

DP-IU Complete Illustrative Solutions

Fall 2009

1. Learning Outcomes:

4A. Describe in detail the basic benefit/product types and their uses,

4B. Recommend designs for products and features to reach particular goals

1A. Describe the steps in the control cycle process within the context of product development;

iv. Actuarial Development

- Assumptions
- Profit Measures
- Regulatory Issues
- Choice of Model

5B Select appropriate assumptions for a given product design, reflecting

- i) product characteristics
- ii) available experience data
- iii) the marketplace
- iv) underwriting
- v) distribution channel characteristics

Solution:

(a) Explain how management's proposal would commit or avoid the common pricing mistakes identified by Atkinson and Dallas:

- It is not appropriate to use the current assumptions for a new product.
- Recommend including policyholder behavior
- Policyholder behavior can be very different from experience
- Should not develop a product using educated guesses for key assumptions
- Assumptions have not accounted for the effect of new target market, distribution system or economic changes
- Not accounting for rational buyer or seller behavior
- Not understanding cost of options a GMWB gives to the owner
- Not accurately reflecting accounting guidelines, reserve standards, taxes, capital, etc. when company tries to get to market too quickly

(b) (i) Recommend changes to the pricing assumptions for lapses and partial withdrawals to reflect the addition of the new benefit. Justify your recommendation
Partial WD's

- Greater and earlier utilization of partial withdrawals
- More withdrawals leads to less assets which lowers profitability
- Recommend adding a dynamic partial withdrawal function
- As in-the-money (ITM) increases, WD activity should increase

Lapses

1. Continued

- GMWB will change the lapse rates
- Existence of GMWB, especially if ITM, may decrease lapse rates
- If market increases, GMWB would not materially affect lapse rates
- Since GMWB is new, spike lapse and ultimate lapse are based on less experience
- Presence of GMWB may delay spike lapse after end of surrender charge period
- Higher lapse rates decrease cost of guarantee (i.e. lapse supported)
- Recommend using dynamic lapse formula
- Can be one sided (only decreases lapse rate) or two sided (increases lapses when out of the money)
- Sample formula: Lapse factor = $\text{Max}[50\%, \text{Min}(100\%, 200\% - \text{GMWB}/\text{AV})]$

(b) (ii) Explain the potential impact of the 2008 Market and Economic events, as described in the case study, on the cost of hedging the new benefit.

- Market volatility increases the cost of the hedging strategy
 - Current environment: higher volatility needs to be included in pricing benefit
- Reinsurers may not completely cover the tail risk
 - Current environment: reinsurers may further tighten their GMWB offerings
- Exposed to risk of reinsurer insolvency or other counterparty risks
 - Current environment: heightens this risk since several financial institutions have failed

(c) Explain the value of using cohorts in pricing this benefit.

- Cohorts allow business to be separated into different reasons policyholders take withdrawals
 - Cohort A – never withdraws
 - Cohort B – withdraw at retirement
 - Cohort C – withdraw immediately
 - Cohort D – withdraw when ITM
- Allows for a better understanding of profitability by incorporating policyholder behavior

(d) Calculate the GMWB after a withdrawal of 37,500 under a:

- Proportionate withdrawal method
- Dollar-for-dollar method

50% reduction in Av

Original GMWB is 5% of 50k000 = \$2,500

(i) Proportionate: $\$2,500 * 50\% = \$1,250$

(ii) Dollar for dollar: $(\$50,000 - \$37,500) * 5\% = \$625$

2. Learning Outcome:

5H Analyze array of profit measures

5I Determine statutory vs. GAAP profit measures

Solution:

(a)

(i) Identify factors that may cause permanent and timing differences

Permanent Differences

- Investment income may be nontaxable or taxed at a lower rate, such as stock dividends, capital gains, municipal bond interest
- Expenses may have tax credits available, such as research & development
- Expenses may not be deductible, such as business entertainment

Temporary Differences

- Treatment for unrealized & realized capital gains & losses
- Carrying value for bonds, mortgages, real estate, other assets
- Amortization for goodwill, deferred acquisition costs, other assets
- Treatment for uncollected amounts
- Assumptions and calculations for reserves
- Calculations for claim liabilities
- Timing for incurring policyholder dividend liabilities
- Liabilities not recognized for tax purposes, such as reserves to stabilize investment returns or offset future capital losses, other contingency reserves, required capital.

(ii) Calculate the permanent differences in the first three years. Show all work.

$$(9.2.6) \text{ PermDiff}(t) = -\text{InvIncome}(t) * \text{NonTaxInvPct}(t)$$

$$\text{NonTaxInvPct}(t) = 1 - \text{TaxInvPct}(t) = 1 - 0.40 = 0.60$$

$$\text{PermDiff}(1) = -20 * 0.60 = -12$$

$$\text{PermDiff}(2) = -15 * 0.60 = -9$$

$$\text{PermDiff}(3) = -10 * 0.60 = -6$$

(iii) Calculate the permanent differences in the first three years. Show all work.

$$(9.2.5) \text{ TimingDiff}(t) = \text{SolvResIncr}(t) - \text{TasResIncr}(t)$$

$$\text{TimingDiff}(1) = (1400 - 0) - (1300 - 0) = 1400 - 1300 = 100$$

$$\text{TimingDiff}(2) = (1200 - 1400) - (1150 - 1300) = -200 + 150 = -50$$

$$\text{TimingDiff}(3) = (1000 - 1200) - (950 - 1150) = -200 + 200 = 0$$

2. Continued

(b) Calculate the deferred tax liability at the end of the third year using the two approaches described by Atkinson and Dallas.

Approach 2:

$$(9.6.6) \text{ DefTaxLiab}(t) = (\text{TaxRes}(t) - \text{EarnRes}(t)) * \text{EarnTaxRate}(t)$$

$$\text{EarnRes}(t) = \text{BenRes}(t) - \text{DACAmort}(t) = \text{BenRes}(t), \text{ since } \text{DACAmort}(t) = 0$$

$$\text{DefTaxLiab}(3) = (950 - 1200) * 0.35 = -87.5$$

Approach 1:

$$(9.6.3) \text{ DefTaxLiab}(t) = \text{DefTaxLiab}(t-1) + \text{DefTaxProv}(t)$$

$$(9.6.2) \text{ DefTaxProv}(t) = \text{AccruedTax}(t) - \text{TaxOnEarn}(t)$$

$$(9.6.1) \text{ AccruedTax}(t) = (\text{PreTaxStockEarn}(t) + \text{PermDiff}(t)) * \text{EarnTaxRate}(t)$$

$$(9.2.4) \text{ PreTaxStockEarn}(t) = \text{ProdCashFlow}(t) + \text{InvIncome}(t) - \text{BenResIncr}(t) - \text{DACAmort}(t)$$

$$(9.2.10) \text{ TaxOnEarn}(t) = \text{TaxableEarn}(t) * \text{EarnTaxRate}(t)$$

