FETE Complete Illustrative Solutions
Fall 2009

1. Learning Objectives:

5b. Describe tests of efficiency and their implications for capital structure, portfolio management, and risk management.

This is a computation question which tests candidate's ability to compute the value of information on allocating resources in an efficient manner to a given assignment.

Solution:

(a)

Project	Skilled	Unskilled	Shortage	Excess	Utility
A	(15)(0.85)(10,000)	(15)(0.15)(–2000)	0	5(–1000)	118,000
B	(20)(0.85)(10,000)	(20)(0.15)(–2000)	0	0	164,000

Project A expected profit = 118,000
Project B expected profit = 164,000
Total profit = 282,000

(b) Report 1 = $40,000, now

Project	Supply/Demand
A	15/15
B	25/25

Project	Skilled	Unskilled	Shortage	Excess	Utility
A	15(0.85)(10,000)	15(0.15)(–2000)	0	0	123,000
B	25(0.85)(10,000)	25(0.15)(–2000)	0	0	205,000

Expected profits for Project A = 123,000 and project B = 205,000
Total profits = 328,000
Value of Report 1 = 328,000 – 282,000 = 46,000 > 40,000

Report 2 = $100,000

Project	Supply/Demand
A	15/15
B	25/25

1. Continued

Project	Skilled	Unskilled	Shortage	Excess	Utility
A	15(10,000)	0	0	0	150,000
B	25(10,000)		0	0	250,000

Profit for project A = 150,000
Profit for project B = 250,000
Total profit = 400,000

Value of report 2 = 400,000 – 282,000 = 118,000 > 100,000

Report 1: Additional profit = 46,000 – 40,000 = 6,000
Report 2: Additional profit = 118,000 – 100,000 = 18,000
Hence, buy report 2.

2. Learning Objectives:

1c. Evaluate the various profitability measures including IRR, NPV and ROE, etc.

1e. Apply the concept of economic capital and describe methodologies for allocating capital within a financial organization.

This question deals with evaluating a project's profitability measures under uncertainty, and the concept of Economic Capital, measuring RAROC covered in Crouhy.

Solution:

(a) $$\text{ROC} = \frac{\text{After} - \text{Tax Income}}{\text{Capital}}$$

- Income is based off of accounting profits
- Capital on Balance Sheet may not reflect true risk of business
- Gives an idea of earnings produced by deploying capital

$$\begin{aligned}
\text{Pre-Tax Earnings} &= \text{Interest on Assets} \\
&\quad + \text{Interest on Assets Backing Capital} \\
&\quad - \text{Expenses} \\
&\quad - \text{Credit Losses} \\
&\quad - \text{Interest credited on Loans} \\
&= 100(0.07) + 3(0.05) - 0.5 - 1 - 100(0.05) \\
&= 0.65\text{M} \\
\text{Taxes} &= 0.2 \times 0.65\text{M} = 0.13\text{M} \\
\text{After-Tax Income} &= 0.65 - 0.13 = 0.52_{\text{M}}
\end{aligned}$$

$$\text{ROC} = \frac{0.52}{3} = 17.33\%$$

(b)(i) $$\text{RAROC} = \frac{\text{Risk-Adjusted Return}}{\text{Risk-Adjusted Capital}}$$

- Adjusts ROC profitability measure for risks taken to produce profits
- Use Economic Capital (EC) as denominator

$$\begin{aligned}
\text{Pre-Tax Earnings} &= \text{Interest on Assets} \\
&\quad + \text{Interest on Assets Backing EC} \\
&\quad - \text{Expenses} \\
&\quad - \text{Credit Losses} \\
&\quad - \text{Interest Credited on Debt}
\end{aligned}$$

- Hold EC to withstand 200 year event, or 99.5^{th} percentile

2. Continued

- $EC = Loss_{99.5\%} - E[Losses] = 6.5M - 1M = 5.5M$

Pre-Tax Earnings $= 100(0.07) + 6.5(0.05) - 0.5 - 1 - 100(0.05) = 0.775\text{M}$

Taxes $= 0.2 \times 0.775 = 0.155\text{M}$

After-Tax Income $= 0.775\text{M} - 0.155\text{M} = 0.62\text{M}$

$$\text{RAROC} = \frac{0.62\text{M}}{5.5\text{M}} = 11.27\%$$

(ii) Because RAROC(11.27%) < Hurdle Rate (15%), the investment did not add value to the company given the risks taken to produce the earnings.

(c)(i) IRR

(+) reflects true value of money
(+) simple and easy to understand – accept project if IRR > Hurdle Rate
(–) ignores riskiness of underlying cash flows
(–) violates value-additive principle
(–) can provide multiple IRR's if CF's change sign

ROC

(+) Earnings and capital numbers are readily available from financial statements
(–) risks undertaken to produce profits may not be reflected well if using regulatory capital
(–) result can be manipulated

RAROC

(+) adjusts return measure to accurately reflect specific risks taken
(+) allows managers to efficiently make risk forward tradeoff
(+) can be used for performance measurement and capital allocation
(–) complicated
(–) no standard method in industry to calculate

(ii) Recommend RAROC since it reflects risks undertaken to produce profits

3. Learning Objectives:

4b. Contrast commonly used equity and interest rate models.
4e. Recommend an equity or interest rate model for a given situation.

This question asks the candidates to compare two of the classic interest rate models and demonstrate some knowledge of the key implementation issue, run time.
Solution:

a) CIR

Advantages: A square root process which implies that the volatility of the interest rate is not independent of the interest rate
Proportional to the square root of the interest rate
The interest rate cannot become negative.
Leads to closed form solutions for the price of treasury bonds and options on them.

Disadvantages: Do not imply a perfect fit to the initial term structure of interest rates and interest rate volatilities that are observed in the market.

HW

Advantages: unlike the Vasicek and CIR processes, 3 of the 4 parameters are time dependent
Thus can estimate interest rates and interest rate volatilities that are observed in the market.

Disadvantages: negative interest rates possible

b) Describe an effective variance reduction technique that would be effective for speeding up the computational process.

Antithetic Variable Technique
Takes the average of two simulated scenarios/parameters (f1 + f2) / 2 because the 1st scenario/parameter may be too high on average & the 2nd too low. The average of the two will return an appropriate parameter. By doing this the # of simulations that need to be completed decreases, thus increasing the speed of the computation.
Control Variate Technique
Importance Sampling
Stratified Sampling
Moment Matching

(c) Explain why the HW model would generate a higher put option price than CIR model.

3. Continued

- A low interest rate in either model helps set a high max bond price over the option's life if it occurs late during the option's life.
- When initial interest rate is higher than the long term mean rate, HW implies a higher probability of a lower interest rate late in the option's life.
- On average, the interest rate will reach a much lower level by the time the option has been outstanding for say 80% of the life.
- At that time, the volatility of the interest rate will be much higher in the HW model
- higher HW volatility implies higher probability of a lower interest rate.

4. Learning Objectives:

3f. Demonstrate mastery of option pricing techniques and theory for equity and interest rate derivatives.

4a. Describe and evaluate equity and interest rate models.

4f. Describe issues and best practices in the estimation or calibration of financial models.

Standard equity models don't capture changing volatility environments; Hardy's RSLN-2 model addresses this in a way useful to practitioners. Parameterizing models is also a major issue; we ask the candidate to explain one powerful technique to do this.

***Comments**: The first and last parts of this question were answered well. The stronger candidates could demonstrate an understanding of the principles of maximum likelihood estimation for dependent data in part (b), even if they did not recall the details of the RSLN estimation procedure. Similarly, in part (c), the stronger candidates described the Bayesian approach, in particular how the prior and the likelihood combine to construct the posterior, and how the MHA achieves this by iterating through the parameters, using a candidate distribution and an acceptance-rejection process.*

Solution:

(a) **RSLN-2 model and its key features:**

The Regime Switching Lognormal Model, with 2 regimes (RSLN-2) is a stochastic process in which there are two sources of randomness.

