Society of Actuaries

Casualty Actuarial Society

Canadian Institute of Actuaries

Exam MFE
Actuarial Models
Financial Economics Segment

Friday,
May 15, 2009
2:00 p.m. – 4:00 p.m.

MFE

INSTRUCTIONS TO CANDIDATES

1. Write your candidate number here ____________. Your name must not appear.

2. Do not break the seal of this book until the supervisor tells you to do so.

3. Tables and numerical values necessary for solving some of the questions on this examination will be distributed by the Supervisor.

4. This examination consists of 20 multiple-choice questions.

5. Each question has equal weight. Your score will be based on the number of questions which you answer correctly. No credit will be given for omitted answers and no credit will be lost for wrong answers; hence, you should answer all questions even those for which you have to guess.

6. A separate answer sheet is inside the front cover of this book. During the time allotted for this examination, record all your answers on side 2 of the answer sheet. NO ADDITIONAL TIME WILL BE ALLOWED FOR THIS PURPOSE. No credit will be given for anything indicated in the examination book but not transferred to the answer sheet. Failure to stop writing or coding your answer sheet after time is called will result in the disqualification of your answer sheet or further disciplinary action.

7. Five answer choices are given with each question, each answer choice being identified by a key letter (A to E). Answer choices for some questions have been rounded. For each question, blacken the oval on the answer sheet which corresponds to the key letter of the answer choice that you select.

8. Use a soft-lead pencil to mark the answer sheet. To facilitate correct mechanical scoring, be sure that, for each question, your pencil mark is dark and completely fills only the intended oval. Make no stray marks on the answer sheet. If you have to erase, do so completely.

9. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.

10. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

11. Clearly indicated answer choices in the test book can be an aid in grading examinations in the unlikely event of a lost answer sheet.

12. Use the blank portions of each page for your scratch work. Extra blank pages are provided at the back of the examination book.

13. When the supervisor tells you to do so, break the seal on the book and remove the answer sheet.

    On side 1 of the answer sheet, space is provided to write and to code candidate information. Complete Blocks A through G as follows:

    (a) in Block A, print your name and the name of this test center;

    (b) in Block B, print your last name, first name and middle initial and code your name by blackening the ovals (one in each column) corresponding to the letters of your name; for each empty box, blacken the small rectangle immediately above the "A" oval;

    (c) write your candidate number in Block C (as it appears on your ticket of admission for this examination) and write the number of this test center in Block D (the supervisor will supply the number);

    (d) code your candidate number and center number by blackening the five ovals (one in each column) corresponding to the five digits of your candidate number and the three ovals (one in each column) corresponding to the three digits of the test center number, respectively. Please be sure that your candidate number and the test center number are coded correctly;

    (e) in Block E, code the examination that you are taking by blackening the oval to the left of "Exam MFE";

    (f) in Block F, blacken the appropriate oval to indicate whether you are using a calculator and write in the make and model number; and

    (g) in Block G, sign your name and write today's date. If the answer sheet is not signed, it will not be graded.

    On side 2 of your answer sheet, space is provided at the top for the number of this examination book. Enter the examination book number, from the upper right-hand corner of this examination book, in the four boxes at the top of side 2 marked "BOOKLET NUMBER".

14. After the examination, the supervisor will collect this book and the answer sheet separately. DO NOT ENCLOSE THE ANSWER SHEET IN THE BOOK. All books and answer sheets must be returned. THE QUESTIONS ARE CONFIDENTIAL AND MAY NOT BE TAKEN FROM THE EXAMINATION ROOM.

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Unless otherwise stated in the question, assume:

- The market is frictionless. There are no taxes, transaction costs, bid/ask spreads, or restrictions on short sales. All securities are perfectly divisible. Trading does not affect prices. Information is available to all investors simultaneously. Every investor acts rationally (i.e. there is no arbitrage).

- The risk-free interest rate is constant.

- The notation is the same as used in *Derivatives Markets*, by Robert L. McDonald.

When using the normal distribution, choose the nearest z-value to find the probability, or if the probability is given, choose the nearest z-value. No interpolation should be used.

> Example: If the given z-value is 0.759, and you need to find $\Pr(Z < 0.759)$ from the normal distribution table, then choose the probability for z-value = 0.76: $\Pr(Z < 0.76) = 0.7764$.

The density function for the standard normal random variable is $\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$.

NORMAL DISTRIBUTION TABLE

Entries represent the area under the standardized normal distribution from -∞ to z, Pr(Z<z)
The value of z to the first decimal is given in the left column. The second decimal place is given in the top row.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Values of z for selected values of Pr(Z<z)							
z	0.842	1.036	1.282	1.645	1.960	2.326	2.576
Pr(Z<z)	0.800	0.850	0.900	0.950	0.975	0.990	0.995

****BEGINNING OF EXAMINATION****

1. You use the usual method in McDonald and the following information to construct a binomial tree for modeling the price movements of a stock. (This tree is sometimes called a forward tree.)