$$(9.2.7) \text{ TaxableEarn}(t) = \text{ProdCashFlow}(t) + \text{InvIncome}(t) - \text{TaxResIncr}(t) + \text{PermDiff}(t)$$

$$\text{TaxableEarn}(1) = 1400 + 20 - (1300 - 0) - 12 = 108$$

$$\text{TaxableEarn}(2) = 200 + 15 - (1500 - 1300) - 9 = 356$$

$$\text{TaxableEarn}(3) = 200 + 10 - (950 - 1150) - 6 = 404$$

$$\text{TaxOnEarn}(1) = 108 * 0.35 = 37.8$$

$$\text{TaxOnEarn}(2) = 356 * 0.35 = 124.6$$

$$\text{TaxOnEarn}(3) = 404 * 0.35 = 141.4$$

$$\text{PreTaxStockEarn}(1) = 1400 + 20 - (1200 - 0) - 0 = 220$$

$$\text{PreTaxStockEarn}(2) = 200 + 15 - (1200 - 1200) - 0 = 215$$

$$\text{PreTaxStockEarn}(3) = 200 + 10 - (1200 - 1200) - 0 = 210$$

$$\text{AccruedTax}(1) = (220 - 12) * 0.35 = 72.8$$

$$\text{AccruedTax}(2) = (215 - 9) * 0.35 = 72.1$$

$$\text{AccruedTax}(3) = (210 - 6) * 0.35 = 71.4$$

$$\text{DefTaxProv}(1) = 72.8 - 37.8 = 35.0$$

$$\text{DefTaxProv}(2) = 72.1 - 124.6 = -52.5$$

$$\text{DefTaxProv}(3) = 71.4 - 141.4 = -70.0$$

$$\text{DefTaxLiab}(1) = 0.0 + 35.0 = 35.0$$

$$\text{DefTaxLiab}(2) = 35.0 - 52.5 = -17.5$$

$$\text{DefTaxLiab}(3) = -17.5 - 70.0 = -87.5$$

2. Continued

Proof (not part of question) that Approach 1 = Approach 2 in this case, substituting:

$$\text{AccruedTax}(t) = (\text{ProdCashFlow}(t) + \text{InvIncome}(t) - \text{BenResIncr}(t) - \text{DACAmort}(t) + \text{PermDiff}(t)) * \text{EarnTaxRate}(t)$$

$$\text{TaxOnEarn}(t) = (\text{ProdCashFlow}(t) + \text{InvIncome}(t) - \text{TaxResIncr}(t) + \text{PermDiff}(t)) * \text{EarnTaxRate}(t)$$

$$\text{DefTaxProv}(t) = \text{AccruedTax}(t) - \text{TaxOnEarn}(t)$$

With $\text{DACAmort}(t) = 0$, and crossing off terms that cancel each other:

$$\text{DefTaxProv}(t) = (-\text{BenResIncr}(t) - -\text{TaxResIncr}(t)) * \text{EarnTaxRate}(t)$$

$$\text{DefTaxProv}(t) = (\text{TaxResIncr}(t) - \text{BenResIncr}(t)) * \text{EarnTaxRate}(t)$$

$$\text{DefTaxLiab}(3) = ((\text{TaxRes}(3) - \text{TaxRes}(0)) - (\text{BenRes}(3) - \text{BenRes}(0))) * \text{EarnTaxRate}(t)$$

With $\text{TaxRes}(0) = \text{BenRes}(0) = 0$, and $\text{EarnTaxRate}(t)$ always the same

$$\text{DefTaxLiab}(t) = (\text{TaxRes}(t) - \text{BenRes}(t)) * \text{EarnTaxRate}(t), \text{ or formula for Approach 2.}$$

(c)

(i) Describe the conditions under which tax loss carry forwards are created. Increase tax loss carry forward amount when tax losses are not offset by tax gains

- Pay zero taxes that year & increase tax loss carry forward by the loss amount
- Gains from profitable products offset losses from unprofitable ones
- Match gains & losses within same tax consolidation group
- Large companies have many gains/losses to offset, while small/fast growing or new companies may build substantial tax loss carry forward amount

Decrease tax loss carry forward amount as future tax gains occur

- Starts and ends at zero

(ii) Analyze their impact on solvency and stockholder earnings. Solvency earnings are based on currently payable taxes

- Therefore tax loss carry forward has an impact on solvency earnings

Stockholder earnings are based on accrued taxes

- Therefore tax loss carry forward has no impact on stockholder earnings

3. Learning Outcomes:

5D. Incorporate expenses (fixed, variable, marginal) and taxes (income and premium) into product pricing

Solution:

- (a) Contrast the two common philosophies used to set pricing assumptions for expenses.
- Fully allocated expenses
 - When pricing, the company distributes all the company's expenses (including overhead) to the expenses used in pricing each product
 - One should be able to reproduce total company expenses by multiplying each expense rate by the appropriate expense units
 - Disadvantage of full cost pricing
 - Pricing Actuary selects the quantities they will sell and the customers they will serve, before identifying the price they can charge
 - This results in the development of a cost-plus pricing strategy that may not be what the customer or distribution is willing to sell
 - Often results in the inability to field competitive prices.
 - Marginal allocated expense
 - When pricing, only expenses relevant to pricing decision are allocated
 - Expenses that would be affected by changes in sales levels would be considered relevant or marginal for pricing decisions.
 - Often leads to more competitive prices
 - Disadvantages of full cost pricing
 - Overlook opportunity cost
 - Treating a single cost as either all relevant or all irrelevant for pricing
 - Averaging total variable costs when estimating the cost of a single unit
 - Relevant costs must be based on expected changes in market value, not book value
- (b) Explain how the choice of expense allocation methodology will affect product pricing when reserves are determined using a Principles-Based Approach.
- Principles Based Reserve: Subjective determination of reserves, attempting to more accurately reflect the real risks that face the company
 - Based on actuary's judgment as to best estimate experience plus a MfAD
 - Principles-based reserves will be introduced prospectively; will only apply only to new business issued after 1/1/2009
 - The existing business will be valued using the old prescribed NAIC basis (formula based)
 - The dual approach will create expense allocation issues
 - PBA reserve will be sensitive to choice of expense allocation methodology

3. Continued

- Pricing for new business premiums will be affected by the reserve levels held under PBR

