SOCIETY OF ACTUARIES
Exam FETE
Financial Economic Theory and Engineering Exam (Finance/ERM/Investment)

Exam FETE

MORNING SESSION

Date: Thursday, October 29, 2009
Time: 8:30 a.m. – 11:45 a.m.

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has a total of 120 points. It consists of a morning session (worth 60 points) and an afternoon session (worth 60 points).
   a) The morning session consists of 9 questions numbered 1 through 9.
   b) The afternoon session consists of 10 questions numbered 10 through 19.

   The points for each question are indicated at the beginning of the question.
2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.
3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.
2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.
3. The answer should be confined to the question as set.
4. When you are asked to calculate, show all your work including any applicable formulas.
5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets since they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate morning or afternoon session for Exam FETE.
6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d’examen pour la version française.

Printed in the U.S.A.
Exam FETE-Front Cover

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TABLE FOR $N(X)$ WHEN $X \geq 0$

The table below shows values of $N(x)$ for $x \geq 0$. This table should be used with interpolation. For example,

$$\begin{aligned} N(0.6278) &= N(0.62) + 0.78[N(0.63) - N(0.62)] \\ &= 0.7324 + 0.78 \times (0.7357 - 0.7324) \\ &= 0.7350 \end{aligned}$$

x	***0.00***	***0.01***	***0.02***	***0.03***	***0.04***	***0.05***	***0.06***	***0.07***	***0.08***	***0.09***
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9986	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE FOR $N(X)$ WHEN $X \leq 0$

The table below shows values of $N(x)$ for $x \leq 0$. This table should be used with interpolation. For example,

$$\begin{aligned} N(-0.1234) &= N(-0.12) - 0.34[N(-0.12) - N(-0.13)] \\ &= 0.4522 - 0.34 \times (0.4522 - 0.4483) \\ &= 0.4509 \end{aligned}$$

x	*0.00*	*0.01*	*0.02*	*0.03*	*0.04*	*0.05*	*0.06*	*0.07*	*0.08*	*0.09*
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−3.0	0.0014	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
−3.6	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
−3.7	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
−3.8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
−3.9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
−4.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Financial Economic Theory and Engineering Formulae Sheet
May 2008

Morning and afternoon exam booklets will include a formula package identical to the one attached to this study note. The exam committee felt that by providing many key formulas, candidates would be able to focus more of their exam preparation time on the application of the formulas and concepts to demonstrate their understanding of the syllabus material and less time on the memorization of the formulas. The formula package was developed sequentially by reviewing the syllabus material for each major syllabus topic. Candidates should be able to follow the flow of the formula package easily. We recommend that candidates use the formula package concurrently with the syllabus material. Not every formula in the syllabus is in the formula package. **Candidates are responsible for all formulas on the syllabus, including those not on the formula sheet.**

Candidates should carefully observe the sometimes-subtle differences in formulas and their application to slightly different situations. For example, there are several versions of the Black-Scholes-Merton option pricing formula to differentiate between instruments paying dividends, tied to an index, etc. Candidates will be expected to recognize the correct formula to apply in a specific situation of an exam question.

Candidates will note that the formula package does not indicate where the formula occurs in the syllabus, nor does it provide names or definitions of the formula or symbols used in the formula. With the wide variety of references and authors of the syllabus, candidates should recognize that the letter conventions and use of symbols may vary from one part of the syllabus to another and thus from one formula to another.

We trust that you will find the inclusion of the formula package to be a valuable study aide that will allow for more of your preparation time to be spent on mastering the learning objectives and learning outcomes.

Copeland, Weston, Shastri, Financial Theory and Corporate Policy

$$S_0 = \sum_{t=1}^{\infty} \frac{Div_t}{(1+k_s)^t}$$

$$\mathrm{Re}v_t + m_t S_t = Div_t + (W\,\&\,S)_t + I_t$$

$$Div_t = \mathrm{Re}v_t - (W\,\&\,S)_t - I_t$$

$$S_0 = \sum_{t=1}^{\infty} \frac{\mathrm{Re}v_t - (W\,\&\,S)_t - I_t}{(1+k_s)^t}$$

$$NI_t = \mathrm{Re}v_t - (W\,\&\,S)_t - dep_t$$

$$\Delta A_t = I_t - dep_t$$

$$S_0 = \sum_{t=1}^{\infty} \frac{\mathrm{Re}v_t - (W\,\&\,S)_t - dep_t - (I_t - dep_t)}{(1+k_s)^t} \quad = \sum_{t=1}^{\infty} \frac{NI_t - \Delta A_t}{(1+k_s)^t}$$

$$NPV = \sum_{t=1}^{N} \frac{FCF_t}{(1+k)^t} - I_0$$

$$NPV = 0 = \sum_{t=1}^{N} \frac{FCF_t}{(1+IRR)^t} - I_0$$

$$k = WACC = weighted\ average\cos t\ of\ capital\ = k_b(1-\tau_c)\frac{B}{B+S} + k_s\frac{S}{B+S}$$

$$\begin{aligned} FCF_{for\ cap.budgeting} &= (\Delta\mathrm{Re}v - \Delta VC - \Delta FCC) - \tau_c(\Delta\mathrm{Re}v - \Delta VC - \Delta FCC - \Delta dep) - \Delta I \\ &= (\Delta\mathrm{Re}v - \Delta VC - \Delta FCC)(1-\tau_c) + \tau_c(\Delta dep) - \Delta I\ = (\Delta\mathrm{Re}v - \Delta VC - \Delta FCC - \Delta dep)(1-\tau_c) + \Delta dep - \Delta I \\ &= EBIT(1-\tau_c) + \Delta dep - \Delta I \qquad \text{earning before interest and taxes} \end{aligned}$$

$$\gamma_g = \left[\prod (1+\gamma_{pt})\right]^{1/N} - 1 \qquad \text{geometric returns}$$

$$\gamma_a = \frac{1}{N}\left[\sum (1+\gamma_{pt})\right] - 1 \qquad \text{arithmetic returns}$$

$$E(R_j) = R_f + \left[E(R_m) - R_f\right]\beta_j$$

$$R_{jt} = E(R_{jt}) + \beta_j \delta_{mt} + \varepsilon_{jt}$$

$$R_{jt} - R_{ft} = (R_{mt} - R_{ft})\beta_j + \varepsilon_{jt}$$

$$R'_{pt} = \gamma_0 + \gamma_1\beta_p + \varepsilon_{pt} \qquad \text{where } \gamma_1 = R_{mt} - R_{ft} \quad R'_{pt} = \text{excess return on portfolio } p = (R_{pt} - R_{ft})$$

$$R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it}$$

$$E(R_i) = E(R_z) + \left[E(R_m) - E(R_z)\right]\beta_i$$

$$\alpha_i = E(R_z)(1-\beta_i)$$

$$E(R_i) - R_f = b_i\left[E(R_m) - R_f\right] + s_i E(SMB) + h_i E(HML)$$

$$\lambda_i = E(R_m) - E(R_z)$$

$$R_p = \left[\prod(1+r_{pt})\right]^{1/T} - 1$$

$$A(R_t) = A(D_t / P_{t-1}) + A(GP_t)$$

$$R_{it} = \hat{\gamma}_{0t} + \hat{\gamma}_{1t}\beta_{it} + \varepsilon_{it}$$

$$R_{it} = \gamma_{0t} + \gamma_{1t}\ln(size)_t + \gamma_{2t}(Book / Market)_i + \varepsilon_{it}$$

$$E(R_i) = E(R_z) + \left[E(R_m) - E(R_z)\right]\beta_i$$

$$E(R_i) = E(R_{z,I}) + \left[E(R_I) - E(R_{z,I})\right]\beta_{i,I}$$

$$\tilde{R}_i = E(\tilde{R}_i) + b_{i1}\tilde{F}_1 + \ldots. + b_{ik}\tilde{F}_k + \tilde{\varepsilon}_i$$

where $\tilde{R}_i =$ random rate of return on the ith asset

$E(\tilde{R}_i) =$ the expected rate of return on the ith asset

$b_{ik} =$ the sensitivity of the ith asset's returns to the kth factor

$\tilde{F}_k =$ the mean zero kth factor common to the returns of all assets

$\tilde{\varepsilon}_i =$ a random zero mean noise term for the ith asset

$$\sum_{i=1}^{n} w_i = 0$$

$$\tilde{R}_p = \sum_{i=1}^{n} w_i \tilde{R}_i = \sum_i w_i E(\tilde{R}_i) + \sum_i w_i b_{i1}\tilde{F}_1 + \ldots + \sum_i w_i b_{ik}\tilde{F}_k + \sum_i w_i \tilde{\varepsilon}_i$$

$w_i = 1/n$ n chosen to be a large number

$\sum_i w_i b_{ik} = 0$ for each factor k

$$\tilde{R}_p = \sum_i w_i E(\tilde{R}_i) + \sum_i w_i b_{i1}\tilde{F}_1 + \ldots + \sum_i w_i b_{ik}\tilde{F}_k$$

$$R_p = \sum_i w_i E(\tilde{R}_i)$$

$$R_p = \sum_i w_i E(\tilde{R}_i) = 0$$

$$E(\tilde{R}_i) = \lambda_0 + \lambda_1 b_{i1} + \ldots + \lambda_k b_{ik}$$

$$E(R_i) - R_f = \lambda_1 b_{i1} + \ldots + \lambda_k b_{ik}$$

$$E(R_i) - R_f = \left[\bar{\delta}_1 - R_f\right] b_{i1} + \ldots + \left[\bar{\delta}_k - R_f\right] b_{ik}$$

$$b_{ik} = Cov(R_i, \delta_k) \big/ Var(\delta_k)$$

where $Cov(R_i, \delta_k) =$ the covariance between the ith asset's returns and the linear transformation of the kth factor

$Var(\delta_k) =$ the variance of the linear transformation of the kth factor

$$C(S_A, S_B, T) = S_A N(d_1) - S_B N(d_2) \qquad \text{where } d_1 = \left[\ln(S_A / S_B) + V^2 T\right] \big/ V\sqrt{T} \qquad d_2 = d_1 - V\sqrt{T}$$

$$V^2 = V_A^2 - 2\rho_{AB} V_A V_B + V_B^2$$

$$Q_s = Q_d - MAX\left[0, Q_d - capacity\right]$$

$$V(\eta) \equiv \sum_m q(m) \underset{a}{MAX} \sum_e p(e|m) U(a,e) - V(\eta_0)$$

$$f_m(p_{1t}, \ldots p_{nt} | \eta_{t-1}^m) = f(p_{1t}, \ldots p_{nt} | \eta_{t-1})$$

$$V(\eta_i) - V(\eta_0) \equiv 0$$

$$p(r - c_2) + (1 - p)(dr - c_2) = p(r/d - c_1) + (1 - p)(r - c_1)$$

$$p = \frac{r(1-d) + c_2 - c_1}{2r - rd - r/d}$$

$r(d-1) > c_2 - c_1$ and $r(1 - 1/d) < c_2 - c_1$

fair game $\varepsilon_{j,t+1} = \dfrac{p_{j,t+1} - p_{jt}}{p_{jt}} - \dfrac{E(p_{j,t+1}|\eta_t) - p_{jt}}{p_{jt}} = 0 = \dfrac{p_{j,t+1} - E(p_{j,t+1}|\eta_t)}{p_{jt}} = 0$

where $p_{j,t+1} =$ the actual price of security j next period

$E(p_{j,t+1}|\eta_t) =$ the predicted end-of-period price of security j given the current information structure η_t

$\varepsilon_{j,t+1} =$ the difference between actual and predicted returns

$$E(\varepsilon_{j,t+1}) = E\left[r_{j,t+1} - E(r_{j,t+1}\left|\eta_t)\right.\right] = 0$$

submartingale $\dfrac{E(p_{j,t+1}\left|\eta_t) - p_{jt}\right.}{p_{jt}} = E(r_{j,t+1}\left|\eta_t) > 0\right.$

martingale $\dfrac{E(p_{j,t+1}\left|\eta_t) - p_{jt}\right.}{p_{jt}} = E(r_{j,t+1}\left|\eta_t) = 0\right.$

random walk $f(r_{1,t+1},...,r_{n,t+1}) = f(r_{1,t+1},...,r_{n,t+1}\left|\eta_t)\right.$

$$\varepsilon_{j,t+1} = r_{j,t+1} - E(r_{j,t+1}\left|r_{jt}, r_{j,t-1},...,r_{j,t-n})\right.$$

$$E\left[(r_{j,t+1} - E(r_{j,t+1}))(r_{jt} - E(r_{jt}))\right] = Cov(r_{j,t+1}, r_{jt}) = \int_{r_{jt}}\left[r_{jt} - E(r_{jt})\right]\left[r_{j,t+1} - E(r_{j,t+1})\right] f(r_{jt})dr_{jt}$$

$$E(R_{jt}\left|\hat{\beta}_{jt}) = R_{ft} + \left[E(R_{mt}\left|\hat{\beta}_{mt}) - R_{ft}\right.\right]\hat{\beta}_{jt}\right.$$

$E(\varepsilon_{jt}) = 0$ where $\varepsilon_{jt} = R_{jt} - E(R_{jt}\left|\hat{\beta}_{jt})\right.$

$E(R_{jt}\left|\hat{\beta}_{jt}) =\right.$ the expected rate of return on the jth asset during this time period, given a prediction of its systematic risk, $\hat{\beta}_{jt}$

$R_{ft} =$ the risk-free rate of return during this time period

$E(R_{mt}\left|\hat{\beta}_{mt}) =\right.$ the expected market rate of return, given a prediction of its systematic risk, $\hat{\beta}_{mt}$

$\hat{\beta}_{mt} =$ the estimated systematic risk of the jth security based on last time period's information structure η_{t-1}

$$R_{jt} = a_j + b_j R_{mt} + \varepsilon_{jt}$$

$$R_{jt} = \alpha_j + \beta_{1j}(R_{mt} - R_{ft}) + \beta_{2j}(RLE_t - RSE_t) + \beta_{3j}(HBTM_t - LBTM_t) + \varepsilon_{jt}$$

the change in earnings per share for the jth firm $\triangle NI_{jt} = \hat{a} + \hat{b}_j \triangle m_t + \varepsilon_{jt}$

where $\triangle m_t =$ the change in the average EPS for all firms (other than firm j) in the market

$$\triangle \hat{N}J_{j,t+1} = \hat{a} + \hat{b}_j \triangle m_{t+1}$$

where $\hat{a}, \hat{b} =$ coefficients estimated from time-series fits of

$R_{jt} = \alpha_j + \beta_{1j}(R_{mt} - R_{ft}) + \beta_{2j}(RLE_t - RSE_t) + \beta_{3j}(HBTM_t - LBTM_t) + \varepsilon_{jt}$

$\triangle m_{t+1} =$ the actual change in market average EPS during the (t+1)th time period

abnormal performance index $API = \dfrac{1}{N}\sum_{j=1}^{N}\prod_{t=1}^{T}(1 + \varepsilon_{jt})$

where $N =$ the number of companies in a portfolio $T = 1, 2, ..., 12$

$\varepsilon_{ij} =$ abnormal performance measured by deviations from the market model

$$V(\alpha) = \frac{1}{(1+r)}\left[\mu(\alpha) - \lambda\right]$$

where $r =$ the risk-free rate of return

$\mu(\alpha) =$ the valuation schedule used by the market to infer the expected end-of-period value from the signal α

$\lambda =$ the market's adjustment for the risk of the project

$$\max imize\, E\left[U(\tilde{W}_1)\right]$$

subject to $W_0 = X + \beta V_M + Y - (1-\alpha)V(\alpha)$

where $W_0 =$ the entrepreneur's initial wealth $\quad V_M =$ the value of the market portfolio

$\beta =$ the fraction of the market portfolio owned by the entrepreneur

$Y =$ the amount invested in the risk-free asset $\quad \alpha =$ the fraction of the project the entrepreneur retains

$\hat{W}_1 =$ the uncertain end of period wealth of the entrepreneur

$$\hat{W}_1 = \alpha(\mu + \hat{\varepsilon}) + \beta\hat{M} + (1+r)Y$$
$$= \alpha\left[\mu + \hat{\varepsilon} - \mu(\alpha) + \lambda\right] + \beta\left[\hat{M} - (1+r)V_M\right] + (1+r)(W_0 - X) + \mu(\alpha) - \lambda$$

where $\hat{M} =$ the gross return of the market portfolio

$$\frac{\partial E(U(\tilde{W}_1))}{\partial\alpha} = E\left[U'(\tilde{W}_1)(\mu + \tilde{\varepsilon} - \mu(\alpha) + \lambda + (1-\alpha)\mu_\alpha)\right] = 0$$

$$\frac{\partial E(U(\tilde{W}_1))}{\partial\beta} = E\left[U'(\tilde{W}_1)(\tilde{M} - (1+r)V_M)\right] = 0 \qquad \text{where } \mu_\alpha = \frac{\partial\mu}{\partial\alpha}$$

$$(1-\alpha)\mu_\alpha = -\frac{E\left[U'(\tilde{W}_1)(\tilde{\varepsilon} + \lambda)\right]}{E\left[U'(\tilde{W}_1)\right]}$$

$$E(D) = \frac{1}{1+r}\left[V(D) + (1-\tau_p)D + \int_D^{\bar{X}}(X - D)f(X)dX + \int_{\underline{X}}^{D}(1+\beta)(X-D)f(X)dX\right]$$

$$= \frac{1}{1+r}\left[V(D) + \mu - \tau_p D - \beta\int_{\underline{X}}^{D}(X-D)f(X)dX\right]$$

$$E(D) = \frac{1}{1+r}\left[V(D) + \frac{t}{2} - \tau_p D - \beta\frac{D^2}{2t}\right]$$

$$V'(D^*) = \tau_p + \beta\frac{D^*}{t}$$

$$V\left[D^*(t)\right] = \frac{1}{r}\left[\frac{t}{2} - \tau_p D^*(t) - \beta\frac{\left[D^*(t)\right]^2}{2t}\right]$$

$$D^*(t) = At$$

$$V\left[D^*(t)\right] = (\tau_p + \beta A)D^*(t)$$

$$A = -\left[\frac{\tau_p}{\beta}\right]\left[\frac{1+r}{1+2r}\right] + \left[\frac{\tau_p}{\beta}\right]\left[\frac{1+r}{1+2r}\right]\sqrt{1+\frac{\beta(1+2r)}{\tau_p^2(1+r)^2}}$$

$$I + D = C + Np_e = C + P_e$$

$$\max_D imize\left[(1-\tau_p)D + p_e M + \frac{Q-M}{Q+N}X\right]$$

$$\max_D imize\, L - \tau_p D + \left[\frac{P+\tau_p D - L}{P+\tau_p D + I - C}\right]X$$

$$\tau_p = (\tau_p + \frac{\partial P}{\partial D})\frac{L+I-C}{(P+\tau_p D + I - C)}X$$

$$D(X) = \frac{1}{\tau_p}\max(I - C + L, 0)\ln X$$

$$P\left[D(x)\right] = C + X - I - \tau_p D(x)$$

$$\frac{\partial P}{\partial D} = \tau_p \frac{P\left[D(x)\right] + \tau_p D(x) - L}{I - C + L}$$

$$V_0^{old} = S + a$$

$V_0^{old} = \frac{P'}{P'+E}(E+S+a+b)$ where P' = market price of old share

$$\frac{P'}{P'+E}(E+S+a+b) \geq S + a$$

$$\frac{P'}{P'+E}(E+b) \geq \frac{E}{P'+E}(S+a) \qquad (E+b) \geq \frac{E}{P'}(S+a)$$

$$P' = S + a + E(\tilde{B}\,|\,issue\,and\,invest)$$

$$(E+b) < \frac{E}{P'}(S+a)\,or\,a > P'(1+\frac{b}{E}) - S$$

manager’s wage before stock split $\quad W^0(z) = \alpha\overline{P}(z) + \beta P - T(m,P)$

manager’s wage with stock split announcement $W^{s}(n,z)=\alpha\hat{P}(n,z)+\beta P-T(n,p)$

$$W_n=\alpha\hat{P}_n-\frac{t_2}{P^{\gamma-1}}=0 \quad n^{*} \text{ maximize wage}$$

$$\hat{P}(n,z)=P\Rightarrow\alpha\hat{P}_nP^{\gamma-1}=t_2$$

$$\hat{P}(n,z)=k\left[n+c(z)\right]^{1/\gamma} \text{ where } k=\left({t_2\gamma}/{\alpha}\right)^{1/\gamma}$$

market value of the firm after the split announcement

$$M(n,z)=\hat{P}(n,z)-T(n,P)=k(1-t_1)\left[n+c(z)\right]^{1/\gamma}-t_2nk^{1-\gamma}\left[n+c(z)\right]^{{1-\gamma}/{\gamma}}$$

$$M(n,z)=\left[k(1-t_1)-t_2k^{1-\gamma}\right]n^{1/\gamma}$$

$$\mu=\frac{M(n,z)}{\bar{M}(z)}=\left[k(1-t_1)-t_2k^{1-\gamma}\right]n^{1/\gamma}\Big/\bar{M}(z)=K\left[\frac{n}{\bar{M}(z)}\right]^{1/\gamma}\left[\bar{M}(z)\right]^{\frac{1}{\gamma}-1}$$

where $\bar{M}(z)=$ pre-split value of the firm, $K=\left[k(1-t_1)-t_2k^{1-\gamma}\right]$

$$\ln u=\ln K-\frac{1}{\gamma}\left[{\bar{M}(z)}\big/{n}\right]+\left(\frac{1}{\gamma}-1\right)\ln\left[\bar{M}(z)\right]$$

$$V_0=(1-t)X_0$$

$$E(X\,|n)=\hat{X}(n)=\frac{X_0s_0+\hat{Y}_m(n)s_m}{s_0+s_m}$$

$$V_1(n)=\hat{X}(n)-T(n)-C$$

where $T(n)=$ the expected total brokerage commission $=E\left[Xt\left(\frac{X}{n}\right)\middle|n\right]$

