Exam APMV Illustrative Solutions
Spring 2008

1.

Learning Objective:

3 – d. Describe the role of rating agencies in evaluating credit risk

This question asks the candidates to assign a rating to a block of business using the internal risk rating system methodology as described in Cruohy Ch. 7.

Solution:

(a)

(i)

The financial assessment criterion examines different statistics of a company in order to estimate its ability to repay a loan.

The statistics examined include:

Interest coverage ratios – EBT/interest expense; EBITDA/interest expense
Cash flow measures – free operating cash flow/total debt
Leverage ratios – current liabilities/current assets; total liabilities/net worth

These ratios are also compared with the same ratios of other companies in an industry as part of the rating process.

This criterion defines the highest possible rating that an issuer and/or debt issue can receive. The impact of other criteria would either leave the rating unchanged or lower it.

(ii)

Purpose: to determine if current management team are qualified to run this particular business.

Criteria:

- Will look at management capability;
- Does the management have experience and knowledge in this particular business;
- Are the financial reports adequate and in time;
- Is there a succession plan;
- What are management's plans in a time of crisis.

RRS will also look at a company's subsidiaries.

1. continued

(b)

LifeCo has good quality financial statements – timely, accurate and reliable.
LifeCo is subject to minimal country risk – most assets and cash flows are in domestic currency.

LifeCo's position within the industry is relatively weak – keen competition with banks, slow growth rate.

The average duration of assets and liabilities at LifeCo are not excessively long.

2.

Learning Objective:

5 – a. Describe and access techniques that can be used to select or build a benchmark for a given portfolio or portfolio management style

This question asks the candidates to describe the concept of a benchmark in performance measurement, and construct an appropriate benchmark to measure the performance of a block of business. The primary material is from Maginn / Tuttle Ch.12.

Solution:

(a)

- a basic point of reference
- collection of securities or risk factors and associated weights that represents persistent and prominent investment characteristics of an asset category or manager's investment process
- at the investment manager level, it forms the basis of the covenant between the manager and the fund sponsor
- in transfer pricing, goal is to isolate effects of each set of tasks or responsibilities for full performance attribution
- passive representation of manager's investment style, while incorporating the key features of the manager's portfolio
- basis for evaluating the success of the manager's investment management style and efforts

(b)

- unambiguous – identities and weights of securities in benchmark must be clearly defined
- investable – must be possible to forgo active management and simply hold the benchmark assets
- measurable – return is readily calculated on a reasonably frequent basis
- appropriate – consistent with manager's investment style of area of expertise
- reflective of current investment opinions – manager has current investment knowledge or securities and factor exposures within the benchmark
  - specified in advance – benchmark defined before evaluation of returns
- owned – investment manager should be aware of and accept accountability for the performance of the benchmark

2. continued

(c) Government – use medium-term Treasury bonds

Corporate Bonds – need to calc portion of corporate bonds in inv-grade vs. non-investment grade

Total Investment grade	Total Non-Investment grade
= 573 + 315.2	= 143.3 + 171.9
= 888.2	= 315.2

Total = 1203.4, investment grade ~ 74%, non-investment grade ~ 26%

Pass thrus and CMOs – average duration = [78.8 (5) + 78.8 (1.3)] / (78.8 + 78.8) = 3.15

So, use Real Estate I since it most closely matches portfolio by allocation and duration.

Cash – use Money Market

Comparison of Returns

GP	BV	% Tot	Book Yield	Benchmark Yield
Govt	71.6	4.90%	5.60%	5.00%
Bonds	1203.4	82.28%	= [573 (6.4%) + 143.3 (6.75%) + 315.2 (6.8%) + 171.9 (7.3%)] / 1203.4 = 6.68%	6.50%

(d) So, the investment manager's portfolio return was slightly higher than that of the benchmark portfolio.

3.

Learning Objective:

4 – c. Describe how liability requirements affect the selection of an investment strategy or the selection of an optimal portfolio

This question addresses management of interest rate risk in a portfolio supporting life insurance liabilities. It draws on the Case Study and material in Fabozzi Ch. 12, 13, 23, 24, and 27.

Solution:

(a)

- The ALM guideline states: The dollar duration of assets less the dollar duration of liabilities must be < 30% of the Book Value of Assets.
- DD (Liabilities) $= 1575 \times 4.1 = 6458$ million
- DD (Assets) $= 1545 \times 4.7 = 7257$ million
- DD (Assets) $-$ DD (Liabilities) = 801 million
- $0.3 \times$ Book Value Assets = 450 million
- 801 million is greater than 450 million; does not comply; there is a violation

(b)

- Government bonds: highly liquid
- Public corporate (investment grade): Liquid. Could be callable. Drop in interest rates would lead to reinvestment risk. Should shift to shorter maturities.
- Private corporate (investment grade): Less liquid. Increase in interest rates could force liquidation at loss. Should reduce exposure and shift to shorter maturities.
- Public corporate (below investment grade): Less liquidity
- Pass-Throughs: Often agency backed so low credit risk. Prepayment risk depending on tranche.
- Cash and short-term: Highly liquidity. Minimal credit risk. Increase allocation would be wise.
- Real estate: Illiquid
- Equity: less than 1% weight

(c)

- Shift allocations of government and corporate bonds to shorter maturities to bring down duration of asset portfolio.
- Reduce allocations to private corporate bonds to reduce liquidity risk
- Increase allocation to cash and short term to avoid possibility of forced liquidation in a rising interest rate environment.
- Need liquid assets to prepare for high surrenders due to expiration of surrender fee and high market interest rate versus crediting rate.

