SOCIETY OF ACTUARIES
Exam FETE
Financial Economic Theory and Engineering Exam (Finance/ERM/Investment)

Exam FETE

AFTERNOON SESSION

Date: Thursday, October 29, 2009
Time: 1:30 p.m. – 4:45 p.m.

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This afternoon session consists of 10 questions numbered 10 through 19 for a total of 60 points. The points for each question are indicated at the beginning of the question.

2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.

3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.

2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.

3. The answer should be confined to the question as set.

4. When you are asked to calculate, show all your work including any applicable formulas.

5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets since they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate morning or afternoon session for Exam FETE.

6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d'examen pour la version française.

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TABLE FOR $N(X)$ WHEN $X \geq 0$

The table below shows values of $N(x)$ for $x \geq 0$. This table should be used with interpolation. For example,

$$\begin{aligned} N(0.6278) &= N(0.62) + 0.78[N(0.63) - N(0.62)] \\ &= 0.7324 + 0.78 \times (0.7357 - 0.7324) \\ &= 0.7350 \end{aligned}$$

x	*0.00*	*0.01*	*0.02*	*0.03*	*0.04*	*0.05*	*0.06*	*0.07*	*0.08*	*0.09*
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9986	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE FOR $N(X)$ WHEN $X \leq 0$

The table below shows values of $N(x)$ for $x \leq 0$. This table should be used with interpolation. For example,

$$N(-0.1234) = N(-0.12) - 0.34[N(-0.12) - N(-0.13)]$$
$$= 0.4522 - 0.34 \times (0.4522 - 0.4483)$$
$$= 0.4509$$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−3.0	0.0014	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
−3.6	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
−3.7	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
−3.8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
−3.9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
−4.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Financial Economic Theory and Engineering Formulae Sheet
May 2008

Morning and afternoon exam booklets will include a formula package identical to the one attached to this study note. The exam committee felt that by providing many key formulas, candidates would be able to focus more of their exam preparation time on the application of the formulas and concepts to demonstrate their understanding of the syllabus material and less time on the memorization of the formulas. The formula package was developed sequentially by reviewing the syllabus material for each major syllabus topic. Candidates should be able to follow the flow of the formula package easily. We recommend that candidates use the formula package concurrently with the syllabus material. Not every formula in the syllabus is in the formula package. **Candidates are responsible for all formulas on the syllabus, including those not on the formula sheet.**

Candidates should carefully observe the sometimes-subtle differences in formulas and their application to slightly different situations. For example, there are several versions of the Black-Scholes-Merton option pricing formula to differentiate between instruments paying dividends, tied to an index, etc. Candidates will be expected to recognize the correct formula to apply in a specific situation of an exam question.

Candidates will note that the formula package does not indicate where the formula occurs in the syllabus, nor does it provide names or definitions of the formula or symbols used in the formula. With the wide variety of references and authors of the syllabus, candidates should recognize that the letter conventions and use of symbols may vary from one part of the syllabus to another and thus from one formula to another.

We trust that you will find the inclusion of the formula package to be a valuable study aide that will allow for more of your preparation time to be spent on mastering the learning objectives and learning outcomes.

Copeland, Weston, Shastri, Financial Theory and Corporate Policy

$$S_0 = \sum_{t=1}^{\infty} \frac{Div_t}{(1+k_s)^t}$$

$$\mathrm{Re}\,v_t + m_t S_t = Div_t + (W \,\&\, S)_t + I_t$$

$$Div_t = \mathrm{Re}\,v_t - (W \,\&\, S)_t - I_t$$

$$S_0 = \sum_{t=1}^{\infty} \frac{\mathrm{Re}\,v_t - (W \,\&\, S)_t - I_t}{(1+k_s)^t}$$

$$NI_t = \mathrm{Re}\,v_t - (W \,\&\, S)_t - dep_t$$

$$\triangle A_t = I_t - dep_t$$

$$S_0 = \sum_{t=1}^{\infty} \frac{\mathrm{Re}\,v_t - (W \,\&\, S)_t - dep_t - (I_t - dep_t)}{(1+k_s)^t} \quad = \sum_{t=1}^{\infty} \frac{NI_t - \triangle A_t}{(1+k_s)^t}$$

$$NPV = \sum_{t=1}^{N} \frac{FCF_t}{(1+k)^t} - I_0$$

$$NPV = 0 = \sum_{t=1}^{N} \frac{FCF_t}{(1+IRR)^t} - I_0$$

$$k = WACC = weighted\ average\ \cos t\ of\ capital\ = k_b(1-\tau_c)\frac{B}{B+S} + k_s\frac{S}{B+S}$$

$$\begin{aligned} FCF_{for\ cap.budgeting} &= (\triangle \mathrm{Re}\,v - \triangle VC - \triangle FCC) - \tau_c(\triangle \mathrm{Re}\,v - \triangle VC - \triangle FCC - \triangle dep) - \triangle I \\ &= (\triangle \mathrm{Re}\,v - \triangle VC - \triangle FCC)(1-\tau_c) + \tau_c(\triangle dep) - \triangle I \ = (\triangle \mathrm{Re}\,v - \triangle VC - \triangle FCC - \triangle dep)(1-\tau_c) + \triangle dep - \triangle I \\ &= EBIT(1-\tau_c) + \triangle dep - \triangle I \qquad \text{earning before interest and taxes} \end{aligned}$$

$$\gamma_g = \left[\prod (1+\gamma_{pt})\right]^{1/N} - 1 \qquad \text{geometric returns}$$

$$\gamma_a = \frac{1}{N}\left[\sum (1+\gamma_{pt})\right] - 1 \qquad \text{arithmetic returns}$$

$$E(R_j) = R_f + \left[E(R_m) - R_f\right]\beta_j$$

$$R_{jt} = E(R_{jt}) + \beta_j \delta_{mt} + \varepsilon_{jt}$$

$$R_{jt} - R_{ft} = (R_{mt} - R_{ft})\beta_j + \varepsilon_{jt}$$

$$R'_{pt} = \gamma_0 + \gamma_1 \beta_p + \varepsilon_{pt} \qquad \text{where } \gamma_1 = R_{mt} - R_{ft} \quad R'_{pt} = \text{excess return on portfolio } p = (R_{pt} - R_{ft})$$

$$R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it}$$

$$E(R_i) = E(R_z) + \left[E(R_m) - E(R_z)\right]\beta_i$$

$$\alpha_i = E(R_z)(1-\beta_i)$$

$$E(R_i) - R_f = b_i\left[E(R_m) - R_f\right] + s_i E(SMB) + h_i E(HML)$$

$$\lambda_i = E(R_m) - E(R_z)$$

$$R_p = \left[\prod (1 + r_{pt})\right]^{1/T} - 1$$

$$A(R_t) = A(D_t / P_{t-1}) + A(GP_t)$$

$$R_{it} = \hat{\gamma}_{0t} + \hat{\gamma}_{1t}\beta_{it} + \varepsilon_{it}$$

$$R_{it} = \gamma_{0t} + \gamma_{1t}\ln(size)_t + \gamma_{2t}(Book/Market)_i + \varepsilon_{it}$$

$$E(R_i) = E(R_z) + \left[E(R_m) - E(R_z)\right]\beta_i$$

$$E(R_i) = E(R_{z,I}) + \left[E(R_I) - E(R_{z,I})\right]\beta_{i,I}$$

$$\tilde{R}_i = E(\tilde{R}_i) + b_{i1}\tilde{F}_1 + \ldots + b_{ik}\tilde{F}_k + \tilde{\varepsilon}_i$$

where $\tilde{R}_i =$ random rate of return on the ith asset

$E(\tilde{R}_i) =$ the expected rate of return on the ith asset

$b_{ik} =$ the sensitivity of the ith asset's returns to the kth factor

$\tilde{F}_k =$ the mean zero kth factor common to the returns of all assets

$\tilde{\varepsilon}_i =$ a random zero mean noise term for the ith asset

$$\sum_{i=1}^{n} w_i = 0$$

$$\tilde{R}_p = \sum_{i=1}^{n} w_i \tilde{R}_i = \sum_i w_i E(\tilde{R}_i) + \sum_i w_i b_{i1}\tilde{F}_1 + \ldots + \sum_i w_i b_{ik}\tilde{F}_k + \sum_i w_i \tilde{\varepsilon}_i$$

$w_i = 1/n$ n chosen to be a large number

$\sum_i w_i b_{ik} = 0$ for each factor k

$$\tilde{R}_p = \sum_i w_i E(\tilde{R}_i) + \sum_i w_i b_{i1}\tilde{F}_1 + \ldots + \sum_i w_i b_{ik}\tilde{F}_k$$

$$R_p = \sum_i w_i E(\tilde{R}_i)$$

$$R_p = \sum_i w_i E(\tilde{R}_i) = 0$$

$$E(\tilde{R}_i) = \lambda_0 + \lambda_1 b_{i1} + ... + \lambda_k b_{ik}$$

$$E(R_i) - R_f = \lambda_1 b_{i1} + ... + \lambda_k b_{ik}$$

$$E(R_i) - R_f = \left[\bar{\delta}_1 - R_f\right] b_{i1} + ... + \left[\bar{\delta}_k - R_f\right] b_{ik}$$

$$b_{ik} = Cov(R_i, \delta_k) / Var(\delta_k)$$

where $Cov(R_i, \delta_k) =$ the covariance between the ith asset's returns and the linear transformation of the kth factor

$Var(\delta_k) =$ the variance of the linear transformation of the kth factor

$$C(S_A, S_B, T) = S_A N(d_1) - S_B N(d_2) \qquad \text{where } d_1 = \left[\ln(S_A / S_B) + V^2 T\right] / V\sqrt{T} \qquad d_2 = d_1 - V\sqrt{T}$$

$$V^2 = V_A^2 - 2\rho_{AB} V_A V_B + V_B^2$$

$$Q_s = Q_d - MAX\left[0, Q_d - capacity\right]$$

$$V(\eta) \equiv \sum_m q(m) \underset{a}{MAX} \sum_e p(e|m) U(a,e) - V(\eta_0)$$

$$f_m(p_{1t}, ...p_{nt} | \eta_{t-1}^m) = f(p_{1t}, ...p_{nt} | \eta_{t-1})$$

$$V(\eta_i) - V(\eta_0) \equiv 0$$

$$p(r - c_2) + (1-p)(dr - c_2) = p(r/d - c_1) + (1-p)(r - c_1)$$

$$p = \frac{r(1-d) + c_2 - c_1}{2r - rd - r/d}$$

$$r(d-1) > c_2 - c_1 \text{ and } r(1 - 1/d) < c_2 - c_1$$

fair game $\varepsilon_{j,t+1} = \dfrac{p_{j,t+1} - p_{jt}}{p_{jt}} - \dfrac{E(p_{j,t+1}|\eta_t) - p_{jt}}{p_{jt}} = 0 = \dfrac{p_{j,t+1} - E(p_{j,t+1}|\eta_t)}{p_{jt}} = 0$

where $p_{j,t+1} =$ the actual price of security j next period

$E(p_{j,t+1}|\eta_t) =$ the predicted end-of-period price of security j given the current information structure η_t

$\varepsilon_{j,t+1} =$ the difference between actual and predicted returns

$$E(\varepsilon_{j,t+1}) = E\left[r_{j,t+1} - E(r_{j,t+1}|\eta_t)\right] = 0$$

submartingale $\frac{E(p_{j,t+1}|\eta_t) - p_{jt}}{p_{jt}} = E(r_{j,t+1}|\eta_t) > 0$

martingale $\frac{E(p_{j,t+1}|\eta_t) - p_{jt}}{p_{jt}} = E(r_{j,t+1}|\eta_t) = 0$

random walk $f(r_{1,t+1},...,r_{n,t+1}) = f(r_{1,t+1},...,r_{n,t+1}|\eta_t)$

$$\varepsilon_{j,t+1} = r_{j,t+1} - E(r_{j,t+1}|r_{jt}, r_{j,t-1},...,r_{j,t-n})$$

$$E\left[(r_{j,t+1} - E(r_{j,t+1}))(r_{jt} - E(r_{jt}))\right] = Cov(r_{j,t+1}, r_{jt}) = \int_{r_{jt}} \left[r_{jt} - E(r_{jt})\right]\left[r_{j,t+1} - E(r_{j,t+1})\right] f(r_{jt}) dr_{jt}$$

$$E(R_{jt}|\hat{\beta}_{jt}) = R_{ft} + \left[E(R_{mt}|\hat{\beta}_{mt}) - R_{ft}\right]\hat{\beta}_{jt}$$

$E(\varepsilon_{jt}) = 0$ where $\varepsilon_{jt} = R_{jt} - E(R_{jt}|\hat{\beta}_{jt})$

$E(R_{jt}|\hat{\beta}_{jt}) =$ the expected rate of return on the jth asset during this time period, given a prediction of its systematic risk, $\hat{\beta}_{jt}$

$R_{ft} =$ the risk-free rate of return during this time period

$E(R_{mt}|\hat{\beta}_{mt}) =$ the expected market rate of return, given a prediction of its systematic risk, $\hat{\beta}_{mt}$

$\hat{\beta}_{mt} =$ the estimated systematic risk of the jth security based on last time period's information structure η_{t-1}

$$R_{jt} = a_j + b_j R_{mt} + \varepsilon_{jt}$$

$$R_{jt} = \alpha_j + \beta_{1j}(R_{mt} - R_{ft}) + \beta_{2j}(RLE_t - RSE_t) + \beta_{3j}(HBTM_t - LBTM_t) + \varepsilon_{jt}$$

the change in earnings per share for the jth firm $\triangle NI_{jt} = \hat{a} + \hat{b}_j \triangle m_t + \varepsilon_{jt}$

where $\triangle m_t =$ the change in the average EPS for all firms (other than firm j) in the market

$$\triangle \hat{NJ}_{j,t+1} = \hat{a} + \hat{b}_j \triangle m_{t+1}$$

where $\hat{a}, \hat{b} =$ coefficients estimated from time-series fits of

$R_{jt} = \alpha_j + \beta_{1j}(R_{mt} - R_{ft}) + \beta_{2j}(RLE_t - RSE_t) + \beta_{3j}(HBTM_t - LBTM_t) + \varepsilon_{jt}$

$\triangle m_{t+1} =$ the actual change in market average EPS during the (t+1)th time period

abnormal performance index $API = \frac{1}{N}\sum_{j=1}^{N}\prod_{t=1}^{T}(1 + \varepsilon_{jt})$

where $N =$ the number of companies in a portfolio $T = 1, 2, ..., 12$

ε_{ij} = abnormal performance measured by deviations from the market model

$$V(\alpha)=\frac{1}{(1+r)}\left[\mu(\alpha)-\lambda\right]$$

where r = the risk-free rate of return

$\mu(\alpha)$ = the valuation schedule used by the market to infer the expected end-of-period value from the signal α

λ = the market's adjustment for the risk of the project

$$\max imize\, E\left[U(\tilde{W}_1)\right]$$

subject to $W_0 = X + \beta V_M + Y - (1-\alpha)V(\alpha)$

where W_0 = the entrepreneur's initial wealth V_M = the value of the market portfolio

β = the fraction of the market portfolio owned by the entrepreneur

Y = the amount invested in the risk-free asset α = the fraction of the project the entrepreneur retains

$\hat{W}_1$ = the uncertain end of period wealth of the entrepreneur

$$\hat{W}_1 = \alpha(\mu+\hat{\varepsilon}) + \beta\hat{M} + (1+r)Y$$
$$= \alpha\left[\mu+\hat{\varepsilon}-\mu(\alpha)+\lambda\right] + \beta\left[\hat{M}-(1+r)V_M\right] + (1+r)(W_0 - X) + \mu(\alpha) - \lambda$$

where $\hat{M}$ = the gross return of the market portfolio

$$\frac{\partial E(U(\tilde{W}_1))}{\partial \alpha} = E\left[U'(\tilde{W}_1)(\mu+\tilde{\varepsilon}-\mu(\alpha)+\lambda+(1-\alpha)\mu_\alpha)\right]=0$$

$$\frac{\partial E(U(\tilde{W}_1))}{\partial \beta} = E\left[U'(\tilde{W}_1)(\tilde{M}-(1+r)V_M)\right]=0 \qquad \text{where } \mu_\alpha = \frac{\partial \mu}{\partial \alpha}$$

$$(1-\alpha)\mu_\alpha = -\frac{E\left[U'(\tilde{W}_1)(\tilde{\varepsilon}+\lambda)\right]}{E\left[U'(\tilde{W}_1)\right]}$$

$$E(D)=\frac{1}{1+r}\left[V(D)+(1-\tau_p)D+\int_D^{\bar{X}}(X-D)f(X)dX+\int_{\underline{X}}^{D}(1+\beta)(X-D)f(X)dX\right]$$

$$=\frac{1}{1+r}\left[V(D)+\mu-\tau_p D-\beta\int_{\underline{X}}^{D}(X-D)f(X)dX\right]$$

$$E(D)=\frac{1}{1+r}\left[V(D)+\frac{t}{2}-\tau_p D-\beta\frac{D^2}{2t}\right]$$

$$V'(D^*)=\tau_p+\beta\frac{D^*}{t}$$

$$V\left[D^*(t)\right]=\frac{1}{r}\left[\frac{t}{2}-\tau_p D^*(t)-\beta\frac{\left[D^*(t)\right]^2}{2t}\right]$$

$$D^*(t) = At$$

$$V\left[D^*(t)\right] = (\tau_p + \beta A)D^*(t)$$

$$A = -\left[\frac{\tau_p}{\beta}\right]\left[\frac{1+r}{1+2r}\right] + \left[\frac{\tau_p}{\beta}\right]\left[\frac{1+r}{1+2r}\right]\sqrt{1+\frac{\beta(1+2r)}{\tau_p^2(1+r)^2}}$$

$$I + D = C + Np_e = C + P_e$$

$$\max_D imize\left[(1-\tau_p)D + p_e M + \frac{Q-M}{Q+N}X\right]$$

$$\max_D imize\, L - \tau_p D + \left[\frac{P+\tau_p D - L}{P+\tau_p D + I - C}\right]X$$

$$\tau_p = (\tau_p + \frac{\partial P}{\partial D})\frac{L+I-C}{(P+\tau_p D + I - C)}X$$

$$D(X) = \frac{1}{\tau_p}\max(I - C + L, 0)\ln X$$

$$P\left[D(x)\right] = C + X - I - \tau_p D(x)$$

$$\frac{\partial P}{\partial D} = \tau_p \frac{P\left[D(x)\right] + \tau_p D(x) - L}{I - C + L}$$

$$V_0^{old} = S + a$$

$$V_0^{old} = \frac{P'}{P'+E}(E+S+a+b) \quad \text{where } P' = \text{market price of old share}$$

$$\frac{P'}{P'+E}(E+S+a+b) \geq S + a$$

$$\frac{P'}{P'+E}(E+b) \geq \frac{E}{P'+E}(S+a) \qquad (E+b) \geq \frac{E}{P'}(S+a)$$

$$P' = S + a + E(\tilde{B} \,|\, issue\, and\, invest)$$

$$(E+b) < \frac{E}{P'}(S+a)\, or\, a > P'(1+\frac{b}{E}) - S$$

manager’s wage before stock split $W^0(z) = \alpha\overline{P}(z) + \beta P - T(m, P)$

manager's wage with stock split announcement $W^s(n,z) = \alpha \hat{P}(n,z) + \beta P - T(n,p)$

$$W_n = \alpha \hat{P}_n - \frac{t_2}{P^{\gamma-1}} = 0 \quad n^* \text{ maximize wage}$$

$$\hat{P}(n,z) = P \Rightarrow \alpha \hat{P}_n P^{\gamma-1} = t_2$$

$$\hat{P}(n,z) = k\left[n + c(z)\right]^{1/\gamma} \text{ where } k = (t_2\gamma/\alpha)^{1/\gamma}$$

market value of the firm after the split announcement

$$M(n,z) = \hat{P}(n,z) - T(n,P) = k(1-t_1)\left[n + c(z)\right]^{1/\gamma} - t_2 n k^{1-\gamma}\left[n + c(z)\right]^{1-\gamma/\gamma}$$

$$M(n,z) = \left[k(1-t_1) - t_2 k^{1-\gamma}\right] n^{1/\gamma}$$

$$\mu = \frac{M(n,z)}{\bar{M}(z)} = \left[k(1-t_1) - t_2 k^{1-\gamma}\right] n^{1/\gamma} \Big/ \bar{M}(z) = K\left[\frac{n}{\bar{M}(z)}\right]^{1/\gamma} \left[\bar{M}(z)\right]^{\frac{1}{\gamma}-1}$$

where $\bar{M}(z) =$ pre-split value of the firm, $K = \left[k(1-t_1) - t_2 k^{1-\gamma}\right]$

$$\ln u = \ln K - \frac{1}{\gamma}\left[\bar{M}(z)/n\right] + (\frac{1}{\gamma} - 1)\ln\left[\bar{M}(z)\right]$$

$$V_0 = (1-t)X_0$$

$$E(X \mid n) = \hat{X}(n) = \frac{X_0 s_0 + \hat{Y}_m(n) s_m}{s_0 + s_m}$$

$$V_1(n) = \hat{X}(n) - T(n) - C$$

where $T(n) =$ the expected total brokerage commission $= E\left[Xt(\frac{X}{n}) \middle| n\right]$