(c) Must find initial Profit Margin

$$\text{Profit Margin} = \text{PV Profits} / \text{PV Premiums}$$

$$\begin{aligned} \text{PV Profits} &= (\text{FYP} + \text{PV RenewPems}) * (1 - \text{Premtax}\%) - \text{FYC} - \\ &\text{TrailComms} - \text{VarAcqExp} - \text{Fixed AcqExp} - \\ &\text{VarMaintExp} - \text{FixedMaintExp} - \text{Benefits} = (500,000 + \\ &1,200,000) * (1 - 0.02) - 50,000 - 6,000 - 50,000 - \\ &500,000 - 20,000 - 200,000 - 800,000 = 40,000 \end{aligned}$$

$$\begin{aligned} \text{PV Premiums} &= \text{FYP} + \text{PV RenewPems} \\ &= 500,000 + 1,200,000 \\ &= 1,700,000 \end{aligned}$$

$$\text{Profit Margin} = 40,000 / 1,700,000 = 2.353\%$$

Solve for $(1 + y)$ such that the profit margin remains the same while reflecting the 5% premium rate reduction

$$\text{Profit margin Old} = \text{PV Profits}^{\text{New}} (\text{Adjusted}) / \text{PV Premiums}^{\text{New}} (\text{Adjusted})$$

$$\begin{aligned} \text{PV Profits}^{\text{New}} (\text{adj}) &= (\text{FYP} + \text{PV RenewPems}) * (1 - \text{Premtax}\%) * (1 + y) * (1 - 0.05) \\ &- (\text{FYC} + \text{TrailComms}) * (1 + y) * (1 - 0.05) - \text{VarAcqExp} * (1 + y) \\ &- \text{FixedAcqExp} - \text{VarMaintExp} * (1 + y) - \text{FixedMaintExp} - \text{Benefits} * (1 + y) \\ &= 1,582,700 * (1 + y) - 53,200 * (1 + y) - 50,000 * (1 + y) - 500,000 - 20,000 * \\ &(1 + y) - 200,000 - 800,000 * (1 + y) \\ &= 659,500 * (1 + y) - 700,000 \end{aligned}$$

$$\begin{aligned} \text{PV Premiums}^{\text{New}} (\text{Adj}) &= \text{FYP} + \text{PV RenewPems} * (1 - 0.05) * (1 + y) \\ &= 1,615,000 * (1 + y) \end{aligned}$$

$$0.02353 = (659,500 * (1 + y) - 700,000) / 1,615,000 * (1 + y)$$

$$(1 + y) = 1.1263$$

$$y = 12.63\% \text{ or the volume increase is } 12.63\%$$

4. Learning Outcomes:

5G. Explaining data quality

5B. Select appropriate assumptions for a given product design, reflecting

i) available experience data

Solution:

(a) Data Quality

- Selection of data
 - Data good for use?
 - What data to use?
 - Data credibility?
 - Internal data vs. external data

- Reliance of data
 - Reliance on others
 - Reliance on others for the use of data

- Review of data
 - Check consistency of data
 - Actuary's liability is limited
 - Do not have to audit the data

- Document anything important about the data
 - Judgment
 - Limitations of the data
 - Any unresolved problems

(b)

Policy #	Face(\$000)	Exposure period	Exposure	Face Exposure
1	1000	7/1 – 12/31	0.5	500
2	500	1/1 – 6/30	0.5	250
3	250	4/1 – 9/30 (death)	0.75	187.5
4	750	4/1 – 12/31	0.75	562.5
5	100	1/1 – 3/31	0.25	25
6	2000	1/1 – 6/30 (death)	1	2000
7	150	3/1 – 6/1	0.25	37.5
Other 1500	2,000,000	1/1 – 12/31	1	2,000,000
		Total	1504	2,003,625

Actual deaths by count = 2

Actual deaths by amount = 2,250,000

Expected by count = $1504 * 2/1,000 = 3.008$

Expected by amount = $2,003,625,000 * 2/1,000 = 4,007,250$

A/E by count = $2/3.008 = 66.5\%$

4. Continued

$$A/E \text{ by amount} = 2,250,000/4,007,250 = 56.1\%$$

Higher face amounts have lower mortality and higher weight for ratios by amount. Therefore not unusual to have lower ratios by amount than by count.

Recommend the face amount approach

Face amount represents the financial impact of the loss and is more common

(c) Assume x = extra lapse %

$$\text{Mort}^{\text{actual}} = \frac{(1 - \text{lapse}^{\text{norm}} - \text{lapse}^{\text{nonsel}}) * \text{Mort}^{\text{norm}} - \text{lapse}^{\text{select}} * \text{mort}^{\text{select}}}{(1 - \text{lapse}^{\text{norm}} - \text{lapse}^{\text{extra}})}$$

$$\text{Lapse}^{\text{select}} = 0.8 * \text{lapse}^{\text{extra}}$$

$$\text{Lapse}^{\text{nonsel}} = 0.2 * \text{lapse}^{\text{extra}}$$

$$2.1 = \frac{(1 - 5\% - 0.2x) * 2 - 0.8x * 1}{(1 - 5\% - x)}$$

$$0.95 * 2.1 - 2.1x = 0.95 * 2 - 0.4x - 0.8x$$

$$0.095 = 0.9x$$

$$x = 10.56\%$$

Maximum lapse rate cannot exceed normal rate plus extra

$$\text{Normal} + \text{extra} = 5\% + 10.56\% = 15.56\%$$

5. Learning Outcomes:

2B. Analyze:

- how the forces of the economy and demographics impact product design
- consumer behavior impact product design.
- how the forces of the marketplace and competition impact product design.

5H. Analyze array of profit measures

5D. Incorporate expenses (fixed, variable, marginal) into product pricing

5F. Identify the most influential assumptions and price products for the particular feature including:

- ii) Equity linked
- iii) Embedded Options

5N. Interpret results of a stochastic model

Solution:

$$B = \text{PV of BS put options for 3 years} = \sum_{t=1, 2, 3} \text{BSP0}(t) \times t - 1pT50 \times 1qd50, t-1$$

$$\text{Put option cost for year one} = \text{BSP0}(1) \times 1qd50 = 0.1 \times 0.002 = 0.0002$$

$$\begin{aligned} \text{Put option cost for year two} &= \text{BSP0}(2) \times 1pT50 \times 1qd51 = 0.12 \times 0.92 \times 0.0025 \\ &= 0.000276 \end{aligned}$$

$$\begin{aligned} \text{Put option cost for year three} &= \text{BSP0}(3) \times 2pT50 \times 1qd52 = 0.15 \times 0.88 \times 0.003 \\ &= 0.000396 \end{aligned}$$

$$B = \text{PV of BS put options for 3 years} = 0.0002 + 0.000276 + 0.000396 = 0.000872$$

$$\text{Annual Cost} = B/aT50:3$$

$$B/aT50:3 = 0.000872/2.8 = 0.0003114$$

$$CF1 = 1pT50 \times FO \times S1 \times md - 1qd50(G - FO \times S1(1 - m))$$

$$\begin{aligned} CF1 &= 0.92 \times 100000 \times 0.8 \times 0.0125 - 0.0003(100000 - 100000 \times 0.8 \times (1 - 0.0125)) \\ &= 242.88 - 42 = 200.88 \end{aligned}$$

- (i) Increase rollup = more expensive
- (ii) Increase persistency = more expensive
- (iii) Decrease mortality = less expensive
- (iv) Increase volatility = more expensive
- (v) Increase fund growth rate expectation = less expensive
- (vi) Increase management fee = more expensive