(i) The length of each period is one year.

(ii) The current stock price is 100.

(iii) The stock's volatility is 30%.

(iv) The stock pays dividends continuously at a rate proportional to its price. The dividend yield is 5%.

(v) The continuously compounded risk-free interest rate is 5%.

Calculate the price of a two-year 100-strike American call option on the stock.

(A) 11.40

(B) 12.09

(C) 12.78

(D) 13.47

(E) 14.16

2. You have observed the following monthly closing prices for stock XYZ:

Date	Stock Price
January 31, 2008	105
February 29, 2008	120
March 31, 2008	115
April 30, 2008	110
May 31, 2008	115
June 30, 2008	110
July 31, 2008	100
August 31, 2008	90
September 30, 2008	105
October 31, 2008	125
November 30, 2008	110
December 31, 2008	115

The following are one-year European options on stock XYZ. The options were issued on December 31, 2007.

(i) An arithmetic average Asian call option (the average is calculated based on monthly closing stock prices) with a strike of 100.

(ii) An up-and-out call option with a barrier of 125 and a strike of 120.

(iii) An up-and-in call option with a barrier of 120 and a strike of 110.

Calculate the difference in payoffs between the option with the largest payoff and the option with the smallest payoff.

(A) 5

(B) 10

(C) 15

(D) 20

(E) 25

3. You are given the following regarding stock of Widget World Wide (WWW):

(i) The stock is currently selling for \$50.

(ii) One year from now the stock will sell for either \$40 or \$55.

(iii) The stock pays dividends continuously at a rate proportional to its price. The dividend yield is 10%.

The continuously compounded risk-free interest rate is 5%.

While reading the Financial Post, Michael notices that a one-year at-the-money European call written on stock WWW is selling for \$1.90. Michael wonders whether this call is fairly priced. He uses the binomial option pricing model to determine if an arbitrage opportunity exists.

What transactions should Michael enter into to exploit the arbitrage opportunity (if one exists)?

(A) No arbitrage opportunity exists.

(B) Short shares of WWW, lend at the risk-free rate, and buy the call priced at \$1.90.

(C) Buy shares of WWW, borrow at the risk-free rate, and buy the call priced at \$1.90.

(D) Buy shares of WWW, borrow at the risk-free rate, and short the call priced at \$1.90.

(E) Short shares of WWW, borrow at the risk-free rate, and short the call priced at \$1.90.

4. Your company has just written one million units of a one-year European asset-or-nothing put option on an equity index fund.

The equity index fund is currently trading at 1000. It pays dividends continuously at a rate proportional to its price; the dividend yield is 2%. It has a volatility of 20%.

The option's payoff will be made only if the equity index fund is down by more than 40% at the end of one year.

The continuously compounded risk-free interest rate is 2.5%

Using the Black-Scholes model, determine the price of the asset-or-nothing put options.

(A) 0.2 Million

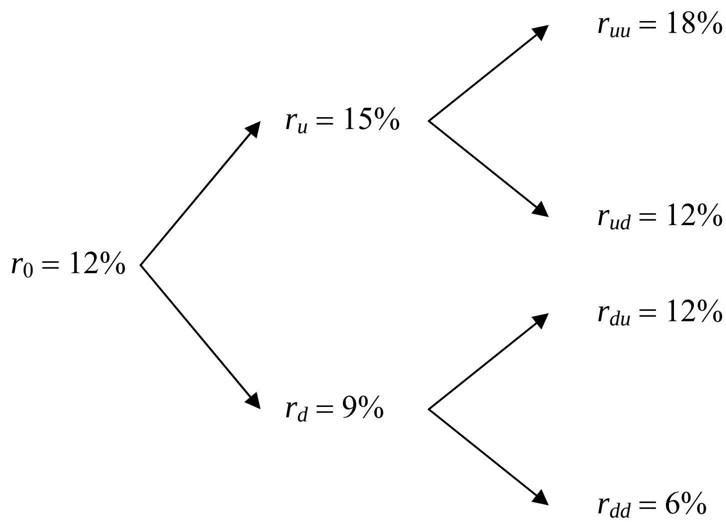
(B) 0.9 Million

(C) 2.7 Million

(D) 3.6 Million

(E) 4.2 Million

5. You are given the following three-period interest rate tree. Each period is one year. The risk-neutral probability of each up-move is 70%. The interest rates are continuously compounded rates.



Consider a European put option that expires in 2 years, giving you the right to sell a one-year zero-coupon bond for 0.90. This zero-coupon bond pays 1 at maturity.

Determine the price of the put option.