$C=$ the cost of executing the split

$$V_1(n)=\hat{X}(T)-T-C \quad \hat{X}(T)=E(X\,|T)$$

$$N(T)={T}/{F}=FT$$

$$V_2(T,\bar{Y})=\frac{X_0s_0+\hat{Y}_ms_m+\bar{Y}FT_s}{s_0+s_m+FT_s}-E\left[Xt\left({X}/{n}\right)\middle|T,\hat{Y}_m,\bar{Y}\right]-C$$

$$E\left[V_2(T)\middle|Y_m\right]=\frac{X_0s_0+\hat{Y}_ms_m+(\frac{X_0s_0+Y_ms_m}{s_0+s_m})FT_s}{s_0+s_m+FT_s}-T-C$$

$$E\left[\bar{Y}\middle|Y_m\right]=\frac{X_0s_0+Y_ms_m}{s_0+s_m} \quad E\left[(Xt(X/n)\middle|T,\hat{Y}_m,\bar{Y})\middle|Y_m\right]=T$$

$$\hat{Y}_m'(T)=\frac{s_0+s_m+FT_s}{S_m}$$

$$Y_m(T)=\frac{s_0+s_m}{s_m}T+\frac{F_s}{2s_m}T^2+K$$

$$(\frac{d\alpha}{dD})\left[V(n)-B(n,D)\right]-\alpha(D)(\frac{\partial B}{\partial D})=0$$

$$(\frac{d^2\alpha}{dD^2})\left[V(n)-B(n,D)\right]-2(\frac{d\alpha}{dD})(\frac{\partial B}{\partial D})-\alpha(D)(\frac{\partial^2 B}{\partial D^2})<0$$

$$\alpha(D^*(n))\left[V(n)-B(n,D^*(n))\right]-\left[V(n)-I\right]=0$$

$$\left[\frac{d^2\alpha}{dD^2}(V-B)-2\frac{d\alpha}{dD}\frac{\partial B}{\partial D}-\alpha\frac{\partial^2 B}{\partial D^2}\right]\frac{dD}{dn}=\frac{d\alpha}{dD}(\frac{\partial B}{\partial n}-\frac{dV}{dn})+\alpha(\frac{\partial^2 B}{\partial n\partial D})$$

$$\hat{\varepsilon}(\alpha,D)=\left[(a-D_0)^2+D^2\frac{1-\alpha}{\alpha^2}\right]$$

$$\bar{D}=X_1\left[\frac{\bar{\alpha}(\varepsilon-\gamma)}{(1-\bar{\alpha})(1-\varepsilon)\gamma}\right]$$

$$\underset{c(s,p),a}{\max} imize\int\int U\left[s-c(s,p)\right]f(s,p|a)\,dsdp$$

$$\text{subject to }\int\int V\left[c(s,p)\right]f(s,p|a)dsdp-G(a)\geq\underline{V}$$

$$\underset{c(s,p),a}{\max} imize\int\int U\left[s-c(s,p)\right]f(s,p|a)dsdp+\lambda\left[\int\int V\left[c(s,p)\right]f(s,p|a)dsdp-G(a)-\underline{V}\right]$$

$$-U'\left[s-c(s,p)\right]+\lambda V'\left[c(s,p)\right]=0 \quad or \quad \frac{U'\left[s-c(s,p)\right]}{V'\left[c(s,p)\right]}=\lambda$$

$$\underset{a}{\max} imize U(k)+\lambda\left[\int\int V\left[c(s,p)\right]f(s,p|a)\,dsdp-G(a)-\underline{V}\right]$$

$$\underset{c(s),a}{\max} imize\int U\left[s-c(s)\right]f(s|a)ds \qquad \text{subject to }\int V\left[c(s)\right]f(s|a)ds-G(a)\geq\underline{V}$$

$$\int V\left[c(s)\right] f_a(s|a)ds - G'(a) = 0$$

$$\max_{c(s),a} imize \int U\left[s - c(s)\right] f(s|a)ds + \lambda\left[\int V\left[c(s)\right] f(s|a)ds - G(a) - \underline{V}\right]$$
$$+\mu\left[\int V\left[c(s)\right] f_a(s|a)ds - G'(a)\right] -U'\left[s - c(s)\right] f(s|a) + \lambda V'\left[c(s)\right] f(s|a) + \mu V'\left[c(s)\right] f_a(s|a)$$

$$\frac{U'\left[s - c(s)\right]}{V'\left[c(s)\right]} = \lambda + \mu\frac{f_a(s|a)}{f(s|a)}$$

$$(\frac{1}{\delta_1})(\delta_0 + \delta_1 c)^{\gamma} = \lambda + \mu\frac{f_a(s|a)}{f(s|a)}$$

$$c(s) = -\frac{\delta_0}{\delta_1} + (\delta_1)^{(\frac{1}{\gamma}-1)}(\lambda + \mu\frac{f_a(s|a)}{f(s|a)})^{\frac{1}{\gamma}}$$

$$\underset{c(s,p),a}{Maximize} \int\int U\left[s - c(s,p)\right] f(s,p|a)dsdp + \lambda\left[\int\int V\left[c(s,p)\right] f(s,p|a)dsdp - G(a) - \underline{V}\right] +$$
$$\mu\left[\int\int V\left[c(s,p)\right] f_a(s,p|a)dsdp - G'(a)\right]$$

$$\frac{U'\left[s - c(s,p)\right]}{V'\left[c(s,p)\right]} = \lambda + \mu\frac{f_a(s,p|a)}{f(s,p|a)}$$

$$\frac{U'\left[s - c(\mathrm{P})\right]}{V'\left[c(\mathrm{P})\right]} = \lambda + \mu_1\frac{f_{a_1}(p|a)}{f(p|a)} + \mu_2\frac{f_{a_2}(p|a)}{f(p|a)} + .. + \mu_n\frac{f_{a_n}(p|a)}{f(p|a)}$$

$$s = \sum_{j=1}^{m} b_j a_j + \varepsilon_s$$

$$p_j = \sum_{j=1}^{m} q_{ij} a_j + \varepsilon_i \quad for\ i = 1,..,k$$

$$c(p_1,...,p_k) = \beta_0 + \sum_{i=1}^{k} \beta_i \tilde{p}_i$$

$$\underset{c(s,p,m),a(m),m(m)}{\max imize}\ E_{s,p,m}\left[U\left[s - c(s,p,m)\right] \| a(m)\right]$$

subject to (for all m) $E_{s,p|m}\left[\left[V\left[c(s,p,m)\right] - G\left[a(m)\right]\right] \| a(m)\right] \geq \underline{V}$

$a(m) =$ a that maximizes $E_{s,Y|m}\left[V\left[c(s,p,m)\right] \| a\right] - G(a)$ for each m

$m(m) =$ the $\hat{m}(m)$ that maximize $E_{s,Y|m}\left[V\left[c(s,p,\hat{m})\right] \| a\right] - G(a)$ for each m

$$\frac{\partial V(X^*)}{\partial X} = \frac{\partial P(X^*)}{\partial X} - \frac{\partial C(X^*)}{\partial X} = 0$$

$$S = \frac{(1-\beta)E(s) - \alpha - \lambda(1-\beta)Cov(s,R_M)}{1+r_f}$$

$$\max_{\alpha,\beta} imize \frac{(1-\beta)E(s) - \alpha - \lambda(1-\beta)Cov(s,R_M)}{1+r_f}$$

subject to $a\left[E(W)+\alpha+\beta E(s)\right] - b\left[Var(W)+2\beta Cov(W,s)+\beta^2 Var(s)\right] \geq \underline{V}$

$$\max_{\alpha,\beta} imize \frac{(1-\beta)E(s) - \alpha - \lambda(1-\beta)Cov(s,R_M)}{1+r_f} +$$
$$\mu(a\left[E(W)+\alpha+\beta E(s)\right] - b\left[Var(W)+2\beta Cov(W,s)+\beta^2 Var(s)\right] - \underline{V})$$

$$\mu a - \frac{1}{1+r_f} = 0$$

$$\mu\left[aE(s) - 2b(Cov(W,s)+\beta Var(s))\right] - \frac{E(s)-\lambda Cov(s,R_m)}{1+r_f} = 0$$

$$a\left[\alpha+\beta E(s)\right] - b\beta^2 Var(s) - \underline{V} = 0$$

$$\beta = \frac{\lambda a Cov(s,R_M)}{2bVar(s)} - \frac{Cov(W,s)}{Var(s)}$$

$$r^* = \frac{i}{1-(1+i)^{-T}}$$

$$B_{new} = \frac{D}{(1+r_f)^t} - (D)\left[P\left(\frac{V+dB}{D+dD},1,T,r_f,\sigma_V\right)\right]$$

$$B_{new} - B = D\left\{-\left[P\left(\frac{V+dB}{D+dD},1,T,r_f,\sigma_V\right)\right] + P\left(\frac{V}{D},1,T,r_f,\sigma_V\right)\right\} = D(-P_X + P_Y)$$

$$\frac{\partial B}{\partial \sigma_V} = -\frac{\partial P(V,D,T,r_f,\sigma_V)}{\partial \sigma_V}$$

$$V = \int_{s_a}^{\infty} q(s)\left[V(s)-I\right]ds$$

$$V_E = \int_{s_a}^{\infty} q(s)\left[V(s)-I-D\right]ds$$

$$V_D = \int_{s_a}^{s_b} q(s)\left[V(s)-I\right]ds + \int_{s_b}^{\infty} q(s)Dds$$

$$V = \int_{s_b}^{\infty} q(s)\left[V(s) - I\right] ds$$

$$V_U = E(F\tilde{C}F) \Big/ \rho$$

where $V_U =$ the present value of unlevered firm(i.e. all equity)

$E(F\tilde{C}F) =$ the perpetual free cash flow after taxes

$\rho =$ the discount rate of an all-equity firm of equivalent risk

$$V_U = \frac{E(F\tilde{C}F)}{\rho} \ or\ V_U = \frac{E(EB\tilde{I}T)(1-\tau_c)}{\rho}$$

$$\tilde{NI} + k_d D = (\mathrm{Re}\tilde{v} - \tilde{VC} - FCC - dep)(1-\tau_c) + k_d D \tau_c$$

$$V^L = \frac{E(EB\tilde{I}T)(1-\tau_c)}{\rho} + \frac{k_d D \tau_c}{k_b}$$

$$B = \frac{k_d D}{k_b}$$

$$V_L = V_U + \tau_c B$$

$$\frac{\triangle V_L}{\triangle I} = \frac{(1-\tau_c)}{\rho} \frac{\triangle E(EB\tilde{I}T)}{\triangle I} + \tau_c \frac{\triangle B}{\triangle I}$$

$$\triangle V_L = \triangle S^0 + \triangle S^n + \triangle B^0 + \triangle B^n$$

$$\frac{\triangle V_L}{\triangle I} = \frac{\triangle S^0}{\triangle I} + \frac{\triangle S^n}{\triangle I} + \frac{\triangle B^0}{\triangle I} + \frac{\triangle B^n}{\triangle I}$$

$$\triangle I = \triangle S^n + \triangle B^n$$

$$\frac{\triangle V_L}{\triangle I} = \frac{\triangle S^0}{\triangle I} + \frac{\triangle S^n + \triangle B^0}{\triangle I} = \frac{\triangle S^0}{\triangle I} + 1$$

$$\frac{\triangle S^0}{\triangle I} = \frac{\triangle V_L}{\triangle I} - 1 > 0$$

$$\frac{(1-\tau_c)\triangle E(EBIT)}{\triangle I} > \rho(1 - \tau_c \frac{\triangle B}{\triangle I})$$

weighted average cost of capital $WACC = \rho(1 - \tau_c \frac{\triangle B}{\triangle I})$

$$WACC = \rho(1-\tau_c \frac{\triangle B}{\triangle V})$$

$$\frac{\triangle NI}{\triangle I} + \frac{\triangle k_d D}{\triangle I} - \frac{\tau_c \triangle (k_d D)}{\triangle I} = (1-\tau_c)\frac{\triangle EBIT}{\triangle I}$$

$$\frac{\triangle V_L}{\triangle I} = \frac{{}^{\triangle NI}\!\big/_{\triangle I} + (1-\tau_c)\,{}^{\triangle (k_d D)}\!\big/_{\triangle I}}{\rho} + \tau_c \frac{\triangle B}{\triangle I}$$

$$\frac{\triangle V_L}{\triangle I} = \frac{\triangle S^0 + \triangle S^n}{\triangle I} + \frac{\triangle B^n}{\triangle I} \quad \triangle B^0 \equiv 0$$

$$\frac{\triangle NI}{\triangle S^0 + \triangle S^n} = \rho + (1-\tau_c)(\rho - k_b)\frac{\triangle B}{\triangle S^0 + \triangle S^n}$$

$$k_s = \rho + (1-\tau_c)(\rho - k_b)\frac{\triangle B}{\triangle S}$$

$$WACC = (1-\tau_c)k_b \frac{B}{B+S} + k_s \frac{S}{B+S}$$

$$WACC = \rho(1-\tau_c \frac{B}{B+S})$$

$$G = V_L - V_U = \tau_c B$$

$$V_U = \frac{E(EBIT)(1-\tau_c)(1-\tau_{ps})}{\rho}$$

payment to shareholders $(EBIT - k_d D)(1-\tau_c)(1-\tau_{ps})$

payment to bondholders after personal taxes $k_d D(1-\tau_{pB})$

total cash payments to supplies of capital $= EBIT(1-\tau_c)(1-\tau_{ps}) - k_d D(1-\tau_c)(1-\tau_{ps}) + k_d D(1-\tau_{pB})$

$$V_L = \frac{E(EBIT)(1-\tau_c)(1-\tau_{ps})}{\rho} + \frac{k_d D\left[(1-\tau_{pB}) - (1-\tau_c)(1-\tau_{ps})\right]}{k_b} = V_U + \left[1 + \frac{(1-\tau_c)(1-\tau_{ps})}{(1-\tau_{pB})}\right]B$$

where $B = {}^{k_d D(1-\tau_{pB})}\!\big/_{k_b}$

$$G = \left[1 - \frac{(1-\tau_c)(1-\tau_{ps})}{(1-\tau_{pB})}\right]B$$

$$(1-\tau_{pB}) = (1-\tau_c)(1-\tau_{ps})$$

$$G = (1 - \frac{(1-\tau_c)}{(1-\tau_{pB})})B$$

$$E(R_j) = R_f + \left[E(R_m) - R_f \right] \beta_j$$

where $E(R_j) =$ the expected rate of return on asset j $R_f =$ the risk-free rate

$E(R_m) =$ the expected rate of return on the market portfolio

$$\beta_j = Cov(R_j, R_m) \Big/ Var(R_m)$$

$$\beta_L = \left[1 + (1-\tau_c)\frac{B}{S} \right] \beta_U$$

$$E(\tilde{R}_{bj}) = R_f + \left[E(\tilde{R}_m) - R_f \right] \beta_{bj}$$

$$k_s = \frac{(\widetilde{EBIT} - \tilde{R}_{bj}B)(1-\tau_c)}{S^L}$$

$$E(k_s) = R_f + \lambda^* Cov(k_s, R_m)$$

$$Cov(k_s, R_m) = \frac{(1-\tau_c)}{S^L} Cov(EBIT, R_m) - \frac{(1-\tau_c)B}{S^L} Cov(R_{bj}, R_m)$$

$$R_f S^L + \lambda^*(1-\tau_c)Cov(EBIT, R_m) - \lambda^*(1-\tau_c)B\left[Cov(R_{bj}, R_m) \right] = (EBIT)(1-\tau_c) - E(R_{bj})B(1-\tau_c)$$

$$R_f V^U + \lambda^*(1-\tau_c)Cov(EBIT, R_m) = E(EBIT)(1-\tau_c)$$

$$V_L = V_U + \tau_c B$$

$$V = (B - P) + S$$

$$E(r_i) = r_f + \left[E(r_m) - r_f \right] \beta_i$$

where $E(r_i) =$ the instantaneous expected rate of return on asset i

$\beta_i = \dfrac{Cov(r_i, r_m)}{Var(r_m)}$ the instantaneous systematic risk of the ith asset