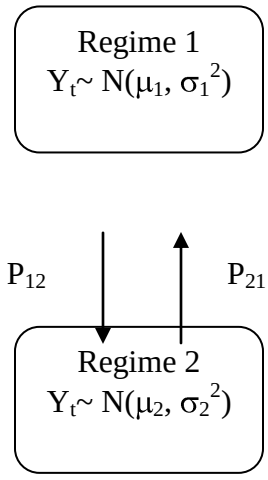
The regime process $\{\rho_t\}$ takes values 1 or 2 and indicates which regime the process is in during the interval (t,t+1). This is a Markov process, that is, the regimes switch randomly, and the probability of switching regimes depends only on the current regime, not on any other feature of the process history.

Given the regime , ρ_t , the log returns $Y_t=\log(S_{t+1}/S_t)$ are independent, and are normally distributed, with mean and variance dependent on the regime. That is

$$Y_t \mid \rho_t = \mu_{\rho_t} + \sigma_{\rho_t}\,\varepsilon_t\ , \qquad \varepsilon_t \sim N(0,1), iid$$

4. **Continued**

In diagram form:



RSLN-2 Characteristics:

- Generally two regimes is sufficient to capture stock index movements
- The volatility is stochastic (in the simplest way) through the regime process
- The resulting return distribution is significantly fatter tailed than the independent lognormal process (GBM)
- There are six parameters, $\{p_{12}, p_{21}, \mu_1, \sigma_1^2, \mu_2, \sigma_2^2\}=\Theta$
- Generally, we observe one low mean high volatility regime, and one more stable higher mean low volatility regime.
- The low vol, higher mean regime is the most frequent. The high vol regime captures relatively short, relatively rare periods of market uncertainty.
- The model captures volatility clustering through the fact that when the process moves to the high vol state, it tends to stay there for several months.
- The regime process captures the association of low means with more volatile periods.
- There is some autocorrelation through the serial dependence on regimes.

(b) **Parameter estimation using MLE method.**

- Take the log return data y_t
- We seek the values of the parameters $p_{12}, p_{21}, \mu_1, \sigma_1^2, \mu_2, \sigma_2^2$ which maximise the log of the likelihood
- The likelihood is the joint density of the sample $\{y_t\}$
- So we maximise

4. Continued

$$\boxed{LL = \sum_{t=1}^{120} \log f(y_t \mid y_1, ..., y_{t-1}, \Theta)}$$

- For each observation we know the density (=normal) for $\rho_t=1$ and for $\rho_t=2$, $f(y_t|\rho_t)$
- We can iteratively construct the likelihood, using

$$f(y_t \mid y_{t-1}, y_{t-2}, \ldots, y_1, \Theta) = \sum_{i=1}^{2} \sum_{j=1}^{2} f(\rho_t = i, \rho_{t-1} = j, y_t \mid y_{t-1}, y_{t-2}, \ldots y_1, \Theta)$$

- We use (eg) Solver in excel to find the parameters giving the maximum value for LL.
- For starting values, use mean and variance of the y_t for both sets of regime parameters, and, for example, $p_{12}=p_{21}=0.5$ for the transition probabilities.

(c) **MCMC method**

- The MCMC method is a Bayesian approach
- We model parameters as random variables, representing our uncertainty about their values
- We assign prior distributions to each parameter with a high variance if our knowledge (pre-data) is not strong; the mean of the prior distribution represents the best estimate of the parameter pre-data.
- Typically use beta distributions for p_{12} and p_{21} , inverse gamma for σ_1 and σ_2 and normal for μ_1, μ_2
- Prior distribution $\pi(\Theta)$ is combined with the likelihood function $f(y_1,...,y_n|\Theta)$ to derive the posterior function $f(\Theta|y_1,...,y_n)$
- We cannot do this analytically for this model. The Metropolis-Hastings method allows us to generate a sample from the joint posterior distribution of the 6 parameters.
- We use a candidate distribution for each parameter, sample from the candidate distribution, iteratively through the parameters, accept the candidate with probability

$$\alpha = \min\left(1, \frac{L_i(\xi, \theta_{\sim i}^{(r,r+1)})\pi(\xi)q(\theta_i^{(r)} \mid \xi)}{L_i(\vartheta_i^{(r)}, \theta_{\sim i}^{(r,r+1)})\pi(\vartheta_i^{(r)})q(\xi \mid \theta_i^{(r)})}\right)$$

- Where ξ is the candidate, π is the prior distribution, L is the likelihood (using candidate in the numerator, using the previous iteration in the denominator) and q is the candidate distribution density function

4. Continued

- We need to throw out burn-in values until the process stabilizes
- We use the means of the posterior distribution for point estimates of parameters
- The full posterior distribution gives uncertainty distribution, jointly for all parameters.

(d) Advantages of the MCMC Method

- We get the full joint density of the parameters, rich information
- We can use the sample from the posterior distribution to generate values from the predictive distribution for Y_t.

Disadvantages of the MCMC method

- It's complex and computationally intensive
- It is difficult to explain the process to end users
- It may take some work to select appropriate candidate distributions

Advantages of MLE

- Well understood
- Easy to do in Excel
- MLE estimates have some nice asymptotic properties, such as unbiased, minimum variance

Disadvantages of MLE

- Asymptotics fail when parameters are near the bounds of their space
- MLE focuses on the center of the distribution; may not work well in tails, exactly where we need the fit to be good. MCMC should give a better picture of the tail uncertainty.
- MLE gives point estimates. We can numerically estimate standard errors, but these are cumbersome, and not very reliable. MCMC gives the full joint distribution.

MLE methods may generate local maxima; not a problem for MCMC.

5. Learning Objectives:

1b. Calculate the cost of capital for a venture or a firm using the most appropriate method for given circumstances and justify the choice of method.

1c. Evaluate various profitability measures including IRR, NPV and ROE, etc.

This questions deals with evaluating a project under various capital budgeting techniques.

Solution:

(a) Strengths (S) and Weaknesses (W)

(i) Paycheck Method
- (S) Simple, straight forward
- (W) Does not consider all cash flow and time value of money

(ii) Accounting Rate of Return
- (S) Simple figures handily available
- (W) Accounting income does not mean to consider all cash flows from the project, neglects time value of money

(iii) Net Present Value
- (S) Connect method, consider all cash flow and time value of money, results are addictive and allow to set from mutually exclusive projects
- (W) Maybe more complicated than approach above

(iv) Internal Rate of Return
- (S) Consider all cash flow, time value of money
- (W) Results of different projects are not addictive

(b) (i) Paycheck Method, given initial cost = 90
Project 1, payback period = 1 year
Project 2, payback period = 2 years
Project 3, payback period = 2 years

Project 1 is chosen

(ii) Accounting Rate of Return

Project 1, $\frac{100+0}{90}-1=11.11\%$

Project 2, $\frac{60+24}{90}-1=-6.\overline{6}\%$

5. Continued

Project 3, $\frac{24+60}{90}-1=-66\overline{|}\%$

Project 1 is chosen

(iii) NPV

Project 1, $\frac{125}{1.1}-\frac{25}{1.1^2}-90=2.9\overline{|}52$

Project 2, $\frac{75}{1.1}+\frac{30}{1.1^2}-90=2.9\overline{|}52$

Project 3, $\frac{30}{1.1}+\frac{75}{1.1^2}-90=-0\overline{|}4$

Project 1 or 2 can be chosen

(iv) IRR

Project 1, $\frac{125}{1+r}-\frac{25}{(1+r)^2}-90=0\Rightarrow r\sim 14.6\%$

Project 2, $\frac{75}{1+r}+\frac{30}{(1+r)^2}-90=0\Rightarrow r\sim 12.8\%$

Project 3, $\frac{30}{1+r}+\frac{75}{(1+r)^2}-90=0\Rightarrow r\sim 9.4\%$

Project 1 is chosen

(c) Projects with positive NPV should be chosen
i.e. Project 1 and 2

(d) (i) Given unlevered: $p=10\%$

New WACC $=10\%\times(1-20\%\times 30\%)$
$=9.4\%$

(ii) Using NPV

Project 1: $\frac{25}{1.094}-\frac{25}{1.094^2}-90=3.3\overline{|}$

5. Continued

Project 2: $\frac{75}{1.094} + \frac{30}{1.094^2} - 90 = 3.62$

Project 3: $\frac{30}{1.094} + \frac{75}{1.094^2} - 90 = 0.09$

Therefore… all 3 projects should be undertaken.