(A) 0.012

(B) 0.018

(C) 0.021

(D) 0.024

(E) 0.029

6. $X(t)$ is an Ornstein-Uhlenbeck process defined by

$$dX(t) = 2[4 - X(t)]dt + 8dZ(t),$$

where $Z(t)$ is a standard Brownian motion.

Let

$$Y(t) = \frac{1}{X(t)}.$$

You are given that

$$dY(t) = \alpha(Y(t))dt + \beta(Y(t))dZ(t)$$

for some functions $\alpha(y)$ and $\beta(y)$.

Determine $\alpha(½)$.

(A) −9

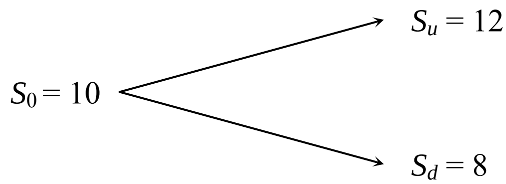
(B) −1

(C) 4

(D) 7

(E) 63

7. The following one-period binomial stock price model was used to calculate the price of a one-year 10-strike call option on the stock.



You are given:

(i) The period is one year.

(ii) The true probability of an up-move is 0.75.

(iii) The stock pays no dividends.

(iv) The price of the one-year 10-strike call is \$1.13.

Upon review, the analyst realizes that there was an error in the model construction and that S_d, the value of the stock on a down-move, should have been 6 rather than 8. The true probability of an up-move does not change in the new model, and all other assumptions were correct.

Recalculate the price of the call option.

(A) \$1.13

(B) \$1.20

(C) \$1.33

(D) \$1.40

(E) \$1.53

8. Assume the Black-Scholes framework. Consider a stock and a derivative security on the stock.

You are given:

(i) The continuously compounded risk-free interest rate, r, is 5.5%.

(ii) The time-t price of the stock is $S(t)$.

(iii) The time-t price of the derivative security is $e^{rt}\ln[S(t)]$.

(iv) The stock's volatility is 30%.

(v) The stock pays dividends continuously at a rate proportional to its price.

(vi) The derivative security does not pay dividends.

Calculate δ, the dividend yield on the stock.

(A) 0.00

(B) 0.01

(C) 0.02

(D) 0.03

(E) 0.04

9. You are given:

(i) The current exchange rate is 0.011\$/¥.

(ii) A four-year dollar-denominated European put option on yen with a strike price of \$0.008 sells for \$0.0005.

(iii) The continuously compounded risk-free interest rate on dollars is 3%.

(iv) The continuously compounded risk-free interest rate on yen is 1.5%.

Calculate the price of a four-year yen-denominated European put option on dollars with a strike price of ¥125.

(A) ¥35

(B) ¥37

(C) ¥39

(D) ¥41

(E) ¥43

10. Two nondividend-paying assets have the following price processes:

$$\frac{\mathrm{d}S_1(t)}{S_1(t)} = 0.08\mathrm{d}t + 0.2\mathrm{d}Z(t)$$

$$\frac{\mathrm{d}S_2(t)}{S_2(t)} = 0.0925\mathrm{d}t - 0.25\mathrm{d}Z(t)$$

where $Z(t)$ is a standard Brownian motion. An investor is to synthesize the risk-free asset by allocating 1000 between the two assets.

Determine the amount to be invested in the first asset, S_1.

(A) 333.33

(B) 444.44

(C) 555.56

(D) 666.67

(E) 750.00

11. Assume the Black-Scholes framework. For $t \geq 0$, let $S(t)$ be the time-t price of a nondividend-paying stock. You are given

(i) $S(0) = 0.5$

(ii) The stock price process is

$$\frac{\mathrm{d}S(t)}{S(t)} = 0.05\mathrm{d}t + 0.2\mathrm{d}Z(t)$$

where $Z(t)$ is a standard Brownian motion.

(iii) $\mathrm{E}[S(1)^a] = 1.4$, where a is a negative constant.

(iv) The continuously compounded risk-free interest rate is 3%.

Consider a contingent claim that pays $S(1)^a$ at time 1.

Calculate the time-0 price of the contingent claim.

(A) 1.345

(B) 1.359

(C) 1.372

(D) 1.401

(E) 1.414

12. You are given:

(i) $C(K, T)$ denotes the current price of a K-strike T-year European call option on a nondividend-paying stock.

(ii) $P(K, T)$ denotes the current price of a K-strike T-year European put option on the same stock.

(iii) S denotes the current price of the stock.

(iv) The continuously compounded risk-free interest rate is r.

Which of the following is (are) correct?

(I) $0 \le C(50, T) - C(55, T) \le 5e^{-rT}$

(II) $50e^{-rT} \le P(45, T) - C(50, T) + S \le 55e^{-rT}$

(III) $45e^{-rT} \le P(45, T) - C(50, T) + S \le 50e^{-rT}$

(A) (I) only

(B) (II) only

(C) (III) only

(D) (I) and (II) only

(E) (I) and (III) only

13. Assume the Black-Scholes framework.

Eight months ago, an investor borrowed money at the risk-free interest rate to purchase a one-year 75-strike European call option on a nondividend-paying stock. At that time, the price of the call option was 8.