CTE satisfies all the criteria for coherence

Quantile ignores the shape of the distribution outside that point

The 95% quantile is the value in a distribution ordered sequentially that occurs 95% of the way through the distribution

$$95\% \text{ Quantile} = 0.06$$

The 95% conditional tail expectation is the average of the last 5% of the distribution.

$$95\% \text{ CTE} = (V96 + V97 + V98 + V99 + V100) / 5$$

$$95\% \text{ CTE} = (0.0067 + 0.0077 + 0.009 + 0.011 + 0.0135) / 5 = 0.00958$$

5. Continued

A price is elastic if the change in price results in a greater than proportional change in demand (absolute value is < 1)

Price Elasticity = % Change in Demand / % Change in Price

$$\text{Price Elasticity} = ((10 - 5)/10)/((33 - 55)/33) = 0.5 / -0.66666 = -0.75$$

Breakeven year = Project Cost / {(VAPrf/Prem + DBPrf/Prem) × IncrSales}

$$\text{BE yr } 33 = 1,000,000 / \{(0.01 + 0) \times 10,000,000\} = 1,000,000 / 100,000 = 10 \text{ years}$$

$$\text{BE yr } 55 = 1,000,000 / \{0.01 + 0.001 \times 5,000,000\} = 1,000,000 / 55,000 = 18.18 \text{ years}$$

EV of Proj = [(VAPrf/Prem + DBPrf/Prem) × AnnPrem] / Hurdle Rate - Proj Cost

$$\begin{aligned} \text{EV of Proj } 33 &= [\{(0.01 + 0) \times 10,000,000\} / 0.15] - 1,000,000 = 100,000 / 0.15 - 1,000,000 \\ &= 666,666 - 1,000,000 = -333,333 \end{aligned}$$

$$\begin{aligned} \text{EV of Proj } 55 &= [\{(0.01 + 0.001) \times 5,000,000\} / 0.15] - 1,000,000 = 55,000 / 0.15 - 1,000,000 \\ &= 333,333 - 1,000,000 = -666,667 \end{aligned}$$

Recommend 33bp: it is the price that maximizes the embedded value of the project

6. Learning Outcomes:

2B. Analyze:

- how the forces of the economy and demographics, impact product design
- consumer behavior impact product design.
- how the forces of the marketplace and competition impact product design.

Solution:

Macro-environment

Economic:

Equity markets declined

Credit spreads widened

Dollar strengthened against foreign currencies

Asset quality problems in general

Financial problems with subprime mortgage backs

Economic recession was both global and domestic

Unemployment rose

Political:

Governments have taken steps to stabilize and stimulate their economy

- Federal Reserve lowered the federal funds rate
- Expanded access to Federal Reserve discount window
- Gave emergency loans to distressed institutions
- Enacted programs to support mortgage back and mortgage lending
- Modified loan terms for distressed borrowers

These modifications may result in inflation or deflation

Task Environment

Markets and Customers

Individual Products

- Life and Protection Solutions
 - Variable Life
 - Universal Life
 - Whole Life
 - Term Life
- Retirement Wealth Strategies
 - Fixed Annuities
 - Variable Annuities
 - Mutual Funds

6. Continued

Institutional Products

- Group Insurance
 - Group Life
 - Non-Medical Health
- Retirement and Savings Products
 - GICs
 - Income and Accumulations Annuities

The target markets for these products are middle income, affluent individuals. They can be small business owners or executives of small and medium companies.

The primary distribution is independent to sell the individual products. Three wholesale models are used:

- Coverage Model and point of Sale Model for life products
- Annuity Model for annuity products

Alternate distributions that are used to sell these products are MetLife Resources and Tower Square.

Institutional products are sold through a dedicated sales force located around the US and serving the UK.

Other Factors in the Marketing System:

Competition

- From other insurance companies, banks, broker dealers, etc...
- Competitor advantage can include:
 - Better product array
 - Better product prices
 - Better claims paying ratings
 - Greater financial resources

7. Learning Outcomes:

5E. Explain how assumption setting reflects investment strategy e.g., hedging

Solution:

(a) **Primary Purposes of Asset Liability Modeling:**

To design an investment strategy that fits the product or liability portfolio

To more accurately predict investment income

To determine the potential effect of different interest rate scenarios

To test strategies used to set credited interest rates

(b) **Liabilities at end of 2008:**

GIC with 2 years remaining for $1000 \times 1.025^3 = 1076.89$

GIC with 4 years remaining for $1000 \times 1.025^5 = 1131.41$

GIC with 6 years remaining for $1000 \times 1.025^7 = 1188.69$

GIC with 9 years remaining for $1000 \times 1.025^{10} = 1280.08$

Macaulay Duration

$$= \left[\sum t \times \text{CashFlow}(t) \times (1+i)^{-t} \right] / \left[\sum \text{CashFlow}(t) \times (1+i)^{-t} \right]$$

Duration of Liabilities at end of 2008 (using 2.5% interest rate):

$$\begin{aligned} &= [2 \times 1076.89 \times 1.025^{-2} + 4 \times 1131.41 \times 1.025^{-4} + 6 \times 1188.69 \times 1.025^{-6} + 9 \times 1280.08 \\ &\times 1.025^{-9}] / [1076.89 \times 1.025^{-2} + 1131.41 \times 1.025^{-4} + 1188.69 \times 1.025^{-6} + 1280.08 \times 1.025^{-9}] \\ &= 5.25 \end{aligned}$$

Assets at end of 2008 (before rebalancing):

Cash (from matured 1 year zero coupon bond invested at Federal Funds rate of 4.25%) = $3000 \times 1.0425 = 3127.50$

10 year zero coupon bond invested at 10 year Treasury rate of 3.91% (9 years left)

Maturity Value = $1000 \times 1.0391^{10} = 1467.48$

Need to rebalance assets to match duration of assets and liabilities.