6. Learning Objectives:

2f. Describe the process, methods and effects of a potential acquisition or reinsurance of a business including its effect on capital structure, return on equity, price/earnings multiples, and share price.

This is a recall and application / analysis question asking candidates to a) outline the detailed components of an Actuarial Appraisal and b) calculate the appraisal value of a firm given the necessary financial information.

Solution:

Step 1: Determine Target Capital

2009 $30 * 2.5 = 75$

2010 $34 * 2.5 = 85$

2011 $40 * 2.5 = 100$

2012 $46 * 2.5 = 115$

Step 2: Determine Interest on Target Capital

2010 $75 * 0.06 = 4.5$

2011 $80 * 0.06 = 5.1$

2012 $100 * 0.06 = 6$

Step 3: Calculate Pre Tax Statutory Income

Step 4: Calculate Pre Tax Statutory Income	**2010**	**2011**	**2012**
= Statutory Profit on Existing and New Business	28.1	27.8	31.6
- unallocated expense	–7.2	–2.3	–1.4
+ Interest on Capital and Surplus	4.5	5.1	6
Pre Tax Statutory Income	25.4	30.6	36.2

Step 5: Calculate Taxable Income	**2010**	**2011**	**2012**
= Total Pre Tax Statutory Income	25.4	30.6	36.2
+ Increase in Statutory Reserves	Change in Stat and Tax Reserves		
– Increase in Tax Reserves	are the same so they offset		
+ Increase in Proxy DAC Asset	15	15	15
– Existing Proxy DAC Tax Asset Runoff			
Taxable Income	40.4	45.6	51.2
Step 6: Calculate tax @ 25%	10.1	11.4	12.8

6. Continued

		2010	2011	2012
Step 7: Calculate After Tax Earnings (Marks for using Pre Tax Statutory Income)				
Pre Tax Statutory Income		25.4	30.6	36.2
Corporate Tax		10.1	11.4	12.8
After Tax Earnings		15.3	19.2	23.4

Step 8: Determine Distributable Earnings		2010	2011	2012
After Tax Earnings		15.3	19.2	23.4
– Change in Required Capital		10	15	15
Distributable Earnings	69.42	5.3	4.2	8.4

Step 9: Determine Discount Rate

Capital Asset Pricing Model

$r = \text{Risk Free Rate} + \beta(\text{Market Rate} - \text{Risk Free Rate})$

$r = 3\% + 0.75*(11\% - 3\%)$ 9.00%

Step 10: Identifying the Actuarial Appraisal Value

= NPV(Distributable Earnings)	69.42	4.8623853	3.535056	6.4863412

Actuarial Appraisal Value = 84.303783

7. Learning Objectives:

4a. Describe and evaluate equity and interest rate models

4f. Describe issues and best practices in the estimation or calibration of financial models

This question tests complicated option pricing material but walks the candidate through in bite-sized pieces. Parts a) through b) set the framework of the model, and c) asks for an outline of the option pricing method to be used. The remainder of the question calculates an example; understanding the formula pieces is necessary but extensive algebra is not.

***Comments**: Very few candidates got through (e) and essentially none through (f). The solution presented is based on the grading outline rather than a candidate paper.*

Solution:

(a)

	Risk-Neutral	Realistic
Arbitrage-Free Interest Rate Model	Current pricing, used with reliable market prices	Not useable since term premium cannot be reliably estimated
Equilibrium Model	Current pricing with market price unreliable or not observable	- Stress testing - Reserve, Adequacy testing and Asset

(b) HW is a risk neutral arbitrage free model so it will do a suitable job pricing assets because it is calibrated with quoted prices.

(c) Steps to follow for valuing coupon-paying bonds.

1. Calculate critical value r^* for which bond price at option maturity equals to strike price K.
2. Value each option on the zero-coupon bonds underlying the coupon-paying bond. Set the strike price K = the bond price at T when $r = r^*$
3. Sum the values of the options on the zero-coupon bonds

(d)

$t = 2.2 \quad \Delta t = 0.3 \quad T = t + \Delta t = 2.5 \quad \sigma = 0.03 \quad R = R^* = 0.0616$

$$\hat{\beta}(t,T) = \frac{\beta(t,T)}{\beta(t,t+\Delta t)}\,\Delta t = \frac{\beta(2.2,2.5)}{\beta(2.2,2.5)}\,(0.3) = 0.3$$

$$\therefore \ln \hat{A}(2.2,2.5) = \ln \frac{P(0,2.5)}{P(0,2.2)} - \ln \frac{P(0,2.5)}{P(0,2.2)} = 0 \quad \Rightarrow \hat{A}(2.2,2.5) = 1$$

7. Continued

$$\therefore P(2.2, 2.5) = \hat{A}(2.2, 2.5)e^{-0.3R}$$
$$= 1 * e^{-0.01848}$$
$$= 0.9817$$

$\therefore$ The required value is $100(0.06/2)(0.9817) = 2.9451$

(e) $\beta(t,T) = \beta(2.2, 3) = \dfrac{1-e^{-a(T-t)}}{a} = \dfrac{1-e^{-0.05(0.8)}}{0.05} = 0.7842$

$\beta(t, t+\Delta t) = \beta(2.2, 2.5) = \dfrac{1-e^{-0.05(0.3)}}{0.05} = 0.2978$

$\left(1+\dfrac{2\%}{2}\right)^2 = e^r \Rightarrow r = 1.99\%$ is continuously compounding rate per year for current term structure

$P(0,T) = P(3) = e^{-3r} = 0.9420$

$P(0, t+\Delta t) = P(2.5) = e^{-2.5r} = 0.9515$

$P(0,t) = P(2.2) = e^{-2.2r} = 0.9572$

Now $\hat{\beta}(t,T) = \dfrac{\beta(2.2,3)}{\beta(2.2,2.5)}(0.3) = 0.7900$

$$\ln \hat{A}(t,T) = \ln\frac{P(0,3)}{P(0,2.2)} - \frac{\beta(2.2,3)}{\beta(2.2,2.5)}\ln\frac{P(0,2.5)}{P(0,2.2)}$$
$$- \frac{\sigma^2}{4a}(1-e^{-2at})\beta(2.2,3)[\beta(2.2,3)-\beta(2.2,2.5)]$$
$$= \ln\frac{0.9420}{0.9572} - \frac{0.7842}{0.2978}\ln\frac{0.9515}{0.9572}$$
$$- \frac{0.03}{4\times 0.05}(1-e^{-2\times 0.05\times 2.2})0.7842[0.7842-0.2978]$$
$$= -0.01592 - 2.6333(-0.00597) - 0.0045 \times 0.19748 \times 0.7842 \times 0.4864$$
$$= -0.0005$$