$E(r_m) =$ the expected instantaneous rate of return on the market portfolio

$r_f =$ the non-stochastic instantaneous annualized rate of return on the risk-free asset

$$dS = \frac{\partial S}{\partial V} dV + \frac{\partial S}{\partial t} dt + \frac{1}{2} \frac{\partial^2 S}{2\partial V^2} \sigma^2 V^2 dt$$

$$\lim_{dt \to 0} \frac{dS}{S} = \frac{\partial S}{\partial V} \frac{dV}{S} = \frac{\partial S}{\partial V} \frac{dV}{V} \frac{V}{S}$$

$$r_S = \frac{\partial S}{\partial V}\frac{V}{S} r_V$$

$$\beta_S = \frac{Cov(r_S, r_m)}{Var(r_m)}, \beta_V = \frac{Cov(r_V, r_m)}{Var(r_m)}$$

$$\beta_S = \frac{\partial S}{\partial V}\frac{V}{S}\frac{Cov(r_V, r_m)}{Var(r_m)} = \frac{\partial S}{\partial V}\frac{V}{S}\beta_V$$

$$S = VN(d_1) - e^{-r_f T} DN(d_2)$$

where $S =$ the market value of equity $V =$ the market value of the firm’s asset $r_f =$ the risk-free rate $T =$ the time to maturity $D =$ the face value of debt (book value)

$$d_1 = \frac{Ln(V/D) + r_f T}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T} \qquad d_2 = d_1 - \sigma\sqrt{T}$$

$$\beta_S = N(d_1)\frac{V}{S}\beta_V$$

$$\beta_S = \frac{VN(d_1)}{VN(d_1) - De^{-r_f T}N(d_2)}\beta_V \quad = \frac{1}{1 - (D/V)e^{-r_f T}\left[N(d_2)/N(d_1)\right]}\beta_V$$

$$k_s = R_f + (R_m - R_f)N(d_1)\frac{V}{S}\beta_V$$

$$k_s = R_f + N(d_1)(R_v - R_f)\frac{V}{S}$$

$$\beta_B = \beta_V \frac{\partial B}{\partial V}\frac{V}{B}$$

$$\frac{\partial B}{\partial V} = N(-d_1) = 1 - N(d_1)$$

$$k_b = R_f + (R_m - R_f)\beta_B$$

$$k_b = R_f + (\rho - R_f)N(d_1)\frac{V}{B}$$

$$k_b\frac{B}{V} + k_s\frac{S}{V} = \left[R_f + (\rho - R_f)N(-d_1)\frac{V}{B}\right]\frac{B}{V} + \left[R_f + N(d_1)(\rho - R_f)\frac{V}{S}\right]\frac{S}{V}$$

$$= R_f\left(\frac{B+S}{V}\right) + (\rho - R_f)\left[N(-d_1) + N(d_1)\right] = R_f + (\rho - R_f)\left[1 - N(d_1) + N(d_1)\right] = \rho$$

$$k_s = \rho + (\rho - k_b)\frac{B}{S}$$

$$\frac{dV}{V} = \mu(V,t)dt + \sigma dW$$

$$\frac{1}{2}\sigma^2 V^2 F_{VV}(V,t) + rVF_V(V,t) - rF(V,t) + F_t(V,t) + C = 0$$

$$\frac{1}{2}\sigma^2 V^2 F_{VV}(V) + rVF_V(V) - rF(V) + C = 0$$

$$F(V) = A_0 + A_1 V + A_2 V^{-(2r/\sigma^2)}$$

$$B(V) = A_0 + A_1 V + A_2 V^{-(2r/\sigma^2)}$$

$$B(V) = C/r + \left[(1-\alpha)V_B - C/r\right](V/V_B)^{-2r/\sigma^2} = (1-p_B)C/r + p_B\left[(1-\alpha)V_B\right] \text{ where } p_B = (V/V_B)^{-2r/\sigma^2}$$

$$DC(V) = \alpha V_B (V/V_B)^{-2r/\sigma^2}$$

$$TB(V) = A_0 + A_1 V + A_2 V^{-2r/\sigma^2}$$

$$TB(V) = 0 = T_c(C/r) - \left[T_c(C/r)\right](V/V_B)^{-2r/\sigma^2}$$

$$V_L(V) = V_U(V) + T_c B(V) - DC(V) = V_U(V) + T_c B - p_B T_c B - \alpha V_B p_B$$

$$M = (1+r)\gamma_0 V_0 + \gamma_1 V_1 \ \ if \ \ V_1 \geq D$$
$$M = (1+r)\gamma_0 V_0 + \gamma_1 (V_1 - c) \ \ if \ \ V_1 < D$$

where $\gamma_0, \gamma_1 =$ positive weights $r =$ the one-period interest rate $V_0, V_1 =$ the current and future value of the firm $D =$ the face value of the debt $c =$ a penalty paid if bankruptcy occurs if $V < D$

$$M_a = \gamma_0 (1+r)\frac{V_{1a}}{1+r} + \gamma_1 V_{1a} \ \ if \ \ D^* < D \leq V_{1a} \ \ \text{(tell the truth)}$$
$$M_a = \gamma_0 (1+r)\frac{V_{1b}}{1+r} + \gamma_1 V_{1a} \ \ if \ \ D < D^* \qquad \text{(lie)}$$

where $D^* =$ the maximum amount of debt that an unsuccessful firm can carry without going bankrupt

$$M_a = \gamma_0 (1+r)\frac{V_{1a}}{1+r} + \gamma_1 (V_{1b} - c) \ \ if \ \ D^* < D \leq V_{1a} \ \ \text{(lie)}$$
$$M_a = \gamma_0 (1+r)\frac{V_{1b}}{1+r} + \gamma_1 V_{1b} \ \ if \ \ D < D^* \qquad \text{(tell the truth)}$$

$$\gamma_0 (V_{1a} - V_{1b}) < \gamma_1 c$$

$$k_u(t+1) = \frac{div_i(t+1) + P_i(t+1) - P_i(t)}{P_i(t)}$$

where $k_u(t+1) =$ the market required rate of return during the time period t

$div_t(t+1) =$ dividends per share paid at the end of time period t

$P_i(t+1) =$ price per share at the end of time period t

$P_i(t) =$ price per share at the beginning of time period t

$$V_i(t) = \frac{Div_i(t+1) + n_i(t)P_i(t+1)}{1 + k_u(t+1)}$$

where $Div_i(t+1) =$ total dollar dividend payment $= n_i(t)div_i(t+1)$

$V_i(t) =$ the market value of the firm $= n_i(t)P_i(t)$

$$\widetilde{EBIT}_i(t+1) + m_i(t+1)\tilde{P}_i(t+1) \equiv \tilde{I}_i(t+1) + \widetilde{Div}_i(t+1)$$

$$\tilde{R}_i(t+1) = \widetilde{Div}_i(t+1) + n_i(t)\tilde{P}_i(t+1)$$

$$\tilde{R}_i(t+1) = \widetilde{Div}_i(t+1) + n_i(t+1)\tilde{P}_i(t+1) - m_i(t+1)\tilde{P}_i(t+1)$$

$$\tilde{R}_i(t+1) = \widetilde{EBIT}_i(t+1) - \tilde{I}_i(t+1) + \tilde{V}_i(t+1)$$

$$\tilde{V}_i(t) = \frac{E\bar{B}IT_i(t+1) - \tilde{I}_i(t+1) + \tilde{V}_i(t+1)}{1 + k_u(t+1)}$$

$$\tilde{Y}_{di} = \left[(\widetilde{EBIT} - rD_c)(1-\tau_c) - rD_{pi} \right](1-\tau_{pi})$$

where $\tilde{Y}_{di} =$ the uncertain income to the ith individual if corporate income is received as dividends

$\widetilde{EBIT} =$ the uncertain cash flows from operations provided by the firm

$r =$ the borrowing rate, which is assumed to be equal for individuals and firm

$D_c =$ the corporate debt $D_{pi} =$ personal debt held by the ith individual

$\tau_c =$ the corporate tax rate $\tau_{pi} =$ the personal income tax rate of the ith individual

$$\tilde{Y}_{gi} = (\widetilde{EBIT} - rD_c)(1-\tau_c)(1-\tau_{gi}) - rD_{pi}(1-\tau_{pi})$$

where $\tilde{Y}_{gi} =$ the uncertain income to the ith individual if corporate income is received as capital gains

$\tau_{gi} =$ the capital gains rate for the ith individual

$$\tilde{Y}_{gi} = \left[(\widetilde{EBIT} - rD_c)(1-\tau_c) - rD_{pi} \right](1-\tau_{gi}) + rD_{pi}(\tau_{pi} - \tau_{gi})$$

$$\tilde{Y}_{gi} / \tilde{Y}_{di} > 1$$

corporate debt $\dfrac{\partial \tilde{Y}_{gi}}{\partial D_c} = -r(1-\tau_c)(1-\tau_{gi})$

personal debt $\dfrac{\partial \tilde{Y}_{gi}}{\partial D_{pi}} = -r(1-\tau_{pi})$

$$R_{jt} - R_{ft} = \delta_0 + \delta_1 \beta_{jt} + \delta_2 \left[(div_{jt} / P_{jt}) - R_{ft} \right] + \tilde{\varepsilon}_{jt}$$

where $\delta_0 =$ a constant $\delta_1 =$ influence of systematic risk on R_{jt}

$\delta_2 =$ influence of dividend payment on R_{jt} $\beta_{jt} =$ the systematic risk of the jth security

$div_{jt} / P_{jt} =$ the dividend yield of the jth security $\tilde{\varepsilon}_{jt} =$ a random error term $R_{ft} =$ the risk-free rate

cost of internal funds $r_A(1-\tau_c)(1-\tau_{di})$

where $r_A =$ the pre-tax return on investments in real assets

$\tau_c =$ corporate effective marginal tax rate $\tau_{di} =$ personal dividend income tax rate of the ith individual

$$EBIT + mP + \triangle B = I + Div$$

$$V_1 = Div_1 + \frac{E(EBIT_2)}{1+k}$$

$$S_1 = V_1 - \triangle B_1 - mP_1 = Div_1 + \frac{E(EBIT_2)}{1+k} - \triangle B_1 - mP_1$$

$$S_1 = EBIT_1 - I_1 + \frac{E(EBIT_2)}{1+k}$$

$$E(S_1) = E_0(EBIT_1) - E_0(I_1) + \frac{E_0\left[f(I_1)\right]}{1+k} = f(I_0) - I_1 + \frac{f(I_1)}{1+k}$$

$$S_1 = EBIT_1 - I_1 + \frac{E_1(EBIT_2)}{1+k} = f(I_0) + \varepsilon_1 - I_1 + \frac{f(I_1) + E_1(\varepsilon_2 | \varepsilon_1)}{1+k} = f(I_0) + \varepsilon_1 - I_1 + \frac{f(I_1) + \gamma\varepsilon_1}{1+k}$$

$$S_1 - E(S_1) = \varepsilon_1 \left[1 + \frac{\gamma}{1+k} \right] = \left[EBIT_1 - E_0(EBIT_1) \right] \left[1 + \frac{\gamma}{1+k} \right]$$

$$\triangle Div_{it} = a_i + c_i(Div_{it}^* - Div_{i,t-1}) + U_{it}$$

where $\triangle Div_{it} =$ the change in dividends

$c_j =$ the speed of adjustment to the difference between a target dividend payment and last year's payout

$Div_{it}^* =$ the target dividend payout $a_i U_{it} =$ a constant and normally distributed random error term

$$P_B - t_g(P_B - P_C) = P_A - t_g(P_A - P_C) + div(1 - t_0)$$

$$\frac{P_B - P_A}{div} = \frac{1 - t_0}{1 - t_g}$$

arbitrage profit $\pi = -P_B + div - t_0 div + P_A + t_0(P_B - P_A)$

$$\pi = (1 - t_0)(P_A - P_B + div)$$

$$DY_i = a_1 + a_2\beta_i + a_3 AGE_i + a_4 INC_i + a_5 DTR_i + \varepsilon_i$$

where $DY_i =$ dividend yield for the ith individual's portfolio

$\beta_i =$ the systematic risk of the ith individual's portfolio $AGE_i =$ the age of the individual

$INC_i =$ the gross family income averaged over the last three years

$DTR_i =$ the difference between the income and capital gain tax rates for the ith individual

$\varepsilon_i =$ a normally distributed random error term

$$\triangle Div_t = \beta_1 Div_{t-1} + \beta_2 NI_t + \beta_3 NI_{t-1} + Z_t$$

where $\triangle Div_t =$ the change in dividends in period t $Div_{t-1} =$ the previous period's dividends

$NI_t =$ this period's earnings $NI_{t-1} =$ last period's earnings

$Z_t =$ unanticipated dividend changes (the error term)

$$R_{jt} = \alpha + \beta_j R_{mt} + \varepsilon_{jt}$$

where $R_{jt} =$ the total return (dividends and capital gains) on the common stock of the jth firm

$\beta_j =$ a constant term $R_{mt} =$ systematic risk $\varepsilon_{jt} =$ the abnormal performance of the jth security

$$P_{it} = a + bDiv_{it} + cRE_{it} + \varepsilon_{it}$$

where $P_{it} =$ the price per share $Div_{it} =$ aggregate dividends paid out

$RE_{it} =$ retained earnings $\varepsilon_{it} =$ the error term

$$\frac{(NI/P)_{it}}{(NI/P)_{kt}} = a_i + b_{it} + \varepsilon_{it}$$

where $(NI/P)_{it} =$ the earnings / price ratio for the firm

$(NI/P)_{kt} =$ the average earnings/price ratio of the industry

$t =$ a time index $\varepsilon_{it} =$ the error term

$$E(\tilde{R}_j) = R_f + \left[E(\tilde{R}_m) - R_f\right]\beta_j$$

$$\tilde{R}_j = \gamma_0 + \left[\tilde{R}_m - \gamma_0\right]\beta_j + \gamma_1 \left[DY_j - DY_m\right] / DY_m + \varepsilon_j$$

where $\tilde{R}_j =$ the rate of return on the jth portfolio

$\gamma_0 =$ an intercept term that should be equal to the risk-free rate R_f according to CAPM

$\tilde{R}_m$ = the rate of return on the market portfolio β_j = the systematic risk of the jth portfolio

γ_1 = the dividend impact coefficient

DY_j = the dividend yield on the jth portfolio, measured as the sum of dividends paid during the previous year divided by the end-of-year price

DY_m = the dividend yield on the market portfolio, measured over the period of 12 months

ε_j = the error term

$$E(\tilde{R}_{jt}) - R_{ft} = a_1 + a_2\beta_j + a_3(DY_{jt} - R_{ft})$$

where $E(\tilde{R}_{jt})$ = the expected before tax return on the jth security

R_{ft} = the before-tax return on the risk-free asset

β_j = the systematic risk of the jth security

a_1 = the constant term a_2 = the marginal effect of systematic risk

a_3 = the marginal effective tax difference between ordinary income and capital gains rates

DY_{jt} = the dividend yield (i.e. dividend divided by price) for the jth security

$$R_{pt} = \lambda_0 + \beta_{1F}\left[MFT + \lambda_1\right] + \beta_{2F}\left[SMB_t + \lambda_2\right] + \beta_{3F}\left[HML_t + \lambda_3\right] + \lambda_4 d_{p,t-1} + \varepsilon_{pt}$$

where MKT = the excess returns on the CRSP value-weighted portfolio

SMB = the difference between average returns on small minus big equity capitalization portfolio

HML = the difference between average return on high minus low book equity to market equity portfolio

$d_{p,t-1}$ = the equally weighted yield of stocks in portfolio p minus the market dividend yield

λ_i = the risk premium corresponding to the ith risk factor

λ_4 = the coefficient on the dividend yield measure

$$P_E N_E = P_0 N_0 - P_T(N_0 - N_E) + \vartriangle W$$

where P_E = the post expiration share price N_E = the number of shares outstanding after repurchase

P_0 = the pre-announcement share price N_0 = the pre-announcement number of shares outstanding

P_T = the tender price $\vartriangle W$ = the shareholder wealth effect attributable to the tender offer

$F_P = 1 - \dfrac{N_E}{N_0}$ fraction of shares repurchased

$$\frac{\vartriangle W}{N_0 P_0} = (1 - F_P)\left(\frac{P_E - P_0}{P_0}\right) + F_P\frac{P_T - P_0}{P_0}$$

Chew, The New Corporate Finance,

None

Hardy, Investment Guarantees

$$\frac{S_{t+w}}{S_t} \sim \mathrm{LN}(w\mu, \sqrt{w}\sigma) \Rightarrow \log\frac{S_{t+w}}{S_t} \sim N(w\mu, w\sigma^2)$$

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi w}} \exp\left\{ -\frac{1}{2} \frac{\left(\log(x) - w\mu\right)^2}{w\sigma^2} \right\}$$

$$\mathrm{E}\left[\frac{S_{t+w}}{S_t} \right] = e^{w\mu + w\sigma^2/2}$$

$$\mathrm{V}\left[\frac{S_{t+w}}{S_t} \right] = e^{2w\mu + w\sigma^2} \left(e^{w\sigma^2} - 1 \right)$$

$$Y_t = \mu + a\left(Y_{t-1} - \mu\right) + \sigma\varepsilon_t$$

$$Y_t = \mu + a\left(Y_{t-1} - \mu\right) + \sigma_t\varepsilon_t$$

$$\sigma_t^2 = \alpha_0 + \alpha_1\left(Y_{t-1} - \mu\right)^2 + \beta\sigma_{t-1}^2$$

$$\pi_1 = \frac{p_{2,1}}{p_{1,2} + p_{2,1}}, \quad \pi_2 = 1 - \pi_1$$

$$p_n(r) = \Pr\left[R_n(0) = r\right] = \pi_1 \Pr\left[R_n(0) = r \middle| \rho_{-1} = 1\right] + \pi_2 \Pr\left[R_n(0) = r \middle| \rho_{-1} = 2\right]$$

$$\sigma^*(R_n) = \sqrt{R_n\sigma_1^2 + (n - R_n)\sigma_2^2}$$

$$F_{S_n}(x) = \Pr(S_n \le x) = \sum_{r=0}^{n} \Pr(S_n \le x | R_n = r)\, p_n(r)$$

$$F_{S_n(x)} = \sum_{r=0}^{n} \Phi\left(\frac{\log x - \mu^*(r)}{\sigma^*(r)} \right) p_n(r)$$

$$f_{S_n}(x) = \sum_{r=0}^{n} \frac{1}{\sigma^*(r)x} \phi\left(\frac{\log x - \mu^*(r)}{\sigma^*(r)} \right) p_n(r)$$ (from errata sheet)

$$y(t) = \exp\left\{ w_y\delta_q(t) + \mu_y + yn(t) \right\} \qquad \text{where } yn(t) = a_y yn(t-1) + \sigma_y z_y(t)$$

$$\mathrm{E}\left[y(t)\right] = e^{\mu_y} \mathrm{E}\left[\exp(w_y\delta_q(t))\right] \mathrm{E}\left[\exp(yn(t))\right]$$

$$M_{\delta_q}(u) = \exp\left(u\mu_q + \frac{u^2(\sigma_q)^2}{2} \right)$$

$$\mathrm{E}\left[y(t)\right] = e^{\mu_y} M_q(w_y) \left[\exp\left(\mu_{yn} + \frac{\sigma_y^2}{2(1 - a_y^2)} \right) \right]$$

$$DM(t) = d_d\delta_q(t) + (1-d_d)DM(t-1)$$

$$\frac{\partial l(\mu,\sigma)}{\partial\mu} = \frac{1}{\sigma}(\sum_{t=1}^{n} y_t - n\mu)$$

$$\frac{\partial l(\mu,\sigma)}{\partial\sigma} = \frac{-n}{\sigma} + \frac{1}{\sigma^3}\sum_{t=1}^{n}(y_t-\mu)^2$$

$$\hat{\sigma} = \sqrt{\frac{\sum_{t=1}^{n}(y_t-\hat{\mu})^2}{n}} \qquad \text{where} \quad \hat{\mu} = \overline{y}$$

$$\frac{\partial^2 l(\mu,\sigma)}{\partial\mu^2} = -\frac{n}{\sigma}$$

$$\frac{\partial^2 l(\mu,\sigma)}{\partial\mu\partial\sigma} = \frac{-1}{\sigma^2}(\sum_{t=1}^{n} Y_t - n\mu)$$

$$\frac{\partial^2 l(\mu,\sigma)}{\partial\sigma^2} = \frac{3}{-\sigma^4}\sum_{t=1}^{n}(Y_t-\mu)^2 + \frac{n}{\sigma^2}$$

$$E\left[\frac{-\partial^2 l(\mu,\sigma)}{\partial\mu^2}\right] = \frac{n}{\sigma}$$

$$E\left[\frac{-\partial^2 l(\mu,\sigma)}{\partial\mu\partial\sigma}\right] = 0$$

$$E\left[\frac{-\partial^2 l(\mu,\sigma)}{\partial\sigma^2}\right] = \frac{2n}{\sigma^2}$$

$$\Sigma \approx \begin{pmatrix} \frac{\hat{\sigma}^2}{n} & 0 \\ 0 & \frac{\hat{\sigma}^2}{2n} \end{pmatrix}$$

$$l(\mu,\sigma,a) = \ln(\sqrt{\frac{1-a^2}{2\pi\sigma^2}}\exp\left\{-\frac{1}{2}\left(\frac{(Y_1-\mu)^2(1-a^2)}{\sigma^2}\right)\right\}) + \sum_{t=2}^{n}\ln(\sqrt{\frac{1}{2\pi\sigma^2}}\exp\left\{-\frac{1}{2}\left(\frac{(Y_t-(1-a)\mu-aY_{t-1})^2}{\sigma^2}\right)\right\})$$

$$= \frac{-n}{2}\ln(2\pi) + \frac{1}{2}\ln(1-a^2) - n\ln\sigma - \frac{1}{2}\left\{\frac{(Y_1-\mu)^2(1-a^2)}{\sigma^2} + \sum_{t=2}^{n}\left(\frac{(Y_t-(1-a)\mu-aY_{t-1})^2}{\sigma^2}\right)\right\}$$