$C =$ the cost of executing the split

$$V_1(n) = \hat{X}(T) - T - C \quad \hat{X}(T) = E(X \mid T)$$

$$N(T) = T/F = FT$$

$$V_2(T,\bar{Y}) = \frac{X_0 s_0 + \hat{Y}_m s_m + \bar{Y} F T_s}{s_0 + s_m + F T_s} - E\left[Xt(X/n) \middle| T, \hat{Y}_m, \bar{Y}\right] - C$$

$$E\left[V_2(T)\middle|Y_m\right]=\frac{X_0s_0+\hat{Y}_ms_m+(\frac{X_0s_0+Y_ms_m}{s_0+s_m})FT_s}{s_0+s_m+FT_s}-T-C$$

$$E\left[\bar{Y}\middle|Y_m\right]=\frac{X_0s_0+Y_ms_m}{s_0+s_m} \quad E\left[(Xt(X/n)\middle|T,\hat{Y}_m,\bar{Y})\middle|Y_m\right]=T$$

$$\hat{Y}_m'(T)=\frac{s_0+s_m+FT_s}{S_m}$$

$$Y_m(T)=\frac{s_0+s_m}{s_m}T+\frac{F_s}{2s_m}T^2+K$$

$$(\frac{d\alpha}{dD})\left[V(n)-B(n,D)\right]-\alpha(D)(\frac{\partial B}{\partial D})=0$$

$$(\frac{d^2\alpha}{dD^2})\left[V(n)-B(n,D)\right]-2(\frac{d\alpha}{dD})(\frac{\partial B}{\partial D})-\alpha(D)(\frac{\partial^2 B}{\partial D^2})<0$$

$$\alpha(D^*(n))\left[V(n)-B(n,D^*(n))\right]-\left[V(n)-I\right]=0$$

$$\left[\frac{d^2\alpha}{dD^2}(V-B)-2\frac{d\alpha}{dD}\frac{\partial B}{\partial D}-\alpha\frac{\partial^2 B}{\partial D^2}\right]\frac{dD}{dn}=\frac{d\alpha}{dD}(\frac{\partial B}{\partial n}-\frac{dV}{dn})+\alpha(\frac{\partial^2 B}{\partial n\partial D})$$

$$\hat{\varepsilon}(\alpha,D)=\left[(a-D_0)^2+D^2\frac{1-\alpha}{\alpha^2}\right]$$

$$\bar{D}=X_1\left[\frac{\bar{\alpha}(\varepsilon-\gamma)}{(1-\bar{\alpha})(1-\varepsilon)\gamma}\right]$$

$$\max_{c(s,p),a} imize\int\int U\left[s-c(s,p)\right]f(s,p|a)\,dsdp$$

$$\text{subject to }\int\int V\left[c(s,p)\right]f(s,p|a)dsdp-G(a)\geq\underline{V}$$

$$\max_{c(s,p),a} imize\int\int U\left[s-c(s,p)\right]f(s,p|a)dsdp+\lambda\left[\int\int V\left[c(s,p)\right]f(s,p|a)dsdp-G(a)-\underline{V}\right]$$

$$-U'\left[s-c(s,p)\right]+\lambda V'\left[c(s,p)\right]=0 \quad or \quad \frac{U'\left[s-c(s,p)\right]}{V'\left[c(s,p)\right]}=\lambda$$

$$\max_{a} imize\,U(k)+\lambda\left[\int\int V\left[c(s,p)\right]f(s,p|a)\,dsdp-G(a)-\underline{V}\right]$$

$$\max_{c(s),a} imize\int U\left[s-c(s)\right]f(s|a)ds \qquad \text{subject to }\int V\left[c(s)\right]f(s|a)ds-G(a)\geq\underline{V}$$

$$\int V\left[c(s)\right] f_a(s|a)ds - G'(a) = 0$$

$$\max_{c(s),a} imize \int U\left[s - c(s)\right] f(s|a)ds + \lambda\left[\int V\left[c(s)\right] f(s|a)ds - G(a) - \underline{V}\right]$$
$$+\mu\left[\int V\left[c(s)\right] f_a(s|a)ds - G'(a)\right] \; -U'\left[s - c(s)\right] f(s|a) + \lambda V'\left[c(s)\right] f(s|a) + \mu V'\left[c(s)\right] f_a(s|a)$$

$$\frac{U'\left[s - c(s)\right]}{V'\left[c(s)\right]} = \lambda + \mu \frac{f_a(s|a)}{f(s|a)}$$

$$(\frac{1}{\delta_1})(\delta_0 + \delta_1 c)^{\gamma} = \lambda + \mu \frac{f_a(s|a)}{f(s|a)}$$

$$c(s) = -\frac{\delta_0}{\delta_1} + (\delta_1)^{(\frac{1}{\gamma}-1)} (\lambda + \mu \frac{f_a(s|a)}{f(s|a)})^{\frac{1}{\gamma}}$$

$$\underset{c(s,p),a}{Maximize} \int\int U\left[s - c(s,p)\right] f(s,p|a)dsdp + \lambda\left[\int\int V\left[c(s,p)\right] f(s,p|a)dsdp - G(a) - \underline{V}\right] +$$
$$\mu\left[\int\int V\left[c(s,p)\right] f_a(s,p|a)dsdp - G'(a)\right]$$

$$\frac{U'\left[s - c(s,p)\right]}{V'\left[c(s,p)\right]} = \lambda + \mu \frac{f_a(s,p|a)}{f(s,p|a)}$$

$$\frac{U'\left[s - c(\mathrm{P})\right]}{V'\left[c(\mathrm{P})\right]} = \lambda + \mu_1 \frac{f_{a_1}(p|a)}{f(p|a)} + \mu_2 \frac{f_{a_2}(p|a)}{f(p|a)} + .. + \mu_n \frac{f_{a_n}(p|a)}{f(p|a)}$$

$$s = \sum_{j=1}^{m} b_j a_j + \varepsilon_s$$

$$p_j = \sum_{j=1}^{m} q_{ij} a_j + \varepsilon_i \quad for \;\; i = 1,..,k$$

$$c(p_1,...,p_k) = \beta_0 + \sum_{i=1}^{k} \beta_i \tilde{p}_i$$

$$\underset{c(s,p,m),a(m),m(m)}{\max imize} \; E_{s,p,m}\left[U\left[s - c(s,p,m)\right] \| a(m)\right]$$
subject to (for all m) $E_{s,p|m}\left[\left[V\left[c(s,p,m)\right] - G\left[a(m)\right]\right] \| a(m)\right] \geq \underline{V}$

$a(m) =$ a that maximizes $E_{s,Y|m}\left[V\left[c(s,p,m)\right] \| a\right] - G(a)$ for each m

$m(m) =$ the $\hat{m}(m)$ that maximize $E_{s,Y|m}\left[V\left[c(s,p,\hat{m})\right] \| a\right] - G(a)$ for each m

$$\frac{\partial V(X^*)}{\partial X} = \frac{\partial P(X^*)}{\partial X} - \frac{\partial C(X^*)}{\partial X} = 0$$

$$S = \frac{(1-\beta)E(s) - \alpha - \lambda(1-\beta)Cov(s, R_M)}{1+r_f}$$

$$\max_{\alpha,\beta} imize \frac{(1-\beta)E(s) - \alpha - \lambda(1-\beta)Cov(s, R_M)}{1+r_f}$$

subject to $a\left[E(W) + \alpha + \beta E(s)\right] - b\left[Var(W) + 2\beta Cov(W,s) + \beta^2 Var(s)\right] \geq \underline{V}$

$$\max_{\alpha,\beta} imize \frac{(1-\beta)E(s) - \alpha - \lambda(1-\beta)Cov(s, R_M)}{1+r_f} +$$
$$\mu(a\left[E(W) + \alpha + \beta E(s)\right] - b\left[Var(W) + 2\beta Cov(W,s) + \beta^2 Var(s)\right] - \underline{V})$$

$$\mu a - \frac{1}{1+r_f} = 0$$

$$\mu\left[aE(s) - 2b(Cov(W,s) + \beta Var(s))\right] - \frac{E(s) - \lambda Cov(s, R_m)}{1+r_f} = 0$$

$$a\left[\alpha + \beta E(s)\right] - b\beta^2 Var(s) - \underline{V} = 0$$

$$\beta = \frac{\lambda a Cov(s, R_M)}{2bVar(s)} - \frac{Cov(W,s)}{Var(s)}$$

$$r^* = \frac{i}{1-(1+i)^{-T}}$$

$$B_{new} = \frac{D}{(1+r_f)^t} - (D)\left[P(\frac{V+dB}{D+dD}, 1, T, r_f, \sigma_V)\right]$$

$$B_{new} - B = D\left\{-\left[P(\frac{V+dB}{D+dD}, 1, T, r_f, \sigma_V)\right] + P(\frac{V}{D}, 1, T, r_f, \sigma_V)\right\} = D(-P_X + P_Y)$$

$$\frac{\partial B}{\partial \sigma_V} = -\frac{\partial P(V, D, T, r_f, \sigma_V)}{\partial \sigma_V}$$

$$V = \int_{s_a}^{\infty} q(s)\left[V(s) - I\right] ds$$

$$V_E = \int_{s_a}^{\infty} q(s)\left[V(s) - I - D\right] ds$$

$$V_D = \int_{s_a}^{s_b} q(s)\left[V(s) - I\right] ds + \int_{s_b}^{\infty} q(s) D ds$$

$$V = \int_{s_b}^{\infty} q(s)\left[V(s) - I\right] ds$$

$$V_U = E(\tilde{FCF}) \big/ \rho$$

where $V_U =$ the present value of unlevered firm(i.e. all equity)

$E(\tilde{FCF}) =$ the perpetual free cash flow after taxes

$\rho =$ the discount rate of an all-equity firm of equivalent risk

$$V_U = \frac{E(\tilde{FCF})}{\rho} \ or\ V_U = \frac{E(\tilde{EBIT})(1-\tau_c)}{\rho}$$

$$\tilde{NI} + k_d D = (\tilde{\mathrm{Re}v} - \tilde{VC} - FCC - dep)(1-\tau_c) + k_d D \tau_c$$

$$V^L = \frac{E(\tilde{EBIT})(1-\tau_c)}{\rho} + \frac{k_d D \tau_c}{k_b}$$

$$B = \frac{k_d D}{k_b}$$

$$V_L = V_U + \tau_c B$$

$$\frac{\triangle V_L}{\triangle I} = \frac{(1-\tau_c)}{\rho} \frac{\triangle E(\tilde{EBIT})}{\triangle I} + \tau_c \frac{\triangle B}{\triangle I}$$

$$\triangle V_L = \triangle S^0 + \triangle S^n + \triangle B^0 + \triangle B^n$$

$$\frac{\triangle V_L}{\triangle I} = \frac{\triangle S^0}{\triangle I} + \frac{\triangle S^n}{\triangle I} + \frac{\triangle B^0}{\triangle I} + \frac{\triangle B^n}{\triangle I}$$

$$\triangle I = \triangle S^n + \triangle B^n$$

$$\frac{\triangle V_L}{\triangle I} = \frac{\triangle S^0}{\triangle I} + \frac{\triangle S^n + \triangle B^0}{\triangle I} = \frac{\triangle S^0}{\triangle I} + 1$$

$$\frac{\triangle S^0}{\triangle I} = \frac{\triangle V_L}{\triangle I} - 1 > 0$$

$$\frac{(1-\tau_c)\triangle E(EBIT)}{\triangle I} > \rho(1-\tau_c \frac{\triangle B}{\triangle I})$$

weighted average cost of capital $WACC = \rho(1-\tau_c \frac{\triangle B}{\triangle I})$

$$WACC = \rho(1-\tau_c \frac{\triangle B}{\triangle V})$$

$$\frac{\triangle NI}{\triangle I} + \frac{\triangle k_d D}{\triangle I} - \frac{\tau_c \triangle(k_d D)}{\triangle I} = (1-\tau_c)\frac{\triangle EBIT}{\triangle I}$$

$$\frac{\triangle V_L}{\triangle I} = \frac{\triangle NI / \triangle I + (1-\tau_c)\triangle(k_d D) / \triangle I}{\rho} + \tau_c \frac{\triangle B}{\triangle I}$$

$$\frac{\triangle V_L}{\triangle I} = \frac{\triangle S^0 + \triangle S^n}{\triangle I} + \frac{\triangle B^n}{\triangle I} \quad \triangle B^0 \equiv 0$$

$$\frac{\triangle NI}{\triangle S^0 + \triangle S^n} = \rho + (1-\tau_c)(\rho - k_b)\frac{\triangle B}{\triangle S^0 + \triangle S^n}$$

$$k_s = \rho + (1-\tau_c)(\rho - k_b)\frac{\triangle B}{\triangle S}$$

$$WACC = (1-\tau_c)k_b \frac{B}{B+S} + k_s \frac{S}{B+S}$$

$$WACC = \rho(1-\tau_c \frac{B}{B+S})$$

$$G = V_L - V_U = \tau_c B$$

$$V_U = \frac{E(EBIT)(1-\tau_c)(1-\tau_{ps})}{\rho}$$

payment to shareholders $(EBIT - k_d D)(1-\tau_c)(1-\tau_{ps})$

payment to bondholders after personal taxes $k_d D(1-\tau_{pB})$

total cash payments to supplies of capital = $EBIT(1-\tau_c)(1-\tau_{ps}) - k_d D(1-\tau_c)(1-\tau_{ps}) + k_d D(1-\tau_{pB})$

$$V_L = \frac{E(EBIT)(1-\tau_c)(1-\tau_{ps})}{\rho} + \frac{k_d D\left[(1-\tau_{pB}) - (1-\tau_c)(1-\tau_{ps})\right]}{k_b} = V_U + \left[1 + \frac{(1-\tau_c)(1-\tau_{ps})}{(1-\tau_{pB})}\right]B$$

where $B = k_d D(1-\tau_{pB}) / k_b$

$$G = \left[1 - \frac{(1-\tau_c)(1-\tau_{ps})}{(1-\tau_{pB})}\right]B$$

$$(1-\tau_{pB}) = (1-\tau_c)(1-\tau_{ps})$$

$$G = (1 - \frac{(1-\tau_c)}{(1-\tau_{pB})})B$$

$$E(R_j) = R_f + \left[E(R_m) - R_f\right]\beta_j$$

where $E(R_j) =$ the expected rate of return on asset j $R_f =$ the risk-free rate

$E(R_m) =$ the expected rate of return on the market portfolio

$$\beta_j = Cov(R_j, R_m) \big/ Var(R_m)$$

$$\beta_L = \left[1 + (1-\tau_c)\frac{B}{S}\right]\beta_U$$

$$E(\tilde{R}_{bj}) = R_f + \left[E(\tilde{R}_m) - R_f\right]\beta_{bj}$$

$$k_s = \frac{(\tilde{EBIT} - \tilde{R}_{bj}B)(1-\tau_c)}{S^L}$$

$$E(k_s) = R_f + \lambda^* Cov(k_s, R_m)$$

$$Cov(k_s, R_m) = \frac{(1-\tau_c)}{S^L}Cov(EBIT, R_m) - \frac{(1-\tau_c)B}{S^L}Cov(R_{bj}, R_m)$$

$$R_f S^L + \lambda^*(1-\tau_c)Cov(EBIT, R_m) - \lambda^*(1-\tau_c)B\left[Cov(R_{bj}, R_m)\right] = (EBIT)(1-\tau_c) - E(R_{bj})B(1-\tau_c)$$

$$R_f V^U + \lambda^*(1-\tau_c)Cov(EBIT, R_m) = E(EBIT)(1-\tau_c)$$

$$V_L = V_U + \tau_c B$$

$$V = (B-P) + S$$

$$E(r_i) = r_f + \left[E(r_m) - r_f\right]\beta_i$$

where $E(r_i) =$ the instantaneous expected rate of return on asset i

$\beta_i = \frac{Cov(r_i, r_m)}{Var(r_m)}$ the instantaneous systematic risk of the ith asset