$\Rightarrow \hat{A}(t,T) = 0.999469$

$\therefore P(t,T) = P(2.2,3) = 0.999382 \times 0.9525 = 0.999469$

$\therefore$ Value required $= [100 + 100(3\%)](0.9520) = 98.054$

7. Continued

(f) Final CF $L = 103$; $K = 98.05441$; $t = 2.2$; $S = 3$; $\sigma = 0.03$; $h = 0.05$

$$P(0,t) = P(0,2.2) = e^{-2.2\times 0.0199} = \left(1+\frac{0.02}{2}\right)^{-4.4} = 0.957163117$$

$$P(0,S) = P(0,3) = e^{-3\times 0.0199} = \left(1+\frac{0.02}{2}\right)^{-6} = 0.942045235$$

$$P(0,S) = P(0,2.5) = e^{-2.5\times 0.0199} = \left(1+\frac{0.02}{2}\right)^{5.0} = 0.951500$$

$$\sigma_p = \frac{\sigma}{a}\left[1-e^{-a(S-T)}\right]\sqrt{1-e^{-2aT}\ /2a}$$

$$= \frac{0.03}{0.05}\left[1-e^{-0.05(3-2.2)}\right]\sqrt{1-e^{-2\times 0.05\times 2.2}\ /(2\times 0.05)}$$

$$= 0.033061091$$

$$h = \frac{1}{\sigma_p}\ln\left(\frac{LP(0,s)}{P(0,T)K}\right)+\frac{\sigma_p}{2}$$

$$= \frac{1}{0.033061091}\times\ln\left(\frac{103\times 0.942045235}{98.05441\times 0.957163117}\right)+\frac{0.033061091}{2} = 1.02333126$$

$$N(h) = N(1.02333126) = 0.846925$$

$$N(h-\sigma_p) = N(0.990270169) = 0.838979$$

$$Call = LP(0,s)N(h) - KP(0,T)N(h-\sigma_p)$$

$$= 103\times 0.942049235\times 0.846925 - 98.05441\times 0.957163117\times 0.838979$$

$$\approx 3.436048$$

2nd to last coupon L = 3; K = 2.945069; 1=2.27 > =2.9; $\sigma = 0.03; a = 0.0$

$$P(0,T) = P(0,2.2) = e^{-0.0199\times 2.2} = \left(1+\frac{0.02}{2}\right)^{-4.4} = 0.957163$$

7. Continued

$$P(0,5) = P(0,2.5) = e^{-0.0199\times 2.5} = \left(1+\frac{0.02}{2}\right)^{-5} = 0.951466$$

$$\breve{p} = \frac{\sigma}{a}\left[1-e^{-a(S-T)}\right]\sqrt{1-e^{-2aT}\ /(2a)}$$

$$= \frac{0.03}{0.05}\left[1-e^{-0.05(2.5-2.2)}\right]\sqrt{1-e^{-2\times 0.05\times 2.2}\ /(2\times 0.05)}$$
$$= 0.012553137$$

$$\ln = \frac{1}{\sigma_p}\ln\left(\frac{LP(0,5)}{P(0,T)K}\right)+\frac{\sigma_p}{2}$$

$$= \frac{1}{0.012553137}\ln\left(\frac{3\times 0.951466}{2.945069\times 0.957163}\right)+\frac{0.012553137}{2} = 1.002864$$

$$N(h) = N(1.002864) = 0.842027$$
$$N(h-\sigma_p) = N(0.990271) = 0.838979$$
$$\text{Call } = LP(0,5)N(h) - KP(0,T)N(h-\sigma_p)$$
$$= 3\times 0.951466\times 0.842027 - 2.945069\times 0.957163\times 0.838979$$
$$= 0.038473$$

Total = 3.47452

8. Learning Objectives:

1d. Define and compare risk metrics used to quantify economic capital and describe their limitations.

1e. Apply the concept of economic capital and describe methodologies for allocating capital within a financial organization.

This question has the candidate explain why Economic Capital should be used, describe ways it can be computed, the advantages or disadvantages of each technique. It then asks for an assessment of the methods for calculating Quantiles and Cumulative Tail Distributions and their suitability.

Solution:

(a) There are several reasons why Windy City should consider using an economic capital approach to evaluating the risk of its new and old products.

- EC requirements are related to the stated financial objectives of Windy City and the constraints that are proprietary to Windy City.
- EC reflects the underlying economics of the business.
- Since the new products are unique, regulatory capital may not cover the risk profile; the industry averages often used for regulatory capital may no longer be suitable.
- Since Windy City is part of IBG, which is an international corporation, and since regulatory capital varies by political jurisdiction, IBG may forfeit gains from economies of scale by basing decisions on local regulatory capital since this amounts to managing IBG's businesses as smaller, independent institutions.
- Since IBG is international in scope, "rule arbitrage" is possible, allowing IBG to strategically shift risks to jurisdictions with different capital requirements.
- EC is more directly comparable across lines of business.

(b) There are several techniques and approaches that Windy City may use to calculate economic capital.

- Full Economic Scenarios are useful for determining EC for all of Windy City's risks combined. IBG would provide economic scenarios for Windy City to use, then aggregate the results from Windy City's testing. This approach may be suitable for Windy City since IBG is already familiar with EC and has been using it in other jurisdictions.
- Stress Tests use a set of highly adverse scenarios as the basis for EC and can be used for single aspect of economic risk since Windy City's new products are equity-linked products. Stress tests may also be used for combinations of risks. This approach may be suitable since Windy City has products with other risks.

8. Continued

- Factor Tables use models or industry tables to develop factors for the various risks but this approach may not be appropriate since it may result in EC close to regulatory capital levels.
- Univariate Stochastic Models use a scenario generator to test one variable (such as the new products' equity risks) with the capital needed being evaluated for each scenario. Diversification adjustments would need to occur outside the model.
- Multivariate Stochastic Models use a scenario generator to test more than one variable at a time such as the combination of equity risks and interest rate risks. This approach may be appropriate since Windy City is subject to several risks and the correlations between the risks may be implicitly included in the models.
- Scenario Generators can be constructed where the equity risk is the random variable and may be risk-neutral or real world. Random number generators are then used to populate a time series for equity risk. The scenario generators may be risk neutral or real world. Scenario generators appear to be appropriate for equity risk.
- Statistical Methods can be used by determining the mean, variance, and covariance of the distribution of gains and losses. Assuming a normal distribution, EC is determined from normal distribution tables. This may or may not be appropriate depending on whether the equity risks are normally distributed.
- Credit Risk Models use models and correlations for default/recovery probability based on historical studies. This approach may not be appropriate since Windy City's new products are not credit risk related.
- Operational Risk Models are loss models based on company or industry data. This approach may not be suitable there may be little data applicable to Windy City's new, unique products.
- Option Pricing Theory may be used by framing the loss situation as an option with option pricing models used to evaluate the EC amounts. This approach may not be appropriate unless the equity risk can be assumed to occur at one point in time.
- Correlation Adjustments may be calculated at a macro or micro level and appear appropriate since there are likely correlations between the new products and the older interest rate products.

(c) Calculations of 92% Quantiles and CTE for each product:

- Product A
  - $V_{92\%}$ = \$10 since worst 8% of scenarios are all \$10
  - CTE(92) = \$10 since average of worst 8% are also \$10

8. Continued

- Product B
  - $V_{92\%}$ = \$0 since only 6 scenarios produced nonzero losses. Therefore the 92^{nd} (and 93^{rd}) losses were 0.
  - CTE(92) = \$11 = (0+0+4+12+17+17+18+20)/8

- Product C
  - $V_{92\%}$ = \$21 using a uniform distribution from 0 to 25 over 50^{th} percentile value of 0
  - CTE(92) = \$23 = average of a uniform distribution from 21 to 25.

Recommend that the CTE should be used since:

- CTEs are "coherent" risk measures while Quantiles are not. Coherent risk measures are:
  - Bounded above by the maximum loss
  - Bounded below by the mean loss
  - Scalar additive and multiplicative
  - Subadditive
- Using Quantiles does not allocate any capital to Product B since the $V_{92\%}$ is 0 but there is some tail risk.
- Quantile measures are on average inadequate if used as the basis for capital
- CTE uses the entire shape of the distribution in excess of the quantile while the Quantile does not.
- CTE is less sensitive to sampling error.