$$\ln S_n \sim N(n\mu,(\sigma h(a,n))^2) \quad \text{where } h(a,n) = \frac{1}{(1-a)}\sqrt{\sum_{i=1}^{n}(1-a^i)^2}$$

$$\ln S_n - n\mu = Z_1 + Z_2 + ... + Z_n = \frac{\sigma}{1-a}\left\{\sum_{i=1}^{n} \varepsilon_i (1-a^{n+1-i})\right\}$$

$$F_{S_n}(x) = Pr\left[S_n \le x\right] = \sum_{r=0}^{n} Pr\left[S_n \le x \middle| R_n = r\right] p_n(r) = \sum_{r=0}^{n} \Phi\left(\frac{\ln x - \mu^*(r)}{\sigma^*(r)}\right) p_n(r)$$

$$f\left(x \middle| x_1,..,x_n\right) = \int_{\theta} f\left(x \middle| \theta\right) \pi\left(\theta \middle| x_1,..,x_n\right) d\theta$$

where $f\left(X \middle| \theta\right)$ is the density of X given the parameter θ

$$\Theta_{\sim i}^{(r+1,r)} = \left(\theta_1^{(r+1)},..,\theta_{i-1}^{(r+1)},\theta_{i+1}^{(r+1)},..,\theta_n^{(r)}\right)$$

$$\alpha = \min\left(1, \frac{L_i\left(\xi, \Theta_{\sim i}^{(r,r+1)}\right)\pi\left(\xi\right)q\left(\theta_i^{(r)} \middle| \xi\right)}{L_i\left(\theta_i^{(r)}\Theta_{\sim i}^{(r,r+1)}\right)\pi\left(\theta_i^{(r)}\right)q\left(\xi \middle| \theta_i^{(r)}\right)}\right)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1(Y_{t-1} - \mu)^2 + \beta\sigma_{t-1}^2$$

$$F_{t^+} = F_{t^-}(1-m) = F_{(t-1)}(1-m)\frac{S_t}{S_{t-1}}$$

$$F_{(t+u)^+} = F_t \frac{S_{t+u}(1-m)^u}{S_t}$$

$$M_t = \left(F_{t^-}\right)m_c = m_c F_{0^-} \frac{s_t\left(1-m\right)^{t-1}}{S_0}$$ *errata sheet*

$$C_n = -{}_n p_x^{\tau}(G - F_n)^+$$

$$C_t = -{}_t p_x^{\tau}\, M_t^d + {}_{t-1|1}q_x^d\left(G - F_t\right)^+$$ *note: M should have d superscript*

$$C_t = -{}_t p_x^{\tau} F_0 - S_t(1-m)^t m_d + {}_{t-1|1}q_x^d\left(G - F_0 - S_t(1-m)^t\right)^+$$ *errata sheet*

$$C_t = {}_{t-1|1}q_x^d\left(G_r - F_t\right)^+ - {}_t p_x^{\tau} M_t \quad \textit{where} \quad n_r < t < n_{r+1}$$

$$C_{n_r} = {}_{n_r-1|1}q_x^d\left(G_r - F_{n_r^-}\right)^+ + {}_{n_r} p_x^{\tau}\left(G_r - F_{n_r^-}\right)^+ - {}_{n_r} p_x^{\tau} M_{n_r}$$

$$P_0 = (K - S_d)\frac{S_u e^{-r} - S_0}{S_u - S_d} = (K - S_d)e^{-r}p^* \qquad \text{where } p^* = \frac{S_u - S_0 e^r}{S_u - S_d}$$

$$A = \Phi^{-1}\left(\frac{1+\beta}{2}\right)\sqrt{N\alpha(1-\alpha)}$$

$$\xi = 1 - \Phi\left(\frac{\log G/S_0 - n(\mu + \log(1-m))}{\sqrt{n}\sigma} \right)$$

$$\Pr\left[F_n + V_\alpha e^{rn} > G \right] \geq \alpha$$

$$V_\alpha = (G - F_{F_n}^{-1}(1-\alpha))\, e^{-rn}$$

$$V_\alpha = (G - F_0 \exp(-z_\alpha \sqrt{n}\sigma + n(\mu + \ln(1-m))))e^{-rn}$$

$$\mathrm{CTE}_\alpha(L) = \frac{(1-\beta')\mathrm{E}\left[\mathrm{X} \middle| X > V_\alpha \right] + (\beta' - \alpha)V_\alpha}{1-\alpha}$$

$$\mathrm{CTE}_\alpha(L) = \mathrm{E}\left[(G - F_n)e^{-rn} \middle| F_n < (G - V_\alpha e^{rn}) \right]$$

$$\mathrm{CTE}_\alpha(L) = e^{-rn}\left\{ G - \frac{e^{n(\mu + \log(1-m) + \sigma^2/2)}}{1-\alpha}\Phi(-z_\alpha - \sqrt{n}\sigma) \right\}$$

$$\mathrm{CTE}_\alpha(X) = \frac{(1-\xi)}{(1-\alpha)}\mathrm{CTE}_\xi(X)$$

$$E\left[L\right] = e^{-rn}\left\{ G(1-\xi) - F_0 \exp(n(\mu + \ln(1-m) + \frac{\sigma^2}{2}))\Phi(A) \right\} \quad \text{where } A = \frac{(\ln G_{F_0} - n(\mu + \ln(1-m)) - n\sigma^2)}{\sqrt{n}\sigma}$$

$$\log(1+i_t)\middle| \rho_t^y = \mu_{\rho_t^y}^y + \phi_{\rho_t^y}^y \left(\log(1+i_{t-1}) - \mu_{\rho_t^y}^y \right) + \sigma_{\rho_t^y}^y \varepsilon_t$$

$$H_0 = B(0,n)\mathrm{E}_Q\left[F_n (ga_{65}(n) - 1)^+ \right]$$

$$H_0 = F_0 \mathrm{E}_Q\left[\left(\frac{ga_{65}^d(0,n)}{B(0,n)} - 1 \right)^+ \right]$$

$$H_t = F_t\left\{ ga_{65}(t)\Phi(d_1(t)) - \Phi(d_2(t)) \right\} \text{ where } d_1(t) = \frac{\log(ga_{65}(t)) + \sigma_y^2(n-t)/2}{\sigma_y\sqrt{n-t}} \quad \text{and} \quad d_2(t) = d_1(t) - \sigma_y\sqrt{n-t}$$

$$\text{PTP :} \quad \max\left[P\left(1 + \alpha\left(\frac{S_n}{S_0} - 1 \right) \right), G \right]$$

where P: single premium, α : participation rate, G: guaranteed payout, S_t : value of the equity index at time t

Annual Ratchet

$$\text{CAR: } P\prod_{t=1}^{n}\left\{ 1 + \max\left(\alpha\left(\frac{S_t}{S_{t-1}} - 1 \right), 0 \right) \right\}$$

SAR: $P\left\{1+\sum_{t=1}^{n}\max\left(\alpha\left(\frac{S_t}{S_{t-1}}-1\right),0\right)\right\}$

CAR with cap rate c: $P\prod_{t=1}^{n}\left\{1+\min\left[\max\left(\alpha\left(\frac{S_t}{S_{t-1}}-1\right),0\right),c\right]\right\}$

SAR with cap rate c: $P\left\{1+\sum_{t=1}^{n}\min\left[\max\left(\alpha\left(\frac{S_t}{S_{t-1}}-1\right),0\right),c\right]\right\}$

High Water Mark: $\max\left[P\left(1+\alpha\left(\frac{S^{\max}}{S_0}-1\right)\right),G\right]$ where $S^{\max}=\max\left(S_0,S_1,..,S_n\right)$

$$H=\left(P\left(1+\alpha\left(\frac{S_n}{S_0}-1\right)\right)-G\right)$$

$$H=\frac{\alpha P}{S_0}\left\{S_n-\frac{S_0}{\alpha}\left(\frac{G}{P}-(1-\alpha)\right)\right\}$$

$$H_0=\frac{\alpha P}{S_0}\left\{S_0e^{-dn}\Phi(d_1)-K^{PTP}e^{-rn}\Phi(d_2)\right\}\text{ where }K^{PTP}=\frac{S_0}{\alpha}\left(\frac{G}{P}-(1-\alpha)\right)$$

$$H_0=\alpha Pe^{-dn}\Phi(d_1)-\left(G-P(1-\alpha)\right)e^{-rn}\Phi(d_2)$$

$$d_1=\frac{\ln\frac{\alpha P}{G-P(1-\alpha)}+\left(r-d+\frac{\sigma^2}{2}\right)n}{\sigma\sqrt{n}}\ ,\ d_2=d_1-\sigma\sqrt{n}$$

$$RP=P\prod_{t=1}^{n}\left\{1+\max\left(\alpha\left(\frac{S_t}{S_{t-1}}-1\right),0\right)\right\}$$

$$H=E_Q\left[e^{-rn}(RP)\right]$$

$$H=PE_Q\left[\prod_{t=1}^{n}e^{-r}\left\{1+\max\left(\alpha\left(\frac{S_t}{S_{t-1}}-1\right),0\right)\right\}\right]$$

$$H=P\prod_{t=1}^{n}\left\{e^{-r}+E_Q\left[e^{-r}\max\left(\alpha\left(\frac{S_t}{S_{t-1}}-1\right),0\right)\right]\right\}$$

$$\alpha E_Q\left[e^{-r}\max\left(S_1-1,0\right)\right]=\alpha\left\{e^{-d}\Phi\left(d_1\right)-e^{-r}\Phi\left(d_2\right)\right\} \qquad \text{where } d_1=\frac{r-d+\frac{\sigma^2}{2}}{\sigma}, \quad d_2=d_1-\sigma$$

$$H=P\left\{e^{-r}+\alpha\left(e^{-d}\Phi\left(d_1\right)-e^{-r}\Phi\left(d_2\right)\right)\right\}^n$$

$$E_Q\left[e^{-r}\left\{1+\max\left(\alpha\left(\frac{S_t}{S_{t-1}}-1\right),e^g-1\right)\right\}\right] \text{ where } e^g\text{ : minimum accumulation factor}$$

$$=E_Q\left[e^{-r}\left\{1+\max\left(\alpha\left(S_1-1\right),e^g-1\right)\right\}\right]$$

$$=E_Q\left[e^{-r}\left\{1+\left(e^g-1\right)+\alpha\max\left(S_1-\left(\frac{e^g-(1-\alpha)}{\alpha}\right),0\right)\right\}\right]$$

$$=e^{g-r}+\alpha BSC\left(K=\frac{e^g-(1-\alpha)}{\alpha},n=1\right)$$

$BSC\left(K,n\right)$: Black-Scholes call-option price with strike K, starting stock price 1.0 and term n years

$$P\left\{\alpha e^{-d}\left(\Phi\left(d_1\right)-\Phi\left(d_3\right)\right)+\left(1-\alpha\right)e^{-r}\left(\Phi\left(d_2\right)-\Phi\left(d_4\right)\right)+e^{g-r}\Phi\left(-d_2\right)+e^{c-r}\Phi\left(d_4\right)\right\}^n$$

$$\text{where } d_1=\frac{\ln\left(\dfrac{1}{\left(\left. e^g-(1-\alpha)\right/\alpha\right)}\right)+r-d+\dfrac{\sigma^2}{2}}{\sigma} \qquad d_2=d_1-\sigma$$

$$\text{where } d_3=\frac{\ln\left(\dfrac{1}{\left(\left. e^c-(1-\alpha)\right/\alpha\right)}\right)+r-d+\dfrac{\sigma^2}{2}}{\sigma} \qquad d_4=d_3-\sigma$$

SAR with life-of-contract guarantee without cap: $P\left\{1+\sum_{t=1}^{n}\alpha\left(\frac{S_t}{S_{t-1}}-1\right)\right\}$

$$H_{t+1}=\alpha\left\{S_{t+1}e^{-d(n-t-1)}\Phi\left(d_1\left(t+1\right)\right)-K^{PTP}e^{-r(n-t-1)}\Phi\left(d_2\left(t+1\right)\right)\right\}$$

$$tc\propto S_{t+1}e^{-d(n-t-1)}\left|\Phi\left(d_1\left(t+1\right)\right)-\Phi\left(d_1\left(t\right)\right)\right|$$

Toole and Herget, Insurance Industry Mergers and Acquisitions

$$r=r_f+\beta(r_m-r_f)$$

where $r =$ expected rate of return on the acquisition
$r_f =$ risk-free rate of return $r_m =$ expected rate of return for the market as a whole
$\beta =$ measure of risk of a company (both debt and equity) relative to the market as a whole

$$r = r^D \frac{D}{D+E} + \frac{E}{D+E}(r_f + \beta^E (r_m - r_f))$$

where $r =$ weighted average cost of capital WACC
$r^D =$ required return on debt $\beta^E =$ beta of a company's stock
$D =$ market value of a company's debt $E =$ market value of a company's equity

cost of $capital_t =$ required $capital_{t-1}$ * (discount rate – after tax earnings $rate_t$)
appraisal cost of capital = NPV(cost of $capital_t$)

NPV(distributable $earning_t$)=Excess $capital_0$+NPV(after tax $earnings_t$ - Insurance in RC_t)
=NPV(after tax earning on the $business_t$) + Excess $capital_0$+NPV(after-tax earning on $capital_t$)-NPV(increase in RC_t)
=NPV(after tax earning on the $business_t$) + Excess $capital_0$
+NPV($RC_{t-1} * i_t$) – (NPV(RC_t) – NPV(RC_{t-1}))
=NPV(after tax earning on the $business_t$) + Excess $capital_0$
+ NPV($RC_{t-1} * i_t$) – ((1+d)NPV(RC_{t-1}) – RC_0 - NPV(RC_{t-1}))
=NPV(after tax earning on the $business_t$) + Excess $capital_0 + RC_0$ -NPV($RC_t * (d - i_t)$)
=value of Inforce and Future Business + adjusted of book value –cost of required capital
where $i_t =$ after tax investment earnings rate on capital $d =$ discount rate RC_t = required capital

Total reserve = $(1 - {}^{1}\!/_{PLDF}) *$ expected loss where $PLDF =$ paid loss development factor

IBNR reserve = $(1 - {}^{1}\!/_{RLDF}) *$ expected loss where $RLDF =$ reported loss development factor

Trigeorgis, Real Options

$$NPV = \sum_{t=1}^{T} \frac{\alpha_t E(c_t)}{(1+r_1)...(1+r_t)} - I$$

$$E(r_j) = r + \beta_j \left[E(r_m) - r \right]$$

$$Expanded\,(strategic)\,net\ present\,value\,(NPV^*) = \left[Direct(\,passive)\,NPV + strategic\,value \right] + \ flexibility\,value$$

$$\frac{d\pi_A}{dK_A} = \frac{\partial \pi_A}{\partial K_A} + \frac{\partial \pi_A}{\partial \alpha_B}\frac{d\alpha^* B}{dK_A}$$

Hull , Options, Futures and Other Derivatives,

$$\vartriangle z = \varepsilon\sqrt{\vartriangle t}$$

$$z(T) - z(0) = \sum_{i=1}^{N} \varepsilon_i \sqrt{\vartriangle t}$$

$$dx = adt + bdz$$

$$dx = a(x,t)dt + b(x,t)dz$$

$$S_T = S_0 e^{\mu T}$$

$$\frac{ds}{S} = \mu dt + \sigma dz$$

$$\Delta S = \mu S \vartriangle t + \sigma S \varepsilon \sqrt{\vartriangle t}$$

$$\frac{\Delta S}{S} \sim \phi(\mu \vartriangle t, \sigma\sqrt{\vartriangle t})$$

$$dG = (\frac{\partial G}{\partial x} a + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial x^2} b^2)dt + \frac{\partial G}{\partial x} bdz$$

$$dS = \mu Sdt + \sigma Sdz$$

$$dG = (\frac{\partial G}{\partial S}\mu S + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial S^2}\sigma^2 S^2)dt + \frac{\partial G}{\partial S}\sigma Sdz$$

$$F = Se^{r(T-t)}$$

$$dF = (\mu - r)Fdt + \sigma Fdz$$

$$dG = (\mu - \frac{\sigma^2}{2})dt + \sigma dz$$

$$\ln S_T \sim \phi\left(\ln S_0 + \left(\mu - \frac{\sigma^2}{2}\right)T, \sigma\sqrt{T}\right)$$

$$E\left(S_T\right) = S_o e^{\mu T}$$

$$\operatorname{var}\left(S_T\right) = S_0^2 e^{2\mu T}\left[e^{\sigma^2 T} - 1\right]$$

$$x = \frac{1}{T}\ln\frac{S_T}{S_0}$$

$$x \sim \phi\left(\mu - \frac{\sigma^2}{2}, \frac{\sigma}{\sqrt{T}}\right)$$

$$s=\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(u_i-\bar{u}\right)^2} \qquad \text{where} \quad u_i=\ln\left(\frac{S_i}{S_{i-1}}\right)$$

$$s=\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}u_i^2-\frac{1}{n(n-1)}\left(\sum_{i=1}^{n}u_i\right)^2}$$

$$dS=\mu S dt+\sigma S dz$$

$$df=\left(\frac{\partial f}{\partial S}\mu S+\frac{\partial f}{\partial t}+\frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)dt+\frac{\partial f}{\partial S}\sigma S dz$$

$$\triangle f=\left(\frac{\partial f}{\partial S}\mu S+\frac{\partial f}{\partial t}+\frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)\triangle t+\frac{\partial f}{\partial S}\sigma S\triangle z$$

$$\Pi=-f+\frac{\partial f}{\partial S}S$$

$$\triangle\Pi=\left(-\frac{\partial f}{\partial t}-\frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)\triangle t$$

$$\frac{\partial f}{\partial t}+rS\frac{\partial f}{\partial S}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 f}{\partial S^2}=rf$$

$$f=e^{-rT}\hat{E}\left(S_T\right)-Ke^{-rT}$$

$$\hat{E}\left(S_T\right)=S_0e^{rT}$$

$$f=S_0-Ke^{-rT}$$

$$c=S_0N\left(d_1\right)-Ke^{-rT}N\left(d_2\right)$$

$$p=Ke^{-rT}N\left(-d_2\right)-S_0N\left(-d_1\right) \qquad \text{where} \quad d_1=\frac{ln\left(S_0/K\right)+\left(r+\sigma^2/2\right)T}{\sigma\sqrt{T}}$$

$$d_2=\frac{ln\left(S_0/K\right)+\left(r-\sigma^2/2\right)T}{\sigma\sqrt{T}}=d_1-\sigma\sqrt{T}$$

$$c=e^{-rT}\left[S_0N\left(d_1\right)e^{rT}-KN\left(d_2\right)\right]$$

$$S\left(t_n\right)-D_n-Ke^{-r\left(T-t_n\right)}\geq S\left(t_n\right)-K$$

$$c+Ke^{-rT}=p+S_0e^{-qT}$$

$$c = S_0 e^{-qT} N\left(d_1\right) - K e^{-rT} N\left(d_2\right)$$

$$p = K e^{-rT} N\left(-d_2\right) - S_0 e^{-qT} N\left(-d_1\right)$$

$$d_1 = \frac{ln\left(S_0 / K\right) + \left(r - q + \sigma^2 / 2\right)T}{\sigma\sqrt{T}} \qquad d_2 = \frac{ln\left(S_0 / K\right) + \left(r - q - \sigma^2 / 2\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

$$dS = (r - q)Sdt + \sigma Sdz$$

$$p = \frac{e^{(r-q)\Delta t} - d}{u - d}$$

$$c = e^{-rT}\left[F_0 N\left(d_1\right) - KN\left(d_2\right)\right]$$

$$p = e^{-rT}\left[KN\left(-d_2\right) - F_0 N\left(-d_1\right)\right]$$

$$c + Ke^{-rT} = p + F_0 e^{-rT}$$

$$f = e^{-rT}\left[\,pf_{\mu} + (1 - p)\, f_d\,\right]$$

$$\frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial F^2}\sigma^2 F^2 = rf$$