$E(r_m) =$ the expected instantaneous rate of return on the market portfolio

$r_f =$ the non-stochastic instantaneous annualized rate of return on the risk-free asset

$$dS = \frac{\partial S}{\partial V}dV + \frac{\partial S}{\partial t}dt + \frac{1}{2}\frac{\partial^2 S}{2\partial V^2}\sigma^2 V^2 dt$$

$$\lim_{dt \to 0}\frac{dS}{S} = \frac{\partial S}{\partial V}\frac{dV}{S} = \frac{\partial S}{\partial V}\frac{dV}{V}\frac{V}{S}$$

$$r_S = \frac{\partial S}{\partial V}\frac{V}{S} r_V$$

$$\beta_S = \frac{Cov(r_S, r_m)}{Var(r_m)}, \beta_V = \frac{Cov(r_V, r_m)}{Var(r_m)}$$

$$\beta_S = \frac{\partial S}{\partial V}\frac{V}{S}\frac{Cov(r_V, r_m)}{Var(r_m)} = \frac{\partial S}{\partial V}\frac{V}{S}\beta_V$$

$$S = VN(d_1) - e^{-r_f T} DN(d_2)$$

where $S =$ the market value of equity $V =$ the market value of the firm's asset $r_f =$ the risk-free rate $T =$ the time to maturity $D =$ the face value of debt (book value)

$$d_1 = \frac{Ln(V/D) + r_f T}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T} \qquad d_2 = d_1 - \sigma\sqrt{T}$$

$$\beta_S = N(d_1)\frac{V}{S}\beta_V$$

$$\beta_S = \frac{VN(d_1)}{VN(d_1) - De^{-r_f T}N(d_2)}\beta_V \quad = \frac{1}{1 - (D/V)e^{-r_f T}\left[N(d_2)/N(d_1)\right]}\beta_V$$

$$k_s = R_f + (R_m - R_f)N(d_1)\frac{V}{S}\beta_V$$

$$k_s = R_f + N(d_1)(R_v - R_f)\frac{V}{S}$$

$$\beta_B = \beta_V \frac{\partial B}{\partial V}\frac{V}{B}$$

$$\frac{\partial B}{\partial V} = N(-d_1) = 1 - N(d_1)$$

$$k_b = R_f + (R_m - R_f)\beta_B$$

$$k_b = R_f + (\rho - R_f)N(d_1)\frac{V}{B}$$

$$k_b\frac{B}{V} + k_s\frac{S}{V} = \left[R_f + (\rho - R_f)N(-d_1)\frac{V}{B}\right]\frac{B}{V} + \left[R_f + N(d_1)(\rho - R_f)\frac{V}{S}\right]\frac{S}{V}$$

$$= R_f(\frac{B+S}{V}) + (\rho - R_f)\left[N(-d_1) + N(d_1)\right] = R_f + (\rho - R_f)\left[1 - N(d_1) + N(d_1)\right] = \rho$$

$$k_s = \rho + (\rho - k_b)\frac{B}{S}$$

$$\frac{dV}{V} = \mu(V,t)dt + \sigma dW$$

$$\frac{1}{2}\sigma^2 V^2 F_{VV}(V,t) + rVF_V(V,t) - rF(V,t) + F_t(V,t) + C = 0$$

$$\frac{1}{2}\sigma^2 V^2 F_{VV}(V) + rVF_V(V) - rF(V) + C = 0$$

$$F(V) = A_0 + A_1 V + A_2 V^{-(2r/\sigma^2)}$$

$$B(V) = A_0 + A_1 V + A_2 V^{-(2r/\sigma^2)}$$

$$B(V) = C/r + \left[(1-\alpha)V_B - C/r\right](V/V_B)^{-2r/\sigma^2} = (1-p_B)C/r + p_B\left[(1-\alpha)V_B\right] \text{ where } p_B = (V/V_B)^{-2r/\sigma^2}$$

$$DC(V) = \alpha V_B (V/V_B)^{-2r/\sigma^2}$$

$$TB(V) = A_0 + A_1 V + A_2 V^{-2r/\sigma^2}$$

$$TB(V) = 0 = T_c(C/r) - \left[T_c(C/r)\right](V/V_B)^{-2r/\sigma^2}$$

$$V_L(V) = V_U(V) + T_c B(V) - DC(V) = V_U(V) + T_c B - p_B T_c B - \alpha V_B p_B$$

$$M = (1+r)\gamma_0 V_0 + \gamma_1 V_1 \ if \ V_1 \ge D$$
$$M = (1+r)\gamma_0 V_0 + \gamma_1 (V_1 - c) \ if \ V_1 < D$$

where $\gamma_0, \gamma_1 =$ positive weights $r =$ the one-period interest rate $V_0, V_1 =$ the current and future value of the firm $D =$ the face value of the debt $c =$ a penalty paid if bankruptcy occurs if $V < D$

$$M_a = \gamma_0(1+r)\frac{V_{1a}}{1+r} + \gamma_1 V_{1a} \ if \ D^* < D \le V_{1a} \text{ (tell the truth)}$$
$$M_a = \gamma_0(1+r)\frac{V_{1b}}{1+r} + \gamma_1 V_{1a} \ if \ D < D^* \text{ (lie)}$$

where $D^* =$ the maximum amount of debt that an unsuccessful firm can carry without going bankrupt

$$M_a = \gamma_0(1+r)\frac{V_{1a}}{1+r} + \gamma_1 (V_{1b} - c) \ if \ D^* < D \le V_{1a} \text{ (lie)}$$
$$M_a = \gamma_0(1+r)\frac{V_{1b}}{1+r} + \gamma_1 V_{1b} \ if \ D < D^* \text{ (tell the truth)}$$

$$\gamma_0(V_{1a} - V_{1b}) < \gamma_1 c$$

$$k_u(t+1) = \frac{div_i(t+1) + P_i(t+1) - P_i(t)}{P_i(t)}$$

where $k_u(t+1) =$ the market required rate of return during the time period t

$div_t(t+1) =$ dividends per share paid at the end of time period t

$P_i(t+1) =$ price per share at the end of time period t

$P_i(t) =$ price per share at the beginning of time period t

$$V_i(t) = \frac{Div_i(t+1) + n_i(t)P_i(t+1)}{1 + k_u(t+1)}$$

where $Div_i(t+1) =$ total dollar dividend payment $= n_i(t)div_i(t+1)$

$V_i(t) =$ the market value of the firm $= n_i(t)P_i(t)$

$$E\tilde{B}IT_i(t+1) + m_i(t+1)\tilde{P}_i(t+1) \equiv \tilde{I}_i(t+1) + \tilde{D}iv_i(t+1)$$

$$\tilde{R}_i(t+1) = \tilde{D}iv_i(t+1) + n_i(t)\tilde{P}_i(t+1)$$

$$\tilde{R}_i(t+1) = \tilde{D}iv_i(t+1) + n_i(t+1)\tilde{P}_i(t+1) - m_i(t+1)\tilde{P}_i(t+1)$$

$$\tilde{R}_i(t+1) = E\tilde{B}IT_i(t+1) - \tilde{I}_i(t+1) + \tilde{V}_i(t+1)$$

$$\tilde{V}_i(t) = \frac{E\bar{B}IT_i(t+1) - \tilde{I}_i(t+1) + \tilde{V}_i(t+1)}{1 + k_u(t+1)}$$

$$\tilde{Y}_{di} = \left[(E\tilde{B}IT - rD_c)(1-\tau_c) - rD_{pi} \right](1-\tau_{pi})$$

where $\tilde{Y}_{di} =$ the uncertain income to the ith individual if corporate income is received as dividends

$E\tilde{B}IT =$ the uncertain cash flows from operations provided by the firm
$r =$ the borrowing rate, which is assumed to be equal for individuals and firm
$D_c =$ the corporate debt $D_{pi} =$ personal debt held by the ith individual
$\tau_c =$ the corporate tax rate $\tau_{pi} =$ the personal income tax rate of the ith individual

$$\tilde{Y}_{gi} = (E\tilde{B}IT - rD_c)(1-\tau_c)(1-\tau_{gi}) - rD_{pi}(1-\tau_{pi})$$

where $\tilde{Y}_{gi} =$ the uncertain income to the ith individual if corporate income is received as capital gains

$\tau_{gi} =$ the capital gains rate for the ith individual

$$\tilde{Y}_{gi} = \left[(E\tilde{B}IT - rD_c)(1-\tau_c) - rD_{pi} \right](1-\tau_{gi}) + rD_{pi}(\tau_{pi} - \tau_{gi})$$

$$\tilde{Y}_{gi} / \tilde{Y}_{di} > 1$$

corporate debt $\dfrac{\partial \tilde{Y}_{gi}}{\partial D_c} = -r(1-\tau_c)(1-\tau_{gi})$

personal debt $\dfrac{\partial \tilde{Y}_{gi}}{\partial D_{pi}} = -r(1-\tau_{pi})$

$$R_{jt} - R_{ft} = \delta_0 + \delta_1 \beta_{jt} + \delta_2 \left[\left(div_{jt} / P_{jt} \right) - R_{ft} \right] + \tilde{\varepsilon}_{jt}$$

where $\delta_0 =$ a constant $\delta_1 =$ influence of systematic risk on R_{jt}

$\delta_2 =$ influence of dividend payment on R_{jt} $\beta_{jt} =$ the systematic risk of the jth security

$div_{jt} / P_{jt} =$ the dividend yield of the jth security $\tilde{\varepsilon}_{jt} =$ a random error term $R_{ft} =$ the risk-free rate

cost of internal funds $r_A(1-\tau_c)(1-\tau_{di})$

where $r_A =$ the pre-tax return on investments in real assets

$\tau_c =$ corporate effective marginal tax rate $\tau_{di} =$ personal dividend income tax rate of the ith individual

$$EBIT + mP + \triangle B = I + Div$$

$$V_1 = Div_1 + \frac{E(EBIT_2)}{1+k}$$

$$S_1 = V_1 - \triangle B_1 - mP_1 = Div_1 + \frac{E(EBIT_2)}{1+k} - \triangle B_1 - mP_1$$

$$S_1 = EBIT_1 - I_1 + \frac{E(EBIT_2)}{1+k}$$

$$E(S_1) = E_0(EBIT_1) - E_0(I_1) + \frac{E_0\left[f(I_1)\right]}{1+k} = f(I_0) - I_1 + \frac{f(I_1)}{1+k}$$

$$S_1 = EBIT_1 - I_1 + \frac{E_1(EBIT_2)}{1+k} = f(I_0) + \varepsilon_1 - I_1 + \frac{f(I_1) + E_1(\varepsilon_2 | \varepsilon_1)}{1+k} = f(I_0) + \varepsilon_1 - I_1 + \frac{f(I_1) + \gamma\varepsilon_1}{1+k}$$

$$S_1 - E(S_1) = \varepsilon_1 \left[1 + \frac{\gamma}{1+k} \right] = \left[EBIT_1 - E_0(EBIT_1) \right] \left[1 + \frac{\gamma}{1+k} \right]$$

$$\triangle Div_{it} = a_i + c_i(Div_{it}^* - Div_{i,t-1}) + U_{it}$$

where $\triangle Div_{it} =$ the change in dividends

$c_j =$ the speed of adjustment to the difference between a target dividend payment and last year's payout

$Div_{it}^* =$ the target dividend payout $a_i U_{it} =$ a constant and normally distributed random error term

$$P_B - t_g(P_B - P_C) = P_A - t_g(P_A - P_C) + div(1 - t_0)$$

$$\frac{P_B - P_A}{div} = \frac{1 - t_0}{1 - t_g}$$

arbitrage profit $\pi = -P_B + div - t_0 div + P_A + t_0(P_B - P_A)$

$$\pi = (1 - t_0)(P_A - P_B + div)$$

$$DY_i = a_1 + a_2\beta_i + a_3 AGE_i + a_4 INC_i + a_5 DTR_i + \varepsilon_i$$

where $DY_i =$ dividend yield for the ith individual's portfolio

$\beta_i =$ the systematic risk of the ith individual's portfolio $AGE_i =$ the age of the individual

$INC_i =$ the gross family income averaged over the last three years

$DTR_i =$ the difference between the income and capital gain tax rates for the ith individual

$\varepsilon_i =$ a normally distributed random error term

$$\triangle Div_t = \beta_1 Div_{t-1} + \beta_2 NI_t + \beta_3 NI_{t-1} + Z_t$$

where $\triangle Div_t =$ the change in dividends in period t $Div_{t-1} =$ the previous period's dividends

$NI_t =$ this period's earnings $NI_{t-1} =$ last period's earnings

$Z_t =$ unanticipated dividend changes (the error term)

$$R_{jt} = \alpha + \beta_j R_{mt} + \varepsilon_{jt}$$

where $R_{jt} =$ the total return (dividends and capital gains) on the common stock of the jth firm

$\beta_j =$ a constant term $R_{mt} =$ systematic risk $\varepsilon_{jt} =$ the abnormal performance of the jth security

$$P_{it} = a + bDiv_{it} + cRE_{it} + \varepsilon_{it}$$

where $P_{it} =$ the price per share $Div_{it} =$ aggregate dividends paid out

$RE_{it} =$ retained earnings $\varepsilon_{it} =$ the error term

$$\frac{(NI/P)_{it}}{(NI/P)_{kt}} = a_i + b_{it} + \varepsilon_{it}$$

where $(NI/P)_{it} =$ the earnings / price ratio for the firm

$(NI/P)_{kt} =$ the average earnings/price ratio of the industry

$t =$ a time index $\varepsilon_{it} =$ the error term

$$E(\tilde{R}_j) = R_f + \left[E(\tilde{R}_m) - R_f\right]\beta_j$$

$$\tilde{R}_j = \gamma_0 + \left[\tilde{R}_m - \gamma_0\right]\beta_j + \gamma_1 \left[DY_j - DY_m\right]\Big/ DY_m + \varepsilon_j$$

where $\tilde{R}_j =$ the rate of return on the jth portfolio

$\gamma_0 =$ an intercept term that should be equal to the risk-free rate R_f according to CAPM

$\tilde{R}_m =$ the rate of return on the market portfolio $\beta_j =$ the systematic risk of the jth portfolio

$\gamma_1 =$ the dividend impact coefficient

$DY_j =$ the dividend yield on the jth portfolio, measured as the sum of dividends paid during the previous year divided by the end-of-year price

$DY_m =$ the dividend yield on the market portfolio, measured over the period of 12 months

$\varepsilon_j =$ the error term

$$E(\tilde{R}_{jt}) - R_{ft} = a_1 + a_2\beta_j + a_3(DY_{jt} - R_{ft})$$

where $E(\tilde{R}_{jt}) =$ the expected before tax return on the jth security

$R_{ft} =$ the before-tax return on the risk-free asset

$\beta_j =$ the systematic risk of the jth security

$a_1 =$ the constant term $a_2 =$ the marginal effect of systematic risk

$a_3 =$ the marginal effective tax difference between ordinary income and capital gains rates

$DY_{jt} =$ the dividend yield (i.e. dividend divided by price) for the jth security

$$R_{pt} = \lambda_0 + \beta_{1F}\left[MFT + \lambda_1\right] + \beta_{2F}\left[SMB_t + \lambda_2\right] + \beta_{3F}\left[HML_t + \lambda_3\right] + \lambda_4 d_{p,t-1} + \varepsilon_{pt}$$

where $MKT =$ the excess returns on the CRSP value-weighted portfolio

$SMB =$ the difference between average returns on small minus big equity capitalization portfolio

$HML =$ the difference between average return on high minus low book equity to market equity portfolio

$d_{p,t-1} =$ the equally weighted yield of stocks in portfolio p minus the market dividend yield

$\lambda_i =$ the risk premium corresponding to the ith risk factor

$\lambda_4 =$ the coefficient on the dividend yield measure

$$P_E N_E = P_0 N_0 - P_T(N_0 - N_E) + \vartriangle W$$

where $P_E =$ the post expiration share price $N_E =$ the number of shares outstanding after repurchase

$P_0 =$ the pre-announcement share price $N_0 =$ the pre-announcement number of shares outstanding

$P_T =$ the tender price $\vartriangle W =$ the shareholder wealth effect attributable to the tender offer

$$F_P = 1 - \frac{N_E}{N_0} \text{ fraction of shares repurchased}$$

$$\frac{\vartriangle W}{N_0 P_0} = (1 - F_P)(\frac{P_E - P_0}{P_0}) + F_P \frac{P_T - P_0}{P_0}$$

Chew, The New Corporate Finance,

None

Hardy, **Investment Guarantees**

$$\frac{S_{t+w}}{S_t} \sim \text{LN}(w\mu, \sqrt{w}\sigma) \Rightarrow \log\frac{S_{t+w}}{S_t} \sim N(w\mu, w\sigma^2)$$

$$f(x)=\frac{1}{x\sigma\sqrt{2\pi w}}\exp\left\{-\frac{1}{2}\frac{\left(\log(x)-w\mu\right)^2}{w\sigma^2}\right\}$$

$$\mathrm{E}\left[\frac{S_{t+w}}{S_t}\right]=e^{w\mu+w\sigma^2/2}$$

$$\mathrm{V}\left[\frac{S_{t+w}}{S_t}\right]=e^{2w\mu+w\sigma^2}\left(e^{w\sigma^2}-1\right)$$

$$Y_t=\mu+a\left(Y_{t-1}-\mu\right)+\sigma\varepsilon_t$$

$$Y_t=\mu+a\left(Y_{t-1}-\mu\right)+\sigma_t\varepsilon_t$$

$$\sigma_t^2=\alpha_0+\alpha_1\left(Y_{t-1}-\mu\right)^2+\beta\sigma_{t-1}^2$$

$$\pi_1=\frac{p_{2,1}}{p_{1,2}+p_{2,1}}\,,\quad \pi_2=1-\pi_1$$

$$p_n(r)=\Pr\left[R_n(0)=r\right]=\pi_1\Pr\left[R_n(0)=r\middle|\rho_{-1}=1\right]+\pi_2\Pr\left[R_n(0)=r\middle|\rho_{-1}=2\right]$$

$$\sigma^*(R_n)=\sqrt{R_n\sigma_1^2+(n-R_n)\sigma_2^2}$$

$$F_{S_n}(x)=\Pr(S_n\le x)=\sum_{r=0}^{n}\Pr(S_n\le x|R_n=r)\,p_n(r)$$

$$F_{S_n(x)}=\sum_{r=0}^{n}\Phi\left(\frac{\log x-\mu^*(r)}{\sigma^*(r)}\right)p_n(r)$$

$$f_{S_n}(x)=\sum_{r=0}^{n}\frac{1}{\sigma^*(r)x}\phi\left(\frac{\log x-\mu^*(r)}{\sigma^*(r)}\right)p_n(r)$$ (from errata sheet)

$$y(t)=\exp\left\{w_y\delta_q(t)+\mu_y+yn(t)\right\}\qquad \text{where } yn(t)=a_y yn(t-1)+\sigma_y z_y(t)$$

$$\mathrm{E}\left[y(t)\right]=e^{\mu_y}\mathrm{E}\left[\exp(w_y\delta_q(t))\right]\mathrm{E}\left[\exp(yn(t))\right]$$

$$M_{\delta_q}(u)=\exp\left(u\mu_q+\frac{u^2(\sigma_q)^2}{2}\right)$$

$$\mathrm{E}\left[y(t)\right]=e^{\mu_y}M_q(w_y)\left[\exp\left(\mu_{yn}+\frac{\sigma_y^2}{2(1-a_y^2)}\right)\right]$$

$$DM(t) = d_d \delta_q(t) + (1-d_d)DM(t-1)$$

$$\frac{\partial l(\mu,\sigma)}{\partial \mu} = \frac{1}{\sigma}(\sum_{t=1}^{n} y_t - n\mu)$$

$$\frac{\partial l(\mu,\sigma)}{\partial \sigma} = \frac{-n}{\sigma} + \frac{1}{\sigma^3}\sum_{t=1}^{n}(y_t - \mu)^2$$

$$\hat{\sigma} = \sqrt{\frac{\sum_{t=1}^{n}(y_t - \hat{\mu})^2}{n}} \qquad \text{where} \quad \hat{\mu} = \overline{y}$$

$$\frac{\partial^2 l(\mu,\sigma)}{\partial \mu^2} = -\frac{n}{\sigma}$$

$$\frac{\partial^2 l(\mu,\sigma)}{\partial \mu \partial \sigma} = \frac{-1}{\sigma^2}(\sum_{t=1}^{n} Y_t - n\mu)$$

$$\frac{\partial^2 l(\mu,\sigma)}{\partial \sigma^2} = \frac{3}{-\sigma^4}\sum_{t=1}^{n}(Y_t - \mu)^2 + \frac{n}{\sigma^2}$$

$$E\left[\frac{-\partial^2 l(\mu,\sigma)}{\partial \mu^2}\right] = \frac{n}{\sigma}$$

$$E\left[\frac{-\partial^2 l(\mu,\sigma)}{\partial \mu \partial \sigma}\right] = 0$$

$$E\left[\frac{-\partial^2 l(\mu,\sigma)}{\partial \sigma^2}\right] = \frac{2n}{\sigma^2}$$

$$\sum \approx \begin{pmatrix} \frac{\hat{\sigma}^2}{n} & 0 \\ 0 & \frac{\hat{\sigma}^2}{2n} \end{pmatrix}$$

$$l(\mu,\sigma,a) = \ln(\sqrt{\frac{1-a^2}{2\pi\sigma^2}} \exp\left\{-\frac{1}{2}\left(\frac{(Y_1-\mu)^2(1-a^2)}{\sigma^2}\right)\right\}) + \sum_{t=2}^{n}\ln(\sqrt{\frac{1}{2\pi\sigma^2}} \exp\left\{-\frac{1}{2}\left(\frac{(Y_t-(1-a)\mu - aY_{t-1})^2}{\sigma^2}\right)\right\})$$

$$= \frac{-n}{2}\ln(2\pi) + \frac{1}{2}\ln(1-a^2) - n\ln\sigma - \frac{1}{2}\left\{\frac{(Y_1-\mu)^2(1-a^2)}{\sigma^2} + \sum_{t=2}^{n}\left(\frac{(Y_t-(1-a)\mu - aY_{t-1})^2}{\sigma^2}\right)\right\}$$

$$\ln S_n \sim N(n\mu, (\sigma h(a,n))^2) \quad \text{where} \ h(a,n) = \frac{1}{(1-a)}\sqrt{\sum_{i=1}^{n}(1-a^i)^2}$$