Today, the stock price is 85. The investor decides to close out all positions.

You are given:

(i) The continuously compounded risk-free rate interest rate is 5%.

(ii) The stock's volatility is 26%.

Calculate the eight-month holding profit.

(A) 4.06

(B) 4.20

(C) 4.27

(D) 4.33

(E) 4.47

14. You are to use a Black-Derman-Toy model to determine $F_{0,2}[P(2, 3)]$, the forward price for time-2 delivery of a zero-coupon bond that pays 1 at time 3.

In the Black-Derman-Toy model, each period is one year. The following effective annual interest rates are given:

r_d = 30%
r_u = 60%
r_{dd} = 20%
r_{uu} = 80%

Determine $1000 \times F_{0,2}[P(2, 3)]$.

(A) 667

(B) 678

(C) 690

(D) 709

(E) 712

15. You are given:

(i) The true stochastic process of the short-rate is given by

$$\mathrm{d}r(t) = [0.008 - 0.1r(t)]\mathrm{d}t + 0.05\mathrm{d}Z(t),$$

where $Z(t)$ is a standard Brownian motion under the true probability measure.

(ii) The risk-neutral process of the short-rate is given by

$$\mathrm{d}r(t) = [0.013 - 0.1r(t)]\mathrm{d}t + 0.05\mathrm{d}\tilde{Z}(t),$$

where $\tilde{Z}(t)$ is a standard Brownian motion under the risk-neutral probability measure.

(iii) For $t \le T$, let $P(r, t, T)$ be the price at time t of a zero-coupon bond that pays \$1 at time T, if the short-rate at time t is r. The price of each zero-coupon bond follows an Itô process:

$$\frac{\mathrm{d}P[r(t),t,T]}{P[r(t),t,T]} = \alpha[r(t),t,T]\mathrm{d}t - q[r(t),t,T]\mathrm{d}Z(t), \quad t \le T.$$

Calculate $\alpha(0.04, 2, 5)$.

(A) 0.041

(B) 0.045

(C) 0.053

(D) 0.055

(E) 0.060

16. You are given the following information about a nondividend-paying stock:

(i) The current stock price is 100.

(ii) The stock-price process is a geometric Brownian motion.

(iii) The continuously compounded expected return on the stock is 10%.

(iv) The stock's volatility is 30%.

Consider a nine-month 125-strike European call option on the stock.

Calculate the probability that the call will be exercised.

(A) 24.2%

(B) 25.1%

(C) 28.4%

(D) 30.6%

(E) 33.0%

17. Assume the Black-Scholes framework. Consider a one-year at-the-money European put option on a nondividend-paying stock.

You are given:

(i) The ratio of the put option price to the stock price is less than 5%.

(ii) Delta of the put option is -0.4364.

(iii) The continuously compounded risk-free interest rate is 1.2%.

Determine the stock's volatility.

(A) 12%

(B) 14%

(C) 16%

(D) 18%

(E) 20%

18. Consider an arbitrage-free securities market model, in which the risk-free interest rate is constant. There are two nondividend-paying stocks whose price processes are

$$S_1(t) = S_1(0)e^{0.1t+0.2Z(t)},$$

$$S_2(t) = S_2(0)e^{0.125t+0.3Z(t)},$$

where $Z(t)$ is a standard Brownian motion and $t \geq 0$.

Determine the continuously compounded risk-free interest rate.

(A) 2%

(B) 5%

(C) 8%

(D) 10%

(E) 20%

19. Consider a one-year 45-strike European put option on a stock S. You are given:

(i) The current stock price, $S(0)$, is 50.00.

(ii) The only dividend is 5.00 to be paid in nine months.

(iii) $\mathrm{Var}[\ln F_{t,1}^{P}(S)] = 0.01 \times t, \quad 0 \le t \le 1.$

(iv) The continuously compounded risk-free interest rate is 12%.

Under the Black-Scholes framework, calculate the price of 100 units of the put option.

(A) 1.87

(B) 18.39

(C) 18.69

(D) 19.41

(E) 23.76

20. Assume that the Black-Scholes framework holds. Consider an option on a stock.

You are given the following information at time 0:

(i) The stock price is $S(0)$, which is greater than 80.

(ii) The option price is 2.34.

(iii) The option delta is -0.181.

(iv) The option gamma is 0.035.

The stock price changes to 86.00. Using the delta-gamma approximation, you find that the option price changes to 2.21.

Determine $S(0)$.

(A) 84.80

(B) 85.00

(C) 85.20

(D) 85.40

(E) 85.80

****END OF EXAMINATION****