$$H_F = e^{-rT} H_A$$

$$H_F = e^{-(r-q)T} H_A$$

$$H_F = e^{-(r-r_f)T} H_A$$

$$N^{'}(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$$

$$\triangle\Pi = \Theta\triangle t + \frac{1}{2}\Gamma\triangle S^2$$

$$\Theta + rS\triangle + \frac{1}{2}\sigma^2 S^2 \Gamma = r\prod$$

$$\triangle = e^{-qT}\left[N(d_1) - 1\right]$$

$$p + S_0 e^{-qT} = c + Ke^{-rT}$$

$$a = e^{\left[f(t) - g(t)\right]\Delta t}$$

$$p = \frac{e^{\left[f(t)-g(t)\right]\Delta t} - d}{u-d}$$

$$\frac{\partial f}{\partial t} + (r-q)S\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf$$

$$\frac{\partial f}{\partial S} = \frac{f_{i,j+1} - f_{i,j}}{\triangle S}$$

$$\frac{\partial f}{\partial S} = \frac{f_{i,j} - f_{i,j-1}}{\triangle S}$$

$$\frac{\partial f}{\partial S} = \frac{f_{i,j+1} - f_{i,j-1}}{2\triangle S}$$

$$\frac{\partial^2 f}{\partial S^2} = \frac{f_{i,j+1} + f_{i,j-1} - 2f_{i,j}}{\triangle S^2}$$

$$a_j f_{i,j-1} + b_j f_{i,j} + c_j f_{i,j+1} = f_{i+1,j} \quad \text{where } a_j = \frac{1}{2}(r-q)j\triangle t - \frac{1}{2}\sigma^2 j^2 \triangle t,\ b_j = 1 + \sigma^2 j^2 \triangle t + r\triangle t$$

$$c_j = -\frac{1}{2}(r-q)j\triangle t - \frac{1}{2}\sigma^2 j^2 \triangle t$$

$$\frac{\partial f}{\partial S} = \frac{f_{i+1,j+1} - f_{i+1,j-1}}{2\triangle S}$$

$$\frac{\partial^2 f}{\partial S^2} = \frac{f_{i+1,j+1} + f_{i+1,j-1} - 2f_{i+1,j}}{\triangle S^2}$$

$$f_{i,j} = a_j^* f_{i+1,j-1} + b_j^* f_{i+1,j} + c_j^* f_{i+1,j+1}$$

$$\text{where } a_j^* = \frac{1}{1+r\triangle t}\left(-\frac{1}{2}(r-q)j\triangle t + \frac{1}{2}\sigma^2 j^2 \triangle t\right),\ b_j^* = \frac{1}{1+r\triangle t}\left(1 - \sigma^2 j^2 \triangle t\right)$$

$$c_j^* = \frac{1}{1+r\triangle t}\left(\frac{1}{2}(r-q)j\triangle t + \frac{1}{2}\sigma^2 j^2 \triangle t\right)$$

$$\alpha_j f_{i,j-1} + \beta_j f_{i,j} + \gamma_j f_{i,j+1} = f_{i+1,j}$$

$$\text{where} \quad \alpha_j = \frac{\triangle t}{2\triangle Z}\left(r - q - \frac{\sigma^2}{2}\right) - \frac{\triangle t}{2\triangle Z^2}\sigma^2 \qquad \beta_j = 1 + \frac{\triangle t}{\triangle Z^2}\sigma^2 + r\triangle t$$

$$\gamma_j = \frac{-\triangle t}{2\triangle Z}\left(r - q - \frac{\sigma^2}{2}\right) - \frac{\triangle t}{2\triangle Z^2}\sigma^2$$

$$\alpha_j^* f_{i+1,j-1} + \beta_j^* f_{i+1,j} + \gamma_j^* f_{i+1,j+1} = f_{i,j}$$

where $\alpha_j^* = \frac{1}{1+r\triangle t}\left[-\frac{\triangle t}{2\triangle Z}(r-q-\frac{\sigma^2}{2})+\frac{\triangle t}{2\triangle Z^2}\sigma^2\right]$

$$\beta_j^* = \frac{1}{1+r\triangle t}(1-\frac{\triangle t}{\triangle Z^2}\sigma^2)$$

$$\gamma_j^* = \frac{1}{1+r\triangle t}\left[\frac{\triangle t}{2\triangle Z}(r-q-\frac{\sigma^2}{2})+\frac{\triangle t}{2\triangle Z^2}\sigma^2\right]$$

$$\sigma_n^2 = \frac{1}{m-1}\sum_{i=1}^{m}\left(u_{n-i}-\bar{u}\right)^2$$

$$\sigma_n^2 = \sum_{i=1}^{m}\alpha_i u_{n-i}^2$$

$$\sigma_n^2 = \gamma V_L + \sum_{i=1}^{m}\alpha_i u_{n-i}^2$$

$$\sigma_n^2 = \lambda\sigma_{n-1}^2 + (1-\lambda)u_{n-1}^2$$

$$\sigma_n^2 = (1-\lambda)\sum_{i=1}^{m}\lambda^{i-1}u_{n-i}^2 + \lambda^m\sigma_{n-m}^2$$

$$\sigma_n^2 = \gamma V_L + \alpha u_{n-1}^2 + \beta\sigma_{n-1}^2$$

$$\sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta\sigma_{n-1}^2$$

$$\sigma_n^2 = \omega + \beta\omega + \beta^2\omega + \alpha u_{n-1}^2 + \alpha\beta u_{n-2}^2 + \alpha\beta^2 u_{n-3}^2 + B^3\sigma_{n-3}^2$$

$$\prod_{i=1}^{m}\left[\frac{1}{\sqrt{2\pi v}}\exp\left(\frac{-u_i^2}{2v}\right)\right]$$

$$\frac{1}{m}\sum_{i=1}^{m}u_i^2$$

$$\sum_{i=1}^{m}\left[-ln\left(v_i\right)-\frac{u_i^2}{v_i}\right]$$

$m\sum_{k=1}^{K}w_k\eta_k^2$ where $w_k = \frac{m+2}{m-k}$

$$\sigma_n^2 = (1-\alpha-\beta)V_L + \alpha u_{n-1}^2 + \beta\sigma_{n-1}^2$$

$$\sigma_n^2 - V_L = \alpha\left(u_{n-1}^2 - V_L\right) + \beta\left(\sigma_{n-1}^2 - V_L\right)$$

$$E\left[\sigma_{n+t}^2\right] = V_L + (\alpha+\beta)^t \left(\sigma_n^2 - V_L\right)$$

$$\text{cov}_n = \frac{1}{m}\sum_{i=1}^{m} x_{n-i} y_{n-i}$$

$$\text{cov}_n = \lambda \,\text{cov}_{n-1} + (1-\lambda)\, x_{n-1} y_{n-1}$$

$$\text{cov}_n = \omega + \alpha x_{n-1} y_{n-1} + \beta \,\text{cov}_{n-1}$$

$$e^{-rT_1}\hat{E}\left[c\,\frac{S_1}{S_0}\right]$$

$$S_0 e^{-qT_2} M\left(a_1, b_1; \sqrt{T_1/T_2}\right) - K_2 e^{-rT_2} M\left(a_2, b_2; \sqrt{T_1/T_2}\right) - e^{-rT_1} K_1 N(a_2)$$

$$a_1 = \frac{ln\left(S_0/S^*\right) + \left(r - q + \sigma^2/2\right)T_1}{\sigma\sqrt{T_1}} \qquad a_2 = a_1 - \sigma\sqrt{T_1}$$

$$b_1 = \frac{ln\left(S_0/K_2\right) + \left(r - q + \sigma^2/2\right)T_2}{\sigma\sqrt{T_2}} \qquad b_2 = b_1 - \sigma\sqrt{T_2}$$

$$K_2 e^{-rT_2} M\left(-a_2, b_2; -\sqrt{T_1/T_2}\right) - S_0 e^{-qT_2} M\left(-a_1, b_1; -\sqrt{T_1/T_2}\right) + e^{-rT_1} K_1 N(-a_2)$$

$$K_2 e^{-rT_2} M\left(-a_2, -b_2; \sqrt{T_1/T_2}\right) - S_0 e^{-qT_2} M\left(-a_1, -b_1; \sqrt{T_1/T_2}\right) - e^{-rT_1} K_1 N(-a_2)$$

$$S_o e^{-qT_2} M\left(a_1, -b_1; -\sqrt{T_1/T_2}\right) - K_2 e^{-rT_2} M\left(a_2, -b_2; -\sqrt{T_1/T_2}\right) + e^{-rT_1} K_1 N(a_2)$$

$$\max(c, p) = c + e^{-q(T_2 - T_1)} \max\left(0, Ke^{-(r-q)(T_2-T_1)} - S_1\right)$$

$$H \le K:\; c_{di} = S_0 e^{-qT} \left(H/S_0\right)^{2\lambda} N(y) - Ke^{-rT} \left(H/S_0\right)^{2\lambda-2} N\left(y - \sigma\sqrt{T}\right)$$

$$\lambda = \frac{r - q + \sigma^2/2}{\sigma^2}$$

$$y = \frac{ln\left[H^2/\left(S_0 K\right)\right]}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}$$

$$c_{do} = c - c_{di}$$

$$H \ge K:\; c_{do} = S_0 N(x_1) e^{-qT} - Ke^{-rT} N\left(x_1 - \sigma\sqrt{T}\right) - S_0 e^{-qT} \left(H/S_0\right)^{2\lambda} N(y_1) + Ke^{-rT} \left(H/S_0\right)^{2\lambda-2} N\left(y_1 - \sigma\sqrt{T}\right)$$

$$c_{di} = c - c_{do}$$

$$x_1 = \frac{\ln\left(S_0/H\right)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T} \qquad\qquad y_1 = \frac{\ln\left(H/S_0\right)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}$$

$$H > K:\ c_{ui} = S_0 N\left(x_1\right)e^{-qT} - Ke^{-rT}N\left(x_1 - \sigma\sqrt{T}\right) - S_0 e^{-qT}\left(H/S_o\right)^{2\lambda}\left[N\left(-y\right) - N\left(-y_1\right)\right]$$
$$+Ke^{-rT}\left(H/S_0\right)^{2\lambda-2}\left[N\left(-y+\sigma\sqrt{T}\right) - N\left(-y_1+\sigma\sqrt{T}\right)\right]$$

$$c_{uo} = c - c_{ui}$$

$$H \geq K:\ p_{ui} = -S_0 e^{-qT}\left(H/S_0\right)^{2\lambda} N\left(-y\right) + Ke^{-rT}\left(H/S_0\right)^{2\lambda-2} N\left(-y+\sigma\sqrt{T}\right)$$

$$p_{uo} = p - p_{ui}$$

$$H \leq K:\ p_{uo} = -S_0 N\left(-x_1\right)e^{-qT} + Ke^{-rT}N\left(-x_1+\sigma\sqrt{T}\right) + S_0 e^{-qT}\left(H/S_0\right)^{2\lambda} N\left(-y_1\right) - Ke^{-rT}\left(H/S_0\right)^{2\lambda-2} N\left(-y_1+\sigma\sqrt{T}\right)$$

$$p_{ui} = p - p_{uo}$$

$$H < K:\ p_{di} = -S_0 N\left(-x_1\right)e^{-qT} + Ke^{-rT}N\left(-x_1+\sigma\sqrt{T}\right) + S_0 e^{-qT}\left(H/S_0\right)^{2\lambda}\left[N\left(y\right) - N\left(y_1\right)\right]$$
$$-Ke^{-rT}\left(H/S_0\right)^{2\lambda-2}\left[N\left(y-\sigma\sqrt{T}\right) - N\left(y_1-\sigma\sqrt{T}\right)\right]$$

$$p_{do} = p - p_{di}$$

$$c_{ELB} = S_0 e^{-qT} N\left(a_1\right) - S_0 e^{-qT}\frac{\sigma^2}{2\left(r-q\right)} N\left(-a_1\right) - S_{\min}e^{-rT}\left(N\left(a_2\right) - \frac{\sigma^2}{2\left(r-q\right)}e^{Y_1}N\left(-a_3\right)\right)$$

$$a_1 = \frac{ln\left(S_0/S_{\min}\right) + \left(r-q+\sigma^2/2\right)T}{\sigma\sqrt{T}} \qquad\qquad a_2 = a_1 - \sigma\sqrt{T}$$

$$a_3 = \frac{ln\left(S_0/S_{\min}\right) + \left(-r+q+\sigma^2/2\right)T}{\sigma\sqrt{T}}$$

$$Y_1 = -\frac{2\left(r-q-\sigma^2/2\right)ln\left(S_0/S_{\min}\right)}{\sigma^2}$$

$$p_{ELB} = S_{\max}e^{-rT}\left(N\left(b_1\right) - \frac{\sigma^2}{2\left(r-q\right)}e^{Y_2}N\left(-b_3\right)\right) + S_0 e^{-qT}\frac{\sigma^2}{2\left(r-q\right)}N\left(-b_2\right) - S_o e^{-qT}N\left(b_2\right)$$

$$b_1 = \frac{ln\left(S_{\max}/S_o\right) + \left(-r+q+\sigma^2/2\right)T}{\sigma\sqrt{T}} \qquad\qquad b_2 = b_1 - \sigma\sqrt{T}$$

$$b_3 = \frac{ln\left(S_{\max}/S_0\right) + \left(r-q-\sigma^2/2\right)T}{\sigma\sqrt{T}}$$

$$Y_2 = \frac{2\left(r-q-\sigma^2/2\right)ln\left(S_{\max}/S_0\right)}{\sigma^2}$$

$$\max\left(0, S_T - S_\tau\right) + \left(S_\tau - K\right)$$

$$r - \frac{1}{2}\left(r - q - \frac{\sigma^2}{6}\right) = \frac{1}{2}\left(r + q + \frac{\sigma^2}{6}\right)$$

$$M_1 = \frac{e^{(r-q)T} - 1}{(r-q)T} S_0$$

$$M_2 = \frac{2e^{\left(2(r-q)+\sigma^2\right)T} S_0^2}{\left(r-q+\sigma^2\right)\left(2r-2q+\sigma^2\right)T^2} + \frac{2S_0^2}{\left(r-q\right)T^2}\left(\frac{1}{2\left(r-q\right)+\sigma^2} - \frac{e^{(r-q)T}}{r-q+\sigma^2}\right)$$

$$\sigma^2 = \frac{1}{T}\ln\left(\frac{M_2}{M_1{}^2}\right)$$

$$V_o e^{-qv^T} N\left(d_1\right) - U_0 e^{-qu^T} N\left(d_2\right)$$

$$d_1 = \frac{ln\left(V_o/U_o\right) + \left(q_U - q_V + \hat{\sigma}^2/2\right)T}{\hat{\sigma}\sqrt{T}} \qquad d_2 = d_1 - \hat{\sigma}\sqrt{T}$$

$$\hat{\sigma} = \sqrt{\sigma_U^2 + \sigma_V^2 - 2\rho\sigma_U\sigma_V}$$

$$dS = (r-q)Sdt + \sigma S^\alpha dz$$

$$\frac{dS}{S} = (r - q - \lambda k)dt + \sigma dz + dp$$

$$\phi(g) = \frac{g^{T/v-1}\,\mathrm{e}^{-g/v}}{v^{T/v}\,\Gamma\left(T/v\right)}$$

$$\ln S_0 + \left(r-q\right)T + \omega + \theta g$$

$$\sigma\sqrt{g}$$

$$\omega = \frac{T}{v}\ln\left(1 - \theta v - \sigma^2 v/2\right)$$

$$dS = (r-q)Sdt + \sigma(t)Sdz$$

$$\frac{dS}{S} = (r-q)dt + \sqrt{V}dz_S$$

$$dV = a(V_L - V)dt + \xi V^{\alpha} dz_V$$

$$dS = (r(t) - q(t))Sdt + \sigma(S,t)Sdz$$

$$\left[\sigma(K,T)\right]^2 = 2\frac{\partial C_{mkt}/\partial T + q(T)C_{mkt} + K\left[r(T)-q(T)\right]\partial C_{mkt}/\partial K}{K^2\left(\partial^2 C_{mkt}/\partial K^2\right)}$$

$$\frac{d\theta}{\theta} = mdt + sdz$$

$$\triangle f_1 = \mu_1 f_1 \triangle t + \sigma_1 f_1 \triangle z$$

$$\triangle f_2 = \mu_2 f_2 \triangle t + \sigma_2 f_2 \triangle z$$

$$\prod = (\sigma_2 f_2) f_1 - (\sigma_1 f_1) f_2$$

$$\triangle\prod = (\mu_1\sigma_2 f_1 f_2 - \mu_2\sigma_1 f_1 f_2)\triangle t$$

$$\frac{\mu_1 - r}{\sigma_1} = \frac{\mu_2 - r}{\sigma_2}$$

$$\frac{df}{f} = \mu dt + \sigma dz$$

$$\frac{\mu - r}{\sigma} = \lambda$$

$$\mu - r = \sum_{i=1}^{n} \lambda_i \sigma_i$$

$$d\theta = \sigma dz$$

$$d\left(\frac{f}{g}\right) = (\sigma_f - \sigma_g)\frac{f}{g}dz$$

$$f_0 = g_0 E_g\left(\frac{f_T}{g_T}\right)$$

$$dg = rgdt$$

$$f_o = g_0 \widehat{E}\left(\frac{f_T}{g_T}\right)$$

$$f_0 = \hat{E}(e^{-\bar{\gamma}T} f_T)$$

$$f_0 = P(0,T)E_T(f_T)$$

$$A(t) = \sum_{i=0}^{N-1} (T_{i+1} - T_i) P(t, T_{i+1})$$

$$s(t) = E_A[s(T)]$$

$$f_o = A(0) E_A\left[\frac{f_T}{A(T)}\right]$$

$$c = P(0,T)E_T[\max(S_T - K, 0)]$$

$$c = e^{-RT} E_T[\max(S_T - k, 0)]$$

$$E_T[\max(S_T - K, 0)] = E_T(S_T)N(d_1) - KN(d_2)$$

$$f_0 = U_0 E_U\left[\max\left(\frac{V_T}{U_T} - 1, 0\right)\right]$$

$$f_0 = V_0 N(d_1) - U_0 N(d_2)$$

European call option on a variable whose value is V

$$c = P(0,T)[F_0 N(d_1) - KN(d_2)]$$

where $d_1 = \dfrac{\ln(F_0/K) + \sigma^2 T/2}{\sigma\sqrt{T}}$ $d_2 = \dfrac{\ln(F_0/K) - \sigma^2 T/2}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$

$F_0 =$ value of F at time zero $K =$ strike price of the option

$P(t,T) =$ price at time t of a zero-coupon bond paying \$1 at time T

$\sigma =$ volatility of F $F =$ forward price of V for a contract maturing T

$T =$ time to maturing of the option $V_T =$ value of V at time T

value of the corresponding put option $p = P(0,T)[KN(-d_2) - F_0 N(-d_1)]$

forward bond price $F_B = \dfrac{B_0 - I}{P(0,T)}$

where $B_0 =$ bond price at time zero

$I =$ present value of coupons that will be paid during the life of the option

volatility of the forward bond price $\sigma_B = D_{y_0} \sigma_y$

where $\sigma_y =$ volatility of the forward bond yield $y_0 =$ initial value of y_F $y_F =$ forward yield

$D =$ modified duration of the bond underlying the option at option maturing

$$c = P(0,T)E_T\left[\max(B_T - K, 0)\right]$$

where $B_T =$ bond price at time T

$E_T =$ expected value in a world that is forward risk neutral with respect to a zero-coupon bond maturing at time T

$$E_T(B_T) = F_B$$

$$\max\left[L - \frac{L(1+R_k\delta_k)}{1+R_k\delta_k}, 0\right]$$

where $\frac{L(1+R_k\delta_k)}{1+R_k\delta_k} =$ value at time t_k of a zero-coupon bond that pays off $L(1+R_k\delta_k)$ at time t_{k+1}

$$E_T(y_T) = y_0 - \frac{1}{2}y_0^2\sigma_y^2 T\frac{G''(y_0)}{G'(y_0)}$$

where $E_T =$ expectations in a world that is forward risk neutral with respect to $P(t,T)$