$$\ln S_n - n\mu = Z_1 + Z_2 + \dots + Z_n = \frac{\sigma}{1-a}\left\{\sum_{i=1}^{n} \varepsilon_i (1-a^{n+1-i})\right\}$$

$$F_{S_n}(x) = Pr\left[S_n \le x\right] = \sum_{r=0}^{n} Pr\left[S_n \le x \middle| R_n = r\right] p_n(r) = \sum_{r=0}^{n} \Phi\left(\frac{\ln x - \mu^*(r)}{\sigma^*(r)}\right) p_n(r)$$

$$f\left(x \middle| x_1, .., x_n\right) = \int_\theta f\left(x \middle| \theta\right) \pi\left(\theta \middle| x_1, .., x_n\right) d\theta$$

where $f\left(X \middle| \theta\right)$ is the density of X given the parameter θ

$$\Theta_{\sim i}^{(r+1,r)} = \left(\theta_1^{(r+1)}, .., \theta_{i-1}^{(r+1)}, \theta_{i+1}^{(r+1)}, .., \theta_n^{(r)}\right)$$

$$\alpha = \min\left(1, \frac{L_i\left(\xi, \Theta_{\sim i}^{(r,r+1)}\right) \pi\left(\xi\right) q\left(\theta_i^{(r)} \middle| \xi\right)}{L_i\left(\theta_i^{(r)} \Theta_{\sim i}^{(r,r+1)}\right) \pi\left(\theta_i^{(r)}\right) q\left(\xi \middle| \theta_i^{(r)}\right)}\right)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 (Y_{t-1} - \mu)^2 + \beta \sigma_{t-1}^2$$

$$F_{t^+} = F_{t^-}(1-m) = F_{(t-1)}(1-m) \frac{S_t}{S_{t-1}}$$

$$F_{(t+u)^+} = F_t \frac{S_{t+u}(1-m)^u}{S_t}$$

$$M_t = \left(F_{t^-}\right) m_c = m_c F_{0^-} \frac{s_t \left(1-m\right)^{t-1}}{S_0}$$ *errata sheet*

$$C_n = -{}_n p_x^\tau (G - F_n)^+$$

$$C_t = -{}_t p_x^\tau M_t^d + {}_{t-1|1} q_x^d \left(G - F_t\right)^+$$ *note: M should have d superscript*

$$C_t = -{}_t p_x^\tau F_0 - S_t (1-m)^t m_d + {}_{t-1|1} q_x^d \left(G - F_0 - S_t (1-m)^t\right)^+$$ *errata sheet*

$$C_t = {}_{t-1|1} q_x^d \left(G_r - F_t\right)^+ - {}_t p_x^\tau M_t \quad \textit{where} \quad n_r < t < n_{r+1}$$

$$C_{n_r} = {}_{n_r-1|1} q_x^d \left(G_r - F_{n_r^-}\right)^+ + {}_{n_r} p_x^\tau \left(G_r - F_{n_r^-}\right)^+ - {}_{n_r} p_x^\tau M_{n_r}$$

$$P_0 = (K - S_d) \frac{S_u e^{-r} - S_0}{S_u - S_d} = (K - S_d) e^{-r} p^* \qquad \text{where } p^* = \frac{S_u - S_0 e^r}{S_u - S_d}$$

$$A = \Phi^{-1}\left(\frac{1+\beta}{2}\right) \sqrt{N\alpha(1-\alpha)}$$

$$\xi = 1 - \Phi\left(\frac{\log G/S_0 - n(\mu + \log(1-m))}{\sqrt{n}\sigma}\right)$$

$$\Pr\left[F_n + V_\alpha e^{rn} > G\right] \geq \alpha$$

$$V_\alpha = (G - F_{F_n}^{-1}(1-\alpha))\, e^{-rn}$$

$$V_\alpha = (G - F_0 \exp(-z_\alpha \sqrt{n}\sigma + n(\mu + \ln(1-m))))e^{-rn}$$

$$\mathrm{CTE}_\alpha(L) = \frac{(1-\beta')\mathrm{E}\left[\mathrm{X} \middle| X > V_\alpha\right] + (\beta' - \alpha)V_\alpha}{1-\alpha}$$

$$\mathrm{CTE}_\alpha(L) = \mathrm{E}\left[(G - F_n)e^{-rn} \middle| F_n < (G - V_\alpha e^{rn})\right]$$

$$\mathrm{CTE}_\alpha(L) = e^{-rn}\left\{G - \frac{e^{n(\mu + \log(1-m) + \sigma^2/2)}}{1-\alpha}\Phi(-z_\alpha - \sqrt{n}\sigma)\right\}$$

$$\mathrm{CTE}_\alpha(X) = \frac{(1-\xi)}{(1-\alpha)}\mathrm{CTE}_\xi(X)$$

$$E\left[L\right] = e^{-rn}\left\{G(1-\xi) - F_0 \exp(n(\mu + \ln(1-m) + \frac{\sigma^2}{2}))\Phi(A)\right\} \quad \text{where } A = \frac{(\ln G_{F_0} - n(\mu + \ln(1-m)) - n\sigma^2)}{\sqrt{n}\sigma}$$

$$\log(1+i_t) \middle| \rho_t^y = \mu_{\rho_t^y}^y + \phi_{\rho_t^y}^y \left(\log(1+i_{t-1}) - \mu_{\rho_t^y}^y\right) + \sigma_{\rho_t^y}^y \varepsilon_t$$

$$H_0 = B(0,n)\mathrm{E}_Q\left[F_n(ga_{65}(n) - 1)^+\right]$$

$$H_0 = F_0 \mathrm{E}_Q\left[\left(\frac{ga_{65}^d(0,n)}{B(0,n)} - 1\right)^+\right]$$

$$H_t = F_t\left\{ga_{65}(t)\Phi(d_1(t)) - \Phi(d_2(t))\right\} \text{ where } d_1(t) = \frac{\log(ga_{65}(t)) + \sigma_y^2(n-t)/2}{\sigma_y\sqrt{n-t}} \quad \text{and} \quad d_2(t) = d_1(t) - \sigma_y\sqrt{n-t}$$

$$\text{PTP}: \quad \max\left[P\left(1 + \alpha\left(\frac{S_n}{S_0} - 1\right)\right), G\right]$$

where P: single premium, α : participation rate, G: guaranteed payout, S_t : value of the equity index at time t

Annual Ratchet

$$\text{CAR: } P\prod_{t=1}^{n}\left\{1 + \max\left(\alpha\left(\frac{S_t}{S_{t-1}} - 1\right), 0\right)\right\}$$

SAR: $P\left\{1+\sum_{t=1}^{n}\max\left(\alpha\left(\frac{S_t}{S_{t-1}}-1\right),0\right)\right\}$

CAR with cap rate c: $P\prod_{t=1}^{n}\left\{1+\min\left[\max\left(\alpha\left(\frac{S_t}{S_{t-1}}-1\right),0\right),c\right]\right\}$

SAR with cap rate c: $P\left\{1+\sum_{t=1}^{n}\min\left[\max\left(\alpha\left(\frac{S_t}{S_{t-1}}-1\right),0\right),c\right]\right\}$

High Water Mark: $\max\left[P\left(1+\alpha\left(\frac{S^{\max}}{S_0}-1\right)\right),G\right]$ where $S^{\max}=\max\left(S_0,S_1,..,S_n\right)$

$$H=\left(P\left(1+\alpha\left(\frac{S_n}{S_0}-1\right)\right)-G\right)$$

$$H=\frac{\alpha P}{S_0}\left\{S_n-\frac{S_0}{\alpha}\left(\frac{G}{P}-(1-\alpha)\right)\right\}$$

$$H_0=\frac{\alpha P}{S_0}\left\{S_0e^{-dn}\Phi(d_1)-K^{PTP}e^{-rn}\Phi(d_2)\right\} \text{ where } K^{PTP}=\frac{S_0}{\alpha}\left(\frac{G}{P}-(1-\alpha)\right)$$

$$H_0=\alpha Pe^{-dn}\Phi(d_1)-\left(G-P(1-\alpha)\right)e^{-rn}\Phi(d_2)$$

$$d_1=\frac{\ln\frac{\alpha P}{G-P(1-\alpha)}+\left(r-d+\frac{\sigma^2}{2}\right)n}{\sigma\sqrt{n}}\ ,\ d_2=d_1-\sigma\sqrt{n}$$

$$RP=P\prod_{t=1}^{n}\left\{1+\max\left(\alpha\left(\frac{S_t}{S_{t-1}}-1\right),0\right)\right\}$$

$$H=E_Q\left[e^{-rn}\left(RP\right)\right]$$

$$H=PE_Q\left[\prod_{t=1}^{n}e^{-r}\left\{1+\max\left(\alpha\left(\frac{S_t}{S_{t-1}}-1\right),0\right)\right\}\right]$$

$$H=P\prod_{t=1}^{n}\left\{e^{-r}+E_Q\left[e^{-r}\max\left(\alpha\left(\frac{S_t}{S_{t-1}}-1\right),0\right)\right]\right\}$$

$$\alpha E_Q\left[e^{-r}\max\left(S_1-1,0\right)\right]=\alpha\left\{e^{-d}\Phi\left(d_1\right)-e^{-r}\Phi\left(d_2\right)\right\} \qquad \text{where } d_1=\frac{r-d+\frac{\sigma^2}{2}}{\sigma}, \quad d_2=d_1-\sigma$$

$$H=P\left\{e^{-r}+\alpha\left(e^{-d}\Phi\left(d_1\right)-e^{-r}\Phi\left(d_2\right)\right)\right\}^n$$

$$E_Q\left[e^{-r}\left\{1+\max\left(\alpha\left(\frac{S_t}{S_{t-1}}-1\right),e^g-1\right)\right\}\right] \text{ where } e^g\text{: minimum accumulation factor}$$

$$=E_Q\left[e^{-r}\left\{1+\max\left(\alpha\left(S_1-1\right),e^g-1\right)\right\}\right]$$

$$=E_Q\left[e^{-r}\left\{1+\left(e^g-1\right)+\alpha\max\left(S_1-\left(\frac{e^g-(1-\alpha)}{\alpha}\right),0\right)\right\}\right]$$

$$=e^{g-r}+\alpha BSC\left(K=\frac{e^g-(1-\alpha)}{\alpha},n=1\right)$$

$BSC\left(K,n\right)$: Black-Scholes call-option price with strike K, starting stock price 1.0 and term n years

$$P\left\{\alpha e^{-d}\left(\Phi\left(d_1\right)-\Phi\left(d_3\right)\right)+\left(1-\alpha\right)e^{-r}\left(\Phi\left(d_2\right)-\Phi\left(d_4\right)\right)+e^{g-r}\Phi\left(-d_2\right)+e^{c-r}\Phi\left(d_4\right)\right\}^n$$

$$\text{where } d_1=\frac{\ln\left(\frac{1}{\left(e^g-(1-\alpha)\big/\alpha\right)}\right)+r-d+\frac{\sigma^2}{2}}{\sigma} \quad d_2=d_1-\sigma$$

$$\text{where } d_3=\frac{\ln\left(\frac{1}{\left(e^c-(1-\alpha)\big/\alpha\right)}\right)+r-d+\frac{\sigma^2}{2}}{\sigma} \quad d_4=d_3-\sigma$$

SAR with life-of-contract guarantee without cap: $P\left\{1+\sum_{t=1}^{n}\alpha\left(\frac{S_t}{S_{t-1}}-1\right)\right\}$

$$H_{t+1}=\alpha\left\{S_{t+1}e^{-d(n-t-1)}\Phi\left(d_1\left(t+1\right)\right)-K^{PTP}e^{-r(n-t-1)}\Phi\left(d_2\left(t+1\right)\right)\right\}$$

$$tc\propto S_{t+1}e^{-d(n-t-1)}\left|\Phi\left(d_1\left(t+1\right)\right)-\Phi\left(d_1\left(t\right)\right)\right|$$

Toole and Herget, Insurance Industry Mergers and Acquisitions

$$r=r_f+\beta(r_m-r_f)$$

where $r =$ expected rate of return on the acquisition
$r_f =$ risk-free rate of return $r_m =$ expected rate of return for the market as a whole
$\beta =$ measure of risk of a company (both debt and equity) relative to the market as a whole

$$r = r^D \frac{D}{D+E} + \frac{E}{D+E}(r_f + \beta^E (r_m - r_f))$$

where $r =$ weighted average cost of capital WACC
$r^D =$ required return on debt $\beta^E =$ beta of a company's stock
$D =$ market value of a company's debt $E =$ market value of a company's equity

cost of $capital_t =$ required $capital_{t-1}$ * (discount rate – after tax earnings $rate_t$)
appraisal cost of capital = NPV(cost of $capital_t$)

NPV(distributable $earning_t$)=Excess $capital_0$+NPV(after tax $earnings_t$ - Insurance in RC_t)
=NPV(after tax earning on the $business_t$) + Excess $capital_0$+NPV(after-tax earning on $capital_t$)-NPV(increase in RC_t)
=NPV(after tax earning on the $business_t$) + Excess $capital_0$
+NPV($RC_{t-1} * i_t$) – (NPV(RC_t) – NPV(RC_{t-1}))
=NPV(after tax earning on the $business_t$) + Excess $capital_0$
+ NPV($RC_{t-1} * i_t$) – ((1+d)NPV(RC_{t-1}) – RC_0 - NPV(RC_{t-1}))
=NPV(after tax earning on the $business_t$) + Excess $capital_0 + RC_0$ -NPV($RC_t * (d - i_t)$)
=value of Inforce and Future Business + adjusted of book value –cost of required capital
where $i_t =$ after tax investment earnings rate on capital $d =$ discount rate RC_t = required capital

Total reserve = $(1 - 1/PLDF) *$ expected loss where $PLDF =$ paid loss development factor

IBNR reserve = $(1 - 1/RLDF) *$ expected loss where $RLDF =$ reported loss development factor

Trigeorgis, Real Options

$$NPV = \sum_{t=1}^{T} \frac{\alpha_t E(c_t)}{(1+r_1)...(1+r_t)} - I$$

$$E(r_j) = r + \beta_j \left[E(r_m) - r \right]$$

$$Expanded\,(strategic)\,net\ present\,value\,(NPV^*) = \left[Direct\,(passive)\,NPV + strategic\,value \right] + flexibility\,value$$

$$\frac{d\pi_A}{dK_A} = \frac{\partial \pi_A}{\partial K_A} + \frac{\partial \pi_A}{\partial \alpha_B} \frac{d\alpha^* B}{dK_A}$$

Hull , Options, Futures and Other Derivatives,

$$\vartriangle z = \varepsilon\sqrt{\vartriangle t}$$

$$z(T) - z(0) = \sum_{i=1}^{N} \varepsilon_i \sqrt{\vartriangle t}$$

$$dx = adt + bdz$$

$$dx = a(x,t)dt + b(x,t)dz$$

$$S_T = S_0 e^{\mu T}$$

$$\frac{ds}{S} = \mu dt + \sigma dz$$

$$\Delta S = \mu S \vartriangle t + \sigma S \varepsilon \sqrt{\vartriangle t}$$

$$\frac{\Delta S}{S} \sim \phi(\mu \vartriangle t, \sigma\sqrt{\vartriangle t})$$

$$dG = (\frac{\partial G}{\partial x} a + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial x^2} b^2)dt + \frac{\partial G}{\partial x} bdz$$

$$dS = \mu S dt + \sigma S dz$$

$$dG = (\frac{\partial G}{\partial S}\mu S + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial S^2}\sigma^2 S^2)dt + \frac{\partial G}{\partial S}\sigma S dz$$

$$F = Se^{r(T-t)}$$

$$dF = (\mu - r)Fdt + \sigma F dz$$

$$dG = (\mu - \frac{\sigma^2}{2})dt + \sigma dz$$

$$\ln S_T \sim \phi\left(\ln S_0 + \left(\mu - \frac{\sigma^2}{2}\right)T, \sigma\sqrt{T}\right)$$

$$E\left(S_T\right) = S_o e^{\mu T}$$

$$\text{var}\left(S_T\right) = S_0^2 e^{2\mu T}\left[e^{\sigma^2 T} - 1\right]$$

$$x = \frac{1}{T}\ln\frac{S_T}{S_0}$$

$$x \sim \phi\left(\mu - \frac{\sigma^2}{2}, \frac{\sigma}{\sqrt{T}}\right)$$

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(u_i - \bar{u}\right)^2} \qquad \text{where} \quad u_i = \ln\left(\frac{S_i}{S_{i-1}}\right)$$

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}u_i^2 - \frac{1}{n(n-1)}\left(\sum_{i=1}^{n}u_i\right)^2}$$

$$dS = \mu S dt + \sigma S dz$$

$$df = \left(\frac{\partial f}{\partial S}\mu S + \frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)dt + \frac{\partial f}{\partial S}\sigma S dz$$

$$\triangle f = \left(\frac{\partial f}{\partial S}\mu S + \frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)\triangle t + \frac{\partial f}{\partial S}\sigma S \triangle z$$

$$\Pi = -f + \frac{\partial f}{\partial S}S$$

$$\triangle\Pi = \left(-\frac{\partial f}{\partial t} - \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)\triangle t$$

$$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 f}{\partial S^2} = rf$$

$$f = e^{-rT}\hat{E}\left(S_T\right) - Ke^{-rT}$$

$$\hat{E}\left(S_T\right) = S_0 e^{rT}$$

$$f = S_0 - Ke^{-rT}$$

$$c = S_0 N\left(d_1\right) - Ke^{-rT}N\left(d_2\right)$$

$$p = Ke^{-rT}N\left(-d_2\right) - S_0 N\left(-d_1\right) \qquad \text{where} \quad d_1 = \frac{ln\left(S_0 / K\right) + \left(r + \sigma^2/2\right)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{ln\left(S_0 / K\right) + \left(r - \sigma^2/2\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

$$c = e^{-rT}\left[S_0 N\left(d_1\right)e^{rT} - KN\left(d_2\right)\right]$$

$$S\left(t_n\right) - D_n - Ke^{-r\left(T-t_n\right)} \geq S\left(t_n\right) - K$$

$$c + Ke^{-rT} = p + S_0 e^{-qT}$$

$$c = S_0 e^{-qT} N(d_1) - Ke^{-rT} N(d_2)$$

$$p = Ke^{-rT} N(-d_2) - S_0 e^{-qT} N(-d_1)$$

$$d_1 = \frac{ln(S_0 / K) + (r - q + \sigma^2 / 2)T}{\sigma\sqrt{T}} \qquad d_2 = \frac{ln(S_0 / K) + (r - q - \sigma^2 / 2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

$$dS = (r-q)Sdt + \sigma Sdz$$

$$p = \frac{e^{(r-q)\Delta t} - d}{u - d}$$

$$c = e^{-rT}\left[F_0 N(d_1) - KN(d_2)\right]$$

$$p = e^{-rT}\left[KN(-d_2) - F_0 N(-d_1)\right]$$

$$c + Ke^{-rT} = p + F_0 e^{-rT}$$

$$f = e^{-rT}\left[pf_\mu + (1-p) f_d\right]$$

$$\frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial F^2}\sigma^2 F^2 = rf$$

$$H_F = e^{-rT} H_A$$

$$H_F = e^{-(r-q)T} H_A$$

$$H_F = e^{-(r-r_f)T} H_A$$

$$N^{'}(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$$

$$\vartriangle\Pi = \Theta\vartriangle t + \frac{1}{2}\Gamma\vartriangle S^2$$

$$\Theta + rS\vartriangle + \frac{1}{2}\sigma^2 S^2 \Gamma = r\prod$$

$$\vartriangle = e^{-qT}\left[N(d_1) - 1\right]$$

$$p + S_0 e^{-qT} = c + Ke^{-rT}$$

$$a = e^{\left[f(t) - g(t)\right]\Delta t}$$

$$p=\frac{e^{\left[f(t)-g(t)\right]\Delta t}-d}{u-d}$$

$$\frac{\partial f}{\partial t}+(r-q)S\frac{\partial f}{\partial S}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 f}{\partial S^2}=rf$$

$$\frac{\partial f}{\partial S}=\frac{f_{i,j+1}-f_{i,j}}{\vartriangle S}$$

$$\frac{\partial f}{\partial S}=\frac{f_{i,j}-f_{i,j-1}}{\vartriangle S}$$

$$\frac{\partial f}{\partial S}=\frac{f_{i,j+1}-f_{i,j-1}}{2\vartriangle S}$$

$$\frac{\partial^2 f}{\partial S^2}=\frac{f_{i,j+1}+f_{i,j-1}-2f_{i,j}}{\vartriangle S^2}$$