Assets available for purchase in 2009:

1 year zero coupon bonds now earning a Federal Funds rate of 0%

Not a recommended investment, would rather hold cash (also earns 0% but more liquid)

10 year zero coupon bonds now earning a Federal Funds rate of 2.25%

7. Continued

Purchase \$y of 10 year zero coupon bonds (and hold the rest as cash) to match duration:

Liability Duration = Asset Duration

$$5.25 = [0 \times (3127.50 - \$y) \times 1.025^{-0} + 9 \times 1467.68 \times 1.025^{-9} + 10 \times (\$y \times 1.0225^{10}) \times 1.025^{-10}] / [(3127.50 - \$y) \times 1.025^{-0} + 1467.68 \times 1.025^{-9} + (\$y \times 1.0225^{10}) \times 1.025^{-10}]$$
$$\$y = 1215$$

Note: The same interest rate should be used to calculate the asset and liability durations

Value of Assets at end of 2008 (after rebalancing):

10 year zero coupon bond invested at 10 year Treasury rate of 2.25% (10 years left): 1215

Cash (earning 0%): $3127.50 - 1215 = 1912.50$

10 year zero coupon bond invested at 10 year Treasury rate of 3.91% (9 years left): $1000 \times 1.0391 = 1039.10$

Expected Asset Yield in 2009

$$= (1912.50 \times 0\% + 1039.10 \times 3.91\% + 1215 \times 2.25\%) / (1912.50 + 1039.10 + 1215)$$
$$= 1.63\%$$

Gross Investment Spread

= Asset Yield – Liability Interest Rate

$$= 1.63\% - 2.50\%$$

$$= -0.87\%$$

(c)

(i) **Exact Matching**

Assets held would result in asset cash flows that exactly match future liability cash flows

Begin by matching final liability cash flow and then work backward to the present time

Not usually practical or even possible to exactly match all future asset and liability cash flows

Rebalancing may be required due to asset prepayments, asset defaults or liability cash flows deviating from expected

In this situation, assets not available for purchase that would exactly match liability cash flows

7. Continued

(ii) **Duration Matching**

Assets held would have the same duration as the liabilities

Small changes in interest rates will have equal effect on assets and liabilities, immunizing the company from interest rate risk

Periodic rebalancing required due to changes in interest rates, asset prepayments, asset defaults or liability cash flows deviating from expected

Although durations are matched, the cash flows of the assets and liabilities may be mismatched

Duration matching is the most feasible for this situation

(iii) **Horizon Matching**

Assets are held such that asset cash flows exactly match liability cash flows for the first several years, and then duration matching is used for the longer-term liability cash flows

As the longer-term cash flows become nearer-term cash flows, the matching process is gradually adjusted to cover these cash flows on an exact matching basis.

In this situation, assets not available for purchase that would exactly match the short-term liability cash flows

8. Learning Outcomes:

2B Analyze:

- how the forces of the marketplace and competition impact product design.

Source: LOMA Chapter 5: Planning Marketing Goals and Strategies: page 110 – 116

Cognitive Skill: Recall, Application

Solution:

- (a)
- (1) Intensive Growth: current products/markets still have potential
 - (i) Market penetration: increase sales of current products to current markets
 - e.g., increase advertising and sales force
 - (ii) Market development: sell current products to new markets
 - e.g., sell products in a different State
 - (iii) Product development: modify or develop new (related) Product for current market
 - e.g., insurer develops a new insurance product for current market
 - (2) Diversified Growth: venture beyond current business areas
 - (i) Horizontal: sell new product (unrelated to current) for current market
 - e.g., insurer sells credit cards to current policyholders
 - (ii) Concentric: sell new product (similar to current) for new market
 - e.g., agency-building insurer sells a new insurance product through direct response
 - (iii) Conglomerate: sell new product (unrelated to current) to a new market
 - e.g., insurer starts selling cars
 - (3) Integrated Growth: taking over another level of current industry or distribution channel
 - (i) Forward: e.g. insurer acquires distribution channel
 - (ii) Backward: e.g. distribution channel acquires an insurance company
 - (iii) Horizontal: e.g. insurer acquires a competitor
- (b)
- (i)
 - (1) Build:
 - (i) Increase market share
 - (ii) Sacrifice immediate earning
 - (2) Hold:
 - (i) Maintain market position
 - (ii) For SBU's with high market share and low market growth

8. Continued

- (3) Harvest:
 - (i) Maximize short term earnings
 - (ii) Reduce resource allocation
 - (iii) Allow market share to decline
- (4) Withdrawal
 - (i) Discontinue business unit
 - (ii) For SBU with weakest growth and investment potential
- (ii) SBU should use the Harvest strategy because it falls into cell C
Market attractiveness is medium and business strength is low.
Should reduce resource allocation and maximize short term earnings.

9. Learning Outcomes:

6A. Manage a product once it is on the books given its pricing.

6C. Recommend changes to non-guaranteed elements for deviations from expected

Solution:

- (a)
- (i) Interest/investment returns
 - Most important source
 - Consider equitable treatment of capital gains
 - Consider investment crediting strategy (but can use either Portfolio or new Money (IYM) approach)
 - Consider treatment of loans (e.g. Fixed vs. variable rates, with or without direct recognition)
 - Consider acceptable assets (i.e. only use actively invested assets)
 - (ii) Mortality
 - Use mortality experience of the dividend class if credible, otherwise consider relationship with company experience
 - (iii) Expenses
 - Need to use equitable allocation
 - (iv) Persistency
 - This has effects on expenses (cost amortization) and mortality (anti-selection)
 - (v) Others
 - Taxes, Reinsurance and Mergers and Acquisitions
 - Must at all times be fair and equitable
- (b)
- (i) Pegging
 - If the new scale produces a lower dividend than the prior year, the company would pay the prior year dividend
 - (ii) Substitution
 - If the new scale is lower, replace the new scale with the old scale
 - This is expensive and only justifiable if the policy is new and the dividends are small
 - (iii) Experience Premium Approach
 - Same as Pegging, but track the cost of pegging and offset against higher dividends later in the scale

9. Continued

These techniques are helpful for improving persistency.

	CY	2010	2011	2012	2013	2014
Pegging	7	7	7	12	18	25
Cost of Pegging	5	3	0	0	0	0
Substitution	8	11	16	24	30	38
Cost of Substitution	6	7	9	12	12	13

- (c)
- (i) State that this is a “Life Insurance Illustration” (not some retirement accumulation product)
 - (ii) Specify the agent showing/using the illustration. This enforces accountability
 - (iii) Format: needs much clearer labeling. For example, are the death benefits “guaranteed” and are the dividends “non-guaranteed”
 - (iv) Narrative Summary: Should include a summary that describes the product and basic features. Again, reiterate that “non-guaranteed elements can change”
 - (v) Numeric Summary: It is unclear what scale is being used. A numeric summary would show a guaranteed scale, illustrated scale, and average scale of various policy values (i.e. premium outlay, death benefit, cash value) for years 1 – 10, 15, 20, and every 5 years thereafter
 - (vi) The agent and customer should sign the illustration indicating the customer understands elements are non-guaranteed and that the agent should sign that he/she explained this to the customer
 - (vii) Show the product name

10. Learning Outcomes:

- 3C. For a given context or product, the candidate will be able to:
Describe how tax regulation can drive (not just constrain) product design

Solution:

- (a) Benefits and Motivations for Securitizing Life Insurance
- Releases embedded value in company's business
 - Benefits
 - Increased liquidity
 - Lower cost of capital
 - Improvements in key performance indicators (e.g., ROE)
 - Source of funding to enhance capital flexibility
 - Release capital from portfolio; or free funds from acquired entity (e.g., closed block).
 - Transfer risks off balance sheets
 - Protect against adverse shifts in interest rates, mortality, lapse, and counterparty risk.
 - Access to funding potential of capital markets.
 - Company needs to provide potential investors with credible in-depth projections of risk/reward profile of business, including cash flows.
 - Improve understanding of business's value drivers.
 - Making asset values and earnings more explicit enhances market credibility.
 - Monetize the value of in force for defined block of business.
 - Required capital relief to bolster solvency.
 - Financing assistance for new business
 - Provide capital to fund acquisitions.
 - Use hedging strategy to transfer extreme mortality risk.
 - Capital solutions for reducing excess XXX reserves.
 - Remove capital from double-tax insurance situation.
- (b) Hurdles and Success Factors for Securitizing Life Insurance
- Securitization process is new and complete.
 - Must understand risk profile of investors
 - Necessary coordination and commitment from many parties
 - Bankers
 - Lawyers
 - Actuaries
 - Accountants
 - Tax advisors
 - Regulators
 - Rating agencies

10. Continued

- Company must consider time, expense, and demands of their management resources required for successful securitization.
- Securitization must be appropriate for capital profile and strategic objectives of company
- Effective explanations and assurances are essential.
- Greater market understanding of how life insurance companies generate revenues.
- Regulatory, accounting, and taxation requirements impose detailed planning and structuring efforts by insurance company.
- Getting approval from regulators is critical.
- Regulators concerned about protection of policyholders' interests.
- Securitizations seen as good for insurance industry because of risk transfer
- Strong yields and good credit ratings caught investors' attention.
- Life insurance's risk/reward profile provides attractive alternative to mortgages and other instruments
- Sophisticated actuarial analysis required to understand risks.
- Deterministic pricing using static assumptions complemented by stochastic modeling incorporating a set of assumptions producing a range of results.
- Key assumptions include:
 - Interest rates
 - Mortality assumptions
 - Lapses
 - Other coverage factors.
- Corporate risk management keen on results generated by sensitivity and stochastic analyses.
- Securitization's success depends on providing clearly defined and reliably quantified cash flows to the market, but can be challenging when securitizing open life portfolios; or new business in open portfolios may not materialize and thus, original forecasts may not be met.
- Insurers will always know their portfolios of risks better than investors, especially those not familiar with insurance risks – as a results:
 - Insurers may try to transfer only unfavorable risks – adverse selection
 - Market may demand sizable risk premium for insurance risks.
 - Investors may not be comfortable investing in and managing insurance risks directly.
- Securitization market will not become popular until there is development of a dedicated class of insurance risk investment managers.

11. Learning Outcomes:

3C. Describe how tax regulation can drive (not just constrain) product design.

Solution:

(a)

For Policy A

- Since the policy is a MEC, it is taxed on a LIFO basis, which means gain comes out and is taxed first before investment in the contract.
- Basis in the contract equals investment in contract which is premiums plus policy loans if the policy loan is included in taxable income
- If a partial withdrawal is “gain”, then the basis is not reduced by the withdrawal
- Basis = \$40,000 + \$10,000 = \$50,000
- Policy loans are treated as a distribution on a MEC to the extent cash value is greater than the investment in contract prior to loan.
- Taxable gain is policy loans + taxable withdrawal = \$10,000 + \$50,000 = \$60,000
- Tax = 30% × taxable gain = 0.30 × \$60,000 = \$18,000
- Penalty tax since policyholder is less than 59.5 which is 10% of gain = 0.10 × 60,000 = 6,000
- Total tax = \$18,000 + \$6,000 = \$24,000

For Policy B

- Policy is not a MEC so taxed on a FIFO basis (taxed on proceeds that exceed the basis in the contract)
- Basis = investment in the contract which is reduced to zero after the withdrawal
- Policy loans are not treated as distributions and are not taxed.
- Taxable gain = withdrawal minus basis = \$50,000 – \$40,000 = \$10,000
- Tax = 30% × \$10,000 = \$3,000
- Since policy is not a MEC, excise tax does not apply.

11. Continued

(b)

For Policy A

- Since the policyholder is age 58, wait at least 1.5 years to age 59.5 to make a withdrawal and eliminate the excise tax of \$6,000

For Policy B

- Since the policy is not a MEC, withdrawal up to the basis and then take policy loans since policy loans are not taxed.
- Partial withdrawal would be \$40,000 with a \$20,000 loan which would reduce taxes to zero
- Recommend taking loans going forward since they are not taxable on a non-MEC

12. Learning Outcomes:

5B Select appropriate assumptions for a given product design, reflecting

- iv) product characteristics
- v) available experience data
- vi) the marketplace
- vii) underwriting
- viii) distribution channel characteristics

5C Explain the effect of each assumption on product pricing.

4B Recommend designs for products and features to reach particular goals

5L Explain how a stochastic model would be used for pricing

5M Describe the advantages and disadvantages of stochastic and deterministic modeling approached

5N Interpret results of a stochastic model.

3C. Describe how tax regulation can drive (not just constrain) product design.

Solution:

(a)

Business factors that affect lapses:

Manner in which policy is sold

Perceived value

Degree of understanding

Ability to pay

Agent's attitude towards persistency

Rates are varying by gender, which is possible

Rates should also vary with the following factors:

By issue age

By policy size (lower policies have higher lapse rates)

By frequency of premium payment

By method of premium payment

By risk class – lower lapse rates among preferred classes

Varies by policy year, which they are doing

Early lapse rates are a little too low

Ultimate lapse rates are too high

Should be between 1% and 2% after duration 7-8

This is a lapse-supported product: overestimating lapse rates will overestimate profits.

Lapse should be lower when the CSV = 0 but shadow AV > 0

(b)

Primary guarantee

- Asset hedge to mitigate the interest rate risk
- Can be very costly

12. Continued

- Use stochastic pricing to understand the risk and decide whether to hedge or not

Secondary Death Benefit guarantee

- AXXX reserves ceded to reinsurers
- To reflect unauthorized reinsurer reserve credits, have to use one of the following:
 - LOC (Letter of credit) secured from a qualifying bank under-prescribed conditions
 - Demands have dramatically increase
 - One-year term arrangement
- Lower capacity limits
- Get capital relief but expose company to LOC pricing risk and econ. Risk of triggering secondary guarantee
- Capital market for securitization arrangements of future cash flows from specified blocks
- Should seek self-affiliated solution as no more capacity with direct reinsurer.