4.

Learning Objectives:

1 – a. Criticize the following modeling methods
1 – d. Describe and evaluate equity and interest rate models
1 – e. Contrast commonly used equity and interest rate models
1 – f. Recommend an equity or interest rate model for a given situation
1 – g. Describe issues and best practices in the estimation or calibration of financial models
2 – e. Explain how numerical methods can be used to effectively model complex assets or liabilities

The candidate is asked to integrate candidates' knowledge of stochastic equity models and parameter estimation techniques in the context of the Case Study. The candidates are expected to compare standard models and techniques and recommend and justify one appropriate for the context. This question draws mainly on Hardy Ch. 2, 3, 4, and 5 and Hardy, Freeland and Till "Validation of Long-Term Equity Return Models for Equity-Linked Guarantees". Other references overlap and complement the main ones.

Solution:

(a)

Non-Trad Life – Universal Life has equity exposure since account credited based on equity index performance.

Equity-Linked GIC's – credit 75% of percentage increase in S&P 500 index, if positive

- equivalent to a short call on the S&P 500

Variable Annuities

- separate accounts grow based on equity index returns. GMDB value depends on performance of the underlying equity funds
- Like a put option on the S&P 500

(b)

(i)

- Log-return in a short time period follows Geometric Brownian Motion
- $\ln(S_T / S_0) \sim N(T\mu, T\sigma^2)$
- Returns in different time periods are independent
- Volatility is constant and no volatility bunching
- Tails of distribution are not fat enough, so doesn't capture extreme events

4. **(b) continued**

(ii)

- Incorporates switching between good and bad states, with different lognormal models with different means and volatilities in each state
- Markov chain for transition from one state to another
- Incorporate stochastic volatility
- Fatter tails than LN
- More realistic and accurate than LN because it captures extreme events

(iii)

- Mean and volatility follow stochastic processes
- Does not fit reality well, especially in tails

(iv)

- Has stochastic volatility, with volatility reverting to a long-term rate
- Can be extended for autocorrelation
- Has volatility bunching
- $\sigma_t^2 = \gamma V_L + \beta\sigma_t^2 + \alpha\mu_{t-1}^2$

(c)

I recommend the RSLN model because it fits reality well in both good and bad market cycles, has volatility bunching and enough probability of extreme events (i.e. fat tails). It's also valid over long periods.

(d)

(i)

- Determine parameters such that the probability of observing the data is highest
- Has limitations, including requirement for large samples, need to be strictly stationary, and doesn't work well near boundaries
- Does give good fit near center of distribution, is asymptotically unbiased and has minimum variance
- Can be used to compare fit of distributions

(ii)

- Choose parameters to match quantile in left tail in order to fully capture more extreme events
- Very important for equity guarantees
- Fits left tail well but sacrifices fit in the rest of the distribution
- Recommended by the CIA for segregated funds
- Can combine with on the mean

4. **(d) continued**

(iii)

- Based on Bayesian statistics, uses prior view on parameter distribution
- Calculate posterior distribution based on prior distribution and observations
- Choose to accept or reject values using Metropolis-Hastings algorithm
- Can discard early simulated values until results are stationary

(e)

I recommend Left-tail because it focuses on the downside, which is relevant for valuation and especially capital modeling.

(f)

- Model is not appropriate for pricing
- For pricing, need arbitrage-free
- Model is equilibrium model calibrated to historical data, not current market prices
- Realistic probabilities can help with understanding full distribution of outcomes

(g)

- CTE of the sum is less than the sum of the CTE's
- This happens because some scenarios are worse for some products than others, so diversification benefits are created
- Better to aggregate by scenario and then calculate an aggregate CTE
- Scenarios should be consistent across all products, otherwise can't aggregate

5.

Learning Objective:

2 – a. Demonstrate mastery of option pricing techniques and theory for equity, interest rate, and credit derivatives

This question asks the candidates to apply basic stock option principles to hedge a simple product. The material is covered in Hull Ch. 14.

Solution:

(a)

Equity Linked GICs offer the return of principal after 5 years plus 75% of the average percentage increase in S&P 500 TR index over that period. Since asset portfolio tracks S&P TR pretty well, we could assume it is equivalent to invest in S&P 500 TR. So to meet the guarantees we need to make sure initial principal is protected, that means buying European put option on S&P 500 TR index.