$a_j f_{i,j-1}+b_j f_{i,j}+c_j f_{i,j+1}=f_{i+1,j}$ where $a_j=\frac{1}{2}(r-q)j\vartriangle t-\frac{1}{2}\sigma^2 j^2\vartriangle t$, $b_j=1+\sigma^2 j^2\vartriangle t+r\vartriangle t$

$$c_j=-\frac{1}{2}(r-q)j\vartriangle t-\frac{1}{2}\sigma^2 j^2\vartriangle t$$

$$\frac{\partial f}{\partial S}=\frac{f_{i+1,j+1}-f_{i+1,j-1}}{2\vartriangle S}$$

$$\frac{\partial^2 f}{\partial S^2}=\frac{f_{i+1,j+1}+f_{i+1,j-1}-2f_{i+1,j}}{\vartriangle S^2}$$

$$f_{i,j}=a_j^* f_{i+1,j-1}+b_j^* f_{i+1,j}+c_j^* f_{i+1,j+1}$$

where $a_j^*=\frac{1}{1+r\vartriangle t}\left(-\frac{1}{2}(r-q)j\vartriangle t+\frac{1}{2}\sigma^2 j^2\vartriangle t\right)$, $b_j^*=\frac{1}{1+r\vartriangle t}\left(1-\sigma^2 j^2\vartriangle t\right)$

$$c_j^*=\frac{1}{1+r\vartriangle t}\left(\frac{1}{2}(r-q)j\vartriangle t+\frac{1}{2}\sigma^2 j^2\vartriangle t\right)$$

$$\alpha_j f_{i,j-1}+\beta_j f_{i,j}+\gamma_j f_{i,j+1}=f_{i+1,j}$$

where $\alpha_j=\frac{\vartriangle t}{2\vartriangle Z}(r-q-\frac{\sigma^2}{2})-\frac{\vartriangle t}{2\vartriangle Z^2}\sigma^2$ $\quad$ $\beta_j=1+\frac{\vartriangle t}{\vartriangle Z^2}\sigma^2+r\vartriangle t$

$$\gamma_j=\frac{-\vartriangle t}{2\vartriangle Z}(r-q-\frac{\sigma^2}{2})-\frac{\vartriangle t}{2\vartriangle Z^2}\sigma^2$$

$$\alpha_j^* f_{i+1,j-1}+\beta_j^* f_{i+1,j}+\gamma_j^* f_{i+1,j+1}=f_{i,j}$$

where $\alpha_j^* = \frac{1}{1+r\triangle t}\left[-\frac{\triangle t}{2\triangle Z}(r-q-\frac{\sigma^2}{2})+\frac{\triangle t}{2\triangle Z^2}\sigma^2\right]$

$$\beta_j^* = \frac{1}{1+r\triangle t}(1-\frac{\triangle t}{\triangle Z^2}\sigma^2)$$

$$\gamma_j^* = \frac{1}{1+r\triangle t}\left[\frac{\triangle t}{2\triangle Z}(r-q-\frac{\sigma^2}{2})+\frac{\triangle t}{2\triangle Z^2}\sigma^2\right]$$

$$\sigma_n^2 = \frac{1}{m-1}\sum_{i=1}^{m}\left(u_{n-i}-\bar{u}\right)^2$$

$$\sigma_n^2 = \sum_{i=1}^{m}\alpha_i u_{n-i}^2$$

$$\sigma_n^2 = \gamma V_L + \sum_{i=1}^{m}\alpha_i u_{n-i}^2$$

$$\sigma_n^2 = \lambda\sigma_{n-1}^2 + (1-\lambda)u_{n-1}^2$$

$$\sigma_n^2 = (1-\lambda)\sum_{i=1}^{m}\lambda^{i-1}u_{n-i}^2 + \lambda^m\sigma_{n-m}^2$$

$$\sigma_n^2 = \gamma V_L + \alpha u_{n-1}^2 + \beta\sigma_{n-1}^2$$

$$\sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta\sigma_{n-1}^2$$

$$\sigma_n^2 = \omega + \beta\omega + \beta^2\omega + \alpha u_{n-1}^2 + \alpha\beta u_{n-2}^2 + \alpha\beta^2 u_{n-3}^2 + B^3\sigma_{n-3}^2$$

$$\prod_{i=1}^{m}\left[\frac{1}{\sqrt{2\pi v}}\exp\left(\frac{-u_i^2}{2v}\right)\right]$$

$$\frac{1}{m}\sum_{i=1}^{m}u_i^2$$

$$\sum_{i=1}^{m}\left[-ln\left(v_i\right)-\frac{u_i^2}{v_i}\right]$$

$m\sum_{k=1}^{K}w_k\eta_k^2$ where $w_k = \frac{m+2}{m-k}$

$$\sigma_n^2 = (1-\alpha-\beta)V_L + \alpha u_{n-1}^2 + \beta\sigma_{n-1}^2$$

$$\sigma_n^2 - V_L = \alpha\left(u_{n-1}^2 - V_L\right) + \beta\left(\sigma_{n-1}^2 - V_L\right)$$

$$E\left[\sigma_{n+t}^2\right] = V_L + (\alpha+\beta)^t\left(\sigma_n^2 - V_L\right)$$

$$\text{cov}_n = \frac{1}{m}\sum_{i=1}^{m} x_{n-i}y_{n-i}$$

$$\text{cov}_n = \lambda\,\text{cov}_{n-1} + (1-\lambda)\,x_{n-1}y_{n-1}$$

$$\text{cov}_n = \omega + \alpha x_{n-1}y_{n-1} + \beta\,\text{cov}_{n-1}$$

$$e^{-rT_1}\hat{E}\left[c\,\frac{S_1}{S_0}\right]$$

$$S_0e^{-qT_2}M\left(a_1,b_1;\sqrt{T_1/T_2}\right) - K_2e^{-rT_2}M\left(a_2,b_2;\sqrt{T_1/T_2}\right) - e^{-rT_1}K_1N(a_2)$$

$$a_1 = \frac{ln\left(S_0/S^*\right) + \left(r-q+\sigma^2/2\right)T_1}{\sigma\sqrt{T_1}} \qquad a_2 = a_1 - \sigma\sqrt{T_1}$$

$$b_1 = \frac{ln\left(S_0/K_2\right) + \left(r-q+\sigma^2/2\right)T_2}{\sigma\sqrt{T_2}} \qquad b_2 = b_1 - \sigma\sqrt{T_2}$$

$$K_2e^{-rT_2}M\left(-a_2,b_2;-\sqrt{T_1/T_2}\right) - S_0e^{-qT_2}M\left(-a_1,b_1;-\sqrt{T_1/T_2}\right) + e^{-rT_1}K_1N(-a_2)$$

$$K_2e^{-rT_2}M\left(-a_2,-b_2;\sqrt{T_1/T_2}\right) - S_0e^{-qT_2}M\left(-a_1,-b_1;\sqrt{T_1/T_2}\right) - e^{-rT_1}K_1N(-a_2)$$

$$S_oe^{-qT_2}M\left(a_1,-b_1;-\sqrt{T_1/T_2}\right) - K_2e^{-rT_2}M\left(a_2,-b_2;-\sqrt{T_1/T_2}\right) + e^{-rT_1}K_1N(a_2)$$

$$\max(c,p) = c + e^{-q(T_2-T_1)}\max\left(0, Ke^{-(r-q)(T_2-T_1)} - S_1\right)$$

$$H \le K:\; c_{di} = S_0e^{-qT}\left(H/S_0\right)^{2\lambda}N(y) - Ke^{-rT}\left(H/S_0\right)^{2\lambda-2}N\left(y-\sigma\sqrt{T}\right)$$

$$\lambda = \frac{r-q+\sigma^2/2}{\sigma^2}$$

$$y = \frac{ln\left[H^2/\left(S_0K\right)\right]}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}$$

$$c_{do} = c - c_{di}$$

$$H \ge K:\; c_{do} = S_0N(x_1)e^{-qT} - Ke^{-rT}N\left(x_1-\sigma\sqrt{T}\right) - S_0e^{-qT}\left(H/S_0\right)^{2\lambda}N(y_1) + Ke^{-rT}\left(H/S_0\right)^{2\lambda-2}N\left(y_1-\sigma\sqrt{T}\right)$$

$$c_{di} = c - c_{do}$$

$$x_1 = \frac{\ln\left(S_0/H\right)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T} \qquad y_1 = \frac{\ln\left(H/S_0\right)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}$$

$$H > K:\ c_{ui} = S_0 N\left(x_1\right)e^{-qT} - Ke^{-rT}N\left(x_1 - \sigma\sqrt{T}\right) - S_0 e^{-qT}\left(H/S_o\right)^{2\lambda}\left[N\left(-y\right) - N\left(-y_1\right)\right]$$
$$+Ke^{-rT}\left(H/S_0\right)^{2\lambda-2}\left[N\left(-y+\sigma\sqrt{T}\right) - N\left(-y_1+\sigma\sqrt{T}\right)\right]$$

$$c_{uo} = c - c_{ui}$$

$$H \geq K:\ p_{ui} = -S_0 e^{-qT}\left(H/S_0\right)^{2\lambda} N\left(-y\right) + Ke^{-rT}\left(H/S_0\right)^{2\lambda-2} N\left(-y+\sigma\sqrt{T}\right)$$

$$p_{uo} = p - p_{ui}$$

$$H \leq K:\ p_{uo} = -S_0 N\left(-x_1\right)e^{-qT} + Ke^{-rT}N\left(-x_1+\sigma\sqrt{T}\right) + S_0 e^{-qT}\left(H/S_0\right)^{2\lambda} N\left(-y_1\right) - Ke^{-rT}\left(H/S_0\right)^{2\lambda-2} N\left(-y_1+\sigma\sqrt{T}\right)$$
$$p_{ui} = p - p_{uo}$$

$$H < K:\ p_{di} = -S_0 N\left(-x_1\right)e^{-qT} + Ke^{-rT}N\left(-x_1+\sigma\sqrt{T}\right) + S_0 e^{-qT}\left(H/S_0\right)^{2\lambda}\left[N\left(y\right) - N\left(y_1\right)\right]$$
$$-Ke^{-rT}\left(H/S_0\right)^{2\lambda-2}\left[N\left(y-\sigma\sqrt{T}\right) - N\left(y_1-\sigma\sqrt{T}\right)\right]$$

$$p_{do} = p - p_{di}$$

$$c_{ELB} = S_0 e^{-qT} N\left(a_1\right) - S_0 e^{-qT}\frac{\sigma^2}{2\left(r-q\right)}N\left(-a_1\right) - S_{\min}e^{-rT}\left(N\left(a_2\right) - \frac{\sigma^2}{2\left(r-q\right)}e^{Y_1}N\left(-a_3\right)\right)$$

$$a_1 = \frac{ln\left(S_0/S_{\min}\right) + \left(r-q+\sigma^2/2\right)T}{\sigma\sqrt{T}} \qquad a_2 = a_1 - \sigma\sqrt{T}$$

$$a_3 = \frac{ln\left(S_0/S_{\min}\right) + \left(-r+q+\sigma^2/2\right)T}{\sigma\sqrt{T}}$$

$$Y_1 = -\frac{2\left(r-q-\sigma^2/2\right)ln\left(S_0/S_{\min}\right)}{\sigma^2}$$

$$p_{ELB} = S_{\max}e^{-rT}\left(N\left(b_1\right) - \frac{\sigma^2}{2\left(r-q\right)}e^{Y_2}N\left(-b_3\right)\right) + S_0 e^{-qT}\frac{\sigma^2}{2\left(r-q\right)}N\left(-b_2\right) - S_o e^{-qT}N\left(b_2\right)$$

$$b_1 = \frac{ln\left(S_{\max}/S_o\right) + \left(-r+q+\sigma^2/2\right)T}{\sigma\sqrt{T}} \qquad b_2 = b_1 - \sigma\sqrt{T}$$

$$b_3 = \frac{ln\left(S_{\max}/S_0\right) + \left(r-q-\sigma^2/2\right)T}{\sigma\sqrt{T}}$$

$$Y_2 = \frac{2\left(r-q-\sigma^2/2\right) ln\left(S_{\max}/S_0\right)}{\sigma^2}$$

$$\max\left(0, S_T - S_\tau\right) + \left(S_\tau - K\right)$$

$$r - \frac{1}{2}\left(r - q - \frac{\sigma^2}{6}\right) = \frac{1}{2}\left(r + q + \frac{\sigma^2}{6}\right)$$

$$M_1 = \frac{e^{(r-q)T} - 1}{(r-q)T} S_0$$

$$M_2 = \frac{2e^{\left(2(r-q)+\sigma^2\right)T} S_0^2}{\left(r-q+\sigma^2\right)\left(2r-2q+\sigma^2\right)T^2} + \frac{2S_0^2}{(r-q)T^2}\left(\frac{1}{2(r-q)+\sigma^2} - \frac{e^{(r-q)T}}{r-q+\sigma^2}\right)$$

$$\sigma^2 = \frac{1}{T}\ln\left(\frac{M_2}{M_1^{\ 2}}\right)$$

$$V_o e^{-q_V T} N\left(d_1\right) - U_0 e^{-q_U T} N\left(d_2\right)$$

$$d_1 = \frac{ln\left(V_o/U_o\right) + \left(q_U - q_V + \hat{\sigma}^2/2\right)T}{\hat{\sigma}\sqrt{T}} \qquad d_2 = d_1 - \hat{\sigma}\sqrt{T}$$

$$\hat{\sigma} = \sqrt{\sigma_U^2 + \sigma_V^2 - 2\rho\sigma_U\sigma_V}$$

$$dS = (r-q)Sdt + \sigma S^{\alpha} dz$$

$$\frac{dS}{S} = (r - q - \lambda k)dt + \sigma dz + dp$$

$$\phi(g) = \frac{g^{T/v-1}\,\mathrm{e}^{-g/v}}{v^{T/v}\,\Gamma\left(T/v\right)}$$

$$\ln S_0 + \left(r-q\right)T + \omega + \theta g$$

$$\sigma\sqrt{g}$$

$$\omega = \frac{T}{v}\ln\left(1 - \theta v - \sigma^2 v/2\right)$$

$$dS = (r-q)Sdt + \sigma(t)Sdz$$

$$\frac{dS}{S} = (r - q)dt + \sqrt{V}dz_S$$

$$dV = a(V_L - V)dt + \xi V^{\alpha} dz_V$$

$$dS = (r(t) - q(t))Sdt + \sigma(S,t)Sdz$$

$$\left[\sigma(K,T)\right]^2 = 2\frac{\partial C_{mkt}/\partial T + q(T)C_{mkt} + K\left[r(T) - q(T)\right]\partial C_{mkt}/\partial K}{K^2\left(\partial^2 C_{mkt}/\partial K^2\right)}$$

$$\frac{d\theta}{\theta} = mdt + sdz$$

$$\triangle f_1 = \mu_1 f_1 \triangle t + \sigma_1 f_1 \triangle z$$

$$\triangle f_2 = \mu_2 f_2 \triangle t + \sigma_2 f_2 \triangle z$$

$$\prod = (\sigma_2 f_2) f_1 - (\sigma_1 f_1) f_2$$

$$\triangle \prod = (\mu_1 \sigma_2 f_1 f_2 - \mu_2 \sigma_1 f_1 f_2)\triangle t$$

$$\frac{\mu_1 - r}{\sigma_1} = \frac{\mu_2 - r}{\sigma_2}$$

$$\frac{df}{f} = \mu dt + \sigma dz$$

$$\frac{\mu - r}{\sigma} = \lambda$$

$$\mu - r = \sum_{i=1}^{n} \lambda_i \sigma_i$$

$$d\theta = \sigma dz$$

$$d\left(\frac{f}{g}\right) = (\sigma_f - \sigma_g)\frac{f}{g}dz$$

$$f_0 = g_0 E_g\left(\frac{f_T}{g_T}\right)$$

$$dg = rgdt$$

$$f_o = g_0 \widehat{E}\left(\frac{f_T}{g_T}\right)$$

$$f_0 = \hat{E}(e^{-\bar{\gamma}T} f_T)$$

$$f_0 = P(0,T)E_T(f_T)$$

$$A(t) = \sum_{i=0}^{N-1}(T_{i+1} - T_i)P(t, T_{i+1})$$

$$s(t) = E_A[s(T)]$$

$$f_o = A(0)E_A\left[\frac{f_T}{A(T)}\right]$$

$$c = P(0,T)E_T[\max(S_T - K, 0)]$$

$$c = e^{-RT}E_T[\max(S_T - k, 0)]$$

$$E_T[\max(S_T - K, 0)] = E_T(S_T)N(d_1) - KN(d_2)$$

$$f_0 = U_0 E_U\left[\max\left(\frac{V_T}{U_T} - 1, 0\right)\right]$$

$$f_0 = V_0 N(d_1) - U_0 N(d_2)$$

European call option on a variable whose value is V

$$c = P(0,T)[F_0 N(d_1) - KN(d_2)]$$

where $d_1 = \dfrac{\ln(F_0/K) + \sigma^2 T/2}{\sigma\sqrt{T}}$ $d_2 = \dfrac{\ln(F_0/K) - \sigma^2 T/2}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$

$F_0 =$ value of F at time zero $K =$ strike price of the option

$P(t,T) =$ price at time t of a zero-coupon bond paying \$1 at time T

$\sigma =$ volatility of F $F =$ forward price of V for a contract maturing T

$T =$ time to maturing of the option $V_T =$ value of V at time T

value of the corresponding put option $p = P(0,T)[KN(-d_2) - F_0 N(-d_1)]$

forward bond price $F_B = \dfrac{B_0 - I}{P(0,T)}$

where $B_0 =$ bond price at time zero

$I =$ present value of coupons that will be paid during the life of the option

volatility of the forward bond price $\sigma_B = D_{y_0}\sigma_y$

where $\sigma_y =$ volatility of the forward bond yield $y_0 =$ initial value of y_F $y_F =$ forward yield

$D =$ modified duration of the bond underlying the option at option maturing

$$c = P(0,T)E_T\left[\max(B_T - K, 0)\right]$$

where $B_T =$ bond price at time T

$E_T =$ expected value in a world that is forward risk neutral with respect to a zero-coupon bond maturing at time T

$$E_T(B_T) = F_B$$

$$\max\left[L - \frac{L(1+R_k\delta_k)}{1+R_k\delta_k}, 0\right]$$

where $\frac{L(1+R_k\delta_k)}{1+R_k\delta_k} =$ value at time t_k of a zero-coupon bond that pays off $L(1+R_k\delta_k)$ at time t_{k+1}

$$E_T(y_T) = y_0 - \frac{1}{2}y_0^2\sigma_y^2 T\frac{G''(y_0)}{G'(y_0)}$$

where $E_T =$ expectations in a world that is forward risk neutral with respect to $P(t,T)$

$\sigma_y =$ forward yield volatility

$$E_T(R_T) = R_0 - \frac{1}{2}R_0^2\sigma_R^2 T\frac{G''(R_0)}{G'(R_0)} = R_0 + \frac{R_0^2\sigma_R^2\tau T}{1+R_0\tau}$$

where $\tau = T^* - T$ $L =$ principal

$R_T =$ zero-coupon interest rate applicable to the period between T and T^*

$$\alpha_V = \rho_{VW}\sigma_V\sigma_W$$

where $\sigma_V =$ volatility of V $\sigma_W =$ volatility of W $\rho_{VW} =$ correlation between V and W

