# Exam M Actuarial Models

# Fall 2009

## Exam M Actuarial Models

Exam M consists of five and a half hours of multiple-choice questions offered in two independent segments. Each segment will be graded separately. In addition, a candidate will not be required to take both segments during the same exam administration period.

The Life Contingencies Segment (Exam MLC) is a three-hour multiple choice examination. The examination is jointly sponsored and administered by the SOA and the Canadian Institute of Actuaries (CIA). The examination is also jointly sponsored by the American Academy of Actuaries (AAA) and the Conference of Consulting Actuaries (CCA).

The Financial Economics Segment (Exam MFE) is a two and a half hour multiple-choice examination and is identical to CAS Exam 3F. The examination is jointly sponsored and administered by the SOA, CAS and the Canadian Institute of Actuaries (CIA). The examination is also jointly sponsored by the American Academy of Actuaries (AAA) and the Conference of Consulting Actuaries (CCA).

The purpose of each segment's syllabus is to develop the candidate's knowledge of the theoretical basis of certain actuarial models and the application of those models to insurance and other financial risks. A thorough knowledge of calculus, probability, and interest theory is assumed. Knowledge of risk management at the level of Exam P is also assumed. In addition, for Exam MFE, candidates are assumed to be familiar with the earlier chapters of the McDonald textbook, which are in the syllabus of Exam FM.

Tables are provided below for the candidate and will be provided to the candidate at the examination. For Exam MLC these include a table of values for the standard normal distribution and illustrative life tables. For Exam MFE these include a table of values from the standard normal distribution and the density and distribution functions for the standard normal and lognormal random variables. Since the tables for Exam MLC and Exam MFE will be provided to the candidate at the examination, candidates will not be allowed to bring copies of the tables into the examination room.

Note: It is anticipated that candidates will have done the relevant exercises in the textbooks.

Check the Updates section of the SOA Web site for any changes to the exam or syllabus.

## LEARNING OUTCOMES – LIFE CONTINGENCIES SEGMENT

- A. Survival models
  - 1. Define survival-time random variables
    - a) for one life, both in the single- and multiple-decrement models;
    - b) for two lives, where the lives are independent or dependent (including the common shock model).
  - 2. Calculate the expected values, variances, probabilities, and percentiles for survival-time random variables.

- 3. Define the continuous survival-time random variable that arises from the discrete survival-time random variable using a:
  - a) uniform distribution;
  - b) constant force of mortality; or
  - c) hyperbolic assumption.
- B. Markov Chain Models
  - 1. Define non-homogeneous and homogeneous discrete-time Markov Chain models and calculate the probabilities of
    - a) being in a particular state;
    - b) transitioning between particular states.
- C. Life insurances and annuities
  - 1. Define present-value-of-benefit random variables defined on survival-time random variables:
    - a) for one life, both in the single- and multiple-decrement models;
    - b) for two lives, where the lives are independent or dependent (including the common shock model).
  - 2. Define and calculate the expected values, variances and probabilities for:
  - a) present-value-of-benefit random variables;
    - b) present-value-of-loss-at-issue random variables, as a function of the considerations (premiums);and
    - c) present-value-of-loss random variables, as a function of the considerations (premiums).
  - 3. Calculate considerations (premiums) for life insurances and annuities,
    - a) using the Equivalence Principle; and
    - b) using percentiles.
  - 4. Calculate liabilities, analyzing the present-value-of-future-loss random variables: a) using the prospective method;
    - b) using the retrospective method;
    - c) using special formulas.
  - 5. Calculate
    - a) gross considerations (expense-loaded premiums);
    - b) expense-loaded liabilities (reserves);
    - c) asset shares.
  - 6. Using recursion, calculate expected values (reserves) and variances of presentvalue-of-future-loss random variables for general fully-discrete life insurances written on a single life.
  - 7. Extend the present-value-of-benefit, present-value-of-loss-at-issue, present-valueof-future-loss random variables and liabilities to discrete-time Markov Chain models, to calculate
    - a) actuarial present values of cash flows at transitions between states;
    - b) actuarial present values of cash flows while in a state;
    - c) considerations (premiums) using the Equivalence Principle;
    - d) liabilities (reserves) using the prospective method.
- D. Poisson processes
  - 1. Define Poisson process and compound Poisson process.
  - 2. Define and calculate expected values, variances, and probabilities for Poisson processes,
    - a) using increments in the homogeneous case;
    - b) using interevent times in the homogeneous case;
    - c) using increments in the non-homogeneous case.
- Note: Co ncepts, principles and techniques n eeded for E xam MLC are covere d in the references li sted below. Can didates and pr ofessional ed ucators may use othe r

references, but candidates should be very familiar with the notation and terminology used in the listed references.

## Textbooks - Life Contingencies Segment \*

#### **OPTION A**

• Actuarial Mathematics (Second Edition), 1997, by Bowers, N.L., Gerber, H.U., Hickman, J.C., Jones, D.A. and Nesbitt, C.J.,

Chapter 3, Chapter 4, Sections 4.1–4.4, Chapter 5, Sections 5.1–5.4, Chapter 6, Sections 6.1(excluding utility-theory approach), 6.2–6.4, Chapter 7, Section 7.1(excluding utility-theory approach), 7.2–7.6, Chapter 8, Sections 8.1–8.4, Chapter 9, Sections 9.1–9.5, 9.6.1, 9.7, 9.9, Chapter 10, Sections 10.1–10.4, 10.5–10.5.1, 10.5.4, 10.6 Chapter 11, Sections 11.1–11.3, Chapter 15, Sections 15.1–15.2.1, 15.4, 15.6–15.6.1.