$\sigma_y =$ forward yield volatility

$$E_T(R_T) = R_0 - \frac{1}{2}R_0^2\sigma_R^2 T\frac{G''(R_0)}{G'(R_0)} = R_0 + \frac{R_0^2\sigma_R^2\tau T}{1+R_0\tau}$$

where $\tau = T^* - T$ $L =$ principal

$R_T =$ zero-coupon interest rate applicable to the period between T and T^*

$$\alpha_V = \rho_{VW}\sigma_V\sigma_W$$

where $\sigma_V =$ volatility of V $\sigma_W =$ volatility of W $\rho_{VW} =$ correlation between V and W

$R =$ forward interest rate for period between T and T^* $\sigma_R =$ volatility of R $W = \frac{1}{(1+R/m)^{m(T^*-T)}}$

$$E_{T^*}(V_T) = E_T(V_T)\exp\left[-\frac{\rho_{VR}\sigma_V\sigma_R R_0(T^*-T)}{1+R_0/m}T\right]$$

$$\alpha_V = \rho_{VW}\sigma_V\sigma_W$$

$$E_X(V_T) = E_Y(V_T)(1+\rho\sigma_V\sigma_W T)$$

value at time t of an interest rate derivative that provides a payoff of f_T at time T $= \hat{E}\left[e^{-\bar{r}(T-t)}f_T\right]$

where $\bar{r} =$ the average value of r in the time interval between t and T

$\hat{E} =$ expected value in the traditional risk-neutral world

$$R(t,T) = -\frac{1}{T-t}\ln\hat{E}\left[e^{-\bar{r}(T-t)}\right]$$

$$dr = m(r)dt + s(r)dz \qquad dr = \mu r dt + \sigma r dz$$

$$dr = a(b-r)dt + \sigma dz$$

$$P(t,T) = A(t,T)e^{-B(t,T)r(t)}$$

$$B(t,T) = \frac{1-e^{-a(T-t)}}{a}$$

$$A(t,T) = \exp\left[\frac{(B(t,T)-T+t)(a^2b-\sigma^2/2)}{a^2} - \frac{\sigma^2 B(t,T)^2}{4a}\right]$$

$$R(t,T) = -\frac{1}{T-t}\ln A(t,T) + \frac{1}{T-t}B(t,T)r(t)$$

$$dr = \theta(t)dt + \sigma dz$$

$$\theta(t) = F_t(0,t) + \sigma^2 t$$

$$P(t,T) = A(t,T)e^{-r(t)(T-t)}$$

where $\ln A(t,T) = \ln\frac{P(0,T)}{p(0,t)} + (T-t)F(0,t) - \frac{1}{2}\sigma^2 t(T-t)^2$

$$dr = \left[\theta(t) - ar\right]dt + \sigma dz = a\left[\frac{\theta(t)}{a} - r\right]dt + \sigma dz$$

$$\theta(t) = F_t(0,t) + aF(0,t) + \frac{\sigma^2}{2a}(1-e^{-2at})$$

$$P(t,T) = A(t,T)e^{-B(t,T)r(t)} \quad \text{where } B(t,T) = \frac{1-e^{-a(T-t)}}{a}$$

$$\ln A(t,T) = \ln\frac{P(0,T)}{P(0,t)} + B(t,T)F(0,t) - \frac{1}{4a^3}\sigma^2(e^{-aT}-e^{-at})^2(e^{2at}-1)$$

$$d\ln r = \left[\theta(t) - a(t)\ln(r)\right]dt + \sigma(t)dz$$

$$df(r) = \left[\theta(t) + \mu - af(r)\right]dt + \sigma_1 dz_1 \qquad du = -budt + \sigma_2 dz_2$$

price at time zero of a call option that matures at time T on a zero-coupon bond maturing at time s

$$LP(0,s)N(h) - KP(0,T)N(h-\sigma_p) \qquad h = \frac{1}{\sigma_p}\ln\frac{LP(0,s)}{P(0,T)K} + \frac{\sigma_p}{2}$$

$$p_{m+1} = \sum_{j=-n_m}^{n_m} Q_{m,j}\exp\left[-(\alpha_m + j\triangle R)\triangle t\right] \text{ where } \alpha_m = \frac{\ln\sum_{j=-n_m}^{n_m} Q_{m,j}e^{-j\triangle R\triangle t} - \ln P_{m+1}}{\triangle t}$$

$$df(r)=\left[\theta(t)-af(r)\right]dt+\sigma dz$$

$$p_{m+1}=\sum_{j=-n_m}^{n_m} Q_{m,j}\exp\left[-g(\alpha_m+j\triangle x)\triangle t\right]$$

$$P(t,T)=\hat{A}(t,T)e^{-\hat{B}(t,T)R}$$

where $\ln\hat{A}(t,T)=\ln\frac{P(0,T)}{P(0,t)}-\frac{B(t,T)}{B(t,t+\triangle t)}\ln\frac{P(0,t+\triangle t)}{P(0,t)}-\frac{\sigma^2}{4a}(1-e^{-2at})B(t,T)\left[B(t,T)-B(t,t+\triangle t)\right]$

$$\hat{B}(t,T)=\frac{B(t,T)}{B(t,t+\triangle t)}\triangle t$$

$$dP(t,T)=r(t)P(t,T)dt+v(t,T,\Omega_t)P(t,T)dz(t)$$

where $P(t,T)=$ price at time t of a zero-coupon bond with principal $1 maturing at time T

$\Omega_t=$ vector of past and present values of interest rates and bond prices at time t that are relevant for determining bond price volatilities at that time

$v(t,T,\Omega_t)=$ volatility of $p(t,T)$ $f(t,T_1,T_2)$ forward rate as seen at time t for the period between time T_1 and T_2

$F(t,T)=$ instantaneous forward rate as seen at time t for a contract maturing at time T

$r(t)$ short-term risk-free interest rate at time t $dz(t)=$ Wiener process driving term structure movements

$$f(t,T_1,T_2)=\frac{\ln\left[p(t,T_1)\right]-\ln\left[p(t,T_2)\right]}{T_2-T_1}$$

$$df(t,T_1,T_2)=\frac{v(t,T_2,\Omega_t)^2}{2(T_2-T_1)}dt+\frac{v(t,T_1,\Omega_t)-v(t,T_2,\Omega_t)}{T_2-T_1}dz(t)$$

$$dF(t,T)=v(t,T,\Omega_t)v_T(t,T,\Omega_t)dt-v_T(t,T,\Omega_t)dz(t)$$

$$m(t,T,\Omega_t)=s(t,T,\Omega_t)\int_t^T s(t,\tau,\Omega_t)d\tau$$

where $m(t,T,\Omega_t)=$ instantaneous drift of $F(t,T)$ $s(t,T,\Omega_t)=$ standard deviation of $F(t,T)$

$$m(t,T,\Omega_t)=\sum_k s_k(t,T,\Omega_t)\int_t^T s_k(t,\tau,\Omega_t)d\tau$$

$$dF_k(t)=\xi_k(t)F_k(t)dz$$

where $F_k(t)=$ forward rate between time t_k and t_{k+1} as seen at time t

$m(t)=$ index for the next reset date at time t, smallest integer such that $t\le t_m(t)$

$\xi_k(t)=$ volatility of $F_k(t)$ at time t $\gamma_k(t)=$ volatility of the zero-coupon bond price $p(t,t_k)$ at time t

$$dF_k(x)=\xi_k(t)\left[v_{m(t)}(t)-v_{k+1}(t)\right]F_k(t)dt+\xi_k(t)F_k(t)dz$$

$$v_i(t) - v_{i+1}(t) = \frac{\delta_i F_i(t)\xi_i(t)}{1+\delta_i F_i(t)}$$

$$\frac{dF_k(t)}{F_k(t)} = \sum_{i=m(t)}^{k} \frac{\delta_i F_i(t)\xi_i(t)\xi_k(t)}{1+\delta_i F_i(t)} dt + \xi_k(t) dz$$

$$\sigma_k^2 t_k = \sum_{i=1}^{k} \Lambda_{k-i}^2 \delta_{i-1}$$

where $\Lambda_i =$ the value of $\xi_i(t)$ when there are i such accrual periods

$\xi_k(t) = \Lambda_{k-m(t)}$ is a step function

$$\frac{dF_k(t)}{F_k(t)} = \sum_{i=m(t)}^{k} \frac{\delta_i F_i(t)\Lambda_{i-m(t)}\Lambda_{k-m(t)}}{1+\delta_i F_i(t)} dt + \Lambda_{k-m(t)} dz$$

$$d\ln F_k(t) = \left[\sum_{i=m(t)}^{k} \frac{\delta_i F_i(t)\Lambda_{i-m(t)}\Lambda_{k-m(t)}}{1+\delta_i F_i(t)} - \frac{(\Lambda_{k-m(t)})^2}{2} \right] dt + \Lambda_{k-m(t)} dz$$

$$F_k(t_{j+1}) = F_k(t_j)\exp\left[(\sum_{i=j+1}^{k} \frac{\delta_i F_i(t)\Lambda_{i-j-1}\Lambda_{k-j-1}}{1+\delta_i F_i(t_j)} - \frac{\Lambda_{k-j-1}^2}{2})\delta_j + \Lambda_{k-j-1}\varepsilon\sqrt{\delta_j} \right]$$

where ε is a random sample $\varepsilon \sim N(0,1)$

$$\frac{dF_k(t)}{F_k(t)} = \sum_{i=m(t)}^{k} \frac{\delta_i F_i(t)\sum_{q=1}^{p}\xi_{i,q}(t)\xi_{k,q}(t)}{1+\delta_i F_i(t)} dt + \sum_{q=1}^{p}\xi_{k,q}(t) dz_q$$

$$F_k(t_{j+1}) = F_k(t_j)\exp\left[(\sum_{i=j+1}^{k} \frac{\delta_i F_i(t_j)\sum_{q=1}^{p}\lambda_{i-j-1,q}\lambda_{k-j-1,q}}{1+\delta_i F_i(t_j)} - \frac{\sum_{q=1}^{p}\lambda_{k-j-1,q}^2}{2})\delta_j + \sum_{q=1}^{p}\lambda_{k-j-1,q}\varepsilon_q\sqrt{\delta_j} \right]$$

$$V(t) = \sum_{q=1}^{p}\left[\sum_{k=0}^{N-1} \frac{\tau_k \beta_{k,q}(t) G_k(t)\gamma_k(t)}{1+\tau_k G_k(t)} \right]^2$$

where $\gamma_k(t) = \dfrac{\prod_{j=0}^{N-1}\left[1+\tau_j G_j(t)\right]}{\prod_{j=0}^{N-1}\left[1+\tau_j G_j(t)\right]-1} - \dfrac{\sum_{i=0}^{k-1}\tau_i \prod_{j=i+1}^{N-1}\left[1+\tau_j G_j(t)\right]}{\sum_{i=0}^{N-1}\tau_i \prod_{j=i+1}^{N}\left[1+\tau_j G_j(t)\right]}$

$$\sqrt{\frac{1}{T_0}\int_{t=0}^{T_0} V(t)dt} \qquad \sqrt{\frac{1}{T_0}\int_{t=0}^{T_0} \sum_{q=1}^{p}\left[\sum_{k=0}^{N-1} \frac{\tau_k \beta_{k,q(t)} G_k(0)\gamma_k(0)}{1+\tau_k G_k(0)} \right]^2 dt}$$

$$\sqrt{\frac{1}{T_0}\int_{t=0}^{T_0} \sum_{q=1}^{p}\left[\sum_{k=0}^{N-1}\sum_{m=1}^{M} \frac{\tau_{k,m} \beta_{k,m,q}(t) G_{k,m}(0)\gamma_k(0)}{1+\tau_{k,m} G_{k,m}(0)} \right]^2 dt}$$

$$\lambda_{j,q} = \frac{\Lambda_j \delta_q \alpha_{j,q}}{\sqrt{\sum_{q=1}^{p} s_q^2 \alpha_{i,q}^2}}$$

$$dF_i(t) = ... + \sum_{q=1}^{p} \xi_{i,q}(t) F_i(t)^{\alpha} \, dz_q$$

$$F_i + \frac{F_i^2 \sigma_i^2 \tau_i t_i}{1 + F_i \tau_i}$$

$$y_i - \tfrac{1}{2} y_i^2 \sigma_{y,i}^2 t_i \frac{G_i''(y_i)}{G_i'(y_i)} - \frac{y_i \tau_i F_i \rho_i \sigma_{y,i} \sigma_{F,i} t_i}{1 + F_i \tau_i}$$

$$V_i + V_i \rho_i \sigma_{W,i} \sigma_{V,i} t_i$$

$$\frac{QL}{n_2} P(0, s_i) N(d_2^*)$$

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best response: $\pi_i(s_i^*, s_{-i}) \geq \pi_i(s_i^{'}, s_{-i}) \forall s_i^{'} \neq s_i^*$

dominated strategy: $\pi_i(s_i^d, s_{-i}) < \pi_i(s_i^{'}, s_{-i}) \forall s_{-i}$

dominant strategy: $\pi_i(s_i^*, s_{-i}) > \pi_i(s_i^{'}, s_{-i}) \forall s_{-i,} \forall s_i^{'} \neq s_i^*$

weakly dominated: $\pi_i(s_i^{''}, s_{-i}) \geq \pi_i(s_i^{'}, s_{-i}) \forall s_{-i} \text{ and } \pi_i(s_i^{''}, s_{-i}) > \pi_i(s_i^{'}, s_{-i}) \text{ for some } s_{-i}$

Nash equilibrium: $\forall i, \ \pi_i(s_i^*, s_{-i}^*) \geq \pi_i(s_i^{'}, s_{-i}^*) \forall s_i^{'}$

pure strategy: $s_i : \omega_i \to a_i$

mixed strategy: $s_i : \omega_i \to m(a_i) \quad where \quad m \geq 0 \quad \int_{A_i} m(a_i) da_i = 1$

completely mixed: $m > 0$

minimax strategies: $\min imize_{s_{-i}} \max imize_{s_i} \pi_i(s_i, s_{-i})$

maximin strategies: $\max imize_{s_i} \min imize_{s_{-i}} \pi_i(s_i, s_{-i})$

$$U(e, w(e)) = \bar{U}$$

$$\max_{e} imize V(q(e) - \tilde{w}(e))$$

$$V'(q(e)-\tilde{w}(e))(\frac{\partial q}{\partial e}-\frac{\partial \tilde{w}}{\partial e})=0$$

$$\frac{\partial q}{\partial e}=\frac{\partial \tilde{w}}{\partial e}$$

$$\frac{\partial \tilde{w}}{\partial e}=-(\frac{\partial U/\partial e}{\partial U/\partial \tilde{w}})$$

$$(\frac{\partial U}{\partial \tilde{w}})(\frac{\partial q}{\partial e})=-(\frac{\partial U}{\partial e})$$

$$U(e^*,q(e^*))=\bar{U}$$

$$\underset{e}{\max} imize U(e,q(e))$$

$$\frac{\partial U}{\partial e}+(\frac{\partial U}{\partial q})(\frac{\partial q}{\partial e})=0$$

$$(\frac{\partial U}{\partial w})(\frac{\partial q}{\partial e})=-\frac{\partial U}{\partial e}$$

$$\underset{w(.)}{\max} imize\, EV(q(\tilde{e},\theta)-w(q(\tilde{e},\theta)))$$

subject to $\tilde{e}=avg\underset{e}{\max}\, EU(e,w(q(e,\theta)))$

$$EU(\tilde{e},w(q(\tilde{e},\theta)))\geq\bar{U}$$

$$C(\tilde{e})=\underset{w(.)}{\min} imum\, Ew(q(\tilde{e},\theta))$$

$$\underset{\tilde{e}}{\max} imize\, EV(q(\tilde{e},\theta)-C(\tilde{e}))$$

$$U(not\,investigate)\leq U(investigate)$$

$$\theta\log(w_1)+(1-\theta)\log(w_2)\leq\left[1-(1-\theta)^2\right]\log(w_1)+(1-\theta)^2\log(w_2)-\alpha$$

$$\theta(1-\theta)\log(\frac{w_1}{w_2})=\alpha$$

$$\log(\bar{w})=\left[1-(1-\theta)^2\right]\log(w_1)+(1-\theta)^2\log(w_2)-\alpha$$

$$w_1=\bar{w}e^{\alpha/\theta}\quad w_2=\bar{w}e^{-\alpha/(1-\theta)}$$

$$\left[1-(1-\theta)^2\right]\bar{w}e^{\alpha/\theta}+(1-\theta)^2\bar{w}e^{-\alpha/1-\theta}$$

$$\bar{\theta}(P) = E\left[\theta \middle| (1+\varepsilon)\theta \le P\right]$$

FET-101-07 None

FET-102-07

$$F = \sum_i \max\left(S_{i0}, S_{iT}\right) = \sum_i S_{iT} + \sum_i \max\left(0, S_{i0} - S_{iT}\right)$$

$$F = \max\left(\sum_i S_{i0}, \sum_i S_{iT}\right) = \sum_i S_{iT} + \max\left(0, \sum_i \left(S_{i0} - S_{iT}\right)\right)$$

FET-105-07 None

FET-106-07

$$dS = \mu S dt + \sigma S dZ$$

$$dr = \mu(r,t) r dt + r\sigma dZ$$

$$\sigma(t,T) = \frac{\sigma\left(\dfrac{\Delta r(t,T)}{r(t,T)}\right)}{\sqrt{\Delta t}}$$

$$\sigma(t,T) = \frac{\sigma(\Delta r(t,T))}{\sqrt{\Delta t}}$$

$$dr = a(b-r)dt + \sigma\sqrt{r}dZ$$

$dr = a(b-r)dt + \sigma dZ$, (a>0)

$$dr = a_1 + b_1(l-r)dt + r\sigma_1 dZ$$

$$dl = (a_2 + b_2 r + c_2 l)dt + l\sigma_2 dW$$

$$dV = M(t,r)dt + \Omega(t,r)dZ$$

$$M(t,r) = V_t + \mu(t,r)V_r + \frac{1}{2}\sigma(t,r)^2 V_{rr}$$

$$\Omega(t,r) = \sigma(t,r)V_r$$

$$d\Pi = (M_1(t,r) - \Delta M_2(t,r))dt + (\Omega_1(t,r) - \Delta\Omega_2(t,r))dZ$$

$$d\Pi = r\Pi dt$$

$$V_t + (\mu(t,r) - \lambda(t,r)\sigma(t,r))V_r + \frac{1}{2}\sigma(t,r)^2 V_{rr} - rV = 0$$

$$P_i^n(1) = 2\left[\frac{P(n+1)}{P(n)}\right]\frac{\delta^i}{(1+\delta^n)} \qquad \delta = e^{-2r(1)\sigma}$$

$$P_i^n(T) = \frac{1}{2}P_i^n(1)\left\{P_i^{n+1}(T-1) + P_{i+1}^{n+1}(T-1)\right\}$$

$$r_i^n(1) = \ln\frac{P(n)}{P(n+1)} + \ln\left(\frac{1}{2}\left(\delta^{\frac{-n}{2}} + \delta^{\frac{n}{2}}\right)\right) + \left(\frac{n}{2} - i\right)\ln\delta$$

Note: Typo in text $r_i^n(1)1 =$ either way will receive full credit.