$R =$ forward interest rate for period between T and T^* $\sigma_R =$ volatility of R $W = \frac{1}{(1+R/m)^{m(T^*-T)}}$

$$E_{T^*}(V_T) = E_T(V_T)\exp\left[-\frac{\rho_{VR}\sigma_V\sigma_R R_0(T^*-T)}{1+R_0/m}T\right]$$

$$\alpha_V = \rho_{VW}\sigma_V\sigma_W$$

$$E_X(V_T) = E_Y(V_T)(1+\rho\sigma_V\sigma_W T)$$

value at time t of an interest rate derivative that provides a payoff of f_T at time T $=$ $\hat{E}\left[e^{-\bar{r}(T-t)}f_T\right]$

where $\bar{r} =$ the average value of r in the time interval between t and T

$\hat{E} =$ expected value in the traditional risk-neutral world

$$R(t,T) = -\frac{1}{T-t}\ln\hat{E}\left[e^{-\bar{r}(T-t)}\right]$$

$$dr = m(r)dt + s(r)dz \qquad dr = \mu r dt + \sigma r dz$$

$$dr = a(b-r)dt + \sigma dz$$

$$P(t,T) = A(t,T)e^{-B(t,T)r(t)}$$

$$B(t,T) = \frac{1-e^{-a(T-t)}}{a}$$

$$A(t,T) = \exp\left[\frac{(B(t,T)-T+t)(a^2b-\sigma^2/2)}{a^2} - \frac{\sigma^2 B(t,T)^2}{4a}\right]$$

$$R(t,T) = -\frac{1}{T-t}\ln A(t,T) + \frac{1}{T-t}B(t,T)r(t)$$

$$dr = \theta(t)dt + \sigma dz$$

$$\theta(t) = F_t(0,t) + \sigma^2 t$$

$$P(t,T) = A(t,T)e^{-r(t)(T-t)}$$

where $\ln A(t,T) = \ln\frac{P(0,T)}{p(0,t)} + (T-t)F(0,t) - \frac{1}{2}\sigma^2 t(T-t)^2$

$$dr = \left[\theta(t) - ar\right]dt + \sigma dz = a\left[\frac{\theta(t)}{a} - r\right]dt + \sigma dz$$

$$\theta(t) = F_t(0,t) + aF(0,t) + \frac{\sigma^2}{2a}(1-e^{-2at})$$

$P(t,T) = A(t,T)e^{-B(t,T)r(t)}$ where $B(t,T) = \frac{1-e^{-a(T-t)}}{a}$

$$\ln A(t,T) = \ln\frac{P(0,T)}{P(0,t)} + B(t,T)F(0,t) - \frac{1}{4a^3}\sigma^2(e^{-aT}-e^{-at})^2(e^{2at}-1)$$

$$d\ln r = \left[\theta(t) - a(t)\ln(r)\right]dt + \sigma(t)dz$$

$df(r) = \left[\theta(t) + \mu - af(r)\right]dt + \sigma_1 dz_1 \qquad du = -budt + \sigma_2 dz_2$

price at time zero of a call option that matures at time T on a zero-coupon bond maturing at time s

$LP(0,s)N(h) - KP(0,T)N(h-\sigma_p) \qquad h = \frac{1}{\sigma_p}\ln\frac{LP(0,s)}{P(0,T)K} + \frac{\sigma_p}{2}$

$p_{m+1} = \sum_{j=-n_m}^{n_m} Q_{m,j}\exp\left[-(\alpha_m + j\triangle R)\triangle t\right]$ where $\alpha_m = \frac{\ln\sum_{j=-n_m}^{n_m} Q_{m,j}e^{-j\triangle R\triangle t} - \ln P_{m+1}}{\triangle t}$

$$df(r) = \left[\theta(t) - af(r)\right]dt + \sigma dz$$

$$p_{m+1} = \sum_{j=-n_m}^{n_m} Q_{m,j} \exp\left[-g(\alpha_m + j \triangle x)\triangle t\right]$$

$$P(t,T) = \hat{A}(t,T)e^{-\hat{B}(t,T)R}$$

where $\ln \hat{A}(t,T) = \ln \frac{P(0,T)}{P(0,t)} - \frac{B(t,T)}{B(t,t+\triangle t)} \ln \frac{P(0,t+\triangle t)}{P(0,t)} - \frac{\sigma^2}{4a}(1-e^{-2at})B(t,T)\left[B(t,T) - B(t,t+\triangle t)\right]$

$$\hat{B}(t,T) = \frac{B(t,T)}{B(t,t+\triangle t)}\triangle t$$

$$dP(t,T) = r(t)P(t,T)dt + v(t,T,\Omega_t)P(t,T)dz(t)$$

where $P(t,T) =$ price at time t of a zero-coupon bond with principal \$1 maturing at time T

$\Omega_t =$ vector of past and present values of interest rates and bond prices at time t that are relevant for determining bond price volatilities at that time

$v(t,T,\Omega_t) =$ volatility of $p(t,T)$ $\quad f(t,T_1,T_2)$ forward rate as seen at time t for the period between time T_1 and T_2

$F(t,T) =$ instantaneous forward rate as seen at time t for a contract maturing at time T

$r(t)$ short-term risk-free interest rate at time t $\quad dz(t) =$ Wiener process driving term structure movements

$$f(t,T_1,T_2) = \frac{\ln\left[p(t,T_1)\right] - \ln\left[p(t,T_2)\right]}{T_2 - T_1}$$

$$df(t,T_1,T_2) = \frac{v(t,T_2,\Omega_t)^2}{2(T_2 - T_1)}dt + \frac{v(t,T_1,\Omega_t) - v(t,T_2,\Omega_t)}{T_2 - T_1}dz(t)$$

$$dF(t,T) = v(t,T,\Omega_t)v_T(t,T,\Omega_t)dt - v_T(t,T,\Omega_t)dz(t)$$

$$m(t,T,\Omega_t) = s(t,T,\Omega_t)\int_t^T s(t,\tau,\Omega_t)d\tau$$

where $m(t,T,\Omega_t) =$ instantaneous drift of $F(t,T)$ $\qquad s(t,T,\Omega_t) =$ standard deviation of $F(t,T)$

$$m(t,T,\Omega_t) = \sum_k s_k(t,T,\Omega_t)\int_t^T s_k(t,\tau,\Omega_t)d\tau$$

$$dF_k(t) = \xi_k(t)F_k(t)dz$$

where $F_k(t) =$ forward rate between time t_k and t_{k+1} as seen at time t

$m(t) =$ index for the next reset date at time t, smallest integer such that $t \le t_m(t)$

$\xi_k(t) =$ volatility of $F_k(t)$ at time t $\qquad \gamma_k(t) =$ volatility of the zero-coupon bond price $p(t,t_k)$ at time t

$$dF_k(x) = \xi_k(t)\left[v_{m(t)}(t) - v_{k+1}(t)\right]F_k(t)dt + \xi_k(t)F_k(t)dz$$

$$v_i(t) - v_{i+1}(t) = \frac{\delta_i F_i(t)\xi_i(t)}{1+\delta_i F_i(t)}$$

$$\frac{dF_k(t)}{F_k(t)} = \sum_{i=m(t)}^{k} \frac{\delta_i F_i(t)\xi_i(t)\xi_k(t)}{1+\delta_i F_i(t)} dt + \xi_k(t) dz$$

$$\sigma_k^2 t_k = \sum_{i=1}^{k} \Lambda_{k-i}^2 \delta_{i-1}$$

where $\Lambda_i =$ the value of $\xi_i(t)$ when there are i such accrual periods

$\xi_k(t) = \Lambda_{k-m(t)}$ is a step function

$$\frac{dF_k(t)}{F_k(t)} = \sum_{i=m(t)}^{k} \frac{\delta_i F_i(t)\Lambda_{i-m(t)}\Lambda_{k-m(t)}}{1+\delta_i F_i(t)} dt + \Lambda_{k-m(t)} dz$$

$$d\ln F_k(t) = \left[\sum_{i=m(t)}^{k} \frac{\delta_i F_i(t)\Lambda_{i-m(t)}\Lambda_{k-m(t)}}{1+\delta_i F_i(t)} - \frac{(\Lambda_{k-m(t)})^2}{2} \right] dt + \Lambda_{k-m(t)} dz$$

$$F_k(t_{j+1}) = F_k(t_j)\exp\left[(\sum_{i=j+1}^{k} \frac{\delta_i F_i(t)\Lambda_{i-j-1}\Lambda_{k-j-1}}{1+\delta_i F_i(t_j)} - \frac{\Lambda_{k-j-1}^2}{2})\delta_j + \Lambda_{k-j-1}\varepsilon\sqrt{\delta_j} \right]$$

where ε is a random sample $\varepsilon \sim N(0,1)$

$$\frac{dF_k(t)}{F_k(t)} = \sum_{i=m(t)}^{k} \frac{\delta_i F_i(t)\sum_{q=1}^{p}\xi_{i,q}(t)\xi_{k,q}(t)}{1+\delta_i F_i(t)} dt + \sum_{q=1}^{p}\xi_{k,q}(t) dz_q$$

$$F_k(t_{j+1}) = F_k(t_j)\exp\left[(\sum_{i=j+1}^{k} \frac{\delta_i F_i(t_j)\sum_{q=1}^{p}\lambda_{i-j-1,q}\lambda_{k-j-1,q}}{1+\delta_i F_i(t_j)} - \frac{\sum_{q=1}^{p}\lambda_{k-j-1,q}^2}{2})\delta_j + \sum_{q=1}^{p}\lambda_{k-j-1,q}\varepsilon_q\sqrt{\delta_j} \right]$$

$$V(t) = \sum_{q=1}^{p}\left[\sum_{k=0}^{N-1} \frac{\tau_k \beta_{k,q}(t) G_k(t)\gamma_k(t)}{1+\tau_k G_k(t)} \right]^2$$

where $\gamma_k(t) = \dfrac{\prod_{j=0}^{N-1}\left[1+\tau_j G_j(t)\right]}{\prod_{j=0}^{N-1}\left[1+\tau_j G_j(t)\right] - 1} - \dfrac{\sum_{i=0}^{k-1}\tau_i \prod_{j=i+1}^{N-1}\left[1+\tau_j G_j(t)\right]}{\sum_{i=0}^{N-1}\tau_i \prod_{j=i+1}^{N}\left[1+\tau_j G_j(t)\right]}$

$$\sqrt{\frac{1}{T_0}\int_{t=0}^{T_0} V(t)dt} \qquad \sqrt{\frac{1}{T_0}\int_{t=0}^{T_0}\sum_{q=1}^{p}\left[\sum_{k=0}^{N-1}\frac{\tau_k \beta_{k,q(t)} G_k(0)\gamma_k(0)}{1+\tau_k G_k(0)} \right]^2 dt}$$

$$\sqrt{\frac{1}{T_0}\int_{t=0}^{T_0}\sum_{q=1}^{p}\left[\sum_{k=0}^{N-1}\sum_{m=1}^{M} \frac{\tau_{k,m}\beta_{k,m,q}(t) G_{k,m}(0)\gamma_k(0)}{1+\tau_{k,m}G_{k,m}(0)} \right]^2 dt}$$

$$\lambda_{j,q} = \frac{\Lambda_j \delta_q \alpha_{j,q}}{\sqrt{\sum_{q=1}^{p} s_q^2 \alpha_{i,q}^2}}$$

$$dF_i(t) = \ldots + \sum_{q=1}^{p} \xi_{i,q}(t) F_i(t)^{\alpha} dz_q$$

$$F_i + \frac{F_i^2 \sigma_i^2 \tau_i t_i}{1 + F_i \tau_i}$$

$$y_i - \tfrac{1}{2} y_i^2 \sigma_{y,i}^2 t_i \frac{G_i''(y_i)}{G_i'(y_i)} - \frac{y_i \tau_i F_i \rho_i \sigma_{y,i} \sigma_{F,i} t_i}{1 + F_i \tau_i}$$

$$V_i + V_i \rho_i \sigma_{W,i} \sigma_{V,i} t_i$$

$$\frac{QL}{n_2} P(0, s_i) N(d_2^*)$$

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best response: $\pi_i(s_i^*, s_{-i}) \geq \pi_i(s_i^{'}, s_{-i}) \forall s_i^{'} \neq s_i^*$

dominated strategy: $\pi_i(s_i^d, s_{-i}) < \pi_i(s_i^{'}, s_{-i}) \forall s_{-i}$

dominant strategy: $\pi_i(s_i^*, s_{-i}) > \pi_i(s_i^{'}, s_{-i}) \forall s_{-i,} \forall s_i^{'} \neq s_i^*$

weakly dominated: $\pi_i(s_i^{''}, s_{-i}) \geq \pi_i(s_i^{'}, s_{-i}) \forall s_{-i}\ and\ \pi_i(s_i^{''}, s_{-i}) > \pi_i(s_i^{'}, s_{-i})\ for\ some\ s_{-i}$

Nash equilibrium: $\forall i,\ \pi_i(s_i^*, s_{-i}^*) \geq \pi_i(s_i^{'}, s_{-i}^*) \forall s_i^{'}$

pure strategy: $s_i : \omega_i \to a_i$

mixed strategy: $s_i : \omega_i \to m(a_i)\quad where\quad m \geq 0 \quad \int_{A_i} m(a_i) da_i = 1$

completely mixed: $m > 0$

minimax strategies: $\min imize_{s_{-i}} \max imize_{s_i} \pi_i(s_i, s_{-i})$

maximin strategies: $\max imize_{s_i} \min imize_{s_{-i}} \pi_i(s_i, s_{-i})$

$$U(e, w(e)) = \bar{U}$$

$$\max_{e} imize V(q(e) - \tilde{w}(e))$$

$$V'(q(e)-\tilde{w}(e))(\frac{\partial q}{\partial e}-\frac{\partial \tilde{w}}{\partial e})=0$$

$$\frac{\partial q}{\partial e}=\frac{\partial \tilde{w}}{\partial e}$$

$$\frac{\partial \tilde{w}}{\partial e}=-(\frac{\partial U/\partial e}{\partial U/\partial \tilde{w}})$$

$$(\frac{\partial U}{\partial \tilde{w}})(\frac{\partial q}{\partial e})=-(\frac{\partial U}{\partial e})$$

$$U(e^*,q(e^*))=\bar{U}$$

$$\underset{e}{\max imize} U(e,q(e))$$

$$\frac{\partial U}{\partial e}+(\frac{\partial U}{\partial q})(\frac{\partial q}{\partial e})=0$$

$$(\frac{\partial U}{\partial w})(\frac{\partial q}{\partial e})=-\frac{\partial U}{\partial e}$$

$$\underset{w(.)}{\max imize}\, EV(q(\tilde{e},\theta)-w(q(\tilde{e},\theta)))$$

subject to $\tilde{e}=avg\,\underset{e}{\max}\, EU(e,w(q(e,\theta)))$

$$EU(\tilde{e},w(q(\tilde{e},\theta)))\geq \bar{U}$$

$$C(\tilde{e})=\underset{w(.)}{\min imum}\, Ew(q(\tilde{e},\theta))$$

$$\underset{\tilde{e}}{\max imize}\, EV(q(\tilde{e},\theta)-C(\tilde{e}))$$

$$U(not\,investigate)\leq U(investigate)$$

$$\theta\log(w_1)+(1-\theta)\log(w_2)\leq\left[1-(1-\theta)^2\right]\log(w_1)+(1-\theta)^2\log(w_2)-\alpha$$

$$\theta(1-\theta)\log(\frac{w_1}{w_2})=\alpha$$

$$\log(\bar{w})=\left[1-(1-\theta)^2\right]\log(w_1)+(1-\theta)^2\log(w_2)-\alpha$$

$$w_1=\bar{w}e^{\alpha/\theta}\qquad w_2=\bar{w}e^{-\alpha/(1-\theta)}$$

$$\left[1-(1-\theta)^2\right]\bar{w}e^{\alpha/\theta}+(1-\theta)^2\,\bar{w}e^{-\alpha/1-\theta}$$

$$\bar{\theta}(P) = E\left[\theta \middle| (1+\varepsilon)\theta \le P\right]$$

FET-101-07 None

FET-102-07

$$F = \sum_i \max\left(S_{i0}, S_{iT}\right) = \sum_i S_{iT} + \sum_i \max\left(0, S_{i0} - S_{iT}\right)$$

$$F = \max\left(\sum_i S_{i0}, \sum_i S_{iT}\right) = \sum_i S_{iT} + \max\left(0, \sum_i \left(S_{i0} - S_{iT}\right)\right)$$

FET-105-07 None

FET-106-07

$$dS = \mu S dt + \sigma S dZ$$

$$dr = \mu(r,t) r dt + r\sigma dZ$$

$$\sigma(t,T) = \frac{\sigma\left(\dfrac{\Delta r(t,T)}{r(t,T)}\right)}{\sqrt{\Delta t}}$$

$$\sigma(t,T) = \frac{\sigma(\Delta r(t,T))}{\sqrt{\Delta t}}$$

$$dr = a(b-r)dt + \sigma\sqrt{r}dZ$$

$dr = a(b-r)dt + \sigma dZ$, (a>0)

$$dr = a_1 + b_1(l-r)dt + r\sigma_1 dZ$$

$$dl = (a_2 + b_2 r + c_2 l)dt + l\sigma_2 dW$$

$$dV = M(t,r)dt + \Omega(t,r)dZ$$

$$M(t,r) = V_t + \mu(t,r)V_r + \frac{1}{2}\sigma(t,r)^2 V_{rr}$$

$$\Omega(t,r) = \sigma(t,r)V_r$$

$$d\Pi = (M_1(t,r) - \Delta M_2(t,r))dt + (\Omega_1(t,r) - \Delta\Omega_2(t,r))dZ$$

$$d\Pi = r\Pi dt$$

$$V_t + (\mu(t,r) - \lambda(t,r)\sigma(t,r))V_r + \frac{1}{2}\sigma(t,r)^2 V_{rr} - rV = 0$$

$$P_i^n(1) = 2\left[\frac{P(n+1)}{P(n)}\right]\frac{\delta^i}{(1+\delta^n)} \qquad \delta = e^{-2r(1)\sigma}$$

$$P_i^n(T) = \frac{1}{2}P_i^n(1)\left\{P_i^{n+1}(T-1) + P_{i+1}^{n+1}(T-1)\right\}$$

$$r_i^n(1) = \ln\frac{P(n)}{P(n+1)} + \ln\left(\frac{1}{2}\left(\delta^{\frac{-n}{2}} + \delta^{\frac{n}{2}}\right)\right) + \left(\frac{n}{2} - i\right)\ln\delta$$

Note: Typo in text $r_i^n(1)1 =$ either way will receive full credit.