9. **Learning Objectives:**

3a. Define the cash flow characteristics of complex derivatives including exotic options, interest rate derivatives, swaps, and other non traditional derivatives.

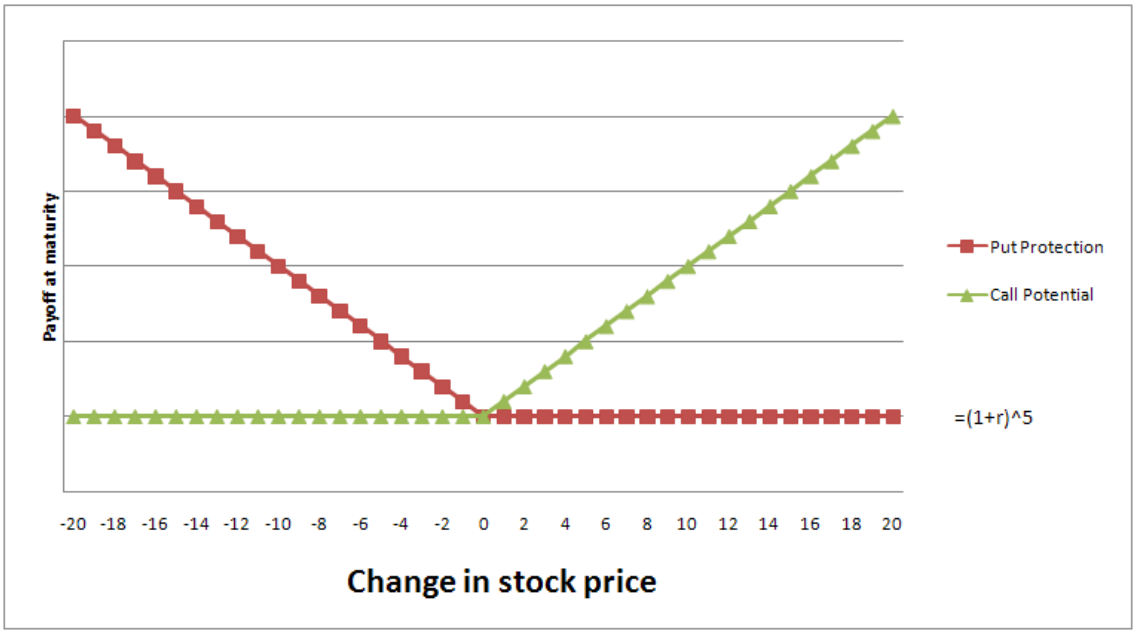
3b. Evaluate the risk/return characteristics of complex derivatives.

3c. Identify embedded options in assets and liabilities.

The question asks the candidate to identify which exotic option might be most appropriate to hedge a certain product, and to relate the capital markets risks to product design and risk management.

Solution:

(a) Graph:



(b)

Advantages

a. Could be very useful in volatile markets
b. Offers a baseline payoff in all markets
c. Allows the customer some time to get a feel for market before making decision

Disadvantages

a. Could be quite expensive
b. 6 months might be too short to be attractive

9. Continued

(c)

A chooser option would work best here
A chooser option has the feature that after a specified period of time, the holder can choose whether the option is a put or a call

d)

Chooser = c + exp(-q*(t2-t1))*max(0, K*exp(-(r-q)*(t2-t1))-S1)

This means that a chooser option can be valued as a call option with strike price K and maturity t2 and exp(-q*(t2-t1)) put options with a strike price of K*exp(-(r-q)*(t2-t1) and maturity t1.

The fewer put options with the lower strike price are clearly worth less than the-at-the money put option portion of the suggestion from the student. Therefore, her suggestion is unnecessarily expensive.

10. Learning Objectives:

5e. Define the elements of a game, including information sets, etc., Nash equilibrium, mixed strategies; explain the prisoners' dilemma and other special cases of a two-person, two-state, single period game, and explain the qualitative implications of repeated games.

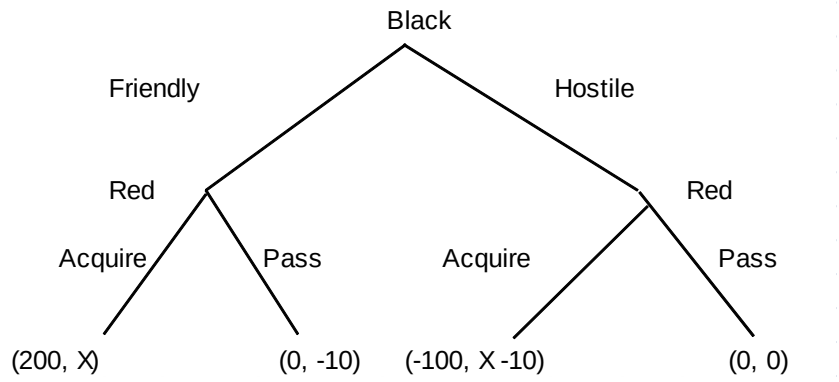
This is a basic game theory question based on the Gibbons article, FET-156-08.

Solution:

(a) I believe that my colleague's description is not completely correct.
This is not a simultaneous game. It is a sequential move game where Black must first decide to treat the takeover as hostile or friendly. Then, Red will make a move.
This is not a game with complete information. Black's payoff is private information and Red does not know of the "poison pill".
This is a two-player game.

(b) Let X be the Black's profit upon acquisition. Payoff in (Red, Black) format.

		Black: Hostile	Black: Friendly
Red	Acquire	-100, X-10	200, X
	Pass	0,0	0, - 10



10. Continued

(c) Hostile is the best strategy when expected payoff from hostile is greater than expected payoff from friendly.

$E[\text{Hostile}] = \gamma\ (X - 10) + (1-\gamma)\ (0)$

$E[\text{Friendly}] = \gamma\ (X) + (1-\gamma)\ (-10)$

$\gamma\ (X - 10) + (1-\gamma)\ (0) > \gamma\ (X) + (1-\gamma)\ (-10)$

$\gamma > 0.5$

11. **Learning Objectives:**

3f. Demonstrate the mastery of option pricing techniques and theory for equity and interest rate derivatives.

This question asks the candidate to demonstrate understanding of Ito's Lemma, the classical option pricing model, the properties of the lognormal distribution, and model risk. This is considered core material for this exam.

Solution:

(a) To Show St ~lognormal, we need ln S_t ~Normal

Let G = G(S, t) = ln S_t

$dS_t = \mu S_t dt + \sigma S_t dZ_t$

By $Itô's$ Lemma: $dG = \left(\frac{\partial G}{\partial S}\mu S + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial S^2}\delta^2 S^2 \right)dt + \frac{\partial G}{\partial S}\delta S dZ$

$$\frac{\partial G}{\partial S} = \frac{\partial}{\partial S} lnS = \frac{1}{S}$$

$$\frac{\partial G}{\partial t} = \frac{\partial}{\partial t} lnS = 0$$

$$\frac{\partial^2 G}{\partial S^2} = \frac{\partial}{\partial S}\left(\frac{\partial}{\partial S} lnS \right) = \frac{\partial}{\partial S}\left(\frac{1}{S} \right) = -\frac{1}{S^2}$$

$$\Rightarrow dG = \left(\frac{\mu S}{S} + 0 + \frac{1}{2}\left(-\frac{1}{S^2} \right)\sigma^2 S^2 \right)dt + \frac{\sigma S}{S} dZ$$

$$= \left(\mu - \frac{1}{2}\delta^2 \right)dt + \sigma dZ$$

$dZ \sim N\ \ 0, \sqrt{dt}$ since Z is wiener process

$\Rightarrow \sigma dZ \sim N\ \ 0, \sqrt{\sigma^{2dt}}$

$\Rightarrow dG \sim \mathrm{N}\left(\mu - \frac{1}{2}\delta^2 \right)dt, \sigma\sqrt{dt}$