$$dr = (f^{'}(0,t) + \sigma^2 t)dt + \sigma dz$$

$$r(n)\sigma^s(n) = \frac{-\frac{1}{2}\ln\left[\delta(n)\delta(n-1)...\delta(1)\right]}{n}$$

$$P_i^n(1) = \left[\frac{P(n+1)}{P(n)}\right]\left[\tfrac{(1+\delta_{n-1}\delta_{n-2}..\delta_1)..(1+\delta_{n-1})2}{(1+\delta_n..\delta_1)..(1+\delta_n)}\right]\delta_n^i$$

$$dr = (f^{'}(0,t) + \sigma^2(t)t + \tfrac{\sigma^{'}(t)}{\sigma(t)}\left[r(t) - f(0,t)\right])dt + \sigma(t)dZ$$

$$P_{i,j}^n(1) = \frac{P(n+1)}{P(n)}\frac{(1+\delta_{n-1}^1..\delta_1^1)(1+\delta_{n-1}^1..\delta_2^1)..(1+\delta_{n+1}^1)2}{(1+\delta_n^1..\delta_1^1)..(1+\delta_n^1\delta_{n-1}^1)(1+\delta_n^1)} \times \frac{(1+\delta_{n-1}^2..\delta_1^2)(1+\delta_{n-1}^2..\delta_2^2)..(1+\delta_{n-1}^2)2}{(1+\delta_n^2..\delta_1^2)(1+\delta_n^2..\delta_2^2)..(1+\delta_n^2)}(\delta_n^1)^i(\delta_n^2)^j$$

$$dr = \left\{f^{'}(t) + \left|\sigma(t)\right|^2 t + \frac{\left|\sigma^{'}(t)\right|\cos\phi(t)}{\left|\sigma(t)\right|\cos\theta(t)}\left[r - f(t)\right]\right\}dt + \sigma(t)dW$$

$$d\ln r = (\theta(t) - \frac{\sigma^{'}(t)}{\sigma(t)}\ln r)dt + \sigma(t)dW$$

$$dr(t) = (\alpha(t) - \beta r(t))dt + \sigma dW(t) \quad \text{where } \alpha(t) = \frac{\partial f(0,t)}{\partial T^*} + \beta f(0,t) + \frac{\sigma^2}{2\beta}(1 - e^{-2\beta t})$$

$$dr = \left[\theta(t) + \mu - ar\right]dt + \sigma_1 dW \qquad du = -budt + \sigma_2 dZ$$

$$dP(t,T^*) = r(t)P(t,T^*)dt + \sigma^p(t,T^*)P(t,T^*)dZ$$

$$df(t,T^*) = \sigma^p(t,T^*)\sigma_{T^*}^p(t,T^*)dt - \sigma_{T^*}^p(t,T^*)dZ$$

$$dP(t,T^*) = r(t)P(t,T^*)dt + \sigma(T^* - t)P(t,T^*)dZ(t,T^*)$$

$$L(t,T^*) = \frac{1}{\triangle}\left(\frac{P(t,T^*)}{P(t,T^* + \triangle)} - 1\right)$$

$$dL(t,T^*) = L(t,T^*)\left[\sum_{j=t^*}^{N^*} \frac{L(t,j\Delta)\Delta}{1+L(t,j\Delta)\Delta}\Lambda(T^* - j\Delta)\Lambda(T^* - t)dt + \Lambda(T^* - t)dZ\right]$$

$$L(k,j+1) = L(k,j)\exp\left[\left(\sum_{i=j+1}^{k} \frac{L(i,j)\Delta}{1+L(i,j)\Delta}\Lambda_{i-j-1}\Lambda_{k-j-1} - \frac{\Lambda^2_{k-j-1}}{2}\right)\Delta + \Lambda_{k-j-1}\sqrt{\Delta}\tilde{Z}\right]$$

where $\sigma^2_j j = \sum_{i=1}^{j} \Lambda^2_{j-i}$

caplet $C_k = L\delta_k P(t_{k+1})\left[F_k N(d_1) - R_x N(d_2)\right]$

where $d_1 = \dfrac{\ln\left[\dfrac{F_k}{R_X}\right] + \sigma_k^2 \dfrac{t_k}{2}}{\sigma_k\sqrt{t_k}} \qquad d_2 = d_1 - \sigma_k\sqrt{t_k}$

$$\text{swaption} = \sum_{i=1}^{mn} \frac{L}{m} P(t_i)\left[R_F N(d_1) - R_X N(d_2)\right] = L * A\left[R_F N(d_1) - R_X N(d_2)\right]$$

where $A = \dfrac{1}{m}\sum_{i=1}^{mn} P(t_i) \quad 1 \le i \le mn$

$$P(k+1,j) = P(k,j)\exp\left[\left(r(k) - \frac{\sigma^2(j-k)}{2}\right)\Delta + \sigma(j-k)\sqrt{\Delta}Z(j-k)\right]$$

$$\sigma^*(T^* - t) = (a + b(T^* - t))\exp(-c(T^* - t)) + d$$

$$L(k,j+1) = L(k,j)\exp\left[\left(\sum_{i=j+1}^{k} \frac{L(i,j)\Delta}{1+L(i,j)\Delta}\Lambda_{i-j-1}\Lambda_{k-j-1} - \frac{\Lambda^2_{k-j-1}}{2}\right)\Delta + \Lambda_{k-j-1}\sqrt{\Delta}Z\right]$$

$$P(T^*,i;T) = \frac{P(T^*+T)}{P(T^*)} \cdot 2 \cdot \frac{\prod_{t=T}^{T+T^*-1} h(t)}{\prod_{t=1}^{T^*-1} h(t)} \delta^{Ti} \quad \text{where } h(t) = \frac{1}{1+\delta^t}$$

FET-108-07

$$V(E) = V(F) - V(D) = V(F) - D_{DF} + P(V(F),D) = C(V(F),D)$$

$$V^*(F) = S + D(1 + \frac{m}{n})$$

$$V_R^{'}(E) = -C + V_R(F) - D + P\left\{V_R(F),D\right\} = -C + V_R - D + P_R$$
$$V_N^{'}(E) = V_N(F) - D + P\left\{V_N(F),D\right\} = V_N - D + P_N$$

$$face\,value + principal\ \ forgiven - default\ put\,assu\min g\,reinvestment = D - (P_N - P_R - NPV) - P_R$$
$$= (D - P_N - B - NPV) + B = value\,of\ regular\,debt + saving\,in\,bankruptcy\cos t$$

FET-109-07

$$RBC = \frac{1}{2}\left[C_0 + C_{4a} + \left[(C_1 + C_{3a})^2 + C_2^2 + C_{3b}^2 + C_{4b}^2 \right]^{\frac{1}{2}} \right]$$

FET-112-07 None

FET-113-07

$$\sigma_Y^2 = \sum_{i=1}^{n} \sigma_{x_i}^2 = \sum_{i=1}^{n} \omega_i^2 \sigma_i^2$$

$$MCaR = k\sigma_r = k\sqrt{\sum_{i=1}^{n} \omega_i^2 \sigma_i^2} = \sqrt{\sum_{i=1}^{n} k^2 \omega_i^2 \sigma_i^2} = \sqrt{\sum_{i=1}^{n} DCaR_i^2}$$

$$Total\, CaR = \sqrt{\sum_{i=1}^{n} CaR_i^2 + \sum_{i=1}^{n} \sum_{i \neq j} CaR_i CaR_j \rho_{ij}}$$

FET-114-07

$$\begin{aligned} NPV &= (1-d)V\left\{S^+\right\} - (C-\mu) - (1+m)V\left\{S^-\right\} \\ &= \mu - (dV\left\{S^+\right\} + mV\left\{S^-\right\}) \end{aligned}$$

$$V\left\{S^+\right\} = \frac{\sigma(n(z) + zN(z))}{(1+r)} \qquad V\left\{S^-\right\} = \frac{\sigma(n(z) - zN(-z))}{(1+r)}$$

FET-115-08 None

FET-138-07

$$c = \int_{W^*}^{\infty} f\left(w\right) dw$$

$$VAR = W_0 \times \alpha\sigma\sqrt{\Delta t}$$

$$se\left(\hat{q}\right) = \sqrt{\frac{c\left(1-c\right)}{T\ f\left(q\right)^2}}$$

FET-139-07 None

FET-141-08 None

FET-142-08 None

FET-143-08

$$Haircut = \frac{XC}{XC + LL}$$ size of the losses

where $XC =$ sum of excess capital

$LL =$ remaining liquid / surrender-able liabilities= total amount of available assets (AA) in excess of 200% RBC available to meet any remaining liquidity demands

FET-144-08

$$leverage = \frac{senior\ debt + excess\ hybrid\ debt\ and\ preferred\ stock}{ECA + senior\ debt + hybrid\ debt + preferred\ stock}$$

$$Hybrid\ Ratio_{U.S.} = \frac{s\tan dard\ \&\ pool's\ qualifying\ hybrid}{U.S.GAAP(consolidated)\ capital + total\ hybrid + total\ senior\ debt}$$

$$Hybrid\ Ratio_{Eurpoe} = \frac{s\tan dard\ \&\ pool's\ qualifying\ hybrid}{Group\ Consolidated\ TAC(excluding\ hybrid) + total\ hybrid + total\ senior\ debt}$$

$$Double\ leverage_{U.S.} = \frac{s\tan dard\ \&\ pool's\ qualifying\ hybrid + total\ senior\ debt + nonqualifying\ hybrid}{U.S.GAAP(consolidated)\ capital + total\ hybrid + total\ senior\ debt}$$

$$Double\ leverage_{Eurpoe} = \frac{s\tan dard\ \&\ pool's\ qualifying\ hybrid}{Group\ Consolidated\ TAC(excluding\ hybrid) + regulatory\ qualifying\ hybrid\ capital}$$

FET-145-08

(equity +franchise) * total shareholder return
=increase in net assets + increase in franchise value + dividend
=increase in franchise value + retained profit + dividend
=franchise * franchise growth rate + equity * return on equity

return on equity = total shareholder return + franchise/equity *(total shareholder return – franchise growth rate)

$$E_0 + F_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+COE)^t} = \sum_{t=1}^{\infty} \frac{D_t + E_t - E_{t-1} - COE * E_{t-1}}{(1+COE)^t} - \sum_{t=1}^{\infty} \frac{E_t}{(1+COE)^t} + \sum_{t=1}^{\infty} \frac{E_{t-1}}{(1+COE)^{t-1}}$$

$$F_0 = \sum_{t=1}^{\infty} \frac{ROE_t - COE}{(1+COE)^t} E_{t-1}$$

$$(1+R_f)\left\{A_0 - L_0 + F_0\right\} = A_0 - L_0 + (1-k_T)\left\{A_0(R_f + m_A - k_A) - L_0(R_f - m_L + k_L)\right\} + F_1$$

where $A_t =$ balance sheet assets at time t $R_A =$ actual asset return

$L_t =$ balance sheet liabilities at time t $R_L =$ actual liability return

$F_t =$ franchise value $R_f =$ risk-free rate

$k_A =$ asset-related expenses as a proportion of A_0 $k_L =$ liability-related expenses as a proportion of L_0

$k_T =$ tax paid as a proportion of pre-tax profit $m_A =$ margin above LIBOR as asset swap

$m_L =$ margin below LIBOR as liability swap

$$(1+R_f)F_0 = (1-k_T)\left\{A_0(m_A - k_A) + L_0(m_L - k_L)\right\} - k_T R_f (A_0 - L_0) + F_1$$

$$(1+R_f)F_0 = A_0(1-k_T)(m_A-k_A)+L_0(1-k_T)(m_L-k_L)+(1-s)F_1-(R_f+s)k_T(A_0-L_0)$$

$$(1+R_f)(A_0-L_0+F_0) = A_0\left\{1+(1-k_T)(R_f-m_A-k_A)\right\}-L_0\left\{1+(1-k_T)(R_f-m_L+k_L)\right\}+(1-s)F_1-sk_T(A_0-L_0)$$

$$(R_f+s-g+sg)F_0 = A_0(1-k_T)(m_A-k_A)+L_0(1-k_T)(m_L-k_L)-(R_f+s)k_T(A_0-L_0)$$

FET-146-08

$D_L = \sum_{x>A} p(x)(x-A)$ where $p(x) =$ probability density for losses $(0 \le x \le \infty)$

$D_A = \sum_{L>y} q(y)(L-y)$ where $q(y) =$ probability density for losses $(0 \le y \le \infty)$

$$D_L = \int_A^\infty (x-A)p(x)dx$$

$$D_A = \int_0^L (L-y)q(y)dy$$

$$d_L = \frac{D_L}{L} = k\phi\left[\frac{-c}{k}\right] - c\Phi\left[\frac{-c}{k}\right]$$

$$d_A = \frac{D_A}{L} = \frac{1}{1-c_A}\left[k_A\phi(\frac{-c}{k_A}) - c_A\Phi(\frac{-c_A}{k_A})\right]$$

where $k_L =$ the cv of losses $k_A =$ the cv of assets $c_A =$ capital / assets ratio

$\Phi(x) =$ the cumulative standard normal distribution $\phi(x) =$ the standard normal density function

$$d_L = \Phi(a) - (1+c)\Phi(a-k)$$

$$d_A = \Phi(b) - \frac{\Phi(b-k_A)}{1-c_A}$$

where $a = ({k}/{2}) - ({\ln(1+c)}/{k})$ $b = ({k_A}/{2}) + ({\ln(1-c_A)}/{k_A})$

$\Phi(x) =$ the cumulative normal distribution

one-period expected policy-holder deficit ratio

$d_1 = \int_{-\infty}^{0} -zp(z)dz$ where $p(z) =$ the density of $\tilde{c}_1$

$\bar{C}_1 =$ the amount of capital at the end of one period

$\tilde{c}_1 = {\tilde{C}_1}/{L_0}$ the amount of capital relative to the original expected loss

$$\bar{c}_1 = c(1+p) + \left[1+c(1+p)\right]\tilde{r} + pc\tilde{b} - \tilde{g}$$

where $\tilde{r}$ *and* $\tilde{g}$ are random variables denoting the annual return on assets and annual rate of change in value of the liabilities
$\tilde{b} =$ incurred loss ratio

$$C = \left[\sum_{i=1}^{n} c_i^2 + \sum_{i \neq j}^{n} \rho_{ij} c_i c_j \right]^{1/2} \text{ total capital}$$

$$D = \sigma\phi(\frac{-\mu}{\sigma}) - \mu\Phi(\frac{-\mu}{\sigma})$$

$$d = \frac{D}{L} = k_T\phi(\frac{-c}{k_T}) - c\Phi(\frac{-c}{k_T})$$

$$d_L = k\phi(\frac{-c}{k}) - c\Phi(\frac{-c}{k})$$

$$d_A = \frac{D_A}{L} = \frac{1}{1-c_A}\left[k_A\phi(\frac{-c_A}{k_A}) - c_A\Phi(\frac{-c_A}{k_A}) \right]$$

$$F = S\Phi(a) - Ee^{-it}\Phi(a - \sigma\sqrt{t})$$

where $a = \dfrac{\ln(S/E) + (i + \sigma^2/2)t}{\sigma\sqrt{t}}$ $\quad S =$ stock price $\quad E =$ exercise price

$$D_L = L\Phi(a) - (1+c)L\Phi(a - \sigma_L)$$

$$d_L = \Phi(a) - (1+c)\Phi(a-k)$$

$$D_L' = A\Phi(a') - L\Phi(a' - \sigma_A)$$

$$d_A = \Phi(b) - \frac{\Phi(b-k_A)}{1-c_A} \qquad \text{where } b = (k_A/2) + (\ln(1-c_A)/k_A)$$

FET-147-08 None

FET-148-08 None

FET-149-08 None

FET-150-08 None

FET-151-08

default put option $V(E) = F + V(A_T) - PV(L) + O$
where $V() =$ market value $\quad PV() =$ the present value $\quad E =$ owner's equity $\quad A =$ the assets $\quad L =$ liabilities
$F =$ the franchise value $\quad A_T =$ tangible assets $\quad O =$ the default put option

FET-152-08 None

FET-153-08 None

FET-154-08 None

FET-155-08

$$\int_{\xi_\rho}^{\infty} \frac{wf(w)dw}{1-\Phi(\xi_\rho)} = CTE(\rho)$$

FET-156-08 None

FET-157-08

$$E_{r_i} = r_f + \beta_i (E_{r_M} - r_F)$$

FET-158-08

$$D(t,T) = \frac{1}{e^{s(t,T)\times(T-t)}} = \frac{1}{e^{\phi(T-t)\times(T-t)}} E\left[\frac{1}{e^{\int_t^T r_s ds}}\right]$$

$$r_s^* = r_s + \phi(s-t) + \phi^{'}(s-t)\times(s-t)$$

$$D(t,T) = \frac{1}{e^{s(t,T)\times(T-t)}} = E\left[\frac{1}{e^{\int_t^T (r_s+\phi(T-t))ds}}\right] = E\left[\frac{1}{e^{\int_t^T r_s^* ds}}\right]$$

FET-159-08

$$dr = (k\theta - (k+\lambda)r)dt + \sigma\sqrt{r}dw^* \quad \text{where } w^*(t) = w(t) + \int_0^t \frac{\lambda}{\sigma}\sqrt{r(s)}ds$$

$$\frac{p(t,TB)}{B(t)} = E^*\left[\frac{1}{B(TB)}\right] \qquad p(t,TB) = E^*\left[\exp(-\int_t^{TB} r(s)ds)\right]$$

$$p(t,TB) = A(t,TB)\exp(-r(t)G(t,TB))$$

$$A(t,TB) = \left[\frac{2\gamma\exp\left[(b+\gamma)\frac{TB-t}{2}\right]}{(\gamma+b)(\exp(\gamma(TB-t))-1)+2\gamma}\right]^{\frac{2c}{\sigma^2}}$$

$$G(t,TB) = \frac{2(\exp(\gamma(TB-t))-1)}{(\gamma+b)(\exp(\gamma(TB-t))-1)+2\gamma} \quad \text{where } b = k+\lambda \quad c = k\theta \quad \gamma = \sqrt{b^2+2\sigma^2}$$

$$C(t) = p(t,TB)\chi^2(2\gamma^*(\varphi+\psi+G(T,TB)), \frac{4c}{\sigma^2}, \frac{2\varphi^2 re^{\gamma(T-t)}}{(\varphi+\psi+G(T,TB))}) -$$

$$Xp(t,T)\chi^2\left[2r^*(\varphi+\psi),\frac{4c}{\sigma^2},\frac{2\varphi^2 re^{\gamma(T-t)}}{(\varphi+\psi)}\right]$$

$$dr=(\phi(t)-\alpha(t)r)dt+\sigma(t)dw^{**} \qquad \phi(t)=\theta(t)+\alpha(t)b-\lambda(t)\sigma(t)$$

$$\frac{x(t)}{B(t)}=E^{**}\left[\frac{x(\tau)}{B(\tau)}\right]$$

$$p(t,TB)=E^{**}\left[\exp(-\int_t^{TB} r(s)ds)\right]$$

$$\alpha(t)=\frac{-\partial^2 G(0,t)\big/\partial t^2}{\partial G(0,t)\big/\partial t}$$

$$\phi(t)=-\alpha(t)\frac{\partial F(0,t)}{\partial t}-\frac{\partial^2 F(0,t)}{\partial t^2}+\left[\frac{\partial G(0,t)}{\partial t}\right]^2\int_0^t\left[\frac{\sigma(\tau)}{\partial G(0,\tau)\big/\partial\tau}\right]^2 d\tau$$

$$C(t)=P(t,TB)N(h)-XP(t,T)N(h-\sigma_p)$$

$$\text{where } h=(\frac{\sigma_p}{2})+(\frac{1}{\sigma_p})\ln\left[\frac{P(t,TB)}{(XP(t,T))}\right]$$

$$\sigma_p^2=\left[G(0,TB)-G(0,T)\right]^2\int_t^T\left[\frac{\sigma(\tau)}{\partial G(0,\tau)/\partial\tau}\right]^2 d\tau$$

$$A(0,t)=\left[\frac{2\gamma\exp\left[(b+\gamma)\frac{t}{2}\right]}{(\gamma+b)(\exp(\gamma t)-1)+2\gamma}\right]^{\frac{2c}{\sigma^2}}$$

$$G(0,t)=\frac{2(\exp(\gamma t)-1)}{(\gamma+b)(\exp(\gamma t)-1)+2\gamma} \qquad \text{where } b=k+\lambda \quad c=k\theta \quad \gamma=\sqrt{b^2+2\sigma^2}$$

$$P\max(P(0,TB),M(0),0,T)=E^*\left[\frac{M(T)}{B(T)}\right]-P(0,TB)=E^*\left[M(T)(\exp(-\int_0^T r(s)ds))\right]-P(0,TB)$$

$$r_{t_i}=r_{t_{i-1}}+(k\theta-(k+\lambda)r_{t_{i-1}})(t_i-t_{i-1})+\sigma\sqrt{r_{t_{i-1}}}\sqrt{(t_i-t_{i-1})}\tilde{\varepsilon}$$

$$PM\widehat{A}X(P(0,TB),M(0),0,T)=(\left\{\frac{1}{N}\sum_{n=1}^{N}M_n(T)\exp\left[-\sum_{i=1}^{m}r_n(t_{i-1})(t_i-t_{i-1})\right]\right\}-P(0,TB))$$

$$PMAX\,(P(0,TB),M(0),0,T) = E^{**}\left[\frac{M(T)}{B(T)}\right] - P(0,TB) = \quad E^{**}\left[M(T)(\exp(-\int_0^T r(s)ds))\right] - P(0,TB)$$

$$r_{t_i} = r_{t_{i-1}} + (\phi(t_{i-1}) - \alpha(t_{i-1})r_{t_{i-1}})(t_i - t_{i-1}) + \sigma\sqrt{r(0)}\sqrt{t_i - t_{i-1}}\,\tilde{\varepsilon}$$