$$dr = (f'(0,t) + \sigma^2 t)dt + \sigma dz$$

$$r(n)\sigma^s(n) = \frac{-\frac{1}{2}\ln\left[\delta(n)\delta(n-1)...\delta(1)\right]}{n}$$

$$P_i^n(1) = \left[\frac{P(n+1)}{P(n)}\right]\left[\frac{(1+\delta_{n-1}\delta_{n-2}..\delta_1)..(1+\delta_{n-1})2}{(1+\delta_n..\delta_1)..(1+\delta_n)}\right]\delta_n^i$$

$$dr = (f'(0,t) + \sigma^2(t)t + \tfrac{\sigma'(t)}{\sigma(t)}\left[r(t) - f(0,t)\right])dt + \sigma(t)dZ$$

$$P_{i,j}^n(1) = \frac{P(n+1)}{P(n)}\frac{(1+\delta_{n-1}^1..\delta_1^1)(1+\delta_{n-1}^1..\delta_2^1)..(1+\delta_{n+1}^1)2}{(1+\delta_n^1..\delta_1^1)..(1+\delta_n^1\delta_{n-1}^1)(1+\delta_n^1)} \times \frac{(1+\delta_{n-1}^2..\delta_1^2)(1+\delta_{n-1}^2..\delta_2^2)..(1+\delta_{n-1}^2)2}{(1+\delta_n^2..\delta_1^2)(1+\delta_n^2..\delta_2^2)..(1+\delta_n^2)}(\delta_n^1)^i(\delta_n^2)^j$$

$$dr = \left\{f'(t) + \left|\sigma(t)\right|^2 t + \frac{\left|\sigma'(t)\right|\cos\phi(t)}{\left|\sigma(t)\right|\cos\theta(t)}\left[r - f(t)\right]\right\}dt + \sigma(t)dW$$

$$d\ln r = (\theta(t) - \frac{\sigma'(t)}{\sigma(t)}\ln r)dt + \sigma(t)dW$$

$$dr(t) = (\alpha(t) - \beta r(t))dt + \sigma dW(t) \quad \text{where } \alpha(t) = \frac{\partial f(0,t)}{\partial T^*} + \beta f(0,t) + \frac{\sigma^2}{2\beta}(1 - e^{-2\beta t})$$

$$dr = \left[\theta(t) + \mu - ar\right]dt + \sigma_1 dW \qquad du = -budt + \sigma_2 dZ$$

$$dP(t,T^*) = r(t)P(t,T^*)dt + \sigma^p(t,T^*)P(t,T^*)dZ$$

$$df(t,T^*) = \sigma^p(t,T^*)\sigma_{T^*}^p(t,T^*)dt - \sigma_{T^*}^p(t,T^*)dZ$$

$$dP(t,T^*) = r(t)P(t,T^*)dt + \sigma(T^* - t)P(t,T^*)dZ(t,T^*)$$

$$L(t,T^*) = \frac{1}{\vartriangle}\left(\frac{P(t,T^*)}{P(t,T^* + \vartriangle)} - 1\right)$$

$$dL(t,T^*) = L(t,T^*)\left[\sum_{j=t^*}^{N^*}\frac{L(t,j\Delta)\Delta}{1+L(t,j\Delta)\Delta}\Lambda(T^*-j\Delta)\Lambda(T^*-t)dt + \Lambda(T^*-t)dZ\right]$$

$$L(k,j+1) = L(k,j)\exp\left[\left(\sum_{i=j+1}^{k}\frac{L(i,j)\Delta}{1+L(i,j)\Delta}\Lambda_{i-j-1}\Lambda_{k-j-1} - \frac{\Lambda_{k-j-1}^2}{2}\right)\Delta + \Lambda_{k-j-1}\sqrt{\Delta}\tilde{Z}\right]$$

where $\sigma_j^2 j = \sum_{i=1}^{j}\Lambda_{j-i}^2$

caplet $C_k = L\delta_k P(t_{k+1})\left[F_k N(d_1) - R_x N(d_2)\right]$

where $d_1 = \dfrac{\ln\left[\dfrac{F_k}{R_X}\right] + \sigma_k^2\dfrac{t_k}{2}}{\sigma_k\sqrt{t_k}}$ $\qquad d_2 = d_1 - \sigma_k\sqrt{t_k}$

swaption $= \sum_{i=1}^{mn}\frac{L}{m}P(t_i)\left[R_F N(d_1) - R_X N(d_2)\right] = L * A\left[R_F N(d_1) - R_X N(d_2)\right]$

where $A = \frac{1}{m}\sum_{i=1}^{mn}P(t_i) \quad 1 \le i \le mn$

$$P(k+1,j) = P(k,j)\exp\left[\left(r(k) - \frac{\sigma^2(j-k)}{2}\right)\Delta + \sigma(j-k)\sqrt{\Delta}Z(j-k)\right]$$

$$\sigma^*(T^*-t) = (a + b(T^*-t))\exp(-c(T^*-t)) + d$$

$$L(k,j+1) = L(k,j)\exp\left[\left(\sum_{i=j+1}^{k}\frac{L(i,j)\Delta}{1+L(i,j)\Delta}\Lambda_{i-j-1}\Lambda_{k-j-1} - \frac{\Lambda_{k-j-1}^2}{2}\right)\Delta + \Lambda_{k-j-1}\sqrt{\Delta}Z\right]$$

$$P(T^*,i;T) = \frac{P(T^*+T)}{P(T^*)}\cdot 2\cdot\frac{\prod_{t=T}^{T+T^*-1}h(t)}{\prod_{t=1}^{T^*-1}h(t)}\delta^{Ti} \quad \text{where } h(t) = \frac{1}{1+\delta^t}$$

FET-108-07

$$V(E) = V(F) - V(D) = V(F) - D_{DF} + P(V(F),D) = C(V(F),D)$$

$$V^*(F) = S + D(1+\frac{m}{n})$$

$$V_R^{'}(E) = -C + V_R(F) - D + P\left\{V_R(F),D\right\} = -C + V_R - D + P_R$$
$$V_N^{'}(E) = V_N(F) - D + P\left\{V_N(F),D\right\} = V_N - D + P_N$$

$$face\,value + principal\ forgiven - default\ put\ assu\min g\ reinvestment = D - (P_N - P_R - NPV) - P_R$$
$$= (D - P_N - B - NPV) + B = value\,of\ regular\ debt + saving\ in\,bankruptcy\cos t$$

FET-109-07

$$RBC = \frac{1}{2}\left[C_0 + C_{4a} + \left[(C_1 + C_{3a})^2 + C_2^2 + C_{3b}^2 + C_{4b}^2 \right]^{\frac{1}{2}} \right]$$

FET-112-07 None

FET-113-07

$$\sigma_Y^2 = \sum_{i=1}^{n} \sigma_{x_i}^2 = \sum_{i=1}^{n} \omega_i^2 \sigma_i^2$$

$$MCaR = k\sigma_r = k\sqrt{\sum_{i=1}^{n} \omega_i^2 \sigma_i^2} = \sqrt{\sum_{i=1}^{n} k^2 \omega_i^2 \sigma_i^2} = \sqrt{\sum_{i=1}^{n} DCaR_i^2}$$

$$Total\,CaR = \sqrt{\sum_{i=1}^{n} CaR_i^2 + \sum_{i=1}^{n} \sum_{i \neq j} CaR_i CaR_j \rho_{ij}}$$

FET-114-07

$$NPV = (1-d)V\left\{S^+\right\} - (C-\mu) - (1+m)V\left\{S^-\right\}$$
$$= \mu - (dV\left\{S^+\right\} + mV\left\{S^-\right\})$$

$$V\left\{S^+\right\} = \frac{\sigma(n(z) + zN(z))}{(1+r)} \qquad V\left\{S^-\right\} = \frac{\sigma(n(z) - zN(-z))}{(1+r)}$$

FET-115-08 None

FET-138-07

$$c = \int_{W^*}^{\infty} f\left(w\right) dw$$

$$VAR = W_0 \times \alpha\sigma\sqrt{\Delta t}$$

$$se\left(\hat{q}\right) = \sqrt{\frac{c\left(1-c\right)}{T\ f\left(q\right)^2}}$$

FET-139-07 None

FET-141-08 None

FET-142-08 None

FET-143-08

$$Haircut = \frac{XC}{XC + LL}$$ size of the losses

where $XC =$ sum of excess capital

$LL =$ remaining liquid / surrender-able liabilities= total amount of available assets (AA) in excess of 200% RBC available to meet any remaining liquidity demands

FET-144-08

$$leverage = \frac{senior\ debt + excess\ hybrid\ debt\ and\ preferred\ stock}{ECA + senior\ debt + hybrid\ debt + preferred\ stock}$$

$$Hybrid\ Ratio_{U.S.} = \frac{s\tan dard\ \&\ pool's\ qualifying\ hybrid}{U.S.GAAP(consolidated)\ capital + total\ hybrid + total\ senior\ debt}$$

$$Hybrid\ Ratio_{Eurpoe} = \frac{s\tan dard\ \&\ pool's\ qualifying\ hybrid}{Group\ Consolidated\ TAC(excluding\ hybrid) + total\ hybrid + total\ senior\ debt}$$

$$Double\ leverage_{U.S.} = \frac{s\tan dard\ \&\ pool's\ qualifying\ hybrid + total\ senior\ debt + nonqualifying\ hybrid}{U.S.GAAP(consolidated)\ capital + total\ hybrid + total\ senior\ debt}$$

$$Double\ leverage_{Eurpoe} = \frac{s\tan dard\ \&\ pool's\ qualifying\ hybrid}{Group\ Consolidated\ TAC(excluding\ hybrid) + regulatory\ qualifying\ hybrid\ capital}$$

FET-145-08

(equity +franchise) * total shareholder return
=increase in net assets + increase in franchise value + dividend
=increase in franchise value + retained profit + dividend
=franchise * franchise growth rate + equity * return on equity

return on equity = total shareholder return + franchise/equity *(total shareholder return – franchise growth rate)

$$E_0 + F_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+COE)^t} = \sum_{t=1}^{\infty} \frac{D_t + E_t - E_{t-1} - COE * E_{t-1}}{(1+COE)^t} - \sum_{t=1}^{\infty} \frac{E_t}{(1+COE)^t} + \sum_{t=1}^{\infty} \frac{E_{t-1}}{(1+COE)^{t-1}}$$

$$F_0 = \sum_{t=1}^{\infty} \frac{ROE_t - COE}{(1+COE)^t} E_{t-1}$$

$$(1+R_f)\left\{A_0 - L_0 + F_0\right\} = A_0 - L_0 + (1-k_T)\left\{A_0(R_f + m_A - k_A) - L_0(R_f - m_L + k_L)\right\} + F_1$$

where $A_t =$ balance sheet assets at time t $\quad R_A =$ actual asset return

$L_t =$ balance sheet liabilities at time t $\quad R_L =$ actual liability return

$F_t =$ franchise value $\quad R_f =$ risk-free rate

$k_A =$ asset-related expenses as a proportion of A_0 $\quad k_L =$ liability-related expenses as a proportion of L_0

$k_T =$ tax paid as a proportion of pre-tax profit $\quad m_A =$ margin above LIBOR as asset swap

$m_L =$ margin below LIBOR as liability swap

$$(1+R_f)F_0 = (1-k_T)\left\{A_0(m_A - k_A) + L_0(m_L - k_L)\right\} - k_T R_f (A_0 - L_0) + F_1$$

$$(1+R_f)F_0 = A_0(1-k_T)(m_A-k_A)+L_0(1-k_T)(m_L-k_L)+(1-s)F_1-(R_f+s)k_T(A_0-L_0)$$

$$(1+R_f)(A_0-L_0+F_0) = A_0\left\{1+(1-k_T)(R_f-m_A-k_A)\right\}-L_0\left\{1+(1-k_T)(R_f-m_L+k_L)\right\}+(1-s)F_1-sk_T(A_0-L_0)$$

$$(R_f+s-g+sg)F_0 = A_0(1-k_T)(m_A-k_A)+L_0(1-k_T)(m_L-k_L)-(R_f+s)k_T(A_0-L_0)$$

FET-146-08

$$D_L = \sum_{x>A} p(x)(x-A) \quad \text{where } p(x) = \text{probability density for losses } (0 \le x \le \infty)$$

$$D_A = \sum_{L>y} q(y)(L-y) \quad \text{where } q(y) = \text{probability density for losses } (0 \le y \le \infty)$$

$$D_L = \int_A^\infty (x-A)p(x)dx$$

$$D_A = \int_0^L (L-y)q(y)dy$$

$$d_L = \frac{D_L}{L} = k\phi\left[\frac{-c}{k}\right] - c\Phi\left[\frac{-c}{k}\right]$$

$$d_A = \frac{D_A}{L} = \frac{1}{1-c_A}\left[k_A\phi(\frac{-c}{k_A}) - c_A\Phi(\frac{-c_A}{k_A})\right]$$

where $k_L =$ the cv of losses $\quad k_A =$ the cv of assets $\quad c_A =$ capital / assets ratio

$\Phi(x) =$ the cumulative standard normal distribution $\quad \phi(x) =$ the standard normal density function

$$d_L = \Phi(a) - (1+c)\Phi(a-k)$$

$$d_A = \Phi(b) - \frac{\Phi(b-k_A)}{1-c_A}$$

where $a = (k/2) - (\ln(1+c)/k)$ $\qquad b = (k_A/2) + (\ln(1-c_A)/k_A)$

$\Phi(x) =$ the cumulative normal distribution

one-period expected policy-holder deficit ratio

$d_1 = \int_{-\infty}^0 -zp(z)dz$ where $p(z) =$ the density of $\tilde{c}_1$

$\bar{C}_1 =$ the amount of capital at the end of one period

$\tilde{c}_1 = \tilde{C}_1 / L_0$ the amount of capital relative to the original expected loss

$$\bar{c}_1 = c(1+p) + [1+c(1+p)]\tilde{r} + pc\tilde{b} - \tilde{g}$$

where $\tilde{r}$ *and* $\tilde{g}$ are random variables denoting the annual return on assets and annual rate of change in value of the liabilities
$\tilde{b} =$ incurred loss ratio

$$C = \left[\sum_{i=1}^{n} c_i^2 + \sum_{i \neq j}^{n} \rho_{ij} c_i c_j \right]^{1/2} \text{ total capital}$$

$$D = \sigma\phi\left(\frac{-\mu}{\sigma}\right) - \mu\Phi\left(\frac{-\mu}{\sigma}\right)$$

$$d = \frac{D}{L} = k_T\phi\left(\frac{-c}{k_T}\right) - c\Phi\left(\frac{-c}{k_T}\right)$$

$$d_L = k\phi\left(\frac{-c}{k}\right) - c\Phi\left(\frac{-c}{k}\right)$$

$$d_A = \frac{D_A}{L} = \frac{1}{1-c_A}\left[k_A\phi\left(\frac{-c_A}{k_A}\right) - c_A\Phi\left(\frac{-c_A}{k_A}\right) \right]$$

$$F = S\Phi(a) - Ee^{-it}\Phi(a - \sigma\sqrt{t})$$

where $a = \dfrac{\ln(S/E) + (i + \sigma^2/2)t}{\sigma\sqrt{t}}$ $S =$ stock price $E =$ exercise price

$$D_L = L\Phi(a) - (1+c)L\Phi(a - \sigma_L)$$

$$d_L = \Phi(a) - (1+c)\Phi(a-k)$$

$$D'_L = A\Phi(a') - L\Phi(a' - \sigma_A)$$

$$d_A = \Phi(b) - \frac{\Phi(b - k_A)}{1 - c_A}$$ where $b = (k_A/2) + (\ln(1-c_A)/k_A)$

FET-147-08 None

FET-148-08 None

FET-149-08 None

FET-150-08 None

FET-151-08

default put option $V(E) = F + V(A_T) - PV(L) + O$
where $V() =$ market value $PV() =$ the present value $E =$ owner's equity $A =$ the assets $L =$ liabilities
$F =$ the franchise value $A_T =$ tangible assets $O =$ the default put option

FET-152-08 None

FET-153-08 None

FET-154-08 None

FET-155-08

$$\int_{\xi_\rho}^{\infty} \frac{wf(w)dw}{1-\Phi(\xi_\rho)} = CTE(\rho)$$

FET-156-08 None

FET-157-08

$$E_{r_i} = r_f + \beta_i (E_{r_M} - r_F)$$

FET-158-08

$$D(t,T) = \frac{1}{e^{s(t,T)\times(T-t)}} = \frac{1}{e^{\phi(T-t)\times(T-t)}} E\left[\frac{1}{e^{\int_t^T r_s ds}}\right]$$

$$r_s^* = r_s + \phi(s-t) + \phi'(s-t)\times(s-t)$$

$$D(t,T) = \frac{1}{e^{s(t,T)\times(T-t)}} = E\left[\frac{1}{e^{\int_t^T (r_s+\phi(T-t))ds}}\right] = E\left[\frac{1}{e^{\int_t^T r_s^* ds}}\right]$$

FET-159-08

$$dr = (k\theta - (k+\lambda)r)dt + \sigma\sqrt{r}dw^* \quad \text{where } w^*(t) = w(t) + \int_0^t \frac{\lambda}{\sigma}\sqrt{r(s)}ds$$

$$\frac{p(t,TB)}{B(t)} = E^*\left[\frac{1}{B(TB)}\right] \qquad p(t,TB) = E^*\left[\exp(-\int_t^{TB} r(s)ds)\right]$$

$$p(t,TB) = A(t,TB)\exp(-r(t)G(t,TB))$$

$$A(t,TB) = \left[\frac{2\gamma \exp\left[(b+\gamma)\frac{TB-t}{2}\right]}{(\gamma+b)(\exp(\gamma(TB-t))-1)+2\gamma}\right]^{\frac{2c}{\sigma^2}}$$

$$G(t,TB) = \frac{2(\exp(\gamma(TB-t))-1)}{(\gamma+b)(\exp(\gamma(TB-t))-1)+2\gamma} \quad \text{where } b = k+\lambda \quad c = k\theta \quad \gamma = \sqrt{b^2+2\sigma^2}$$

$$C(t) = p(t,TB)\chi^2(2\gamma^*(\varphi+\psi+G(T,TB)), \frac{4c}{\sigma^2}, \frac{2\varphi^2 r e^{\gamma(T-t)}}{(\varphi+\psi+G(T,TB))}) -$$

$$Xp(t,T)\chi^2\left[2r^*(\varphi+\psi),\frac{4c}{\sigma^2},\frac{2\varphi^2 re^{\gamma(T-t)}}{(\varphi+\psi)}\right]$$

$$dr=(\phi(t)-\alpha(t)r)dt+\sigma(t)dw^{**}\quad \phi(t)=\theta(t)+\alpha(t)b-\lambda(t)\sigma(t)$$

$$\frac{x(t)}{B(t)}=E^{**}\left[\frac{x(\tau)}{B(\tau)}\right]$$

$$p(t,TB)=E^{**}\left[\exp(-\int_t^{TB} r(s)ds)\right]$$

$$\alpha(t)=\frac{-\partial^2 G(0,t)\big/\partial t^2}{\partial G(0,t)\big/\partial t}$$

$$\phi(t)=-\alpha(t)\frac{\partial F(0,t)}{\partial t}-\frac{\partial^2 F(0,t)}{\partial t^2}+\left[\frac{\partial G(0,t)}{\partial t}\right]^2\int_0^t\left[\frac{\sigma(\tau)}{\partial G(0,\tau)\big/\partial\tau}\right]^2 d\tau$$

$$C(t)=P(t,TB)N(h)-XP(t,T)N(h-\sigma_p)$$

where $h=(\frac{\sigma_p}{2})+(\frac{1}{\sigma_p})\ln\left[\frac{P(t,TB)}{(XP(t,T))}\right]$

$$\sigma_p^2=\left[G(0,TB)-G(0,T)\right]^2\int_t^T\left[\frac{\sigma(\tau)}{\partial G(0,\tau)/\partial\tau}\right]^2 d\tau$$

$$A(0,t)=\left[\frac{2\gamma\exp\left[(b+\gamma)\frac{t}{2}\right]}{(\gamma+b)(\exp(\gamma t)-1)+2\gamma}\right]^{\frac{2c}{\sigma^2}}$$

$$G(0,t)=\frac{2(\exp(\gamma t)-1)}{(\gamma+b)(\exp(\gamma t)-1)+2\gamma}\qquad \text{where } b=k+\lambda\quad c=k\theta\quad \gamma=\sqrt{b^2+2\sigma^2}$$

$$P\max(P(0,TB),M(0),0,T)=E^*\left[\frac{M(T)}{B(T)}\right]-P(0,TB)=E^*\left[M(T)(\exp(-\int_0^T r(s)ds))\right]-P(0,TB)$$

$$r_{t_i}=r_{t_{i-1}}+(k\theta-(k+\lambda)r_{t_{i-1}})(t_i-t_{i-1})+\sigma\sqrt{r_{t_{i-1}}}\sqrt{(t_i-t_{i-1})}\tilde{\varepsilon}$$

$$PM\widehat{A}X(P(0,TB),M(0),0,T)=(\left\{\frac{1}{N}\sum_{n=1}^{N}M_n(T)\exp\left[-\sum_{i=1}^{m}r_n(t_{i-1})(t_i-t_{i-1})\right]\right\}-P(0,TB))$$

$$PMAX(P(0,TB),M(0),0,T) = E^{**}\left[\frac{M(T)}{B(T)}\right] - P(0,TB) = \quad E^{**}\left[M(T)(\exp(-\int_0^T r(s)ds))\right] - P(0,TB)$$

$$r_{t_i} = r_{t_{i-1}} + (\phi(t_{i-1}) - \alpha(t_{i-1})r_{t_{i-1}})(t_i - t_{i-1}) + \sigma\sqrt{r(0)}\sqrt{t_i - t_{i-1}}\tilde{\varepsilon}$$

