (1 point) Show the normal form representation of this game.
- (b) (1 point) Define a Nash Equilibrium.
- (c) (1 point) Identify all pure-strategy Nash Equilibria for this game.
- (d) (1 point) Calculate CB's expected payoff for  $A'$  and  $B'$ .

Assume that GNI does not choose R.

- (e) (1 point) Determine the impact on CB's strategy.
- (f) (1 point) Determine the impact, if any, to the Nash Equilibria.

## **16.** (9 points)

- (a) (1 point) Define in words the five major Greeks in the equity option world:
- (i) Delta
  - (ii) Gamma
  - (iii) Vega
  - (iv) Rho
  - (v) Theta
- (b) (2 points) Determine whether these Greeks are positive, negative or zero for stocks, European calls, and European puts.
- (c) (2 points) You hold a trading book consisting of many long/short positions in Tempranillo Corp. stock and options. You are analyzing two possible scenarios for Tempranillo Corp. stock and want to make money by adjusting your trading book.

Scenario	Market Conditions
1	Swift downward price movement; rising implied volatility
2	No price movement; falling implied volatility

For each of the above scenarios, determine whether a positive or negative delta, gamma, or vega in your trading book would produce a profit.

Now consider the table below which summarizes your positions at the prior day's close of trading:

Option	Position (# of units)	Market Value	\$delta/\$	\$gamma/\$	\$theta/day	\$vega/point
Call totals	-600		-736	-19.6	+0.5	-0.8
Put totals	400		-618	+2.2	+0.2	-0.1
Stock	1,200		1,200	0.0	0.0	0.0
Total			-154	-17.4	+0.7	-0.9

Where \$delta/\$ is the change in the \$ value of your portfolio per \$1 change in the underlying asset or index.

Today Tempranillo Corp.'s stock price dropped \$4 and implied volatility on all options rose 2 percentage points.

## **16. Continued**

- (d) (*3 points*) Estimate the value change in your trading book at closing today.
- (e) (*1 point*) Explain how the value of your trading book will change if Tempranillo Corp. stock falls 50%.

- 17.** (7 points) You are evaluating the following three long-term equity models for applications in the risk management of investment guarantees embedded in insurance products.

Model 1: A GARCH(1,1) model.

Model 2: A Regime-Switching Lognormal model with two regimes (RSLN-2)

Model 3: A Regime-Switching Draw-Down model with two regimes (RSDD-2)

- (a) (2 points) Describe the volatility process in words for each of these models, explaining how the process is stochastic.

You are given the maximum log-likelihoods and the  $p$ -values of the Jarque-Bera test for the three models, which have been fitted to 380 monthly observations of the index.

	Maximum log-likelihood	$p$ -value of the Jarque-Bera test
GARCH(1,1)	651.3	<0.001
RSLN-2	663.2	0.405
RSDD-2	664.3	0.612

- (b) (2 points) Determine which model provides the best overall fit to the equity index in question, based on this information. Justify your selection.

The fitted parameters of the GARCH(1,1) model and RSLN-2 models, per month, are:

	$\mu$	$\alpha_0$	$\alpha_1$	$\beta$
GARCH(1,1)	0.0064	0.00692 <sup>2</sup>	0.121	0.869

	$\mu_1$	$\mu_2$	$\sigma_1$	$\sigma_2$	$p_{12}$	$p_{21}$
RSLN-2	0.013	-0.029	0.035	0.070	0.045	0.245

## **17. Continued**

- (c) (3 points)
- (i) Calculate the long run volatility for this GARCH model expressed as an annual rate.
  - (ii) Your colleague calculates the long run volatility for the RSLN model to be 14.7% per year. She states:

“If the long run GARCH volatility is bigger than the long run RSLN-2 volatility, it would be more prudent to use the GARCH model for measuring the potential liability under our Guaranteed Minimum Maturity Benefit.”

Outline the points you would make in reply. You should state whether or not you agree with your colleague. You should assume that her calculation is correct.

- 18.** (5 points) The head of Asset-Liability Management at Chardonnay Insurance company, Amy, is using a 1-factor LIBOR market model to compute the value of a 1-year interest rate cap struck at 3%, which resets every 3 months and makes a payment at the end of each period based on the rates at the beginning of that period. The initial yield curve is flat at 3%.

Amy simulates a LIBOR Market Model. The following table contains the outcome from a single scenario:

	LIBOR Forward Rates At Forward Fixing Time			
Time	0.00	0.25	0.50	0.75
0.00	3.000%	3.000%	3.000%	3.000%
0.25		2.873%	2.887%	2.902%
0.50			3.1349%	3.138%
0.75				3.047%

- (a) (2 points) Calculate the payoff of the 1-year interest rate cap with a \$500 million notional using the outcome from this scenario.

Amy has now done 1,000 simulations of the cap payoffs with this model and is considering using them to:

- (i) Estimate the price of the cap
  - (ii) Determine the error of the estimated price
  - (iii) Calculate the sensitivity of the price to movement in the short rate
  - (iv) Calculate CTE(98)
- (b) (3 points) Evaluate the appropriateness of using these simulations for each of applications (i) through (iv), and, when appropriate, describe how to proceed.

**\*\*END OF EXAMINATION\*\***

**Afternoon Session**

**USE THIS PAGE FOR YOUR SCRATCH WORK**

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