FET-160-08 None

FET-161-08 None

FET-162-08 None

FET-163-08

share value of taking the project = $\dfrac{PV\ of\ assets\ in\ place + PV\ of\ new\ investment}{number\ of\ original\ shares + number\ of\ new\ shares}$

share value of not taking the project = $\dfrac{PV\ of\ assets\ in\ place}{number\ of\ original\ shares}$

share value : financing project with riskless debt = $\dfrac{value\ of\ original\ assets + NPV\ of\ new\ project}{number\ of\ shares}$

FET-164-08

$Risk - weighted\ amount = \sum Assets * WA + \sum credit\ equivalent * WCE$ where $WA =$
risk capital weighted by asset categories
$WCE =$ weighted by credit equivalents by type of counter party

FET-165-08

$$T = (E + P)(1 + r_i) - L$$

$$E(T) = (E + P - R(I,S))(1 + E(r_i)) - E(L(a) + hC(I,S) - a$$

$$\frac{\partial E(T)}{\partial a} = \frac{-\partial E(L(a))}{\partial a} - 1 + h\frac{\partial C}{\partial I}\frac{\partial I}{\partial L}\frac{\partial L}{\partial a} = 0$$

$$T = \left\{E + D\right\}(1 + E(r)) - E(L(a)) - D(1 + r) + hC(I,S) - a$$

$$\frac{\partial E(T)}{\partial a} = -\frac{\partial E(L(a))}{\partial a} - 1 + h\frac{\partial C}{\partial I}\frac{\partial I}{\partial L}\frac{\partial L}{\partial a} = 0$$

Recommended Approach for Setting Regulatory Risk-Based Capital Requirements for Variable Annuities and Similar Products,

None

Smith, Investor & Management Expectations of the "Return on Equity" Measure vs. Some Basic Truths of Financial Accounting

$$E_t - EV_t = \sum_{x=1}^{t}\left[(ROE_x - IRR) * E_{x-1} * (1+IRR)^{(t-x)}\right]$$

where $E_t =$ equity at time t $EV_t =$ embedded value at time t, using discount rate IRR

$IRR =$ pricing internal rate of return after target surplus

$ROE_x =$ return on equity at time $x =$ earnings in period $x \big/ E_{x-1}$

Bodoff, Capital Allocation by Percentile Layer

percentile layer of capital $(\alpha, \alpha + j) =$ required capital at percentile $(\alpha + j) -$ required capital at percentile (α)

layer of capital $(a, a+b) =$ capital equal to amount $(a+b) -$ capital equal to amount (a)

$VaR(x) =$ total required capital $= \sum_{\alpha=0}^{k-j}\left[x(\alpha + j) - x(\alpha)\right]$

$x(\alpha) =$ loss amount at percentile α $j =$ selected percentile increment

$$\int_{x=y}^{x=\infty} f(x) \big/ (1-F(y))\, dx$$ where $x =$ loss amount $y =$ the capital

$$\int_{y=0}^{y=VaR(99\%)} \int_{x=y}^{x=\infty} f(x) \big/ 1-F(y)\, dxdy$$

$$\int_{y=0}^{y=x} f(x) \big/ (1-F(y))\, dy$$

$$\int_{y=0}^{y=VaR(99\%)} f(x) \big/ (1-F(y))\, dy$$

$$\int_{x=x(0\%)}^{x=\infty} \int_{y=0}^{y=\min(x, VaR(99\%))} f(x) \big/ (1-F(y))\, dydx$$

Allocated capital to loss event x $AC(x) = \int_{y=0}^{y=x} f(x) \big/ (1-F(y))\, dy$

$$AC(x) = f(x)\int_{y=0}^{y=x} 1 \big/ (1-F(y))\, dy$$

$$AC(x) = f(x) \int_{y=0}^{y=VaR(99\%)} 1 \big/ (1-F(y))\, dy$$

$$\frac{d}{dx\{AC(x)\}} = \frac{d}{dx\left\{ f(x)\int\limits_{y=0}^{y=x} {}^{1}\!\big/_{(1-F(y))}\,dy \right\}} = f(x)\frac{d}{\left\{dx\int\limits_{y=0}^{y=x} {}^{1}\!\big/_{(1-F(y))}\,dy\right\} + \int\limits_{y=0}^{y=x} {}^{1}\!\big/_{(1-F(y))}\,dy\;{}^{d}\!\big/_{dx\{f(x)\}}} =$$

$${}^{f(x)}\!\big/_{(1-F(x))} + \int\limits_{y=0}^{y=x} {}^{1}\!\big/_{(1-F(y))}\,dyf'(x)$$

$$rf(x)\int\limits_{y=0}^{y=x} {}^{1}\!\big/_{(1-F(y))}\,dy \quad r = \text{required rate of return on capital}$$

$$r\int\limits_{y=0}^{y=x} {}^{1}\!\big/_{(1-F(y))}\,dy$$

$$x + r\int\limits_{y=0}^{y=x} {}^{1}\!\big/_{(1-F(y))}\,dy$$

$$x\left[1 + r({}^{1}\!\big/_{x})\int\limits_{y=0}^{y=x} {}^{1}\!\big/_{(1-F(y))}\,dy\right]$$

premium net of expenses = expected loss + cost of capital

$$P = E[L] + r*(allocated\ capital - contributed\ capital)$$

where $P =$ premium net of expenses $E[L] =$ expected loss $r =$ required rate of return on capital

$$P = E[L] + {}^{r}\!\big/_{(1+r)} * (allocated\ capital - E[L])$$

$$P(x) = xf(x) + {}^{r}\!\big/_{1+r}\left[f(x)\int\limits_{y=0}^{y=x} {}^{1}\!\big/_{(1-F(y))}\,dy - xf(x)\right]$$

$$P(x) = f(x)\left\{x + {}^{r}\!\big/_{1+r}\left[\int\limits_{y=0}^{y=x} {}^{1}\!\big/_{(1-F(y))}\,dy - x\right]\right\}$$

$$x + {}^{r}\!\big/_{(1+r)}\left[\int\limits_{y=0}^{y=x} {}^{1}\!\big/_{(1-F(y))}\,dy - x\right]$$

$$P(x) = xf(x)\left\{1 + {}^{r}\!\big/_{(1+r)}\left[({}^{1}\!\big/_{x})\int\limits_{y=0}^{y=x} {}^{1}\!\big/_{(1-F(y))}\,dy - 1\right]\right\}$$

$$(r/(1+r))(\int_{y=0}^{y=x} 1/(1-F(y))\,dy - x)$$

$$AC(x) = (1/\theta)\exp(-x/\theta)\int_{y=0}^{y=x}\exp(x/\theta)\,dy$$

$$AC(x) = 1 - \exp(-x/\theta)$$

$$d/dx\{AC(x)\} = (1/\theta)\exp(-x/\theta)$$

$$1 + r(1/x)\theta(\exp(x/\theta) - 1)$$

Hardy, Freeland and Till, Valuation of Long-Term Equity Return Models for Equity-Linked Guarantees

$$Y_t \left| F_{t-1} = \mu + \sigma_t z_t \right. \text{ where } z_t \approx N(0,1), \forall t$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 (Y_{t-1} - \mu)^2 + \beta\sigma_{t-1}^2$$

$$Y_t \left| F_{t-1} = Q_1 \ w.p.q \qquad = Q_2 \ w.p.(1-q) \right.$$

$$\text{where } Q_1 \left| F_{t-1} = \mu_1 + \sigma_t z_t \right. \qquad \sigma_t^2 = \alpha_{1,0} + \alpha_{1,1}(Y_{t-1} - \mu_1)^2 + \alpha_{1,2}(Y_{t-2} - \mu_1)^2$$

$$Q_2 \left| F_{t-1} = \mu_2 + \alpha_{2,0} z_t \right.$$

$$r_{t,1} = r_t \left| (\rho_t = 1) = \frac{y_t - \mu_1}{\sigma_1} \right.$$

$$r_{t,2} = r_t \left| (\rho_t = 2) = \frac{y_t - \mu_2}{\sigma_2} \right.$$

$$r_t = I_{\{P(1)>0.5\}} r_{t,1} + \left(1 - I_{\{P(1)>0.5\}}\right) r_{t,2}$$

****BEGINNING OF EXAMINATION****
Morning Session

1. *(4 points)* You are the Chief Information Officer of the Sherwood Forest Insurance Company. You have 40 system analysts available to allocate to your projects. Your current approach is to allocate analysts equally to the available projects. Rich & Poor's Consulting firm has approached you to offer their services in allocating the analysts more effectively. You are given the following:

The probability of an analyst being skilled for the project is 85%.

Outcome	Utility $= U(a, e)$
Analyst skilled for the project	10,000 profit
Analyst not skilled for the project	2,000 loss
Shortage (not enough analysts allocated)	0
Excess (no work for analysts allocated)	1,000 loss

Project	Analysts required
A	15
B	25

The cost of Rich & Poor's "Report 1" which perfectly matches the supply and demand of analysts to each project is 40,000.

The cost of Rich & Poor's "Report 2" which perfectly matches the supply and demand of analysts to each project and also perfectly matches the analyst skill to each project is 100,000.

(a) *(1 point)* Calculate the expected profit from randomly assigning 20 analysts to each of projects A and B.

(b) *(3 points)* Evaluate the costs of the Rich & Poor's reports and determine which you would purchase.

2. *(7 points)* Palm Tree Company starts a bank and borrows $100M in deposits, investing this money in a portfolio of risky loans. Palm Tree internally calculates Economic Capital to withstand a 1-in-200 year credit loss event on the loans over a 12-month horizon. The bank has annual expenses of $0.5M. All best estimate assumptions below are realized in the first year of operation for the bank.

Palm Tree's Hurdle Rate	15%
Palm Tree's Effective Tax Rate	20%
Borrowing Rate on Deposits	5%
Loan Portfolio Gross Earned Rate	7%
Regulatory Capital	$3M
Earned Rate on Assets Backing Capital	5%
Expected Annual Credit Losses on Loan Portfolio	$1M

Percentile annual credit losses	50	95	99	99.5	99.9
Amount of loss	$0.8M	$5.0M	$6.0M	$6.5M	$8.0M

(a) *(1 point)* Define and calculate the Return on Capital, assuming the bank holds regulatory capital.

(b) *(3 points)*

(i) Define and calculate the Risk Adjusted Return on Capital (RAROC), assuming the bank allocates Economic Capital internally.

(ii) Determine whether Palm Tree's bank investment added value to the firm during the first year.

(c) *(3 points)*

(i) Compare the advantages and disadvantages of IRR, Return on Capital, and RAROC as measures of performance.

(ii) Recommend which of these measures the firm should use for performance measurement.

3. *(6 points)* You run a Monte Carlo simulation to price a lookback put option on a 5-year Treasury bond using both a Cox-Ingersoll-Ross (CIR) and a Hull-White (HW) model. To compare prices, the HW model and the CIR model are fitted such that the initial interest rate volatility is the same. It takes an hour to get the results. The results are as follows:

Initial Interest Rate:	0.07
Long run mean for Interest Rate:	0.05
CIR option price:	0.7028
HW option price:	0.8131

(a) *(2 points)* Compare the two interest rate models.

(b) *(2 points)* Describe an effective variance reduction technique for speeding up the computational process.

(c) *(2 points)* Explain why, under these conditions, the HW model would generate a higher put option price than the CIR model.

4. *(8 points)* You report to the actuary managing an equity-linked portfolio. She has asked you to estimate parameters for the Regime Switching Log normal model with two regimes (RSLN-2), using 10 years of monthly stock price data for an index used in your portfolio.

(a) *(1 point)* Define the RSLN-2 model and describe its key features.

(b) *(3 points)* Explain how to use the data to estimate the RSLN parameters using the maximum likelihood estimation (MLE) method.

(c) *(3 points)* Explain in summary form how the Markov Chain Monte Carlo (MCMC) method may be used to determine the RSLN parameters.

(d) *(1 point)* Describe the advantages and disadvantages of the MCMC approach, compared with the MLE approach.

5. *(7 points)* A firm with no debt has the following three short term investment projects to consider, each of which has an initial outlay of $90 with cash flows over two periods:

	Period	Cash Flow	After Tax Net Income
Project 1			
	Year 1	125	100
	Year 2	- 25	0
Project 2			
	Year 1	75	60
	Year 2	30	24
Project 3			
	Year 1	30	24
	Year 2	75	60

Assume the weighted average cost of capital is 10%.

(a) *(1 point)* Describe the strengths and weaknesses of the following capital budgeting techniques:

(i) Payback Method

(ii) Accounting Rate of Return

(iii) Net Present Value

(iv) Internal Rate of Return

(b) *(3 points)* Evaluate the preferred project, if any, under each of the capital budgeting techniques assuming only one project can be selected.

(c) *(1 point)* Recommend which projects to proceed with, assuming the firm has the capacity to make all investments and desires to maximize shareholder's wealth.

(d) *(2 points)* Assuming a marginal tax rate of 20%, the firm decides it can increase its debt to 30% without affecting its ability to borrow funds;

(i) Calculate the company's new weighted average cost of capital.

(ii) Describe how this affects the decision made in (c) above.

6. *(6 points)* XYZ Insurance is considering an acquisition of ABC Insurance, and is conducting an actuarial appraisal of ABC.

You are given the following:

Risk free rate	3%
Company risk relative to market	75%
Market expected return	11%
Company target capital ratio	250%
Interest on Capital, Surplus, and AVR	6%
ABC adjusted book value at December 31, 2009	$69.42 million
Corporate Income Tax rate	25%

	2009	2010	2011	2012
Minimum Required Capital	30	34	40	46
Statutory Profit on Existing and New Business		28.1	27.8	31.6
Unallocated Expense		7.2	2.3	1.4
Proxy DAC Tax Asset	70	85	100	115

The calculation of Corporate Income Taxes takes into account a Deferred Acquisition Cost component.

The Statutory Reserve and the Tax Reserve are equal.

Calculate the Actuarial Appraisal Value of ABC Insurance at December 31st, 2009, using CAPM, and reflecting 3 years of future cash flows.

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7. *(11 points)*

(a) *(2 points)* Compare and contrast equilibrium and arbitrage-free interest rate models for both realistic and risk-neutral scenarios.

(b) *(1 point)* Assess whether the Hull-White model is suitable for valuing bond options.

(c) *(1 point)* Describe how an option on a coupon-paying bond can be decomposed into options on a series of zero-coupon bonds in the context of a single-factor interest rate model.

You need to price a 2.2-year European call option on a default-free bond that will mature in 3 years using the Hull-White one-factor model. The strike price is the cash price that will be paid for the bond.

Pricing parameters	
a	0.05
σ	0.03
Current term structure	2% semi-annual compounding (flat curve)
Bond coupon rate	6% semi-annual
Bond principal	100
Option strike	101

Suppose $\Delta t = 0.3$ years and R is the level continuous rate in the Δt period. The bond price in the Hull-White model based on R is as follows:

$$P(t,T) = \hat{A}(t,T)e^{-\hat{B}(t,T)R}$$

Where

$$\ln \hat{A}(t,T) = \ln\frac{P(0,T)}{P(0,t)} - \frac{B(t,T)}{B(t,t+\Delta t)}\ln\frac{P(0,t+\Delta t)}{P(0,t)} - \frac{\sigma^2}{4a}\left(1-e^{-2at}\right)B(t,T)\left[B(t,T)-B(t,t+\Delta t)\right]$$

And

$$\hat{B}(t,T) = \frac{B(t,T)}{B(t,t+\Delta t)}\Delta t$$

You are given that the critical value of interest rate for which the price of the coupon-bearing bond equals the strike price of the option on the bond at option maturity is $R^* = 6.16\%$ with continuous compounding.

(d) *(2 points)* Calculate the value at $t = 2.2$ of the coupon due at time $T = 2.5$ given $R = R^*$.

7. **(Continued)**

(e) *(3 points)* Calculate the value at $t = 2.2$ of the coupon and principal due at time $T = 3.0$ given $R = R^*$.

Given the following for Hull-White models:

$$h = \frac{1}{\sigma_p} \ln \frac{L\,P(0,s)}{P(0,T)K} + \frac{\sigma_p}{2} \qquad \sigma_p = \frac{\sigma}{a}\left[1 - e^{-a(s-T)}\right]\sqrt{\left(1 - e^{-2aT}\right)/2a}$$

Where *L* is the principal of the bond, *K* is its strike price for call option that matures at time *T* on a zero-coupon bond maturing at time *s*.

(f) (*2 points*) Determine the value of the option on the coupon paying bond at time $t = 0$.

8. *(7 points)* You are the Chief Risk Officer of Windy City Insurance Company in the US, the insurance arm of international banking corporation IBG. Windy City has traditionally issued products containing only interest rate risk but is now introducing three new, unique equity index-linked products to the US market during a volatile environment for US equities. As a result, Windy City's current practice of evaluating risk based on regulatory capital has been deemed inadequate; IBG has been using economic capital for several years in many other jurisdictions.

(a) *(1 point)* Outline a report to Windy City's Board summarizing reasons to consider using an economic capital approach.

(b) *(3 points)*

- (i) Summarize the current techniques and approaches that Windy City may use to calculate economic capital for these products.
- (ii) Determine the suitability of each approach for the new Windy City products.

IBG has requested that economic capital be calculated on the basis of "Excess Losses" which it defines as economic losses per unit of coverage in excess of the local jurisdiction's statutory reserves. Windy City's modeling actuary has reported to you the following independent excess loss distributions for the three products:

Product A – 0 excess losses in 30% of the scenarios, excess losses of \$5 per unit in 35% of the scenarios and \$10 per unit in 35% of the scenarios.

Product B – 0 excess losses in 94 out of 100 scenarios and nonzero excess losses per unit in 6 of 100 scenarios of: \$4, \$12, \$17, \$17, \$18, \$20

Product C – Above a median excess loss per unit of \$0, excess losses per unit were uniformly distributed from \$0 to \$25.

(c) *(3 points)*

- (i) Calculate both $V_{92\%}$ and the $E\left(L \mid L > V_{92\%}\right)$ of the Excess Loss Distribution for each product.
- (ii) Recommend and justify which measure should be used as the basis for allocation of economic capital for Windy City.

9. *(4 points)* Your company's marketing director has suggested a new product. The product consists of a fixed interest rate for 5 years and, in addition, in six months the policyholder selects between two types of equity participation:

- Upside potential from a call option, with strike equal to the index level at inception
- Downside protection from a put option, with strike equal to the index level at inception

The notional amount for the interest rate and equity portions of the guarantee are equal.

(a) *(1 point)* Graph the payoffs at maturity for each of these two alternatives.

(b) *(1 point)* Describe the advantages and disadvantages of this product to the policyholder.

(c) *(1 point)* Describe the features of the exotic option which would work best to hedge the equity market risk present in this product.

An actuarial student suggests buying, at issue, exchange-traded at-the-money call and put options with 5-year maturities to hedge this product. Assume that the dividend yield is non-negative, and the dividend yield is less than the risk-free rate.

(d) *(1 point)* Demonstrate how this suggestion is less efficient than the exotic option in (c).

****END OF EXAMINATION****
Morning Session