FET-160-08 None

FET-161-08 None

FET-162-08 None

FET-163-08

share value of taking the project = $\dfrac{PV\ of\ assets\ in\ place + PV\ of\ new\ investment}{number\ of\ original\ shares + number\ of\ new\ shares}$

share value of not taking the project = $\dfrac{PV\ of\ assets\ in\ place}{number\ of\ original\ shares}$

share value : financing project with riskless debt = $\dfrac{value\ of\ original\ assets + NPV\ of\ new\ project}{number\ of\ shares}$

FET-164-08

$Risk - weighted\ amount = \sum Assets * WA + \sum credit\ equivalent * WCE$ where $WA =$
risk capital weighted by asset categories
$WCE =$ weighted by credit equivalents by type of counter party

FET-165-08

$$T = (E+P)(1+r_i) - L$$

$$E(T) = (E + P - R(I,S))(1 + E(r_i)) - E(L(a) + hC(I,S) - a$$

$$\frac{\partial E(T)}{\partial a} = \frac{-\partial E(L(a))}{\partial a} - 1 + h\frac{\partial C}{\partial I}\frac{\partial I}{\partial L}\frac{\partial L}{\partial a} = 0$$

$$T = \{E + D\}(1 + E(r)) - E(L(a)) - D(1+r) + hC(I,S) - a$$

$$\frac{\partial E(T)}{\partial a} = -\frac{\partial E(L(a))}{\partial a} - 1 + h\frac{\partial C}{\partial I}\frac{\partial I}{\partial L}\frac{\partial L}{\partial a} = 0$$

Recommended Approach for Setting Regulatory Risk-Based Capital Requirements for Variable Annuities and Similar Products,

None

Smith, Investor & Management Expectations of the "Return on Equity" Measure vs. Some Basic Truths of Financial Accounting

$$E_t - EV_t = \sum_{x=1}^{t}\left[(ROE_x - IRR) * E_{x-1} * (1+IRR)^{(t-x)}\right]$$

where $E_t =$ equity at time t $\quad EV_t =$ embedded value at time t, using discount rate IRR

$IRR =$ pricing internal rate of return after target surplus

$ROE_x =$ return on equity at time $x =$ earnings in period x / E_{x-1}

Bodoff, Capital Allocation by Percentile Layer

percentile layer of capital $(\alpha, \alpha + j) =$ required capital at percentile $(\alpha + j) -$ required capital at percentile (α)

layer of capital $(a, a+b) =$ capital equal to amount $(a+b) -$ capital equal to amount (a)

$VaR(x) =$ total required capital $= \sum_{\alpha=0}^{k-j}\left[x(\alpha + j) - x(\alpha)\right]$

$x(\alpha) =$ loss amount at percentile α $\quad j =$ selected percentile increment

$$\int_{x=y}^{x=\infty} f(x) / (1 - F(y))\, dx \quad \text{where } x = \text{loss amount} \quad y = \text{the capital}$$

$$\int_{y=0}^{y=VaR(99\%)} \int_{x=y}^{x=\infty} f(x) / (1 - F(y))\, dx dy$$

$$\int_{y=0}^{y=x} f(x) / (1 - F(y))\, dy$$

$$\int_{y=0}^{y=VaR(99\%)} f(x) / (1 - F(y))\, dy$$

$$\int_{x=x(0\%)}^{x=\infty} \int_{y=0}^{y=\min(x, VaR(99\%))} f(x) / (1 - F(y))\, dy dx$$

Allocated capital to loss event x $AC(x) = \int_{y=0}^{y=x} f(x) / (1 - F(y))\, dy$

$$AC(x) = f(x) \int_{y=0}^{y=x} 1 / (1 - F(y))\, dy$$

$$AC(x) = f(x) \int_{y=0}^{y=VaR(99\%)} 1 / (1 - F(y))\, dy$$

$$\frac{d}{dx\{AC(x)\}} = \frac{d}{dx\left\{ f(x)\int_{y=0}^{y=x} 1/(1-F(y))\,dy \right\}} = f(x)\frac{d}{\left\{dx\int_{y=0}^{y=x} 1/(1-F(y))\,dy\right\} + \int_{y=0}^{y=x} 1/(1-F(y))\,dy \; d/dx\{f(x)\}} =$$

$$f(x)/(1-F(x)) + \int_{y=0}^{y=x} 1/(1-F(y))\,dy f'(x)$$

$$rf(x)\int_{y=0}^{y=x} 1/(1-F(y))\,dy \quad r = \text{required rate of return on capital}$$

$$r\int_{y=0}^{y=x} 1/(1-F(y))\,dy$$

$$x + r\int_{y=0}^{y=x} 1/(1-F(y))\,dy$$

$$x\left[1 + r(1/x)\int_{y=0}^{y=x} 1/(1-F(y))\,dy\right]$$

premium net of expenses = expected loss + cost of capital

$$P = E[L] + r*(allocated\ capital - contributed\ capital)$$

where $P =$ premium net of expenses $E[L] =$ expected loss $r =$ required rate of return on capital

$$P = E[L] + r/(1+r)*(allocated\ capital - E[L])$$

$$P(x) = xf(x) + r/1+r\left[f(x)\int_{y=0}^{y=x} 1/(1-F(y))\,dy - xf(x)\right]$$

$$P(x) = f(x)\left\{x + r/1+r\left[\int_{y=0}^{y=x} 1/(1-F(y))\,dy - x\right]\right\}$$

$$x + r/(1+r)\left[\int_{y=0}^{y=x} 1/(1-F(y))\,dy - x\right]$$

$$P(x) = xf(x)\left\{1 + r/(1+r)\left[(1/x)\int_{y=0}^{y=x} 1/(1-F(y))\,dy - 1\right]\right\}$$

$$(\tfrac{r}{(1+r)})(\int_{y=0}^{y=x} \tfrac{1}{(1-F(y))}\,dy - x)$$

$$AC(x) = (\tfrac{1}{\theta})\exp(-\tfrac{x}{\theta})\int_{y=0}^{y=x} \exp(\tfrac{x}{\theta})dy$$

$$AC(x) = 1-\exp(-\tfrac{x}{\theta})$$

$$\tfrac{d}{dx\{AC(x)\}} = (\tfrac{1}{\theta})\exp(-\tfrac{x}{\theta})$$

$$1+r(\tfrac{1}{x})\theta(\exp(\tfrac{x}{\theta})-1)$$

Hardy, Freeland and Till, Valuation of Long-Term Equity Return Models for Equity-Linked Guarantees

$$Y_t | F_{t-1} = \mu + \sigma_t z_t \text{ where } z_t \approx N(0,1), \forall t$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 (Y_{t-1} - \mu)^2 + \beta \sigma_{t-1}^2$$

$$Y_t | F_{t-1} = Q_1 \; w.p.q \qquad = Q_2 \; w.p.(1-q)$$

$$\text{where } Q_1 | F_{t-1} = \mu_1 + \sigma_t z_t \qquad \sigma_t^2 = \alpha_{1,0} + \alpha_{1,1}(Y_{t-1} - \mu_1)^2 + \alpha_{1,2}(Y_{t-2} - \mu_1)^2$$

$$Q_2 | F_{t-1} = \mu_2 + \alpha_{2,0} z_t$$

$$r_{t,1} = r_t \Big| (\rho_t = 1) = \frac{y_t - \mu_1}{\sigma_1}$$

$$r_{t,2} = r_t \Big| (\rho_t = 2) = \frac{y_t - \mu_2}{\sigma_2}$$

$$r_t = I_{\{P(1)>0.5\}} r_{t,1} + \left(1 - I_{\{P(1)>0.5\}}\right) r_{t,2}$$

****BEGINNING OF EXAMINATION****
Afternoon Session
Beginning with Question 10

10. *(5 points)* Red Company has informally notified Black Company that it may be interested in acquiring it.

Black's corporate charter and employment agreements with key management include a "poison pill" providing that any outside firm completing a "hostile" takeover will pay an additional 300 to complete the transaction. The poison pill provides that Black's Board of Directors has the ability to determine whether any buyout offer is "hostile" or "friendly." This determination has no cost to Black.

Red is anticipating a profit of 200 from the acquisition, as it does not know of the "poison pill."

A successful acquisition may result in significant but unknown personal profit for Black's Board of Directors.

The Board will incur costs of 10 if they decide the takeover is "hostile" and the deal closes, or if they decide the takeover is "friendly" but the deal does not close.

Your colleague describes the situation as a two-player, simultaneous move game with complete information.

(a) *(1 point)* Critique your colleague's description, and provide an alternate one if you believe your colleague's is wrong.

(b) *(2 points)* Construct a payoff table for this situation and provide a diagram of it in normal form.

Given γ is the probability that Red will go through with the deal;

(c) *(2 points)* Determine the range of values of γ that would force the Board to conclude that declaring the deal is "hostile" is the best strategy.

11. *(8 points)* You are given the following diffusion process for a stock price:

$dS_t = \mu S_t dt + \sigma S_t dZ_t$, where Z_t is a Wiener process

(a) *(2 points)* Using Ito's lemma, show that S_t is lognormally distributed.

You are given the following information:

S_0	\$30
μ	10%
σ	30%

(b) *(1 point)* Calculate the expected stock price and the standard deviation of the stock price 9 months from now.

(c) *(2 points)* Calculate the 95% confidence interval for the stock price 9 months from now.

(d) *(1 point)* Calculate the probability that 9 months from now the stock price will have fallen by 50%.

You are given that 9 months later, the stock price has actually fallen by 50%.

(e) *(2 points)* Assess your model in light of this experience.

12. (*7 points*) Insurance company Multicorp is considering a $500 million acquisition of P&C liabilities which it would support with risk free assets. Multicorp is reviewing its risk based capital requirements in light of this new acquisition. The Expected Policyholder Deficit (EPD) is the primary objective for the capital. The capital requirements for Multicorp's existing balance sheet are shown below, along with correlations based on 10 years of historical data.

Balance Sheet Item	Required Capital ($ millions)
A1 (asset)	350
A2 (asset)	600
L1 (liability)	1,500
L2 (liability)	1,125

	A1	A2	L1	L2
A1	1.0	0.2	-0.3	-0.1
A2		1.0	0.1	0.2
L1			1.0	0.6
L2				1.0

(a) *(1 point)* Calculate an estimate of the total capital requirement based on the balance sheet information for the two existing asset classes (A1 and A2) and two liability classes (L1 and L2).

(b) *(2 points)*

(i) Quantify the diversification benefit inherent in the business.

(ii) Identify any issues surrounding this estimate of diversification benefit.

Multicorp wants to estimate how much capital the new liabilities require on a standalone basis (excluding new business).

(c) *(2 points)* Show how selecting capital to produce a target expected policyholder deficit could be represented as an option by identifying the equivalencies between the stock option and the EPD valuation.

An option pricing model was fit to some historical data provided by the company selling the new liabilities. The option pricing model assumes a stock with the same distribution as the P&C liabilities (excluding new business). The table below summarizes some output from the model. The present value of $1 of risk free assets maturing one year from now is 0.98.

Option	Expiry	Exercise Price	Current Stock Price	Option Price
Put	1 year	70	50	18.7805
Call	1 year	65	50	0.4175

(d) *(2 points)* Illustrate two alternative capital requirement amounts using the option pricing data for the new liabilities assuming each of two different target EPD amounts.

13. *(8 points)* Roger Giggs, the Chief Financial Officer of Accra Company, has been in discussions with the Company's financial advisors about refinancing the capital of the Company to fund a new project. The cost of this project is 2,000 million and is expected to generate a return of 7.5%. The current debt financing consists of 900 million bonds with average yield to maturity of 5.5%. The current equity financing consists of 500 million equity shares with a share price of 15. The Company also has 600 million of retained earnings that can be used to invest in a project.

The following market and Company data was provided by Accra's financial advisors:

Accra's current systematic risk	1.5
The current risk free rate	4.0%
Market risk premium	2.5%
Standard deviation of Return on Stock	40.55%
Corporate income tax rate	25%

Assume that the firm's share price will be unaffected by the refinancing. The following two options were recommended by the financial advisors:

Option 1: Repay current borrowings of 900 million and raise new capital through a non-convertible non-callable bond issue of 2,300 million with an average yield of 5.0%.

Option 2: Repay current borrowings and raise 2,300 million by issuing convertible zero-coupon bonds with total face value of 2,100 million, with an average term to maturity of 2 years. They convert to 120 million shares.

As Deputy Chief Financial Officer, you are asked to prepare a report for Mr. Giggs, which should include the following:

(a) *(1 point)* Assess the firm's current cost of equity and weighted average cost of capital.

(b) *(2 points)* Determine the firm's cost of equity and weighted average cost of capital under Option 1.

(c) *(2 points)* Determine the market value of the convertible bond under Option 2, using a binomial model.

(d) *(1 point)* Determine the weighted average cost of capital under Option 2.

(e) *(1 point)* Compare and contrast the uses of non-convertible debt and convertible debt in financing projects.

(f) *(1 point)* Recommend the appropriate option for the Company.

14. *(4 points)* Myron Dodd is CEO and Chairman of the Board of Directors for Pink Sands Company, a large publicly traded corporation with a debt-to-asset ratio of 5%. Dodd's target compensation is 10% fixed and 90% variable. The variable component is made up of 1-year at-the-money call options on Pink Sands stock. The number of call options granted to Dodd each year is tied to Pink Sands' reported earnings for the year.

A colleague has identified the separation of ownership and control as an important principal-agent problem.

(a) *(1 point)* Explain this principal-agent problem as it relates to Pink Sands Company.

(b) *(2 points)* Determine the strengths and weaknesses of how agency costs are addressed at Pink Sands Company.

(c) *(1 point)* Recommend changes to Dodd's compensation structure to better address agency costs.

15. *(7 points)* Mountain High is a well-established firm. Tiny Giant, one of its subsidiaries, is looking to expand. Tiny has the following probability distribution for future earnings:

Outcome	Probability	PV of Future Earnings
A	0.50	150
B	0.20	200
C	0.30	400

Tiny Giant currently has a market capitalization of 200 and will be raising 180 by issuing new shares. All shares are non-convertible.

It would like to gain funding for one of the following new projects:

<u>Option 1</u>
Cost: 200

Transaction costs in case of bankruptcy: 100

Outcome	Probability	PV of Earnings
Low	0.5	170
High	0.5	275

<u>Option 2</u>
Cost: 230

Transaction costs in case of bankruptcy: 100

Outcome	Probability	PV of Earnings
Low	0.5	50
Medium	p	256
High	$1-0.5-p$	371

The main managers at Mountain High are currently on a team-building trip and have not seen any of the above figures. Fortunately, before they left, they pre-approved funding in an amount up to Tiny Giant's expected PV of future earnings.

(a) *(1 point)* Determine which option(s) Tiny Giant could pursue.

(b) *(6 points)* Show that the overall expected equity of Option 1 and Option 2 are equal when $p = 0.3$.

16. *(6 points)* Your company's variable annuity hedging program uses a series of put options to hedge the embedded equity market risk. The recent economic crisis has resulted in extremely high equity market volatility. Senior management believes that this high volatility is temporary. The volatility assumptions used in your hedging program are determined using a sample standard deviation over the past 6 months.

(a) (*1 point*) Identify the type of volatility smile that exists for equity options, and provide reasons why it exists.

(b) (*1 point*) Describe the impact of the volatility smile on the number of put options needed to hedge your equity market risk.

(c) (*2 points*) Discuss the impact of the higher volatility on:

(i) Your current hedging program.

(ii) A dynamic hedging program.

(d) (*2 points*) Critique your company's current approach for estimating volatility and recommend possible improvements.

17. *(7 points)* ACME manufactures anvils. The Company has $110 million in assets, of which $10 million is invested in cash and short-term, government-issued securities, which are regarded as risk-free. The cash and short-term government securities will earn no interest income over the next year. The remaining $100 million in assets is committed to research and development of a single risky project – the development of a state-of-the-art anvil.

ACME currently has one class of debt. All of this debt (a total of $80 million in face value) will mature in exactly one year from today.

ACME's corporate income tax rate is zero.

There are three plans available for ACME, as follows:

<u>The Current Plan</u>
The current plan is to focus on the single risky project – the development of a new anvil.

<u>The CFO's Plan</u>
The CFO prefers sticking with the single risky project but, in addition, paying an immediate dividend of 10 million to shareholders.

<u>The CEO's Plan</u>
The CEO prefers to undertake a second project that would develop rockets. If successful, this project would complement the anvils and would enhance sales of both the anvils and the rockets. No dividend would be paid. This plan would require issuing an additional 50 million in debt, which would be senior to the existing debt.

		Payoff in 1 Year in Millions		
Scenario	Probability	Current Plan	CFO's Plan	CEO's Plan
1	0.05	200	200	350
2	0.25	120	120	170
3	0.70	70	70	100

(a) *(1 point)* Describe sources of conflict between bondholders and stockholders.

(b) *(2 points)* Evaluate the conflicts created by the CFO's and CEO's plans.

(c) *(2 points)* Determine the plan of maximum benefit to the bondholders considering the expected payoffs.

(d) *(2 points)* Determine the plan of maximum benefit to the shareholders.

18. *(4 points)* Company ABC has released its quarterly financial information which revealed an extra $10 million of operating cash flow. Managers believe that this increased cash flow will persist into the future. The company is planning a large scale project in the next two years and requires extra capital to help fund it. They have no other major projects planned until then. The company's debt to equity ratio is already high so managers would like to acquire capital by issuing new shares. However, there is a concern that the company's stock price is currently too low and managers are looking at ways to increase it before the issue.

(a) *(2 points)* Explain actions that managers can take in order to increase the share price of the company.

(b) *(1 point)* Determine whether the management options in part (a) contradict the assumption of the rational investor.

(c) *(1 point)* Determine the appropriate course of action for the company.

19. *(4 points)* As a Term Life Pricing Actuary for CCC Life you are evaluating a policy design which provides a reduced death benefit for those smokers who lied about their smoking status in their application for this one-year term life insurance contract. You are considering using game theoretic methods to build a separating equilibrium to encourage smokers to be honest on their applications.

Average issue age	35
Gender Distribution	50% male and 50% female
Annual mortality rate for non-smokers	0.1%
Annual mortality rate for smokers	0.2%
Load for Premium	30%
Discount Rate	0%
Average Face Amount	1 million
Annual lapse rate	0%
Annual Premium for non-smokers	1,300
Annual Premium for smokers	2,600

Assume that the premium is paid at issue in full and the death claims will be paid at policy year end.

(a) *(1 point)* Define the following:

(i) Nash equilibrium

(ii) Separating equilibrium

(iii) Pooling equilibrium

(b) *(1 point)* Show that there is no benefit to non-smokers lying on their application.

(c) *(2 points)* Using the separating equilibrium, calculate the death benefit as a percentage of Face Amount that CCC Life will pay for those smokers who lied about their smoking status in their application for the one-year-term life insurance contract.

****END OF EXAMINATION****
Afternoon Session