(c)

Benefits of stochastic pricing

- Serves similar purpose as traditional pricing but provides a more fulsome understanding of risks (cost/benefit)
- Important to model the tail of the distribution
- Distributions allow for assessment of likelihood and acceptability of profit (or loss) relative to targets

Interpreting the results

- Distribution of stochastic results is not in line with deterministic results
- Risks are higher than expected
- Stochastic results are lower than 0 in more than 50% of the scenarios
- Stochastic results are lower than the deterministic results in more than 70% of the scenarios
- Results in the tail can be catastrophic (-80,000)

Recommendations

- Strengthen secondary guarantee so that volatility is reduced
- Increase profit target

Establishing reserves using stochastic pricing

- Evaluating additional reserves same as evaluating embedded option in a market consistent framework.
- Generally arbitrage free rate scenarios where forward rates are driven by the current yield curve

12. Continued

- interest rate volatility assumptions gathered from swaptions and other interest rate derivatives
- PV of claims paid less stipulated premiums after primary AV goes to zero are averaged over stochastic paths
- Embedded option cash flows are discounted at one-year rates along each stochastic path
- Results are added to the base reserve at each valuation date

(d) Calculate maximum cumulative premiums in first five years...

$$\text{GSP} = \text{Death Benefit} \times A_{5:45} / (1 - \text{Pload}) \text{ at } 6\%$$

$$\text{GSP} = 100,000 \times 0.27 / (1 - 0.05)$$

$$\text{GSP} = \$28,421.05$$

$$\text{GLP} = \text{Death benefit} \times A_{55:45} / [a_{55:45} \times (1 - \text{Pload})] \text{ at } 4\%$$

$$\text{GLP} = 100,000 \times 0.32 / [15 \times (1 - 0.05)]$$

$$\text{GLP} = 2,245.61$$

$$\text{After 5 years, cumulative GLP} = 5 \times \text{GLP}$$

$$\text{Cum GLP} = 5 \times 2,245.61 = \$11,228.05$$

Corridor factor does not come into play as AV is too low

Maximum payable in the contract is Max (GSP, Sum of GLP's)

Maximum payable is \$28,421.05

$$7\text{-Pay Premium} = \text{Death benefit} \times \text{NSP at } 4\% / a_7$$

$$7\text{-Pay Premium} = \text{Death benefit} \times A_{55:45} / a_{55:7} \text{ (both at } 4\%)$$

$$7\text{-Pay Premium} = 100,000 \times 0.32 / 6 = \$5,333.33$$

$$\text{After 5 years, cumulative 7-Pay} = 5 \times 5,333.33 = \$26,666.65$$

13. Learning Outcomes:

5B. Select appropriate assumptions for a given product design, reflecting

- i) product characteristics
- ii) available experience data
- iii) the marketplace
- iv) underwriting
- v) distribution channel characteristics

Solution:

(a)

Theory One

Mortality rates will continue to increase with age, but at a decreasing rate

Pattern Method

Let the natural pattern continue until it reaches one; this age can be used as the ultimate age

Theory Two

There is a natural limit to the life span, so rate will jump to one at this ultimate age

Forced Method

Select an ultimate age and set the mortality rate equal to one; this will cause a discontinuity in the rates

Theory Three

Mortality rates are asymptotic to a rate much less than one

Less-than-one Method

Select an ultimate age and use that mortality rate for all subsequent ages even though it will be less than one

Recommendation

I recommend the less-than-one method for the following reasons:

It is commonly used in the industry today

It is more conservative, which is important since this product is focusing on an older population

Studies have shown mortality rates at age 120 are significantly less than one

Note: Other methods could have been recommended provided appropriate justification was given.

13. Continued

(b)

Should not use a 35% multiple for all rates; this will result in mortality rates at older ages that are too low

Should consider using a more modern table since there has been much mortality improvement since the 75-80 table was developed

Issue ages greater than 70 were not included in the 75-80 table; this is important since targeting an older market

Some studies have shown preferred mortality wears off over time, while other studies have shown preferred is similar to gender and does not wear off

Perhaps consider using a long select period rather than one that does not wear off

Could use experience from Framingham or NHANES studies

Using industry average mortality can be dangerous because mortality varies greatly from company to company

Companies have different underwriting standards and target markets

Mortality improvement should vary by age and risk class

Need many years of deaths to construct a reliable mortality table

(c)

The slope of the underlying 75-80 table may not match the slope of the company experience

Recent industry tables have very different slopes

The analysis should be done over many years

Should also account for mortality improvement

Should examine the ratio of preferred to residual mortality

Compare the actual/expected ratios for the two groups

Then the results will be independent of the underlying table

If the preferred to residual ratio converges to one over time, then the preferred is wearing off

Thus can directly analyze if the preferred is wearing off

14. Learning Outcomes:

5B. Select appropriate assumptions for a given product design, reflecting

- ix) product characteristics (FIX BULLETS)
- x) available experience data
- xi) the marketplace
- xii) underwriting
- xiii) distribution channel characteristics

2B. Analyze:

- the advantage/disadvantages of different internal drivers
- how the forces of the economy and demographics, impact product design
- consumer behavior impact product design.
- how the forces of the marketplace and competition impact product design.

2C. Describe the questions to ask the sales and marketing (incl. Agents, brokers and direct marketing)

Solution:

(a) For ABC, the advantages are:

- It is an under-served market
- Less costly acquisition costs for the Company
- Can be expanded to include employee dependents
- Can be expanded to other products
- Can reach more potential clients with a variety of products on a cost effective basis
- Helps employers hold costs on employee benefits
- Attractive to group providers to broaden offerings with existing clients
- Admin support with employer using payroll deduction

For Agents, the advantages are:

- Daytime selling opportunities
- Large prospecting lists
- Fresh “door opener” conversations with decision makers
- Increase in face to face selling situations through one contact

For Consumer/Employee, the advantages are:

- Employees get better products and rates
- Non-intrusive sales process
- Employer has endorsed insurance company and value of product offering
- Payroll deduction to pay premiums
- Usually guaranteed issue or simplified underwriting so process is streamlined/easier
- Portability of coverage

14. Continued

(b) Reasons for deviation are:

- Anti-selective lapses to sex distinct rates
- Agent initiated lapses to higher commission product
- Agent initiated lapses of healthier risks to products with stricter underwriting
- Use of unisex rates subject to normal mix of business/gender risk
- Union groups will usually have higher male content where data used is closer to 50/50
- Union mortality can be different than mortality used to derive expected

Actions to manage risk are:

- Monitor sales volumes in relation to male/female proportion
- Monitor quality of union groups against expected
- Monitor actual sales against pricing expectation