$\Rightarrow Gt - G_0 \sim N\left(\left(\mu - \frac{1}{2}6^2 \right)t, \sigma\sqrt{t} \right)$

$G_t - G_0 = lnS_t - lnS_0 = ln\frac{S_t}{S_0}$

11. Continued

We have proved that $G_t = lnS_t \sim$ Normal dist$_n$
so S_t follows lognormal Dist$_n$

(b)

$$E\ S_t\ = S_0 e^{\mu t}$$

$$\text{var}\ S_t\ = S_0{}^2 e^{2\mu t}\ \ e^{\sigma^2 t} - 1$$

$$S_0 = 30 \quad \mu = 10\% \quad \sigma = 30\% \quad t = \frac{9}{12} = 0.75$$

$$\Rightarrow E\ S_{0.75}\ = 30e^{\ 0.1\ \ 0.75}\ = \$32.3365$$

$$\text{var}\ S_{0.75}\ = 30^2 e^{2\ 0.1\ \ 0.75}\ \ e^{\ 0.3^{\ 2}\ 0.75}\ - 1$$

$$= 73.0181$$

$$\text{standard deviation} = \sqrt{73.0181} = 8.5451$$

(c)

95% Confidence Interval =

$$lnS_{0.75} \sim N\left(lnS_0 + \left(\mu - \frac{\sigma^2}{2} \right) T, \sigma\sqrt{T} \right)$$

$$\sim \text{N}\left(\ln 30 + \left(10\% - \frac{30\%^{\ 2}}{2} \right) 0.75, 30\%\sqrt{0.75} \right)$$

$$\sim N\ \ 3.4424, 0.2598$$

95% CI for $lnS_{0.75}$: $3.4424 \pm 1.96\ \ 0.2598$

$$\Rightarrow\ \ 2.9332, 3.9517$$

$\Rightarrow$ 95% CI for $S_{0.75}$:

$$\Rightarrow \left(e^{3.9517}, e^{2.9332} \right)$$

$$\Rightarrow \left(18.7881, 52.0222 \right)$$

11. Continued

(d)

$$
\begin{aligned}
&P\left[S_{0.75} < S_0, 50\%\right] \\
&= P\left(\frac{S_{0.75}}{S_0} < 50\%\right) \\
&= P\left(ln\frac{S_{0.75}}{S_0} < ln0.5\right) \\
&ln\frac{S_{0.75}}{S_0} \sim \mathrm{N}\left(\left(\mu - \frac{\sigma^2}{2}\right)0.75, \sigma\sqrt{0.75}\right) \\
&= P\left(Z < \frac{ln0.5 - \left(\mu - \frac{\sigma^2}{2}\right)0.75}{\sigma\sqrt{0.75}}\right) \\
&= P\left[Z < -2.8267\right] \\
&= 1 - 0.9977 \\
&= 0.23\%
\end{aligned}
$$

(e)

The lognormal model has lighter tail than the real world, which means it underestimates the probability of extreme events such as exceptionally high or low stock prices.

Since the probability that stock prices drop by more than 50% is 0.23% using the model it is not very likely to happen. This possibly implies that the model has something wrong.

Lognormal model works well in short-time interval but does not generate reliable results in the long run.

It is also possible that the parameters we used calculate the probability is biased.

Left-Tail calibration can be used to increase left-tail fatness.

Regime-swithing model can also be considered.

12. Learning Objectives:

1d. Define and compare risk metrics used to quantify economic capital and describe their limitations.

1e. Apply the concept of economic capital and describe methodologies for allocating capital within a financial organization.

3f. Demonstrate mastery of option pricing techniques and theory for equity and interest rate derivatives.

This question deals with developing RBC based on a targeted Expected Policyholder Deficit. It requires consideration of how diversification is reflected when combining RBC measures associated with different balance sheet items and the dependence on historical correlations. It requires an understanding of how option pricing theory can be used to develop an RBC measure.

Solution:

(a)

$$C = [\sum_{i=1} C_1^{\,2} + \sum_{i \neq j}^{n} \rho_{ij} C_i C_j]^{n/2}$$

$$C = \text{Sqrt}(\ 350^2 + 600^2 + 1500^2 + 1125^2\ +\ 2 \times 0.2 \times 350 \times 600$$

$$+\ 2 \times 0.3 \times 350 \times 1500\ +\ 2 \times .1 \times 350 \times 1125\ +\ 2 \times -0.1 \times 600 \times 1500$$

$$+\ 2x - 0.2 \times 600 \times 1125\ +\ 2 \times .6 \times 1500 \times 1125\ )$$

Required capital = C

(b)

(i) C with no diversification = $\sum C_i^{\,n} =_1 \$3,575M$

So diversification benefit = 3575- C (from (a))

(ii) One issue with this estimate of diversification benefit is that it is based on 10 years of historical correlations. These recent past correlations may not be indicative of future correlations especially if the EPD is weighted towards tail events which are not captured by the historical data.

Also the square root rule is an approximation with an error that depends on whether or not the distribution of losses is normal and the size of the EPD ratio.

12. Continued

(c) The value of the Expected Policyholder Deficit is equivalent to a call option issued by the policyholders to the owners of the company which permits them to pay only the assets of the company in the event that liabilities exceed assets of the company. The equivalencies are illustrated as follows:

Stock Option	**Expected Policyholder Deficit**
S_0 = Current Stock Price	L_0 = Current Liability Value
S_1 = Stock Price in One Year	L_1 = Liability Value in One Year
E1 = Exercise Price	A_1 = Asset Value in One Year
E_0 = Present Value of Exercise Price	A_0 = Current Asset Value
E_0-S_0	C_0= Current Capital Vaule
Max [0_1S_1-E_1] = Option Value When Exercised	Max [0_1L_1-A_1] = Policyholder Deficit in One Year

(d)

The put price must be translated to a call price using put/call parity.
Call Price = Put Price + $S_0 – E_0$

Capital Ratio = $C_0/L_0 = (A_0-L_0)/L_0$ is equivalent to $(E_0-S_0)/S_0$
So C_0/\$500 M – (70× .98 - 50)/50
So C_0 = \$372 M

EPD Ratio = EPD/L_0 is equivalent to Call price/stock price = .1805/50
So EPD = \$1.805 M

So a target EPD of \$1.805 M requires starting capital of \$372 M.

13. Learning Objectives:

2a. Describe the steps necessary to obtain funds for a given project or firm from any specified source, and be able to recommend a specific approach to raising capital in a given situation.

2d. Evaluate alternate options for utilizing capital and recommend the most appropriate use in a given situation.

2e. Apply real options analysis to recommend and evaluate firm decisions on capital utilization

This is an application, analysis and synthesis question asking candidates to a) Apply real options analysis to calculate value of convertible debt, b) understand the workings of convertible debt, c) analyze the capital financing options and recommend the most appropriate use. This is considered core material for this exam, but in (b) most candidates did not calculate the revised levered beta correctly, and in (c) only the strongest candidates scored well.