## **OPTION B**

• *Models for Quantifying Risk,* Third Edition, 2009, by Cunningham, R., Herzog, T. and London, R.L.

Chapters 3–9 Chapter 10, excluding Section 10.7

**NOTE:** Candidates may also use the second Edition of *Models for Quantifying Risk.* The same chapter references apply.

#### \*Any textbook errata are included below

## Study Notes - Life Contingencies Segment Code Title

MLC-24-05	Multi-State Transition Models with Actuarial Applications (Second printing
	with minor corrections, October 2007)
MLC-25-05	Section 8.5 from the second printing of Actuarial Mathematics, Second
	Edition (to be used with text option A only) Second Printing
MLC-28-08	Poisson Processes (and mixture distributions)

## Other Resources

Exam MLC Tables

Candidates using the Second Edition of *Models for Quantifying Risk* will need to supplement the text with the Errata Package available on the Actex web site <u>www.actexmadriver.com</u>

<u>Notational differences</u> between *Actuarial Mathematics (AM)* and *Models for Quantifying Risk* (MQR) for candidates taking MLC

All released exam papers, since 2000 can be found here.

Exam MLC Sample Questions and Solutions

## LEARNING OUTCOMES – FINANCIAL ECONOMICS SEGMENT

A. Interest rate models

- 1. Evaluate features of the Vasicek and Cox-Ingersoll-Ross bond price models.
- 2. Explain why the time-zero yield curve in the Vasicek and Cox-Ingersoll-Ross bond price models cannot be exogenously prescribed.
- 3. Construct a Black-Derman-Toy binomial model matching a given time-zero yield curve and a set of volatilities.
- B. Rational valuation of derivative securities
  - 1. Use put-call parity to determine the relationship between prices of European put and call options and to identify arbitrage opportunities.
  - 2. Calculate the value of European and American options using the binomial model.
  - 3. Calculate the value of European and American options using the Black-Scholes option-pricing model.
  - 4. Interpret the option Greeks.
  - 5. Explain the cash flow characteristics of the following exotic options: Asian, barrier, compound, gap, and exchange.
  - 6. Explain the properties of a lognormal distribution and explain the Black-Scholes formula as an expectation with respect to a lognormal distribution.
  - 7. Explain what it means to say that stock prices follow a diffusion process.
  - 8. Apply Itô's lemma in the one-dimensional case.
  - 9. Apply option pricing concepts to actuarial problems such as equity-linked insurance.

#### C. Simulation

- 1. Simulate lognormal stock prices.
- 2. Use variance reduction techniques to accelerate convergence.
- D. Risk management techniques
  - 1. Explain and demonstrate how to control risk using the method of delta-hedging.
- **Note**: Concepts, principles and techniques needed for Exam MFE are covered in the reference listed below. Candidates and professional educators may use other references, but candidates should be very familiar with the notation and terminology used in the listed references.

## Texts – Financial Economics Segment \*

Derivatives Markets (Second Edition), 2006, by McDonald, R.L.,

Chapter 9,

Chapter 10, (excluding "Options on Commodities" on page 334),

Chapter 11, Sections 11.1 – 11.4, Appendices 11.A and 11.B,

Chapter 12, Sections 12.1–12.5, Appendix 12.A,

Chapter 13, including Appendix 13.B,

Chapter 14,

Chapter 18,

Chapter 19, Sections 19.1–19.5

Chapter 20, Sections 20.1–20.6 (up to but excluding "Multivariate Itô's Lemma" on pages 665-666) and 20.7 (up to but excluding "Valuing a Claim on S<sup>a</sup>Q<sup>b</sup> on pages 670-672 and excluding "Finding the lease rate" on top one-half of page 669),

Chapter 21, Sections 21.1 - 21.2 (excluding "What If the Underlying Asset Is Not an Investment Asset" on pages 688-690) and 21.3 (excluding "The Backward Equation" on pages 691-692, and excluding the paragraph on page 692 that begins "If a probability..." and through the end of the section),

Chapter 22, Section 22.1 (but with only those definitions in Tables 22.1 and 22.2 that are relevant to Section 22.1),

Chapter 23, Sections 23.1 - 23.2 (up to but excluding "Exponentially Weighted Moving Average" on page 746 and through the end of the section),

Chapter 24, Sections 24.1–24.5 (up to but excluding "Forward rate agreements" on pages 806-808),

Appendix B.1, Appendix C and including relevant Errata (see below).

Unless otherwise stated chapter appendices are not included in the required readings from this text.

## \*Any textbook errata are included below.

## **Other Resources - Financial Economics Segment**

Exam MFE/Exam 3F Tables

Some Remarks on Derivatives Markets

*Derivatives Markets, Errata* 2006 Second Edition, by R. McDonald, <u>http://www.kellogg.northwestern.edu/faculty/mcdonald/htm/typos2e.html</u>

<u>All released exam papers</u>, since 2000 can be found here.

Exam MFE/3F Sample Questions and Solutions (1-60)