(c)

$$ml(35, 0) = 130,000$$

$$ml(35, 1) = ml(35, 0) * (1 - mq(35, 1)) = 130,000 * (1 - .01175) = 128,472.5$$

$$ml(35, 2) + ml(35, 1) * (1 - mq(35, 2)) = 128,472.5 * (1 - .0125) + 126,866.6$$

$$ml(35, 3) = ml(35, 2) * (1 - mq(35, 3)) = 126,866.6 * (1 - .01335) = 125,172.9$$

$$fl(35, 0) = 120,000$$

$$fl(35, 1) = fl(35, 0) * (1 - fq(35, 1)) = 120,000 * (1 - .0025) = 119,700$$

$$fl(35, 2) = fl(35, 1) * (1 - fq(35, 2)) = 119,700 * (1 - .0023) = 119,424.7$$

$$fl(35, 3) = fl(35, 2) * (1 - fq(35, 3)) = 119,424.7 * (1 - .0024) = 119,138.1$$

$$l(35, 0) = ml(35, 0) + fl(35, 0) = 130,000 + 120,000 = 250,000$$

$$l(35, 1) = ml(35, 1) + fl(35, 1) = 128,472.5 + 119,700 = 248,172.5$$

$$l(35, 2) = ml(35, 2) + fl(35, 2) = 126,866.6 + 119,424.7 = 246,291.3$$

$$l(35, 3) = ml(35, 3) + fl(35, 3) = 125,172.9 + 119,138.1 = 244,311$$

$$q(35, 1) = (l(35, 0) - l(35, 1)) / l(35, 0) = (250,000 - 248,172.5) / 250,000 = .00731$$

$$q(35, 2) = (l(35, 1) - l(35, 2)) / l(35, 1) = (248,172.5 - 246,291.3) / 248,172.5 = .00758$$

$$q(35, 3) = (l(35, 2) - l(35, 3)) / l(35, 2) = (246,291.3 - 244,311) / 246,291.3 = .00804$$

Estimated mortality rate = $Q = .00731$

Estimated claims for $n = 250,000$ policies = $E = nq = 250,000 * .00731 = 1,827.5$

Variance of expected claims for $n = 250,000$ policies = $\text{Var} = npq$

$$= 250,000 * .00731 * (1 - .00731) = 1,814.14$$

95% confidence interval of expected claims = $E \pm 1.96 * \sqrt{\text{Var}} = 1,827.5 \pm$

$$1.96 * (\sqrt{1,814.14}) = 1,827.5 \pm 83.482$$

14. Continued

= Interval of 1,744.018, 1,910.982

95% confidence interval of mortality rate = $(E \pm 1.96 * \sqrt{Var}) / n$

= $(1,827.5 \pm 1.96 * (\sqrt{1,814.14})) / 250,000 = .00731 \pm .0003339$

= Interval of .0069761, .0076439

(d) Ways to enhance credibility of study are:

- Combine multiple years of issue
- Group issue years and durations together
- Grouped results would then be smoothed to produce final experience mortality table
- Do an actual to expected analysis

15. Learning Outcomes:

- 4A. Describe in detail the basic benefit/product types and their uses,
5B. Select appropriate assumptions for a given product design, reflecting
- (i) product characteristics
 - (ii) available experience data
 - (iii) the marketplace
 - (iv) underwriting
 - (v) distribution channel characteristics
- 5C. Explain the effect of each assumption on product pricing.

Solution:

- (a)
- (i) Market Applications
 - Survivorship insurance is very attractive for clients with an illiquid estate
 - Can provide the funds to pay estate taxes
 - Business reasons
 - Family owned businesses last survivor can be used to pay the taxes associated with transferring the business to the children
 - Provides funds for children to purchase their parents interest in the business upon the death of the second parent
 - Can be used for key person insurance as well
 - (ii) Features and Riders
 - Survivorship insurance pays death benefit upon second death of an insured pair of lives
 - Premium required to fund last survivor policy much less than the cost of insuring each life separately
 - Flexibility very important
 - Whole life/term rider combos helped add flexibility
 - Premium Flexibility
 - Low annual premiums to limited payment premium as cash value build up is not a focus
 - Sometimes the insured will make a gift of the premium to the owner – often as irrevocable trust or children
 - Automatic increase in death benefit
 - Provide for increase resulting from inflation and increased value of insurable interest
 - Consider additional cost and anti selection
 - Policy Split Rider
 - Usually under specified events can split the policy into two single life policies
 - Such as divorce or change in the tax law
 - Sum of the face amount cannot exceed the existing face amount

15. Continued

- New policies based on original ages and risk classification
- May not be a charge but then evidence of insurability will be required at time of split
- Estate Preservation Rider
 - Additional coverage during first three-four years
 - Developed specifically for US tax environment
 - In many instances the beneficiary or a trust is the owner so that the proceeds are not included in the estate for estate tax purposes
 - If death occurs within three years of transfer to Trust to policy then it would be included in the estate which would negate the purpose as an estate tax tool
 - Therefore some companies developed benefits that roughly double the death benefit if the death occurs within the first four years of the policy
- First-to-die Term Rider
 - Provides some death benefit at the first death
 - Declining benefit designed to complete funding of the policy at the first death
 - Could also “vanish” the contract for the surviving spouse

(b) Using Fraseirization

$qx(t)$	$qy(t)$	$px(t)$	$py(t)$	tpx	tpy	$tpxy$	$pxy(t)$	$qxy(t)$	claims	pv claims
0.0100	0.0200	0.9900	0.9800	0.9900	0.9800	0.9998	0.9998	0.0002	2000.00	1,923
0.0150	0.0300	0.9850	0.9700	0.9752	0.9506	0.9988	0.9990	0.0010	10275.90	9,501
0.0250	0.0450	0.9750	0.9550	0.9508	0.9078	0.9955	0.9967	0.0033	33101.68	29,427
0.0400	0.0600	0.9600	0.9400	0.9127	0.8534	0.9872	0.9917	0.0083	82585.46	70,594
0.0550	0.0750	0.9450	0.9250	0.8625	0.7894	0.9710	0.9836	0.0164	161594.24	132,819
									sum	244,264

Calculate px 's: $px(t) = 1 - qx(t)$

Calculate survivorship factors: $tpx = px(t-1) * px(t)$

Calculate joint survivorship factors: $tpxy = tpx + tpy - tpx * tpy$

Calculate joint px 's: $pxy(t) = tpxy / t - 1pxy$

Calculate joint qx 's: $qxy(t) = 1 - pxy(t)$

Calculate claims: $claims(t) = \$10mil * qxy(t) * t-1pxy$

Calculate PV claims: sum from 1 to 5 of $claims(t) * v^t$

15. Continued

Equivalent Age Calcs:

$qx(t)$	tpx	claims	PV claims
0.0010	0.9990	10000.00	9,615
0.0020	0.9970	19980.00	18,473
0.0040	0.9930	39880.08	35,453
0.0060	0.9871	59580.84	50,930
0.0100	0.9772	98705.59	81,129
			195,600

$$tpx = (1 - qx(t))^t - 1px$$

Claims(t) and PV claims(t) calculated using formulae above

- (c) Mortality assumption improvements
- Degree of underwriting concessions should be factored in
 - Consider whether credible basis for estimating female mortality at advanced issue ages.
 - Contagion should be considered
 - For joint accident risk
 - For broken heart syndrome
 - Should not assume exact independence when using exact age assumptions
 - Using single life experience may not be good source for assumptions
 - Other factors where they might deviate from single life experience
 - Impact for all business medically underwritten
 - Socio-economic class of lives insured
 - Impact of very low lapses on long term mortality
 - Implications of virtually all issues to married couples
 - If widowed, higher than aggregate mortality
 - If married, lower than aggregate mortality
 - Some companies assume substandard mortality in the 12 months following the death of the first insured