Sources: (a) Copeland, Weston, Shastri, Financial Theory and Corporate Policy, 4th Edition, Ch. 15 Capital Structure and the Cost of Capital, Pages 569 and 576-578
(b) Chew, D.H. The New Corporate Finance: Where Theory Meets Practice, Ch. 18, Page 282
(c) Doherty, Integrated Risk Management, Ch. 13 Pages 467-474
(d) Megginson, W.L. Corporate Finance Theory, Ch. 9, Pages 399-408

Solution:

(a)

Current cost of debt rd = 5.5%
Rf = risk free interest rate, wd = weighting of debt
Current cost of equity re = rf + beta * market risk premium
= 4% + 1.5*2.5%=7.75%
WACC = wd*rd*(1 – tax rate) + (1 – wd) * re
wd = 900/(900 + 15 * 500) = 10.71%
WACC = 10.71% * 5.5% * (1 – 25%) + (1 – 10.71%)*7.75%
WACC = 7.36%

13. Continued

(b)

Cost of debt = 5%
Beta unlevered = Beta original * S/(S + B * (1 – tax rate))
Beta unlevered = 1.5 * 7,500/ (7,500 + 900 * (1-25%))
Beta unlevered = 1.3761
Beta levered now = Beta unlevered * (S + (1 – tax rate) * B)/S
Beta levered now = 1.3761 * (7,500 + (1-25%) * 2,300)/ 7,500
Beta levered now = 1.6927
Cost of equity re = rf + beta levered now * market risk premium
= 4% + 1.6927 * 2.5% = 8.23%
WACC = wd * rd * (1 – tax rate) + (1 – wd) * re
WACC = 23.47% * 5% * (1-25%) + (1 – 23.47%) * 8.23 = 7.18%

(c)

Value of the firm at time 0 after issuance of convertible debt V(0):

V(0) = \$15 * 500 + \$2,300 million = \$9,800 million

u = exp (40.55%) = 1.5
d = 1/1.5 = 0.6667

% owned if Converted = q = 120/(120 + 500) = .19355

pn = risk neutral probability = (1 + rf – d)/(u – d) = 0.448

Value of the Company after Issuance of the convertible debt	0	1	2
	V(0)	V(1)	V(3)
		V(2)	V(4)
			V(5)

V(1) = \$9,800 * 1.5 = \$14,701 million
V(2) = \$9,800 / 1.5 = \$6,533 million
V(3) = V(1) * 1.5 = \$22,052 million
V(4) = \$9,800 million
V(5) = V(2) / 1.5 = \$4,355 million

Value of Convertible Debts	0	1	2
	CB (0)	CB(1)	CB(3)
		CB(2)	CB(4)
			CB(5)

13. Continued

CB(3) = MAX(V(3) * q, F) = 4,268 million
CB(4) = MAX (V(4) * q, F) = 2,100 million
CB(5) = MAX(V(5) * q, F) = 2,100 million
CB(1) = (pn * CB(3) + (1 – pn) * CB(4))/(1 + rf) = 2,953 million
CB(2) = (pn * CB(4) + (1 – pn) * CB(5))/(1 + rf) = 2,019 million
CB(0) = (pn * CB(1) + (1 – pn) * CB(2))/(1 + rf) = 2,344 million

Market value of convertible bond = 2,344 million

(d)

S = $9,800 - $2,344 = $7,456 million
Beta levered = 1.3761 * (7,456 + (1 – 25%) * 2,344) / 7,456 = 1.7006
WACC = 4% + 1.7006 * 2.5 = 8.25%

(e)

Convertible debt help to align the interests of shareholders and
Bondholder, bondholders can participate on the upside
Convertible debt reduce bankruptcy risk, since less risky projects are selected
Convertible debt reduce the under-investment problem

(f)

Option 1's WACC is lower than the return generated.
Option 2's WACC is higher than the return generated,
So Option 1 should be chosen.

14. Learning Objectives:

1j. Identify sources of agency costs and methods to address them.

This question deals with the principal-agent problem and analyzing a specific example.

Solution:

$$\frac{D}{A} = .05 \rightarrow low$$

(a)

Principal –Agent Problem

- Interests don't always align between owners (shareholders) and managers
- Different time horizons
- Different risk tolerances
- Effort given by the agency(manager)
- Need to align manager incentives with shareholders through compensation tied to performance, but don't want entire compensation based on company results since some is outside manager's control
- Agent can act selfishly for other stakeholders

(b)

Strengths

- Has both fixed and variable component
- Variable component based on stock helps align with shareholders' number of options based on earnings, also in shareholders' interest

Weaknesses

- Variable portion isn't risk adjusted
- Strike price for call options should be indexed up by a target rate of return
- Variable part is also based on reported earnings which can be subject to manipulation
- Only one year calls (short horizon)

(c)

- Maturity on call options should be longer than 1 year ties with long term incentive of shareholders
- Number of call options shouldn't be based on reported earnings
- Payoff on call options should happen when stock returns beat a target rate of return (cost of capital)
    - E.g. strike price increases each year at the cost of capital
- Should account for riskiness of manager's actions
- Increasing debt amount would put more pressure on CEO to perform since chance of bankruptcy is increased, also increases monitoring by creditors

15. Learning Objectives:

2a. Describe the steps necessary to obtain funds for a given project or firm from any specified source, and be able to recommend a specific approach for a given set of circumstances.

This question builds on the candidate's understanding of the split of a firm's value into old debt, new debt, and equity. It then asks the candidate to consider other ways to raise funds for a project.

The question was worded as new "equity", though it would have been more natural to word it is new "debt", which would be non-convertible into equity. The exam committee made adjustments to the grading outline for this question as well as reflecting this in setting the exam pass mark. Candidates are reminded that the cover of the exam books state "The answer should be confined to the question as set." Candidates should do the best they can with the questions as written; if you write nothing we can give you no points.

Solution:

(a)

Current Expected Future Earnings
= (0.5)(150) + (0.2)(200) + (0.3)(400)
= 235

Therefore, Tiny giant could pursue either option 1 (200 < 235) or option 2 (230 < 235).

(b)

Option 1

	Earnings			Debt			Bankruptcy	Equity
Pr	Old	New	Tot	Old	New	Tot		
0.25	150	170	320	200	20	220	100	0
0.10	200	170	370	200	70	270	100	0
0.15	400	170	570	200	180	380	100	90
0.25	150	275	425	200	180	380	0	45
0.10	200	275	475	200	180	380	0	95
0.15	400	275	675	200	180	380	0	295
								78.5

15. Continued

Option 2

	Earnings			Debt			Bankruptcy	Equity
Pr	Old	New	Tot	Old	New	Tot		
0.25	150	50	200	100	0	100	100	0
0.10	200	50	250	150	0	150	100	0
0.15	400	50	450	200	180	380	100	0
0.15	150	256	406	200	180	380	0	26
0.09	200	256	456	200	180	380	0	76
0.06	400	256	656	200	180	380	0	276
0.10	150	371	521	200	180	380	0	141
0.06	200	371	571	200	180	380	0	191
0.04	400	371	771	200	180	380	0	391
								78.5

It has been shown that the overall equity is the same under both Option 1 and Option 2.

16. Learning Objectives:

4g. Describe how option pricing models can be modified or alternative techniques that can be used to deal with option pricing techniques' limitations.

The candidate is asked to discuss the impact of volatility skew and extreme market volatility on a VA hedging program.

Solution:

(a) Identify the type of volatility smile that exists for equity options, and provide reasons why it exists.

- Equity options generally display a volatility skew (i.e. the implied volatility is higher for low strike prices

- The Skew corresponds to an implied probability distribution that has a heavier left tail than the lognormal distribution
- The skew exists because investors expect a market decrease more than a market increase
    - Leverage: equities are riskier as firm value decreases and leverage increases
    - Crashphobia: investors are afraid of a market crash

(b) Describe the impact of the volatility smile on the number of put options needed to hedge your equity market risk.

- The delta of a put option is given by $\frac{\partial p_{BS}}{\partial S} + \frac{\partial p_{BS}}{\partial \sigma_{imp}} \frac{\partial \sigma_{imp}}{\partial S}$

16. Continued

- Given a volatility skew, volatility is a decreasing function of K/S. Hence, implied volatility increases as asset prices increases, or $\frac{\partial \sigma_{imp}}{\partial S} > 0$
- The delta of a put option (given a volatility skew), is higher than the delta given by the Black-Scholes assumption (i.e. lognormal distribution)
- To delta hedge the VA equity exposure, you would require less put options relative to the amount of options predicted by Black-Scholes assumptions

(c) Discuss the impact of the higher volatility on:

(i) Your current hedging program:

- Higher implied volatility will increase option prices
- For hedges already purchased, their market value will increase, but there is no additional cost
- Options that need to be purchased will be more expensive, and increase the cost of the hedging program

(ii) A dynamic hedging program:

- As a dynamic hedging program rebalances to remain neutral Greeks, increased market volatility will result in increased transaction costs
- Options used in a dynamic hedging program would be more expensive
- A delta-only hedge program would not be very effective during extreme market volatility
- A program that hedges more Greeks (e.g. gamma & vega) and has stricter rebalancing limits would be more effective during extreme market volatility

(d) Critique your company's current approach for estimating volatility and recommend a new method.

- The current method to estimate volatility is the sample standard deviation over 6 months. This method has several weaknesses:
  - All data is equally weighted, even though more recent data may be more relevant.
  - Does not incorporate reversion to a long-term mean.
  - The sampling period is only 6 months long. A longer sampling period may provide more stable data.

16. Continued

- The following methods are valid recommendations:
  - GARCH (1,1): $\sigma_n^2 = \gamma V_{1.} + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2$:
    - Model gives greater weight to more recent data
    - Model incorporates mean reversion (V_L is the long-term variance)
    - Extend sampling period to 1 year or longer
  - Market implied volatility/VIX for estimating volatility:
    - Market-implied volatility of equity options may be solved for given option prices observed on the market

17. Learning Objectives:

5d. Define principal-agency theory and explain how it affects capital structure, portfolio management and risk management.

Source: Copeland, Weston, Shastri, Financial Theory and Corporate Policy, 4th Edition, Chapter 12 (Information Asymmetry and Agency Theory), pages 456-460

This question highlights the different motivations and concerns of shareholders and bondholders.

Solution:

(a) Dividend Payout – unexpected changes in dividend payouts can reduce asset base or result in wealth transfer to shareholders.

Claim Dilution – new debt can be issued senior to existing debt, thereby increasing riskiness of existing debt.

Asset Substitution – management will shift into riskier projects in order to increase firm risk. Shareholders can participate in gains on the upside but have limited loses on the downside.

Underinvestment – firm will not invest in profitable project if benefits disproportionately accrue to bondholders.

(b) CFO's Plan

- Main conflict is dividend payout. After payout of $10m, debt is no longer covered in the adverse scenario.

CEO's Plan

- Issue #1: Claim dilution. Existing debt will become subordinate.
- Issue #2: Asset substitution. Taking on riskier project will increase existing debt's riskiness. Original bondholders do not lose under top two scenarios. In adverse scenario, assets to pay bondholders = 100 + 10 – 50 = 60. Therefore, expected loss = $0.70 \times (80 - 60)$ = $14mil.

17. Continued

(c) Payoff to original debtholders under the three plans:

	Current Plan	CFO Plan	CEO Plan
Scenario 1	200+10>80=>80	200>80=>80	350+10 50>80=>80
Scenario 2	120+10>80=>80	120>80=>80	170=10-50>80=>80
Scenario 3	70+10cash=80=>80	70<80=>70(default)	100+10-50<80=>60(default)

Expected bond payoffs:

Current Plan = \$80
CFO Plan = 0.05*80 + 0.25*80 + 0.7*70 = 73, or an expected loss of \$7
CEO Plan = 0.05*80 + 0.25*80 + 0.7*60 = 66, or an expected loss of \$14

Therefore, the plan of maximum benefit to bondholders is the current plan (original investment), i.e. risk-free debt.

(d) Payoff to shareholders under the three plans:

	Current Plan	CFO Plan	CEO Plan
Scenario 1	200+10-80=130	200-80=120	350+10-80-50=230
Scenario 2	120+10-80=50	120-80=40	170+10-80-50=50
Scenario 3	70+10-80=0	70-80<0 =>0 (default)	100+10-50-50<0=>0 (default)

Expected shareholder payoffs:

Current Plan = 0.05*130 + 0.25*50 + 0.7*0 = \$19 (at time 1)
CFO Plan = 0.05*120 + 0.25*40 + 0.7*0 = \$16 (at time 1). With the \$10 dividend at time 0, the CFO Plan pays a total of \$26.
CEO Plan = 0.05*230 +0.25*50 + 0.7*0 = \$24 (at time 1)

The best plan for shareholders is the CFO Plan.

18. Learning Objectives:

2h. Describe how behavioral characteristics and biases of users and providers of capital can affect the capital structure.

This is a recall and application question asking candidates to describe actions that a firm can take to increase the stock price, and then determine which option is the best for management to take.

Solution:

(a) Issue Debt: Managers would not increase debt if they thought it would be hard to pay back. Signals strong future prospects (generating taxable earnings to cover debt service costs) and will have positive impact on stock price.

Increase Dividends: Managers would not increase dividends if they believed the risk was high that they might have to decrease them in the future; signals that cashflows are higher than expected. As long as the firm is able to increase the dividend without cutting back on investment, then the increased dividend is a good thing.

Share Repurchase: Market views share repurchase the same as a dividend increase.

Manage Earnings: Increase current period earnings at the expense of future earnings.

(b) Issue Debt: Neutral to the rational investor since dividends and debt service costs are both tax deductible.

Increase Dividends: Contradicts assumption of rational investor – not tax beneficial versus share repurchase; taxed at a higher rate than capital gains.

Share Repurchase: Acceptable to the rational investor since taxed more like capital gains.

Manage Earnings: Contradicts assumption of the rational investor. The rational investor would assume that markets are efficient and management would not manipulate earnings because the markets would not react.

18. Continued

(c) The best option for management to take is to announce a dividend increase and use the additional cashflow to fund the dividend. This will create the short term share price increase the firm is looking for.

If the company is not successful in increasing the stock price to the desired level, it would defer taking on the large project until the stock price has recovered to a reasonable level.

19. Learning Objectives:

5c. Explain information asymmetry and how it can affect financial markets, especially insurance markets.

This is a recall and application question asking candidates to explain topics related to information asymmetry applications in game theoretic methods. The expected answer was based on the content of Information Asymmetry in Chapter 12 of Copeland's textbook, FET-157-08: Finance Applications of Game Theory by Allen and Morris, and FET-156-08: An introduction to Applicable Game Theory by Gibbons. Credit was also given for answers that express the understanding of different kinds of equilibriums in game theory, and the application of the model from Ross [1977] to a simple example of an insurance contract.

Solution:

(a) (i) Nash equilibrium
- Each player's predicted strategy is that player's best response to the predicted strategy of the other player.
- No single player wants to derivate from her strategies.

(ii) Separating equilibrium
- Gain to the low quality firms from mimicking the actions of a high-quality firm is lower than the cost of the signaling falsely.

(iii) Pooling equilibrium
- Gain to low-quality firms from mimicking the actions of a high-quality firm is higher than the cost of the signaling falsely.

(b) For non-smokers, the expected payout is
= face amount x 0.1% - 1300 if applied as a non-smoker.
= -300
= face amount x 0.1% - 2600 if applied as a smoker
= -1600

Since -300> -1600, non-smokers will always tell the truth.

19. Continued

(c) For smokers, the expected payout is
= face amount x P% x 0.2% - 1300 if applied as a non-smoker
= 2000 x P% - 1300
= face amount x 0.2% - 2600 if applied as a smoker honestly
=2000 – 2600 = -600

In order for smokers to tell the truth, -600 > 2000 x P% - 1300
= 2000 x P% < 700
P% < 35%

Death benefit should be lower than 35% of Face amount if the smoker lied.