The strike price = 1590 / 1.06 = 1500

Term: 4.5 years

(b)

The notional of the option is $55\text{M} \times 0.75 = 41.25\,\text{M}$

(c)

Using Black-Scholes formula to price the put option

$S_0 = 1590 \quad K = 1500 \quad r = 12\% \quad vol = 18\% \quad T = 4.5 \quad q = 0$

$p = 1500 \times \exp(-0.12 \times 4.5) N(-d_2) - 1590 N(-d_1)$

$d_1 = 1.9486 \quad d_2 = 1.5668$

$p = 11.16$

The number of put contracts 41.25M / 1590 = 25,943.4

Total cost $= 11.16 \times 25,943.4 = 289,506$

(d)

If index drops from 1590 to 1500, the return of the portfolio

$= (1590 - 1500)/1590 = -5.66\%$

The principal of the portfolio is $55\text{M} \times (1 - 5.66\% \times 0.75) = 52,665,094$

If the S&P 500 TR index drops to 1350, the payoff of put contracts is $25,943.4 \times (1500 - 1350) = 3,891,509$, the portfolio will worth

$55\text{M} \times (1 - ((1590 - 1350)/1590) \times 0.75) = 48,773,585$

The total portfolio will worth $3,891,509 + 48,773,585 = 52,665,094$

The principal is protected

6.

Learning Objectives:

5 – c. Describe and access performance measurement methodologies for investment portfolios
5 – d. Recommend a performance measurement methodology

This question asks the candidates to describe the concept of transfer pricing, the advantages and disadvantages of using transfer pricing for a life insurance company and calculate the financial performance with transfer pricing adjustment. The material is covered in 8V-314.

Solution:

(a)

- To assign a "price" or charge for funds transferred internally
- Involve one or more benchmarks
- Establishes a rate between different parts of a company to help determine the separate performance
- Implementing full performance attribution
- Applicable to any combination of assets and/or liabilities
- Choice of structure depends on how important it is to separate impacts of different components

With one benchmark,

- Components can be: investment area's actual assets; loans to investment area/from product lines;
- And product line insurance liabilities
- The strategy can isolate product performance from investment performance

With two benchmarks,

- Components can be: investment area's actual assets: loans to ALM/from product lines; loans to ALM/from product lines; product line insurance liabilities can use similar structure as in the banking industry.

6. continued

(b)

Advantages

- Establishes benchmark for asset portfolio to promote better investment decisions over time
- Promotes measurement of asset performance on a risk-adjusted basis
- Creates meaningful performance measures
- Provides a viable investment alternative when returns on actual assets don't match the performance of the benchmark portfolio over time
- Concentrate all ALM risk into one unit, promoting a more coordinated approach to managing this risk
- Permits separate measurement of impact of any residual asset/liability mismatches which should ultimately improve risk management decisions
- Isolation of asset and liability risks
- Facilitates comparisons of actual versus expected profitability

Disadvantages

- Structure the system properly
- Possible inappropriate use in decision-making context
- Method can't be the only tool for ALM, investment management, profitability analysis, etc.
- Hard to determine proper market rate curve to use
- Need to account for duration and convexity of underlying products and investments
- Possible issue from making decisions on data too short-term in nature
- Not easy to implement and communicate
- Can be time-consuming without affecting the bottom line of the company

6. continued

(c) Prior to Transfer Pricing Adjustment

Income item	A	B	C
Interest of loan	= \$1MM×7.25% = \$72,500.00	=\$22MM×7.25% = \$1,595,000.00	= 0 = 0
Investment income	= 0	= 0	= \$8MM×11.25% = \$900,000
Interest paid to deposit	= (\$30MM)×3.00% = (\$900,000)	= (\$1MM)×3.00% = (\$30,000)	= 0 = 0
Expenses	= (\$175,000)	= (\$405,000)	= (\$550,000)
Net Income	= (\$1,002,500)	= \$1,160,000	= \$350,000

After Transfer Pricing Adjustment (TPA)

Income item	A	B	C
Amount Lent / Borrowed	= -(sum of amount for B, C) = \$29 MM	= \$1MM - \$22M = (-\$21 MM)	= (-\$8 MM) = (-\$8 MM)
Transfer Pricing Charge Income / (Loss)	=\$29 MM x TPA %	= (-\$21 MM) x PA%	= (-\$8 MM) x TPA%

For A's revised net income to be 597,720, we need TPA% such that:

Change in A's net income	= amount due to TPA adjustment, i.e.,
597,720 – (1,002,5000)	= \$29 MM x TPA%
1,600,220	= 29,000,000 x TPA%
TPA%	= 1,600,220 / 29,000,000
TPA%	= 5.518%

So,

Income	A	B	C
Initial Net Income	(\$1,002,500)	\$1,160,000	\$350,000
Transfer Pricing Charge Income / Loss	= \$29MM x 5.518% = \$1,600,220	= (-\$21 MM) x 5.518% = (-\$1,158,780)	= (-\$8 MM) x 5.518% = (-\$441,440)
Revised Net Income	= \$597,720	= \$1,220	= (-\$91,440)

7.

Learning Objective:

6 – d. Contrast expectations of future investment performance with historical performance

This question asks the candidates to explain different types of the ERP estimates as each type will produce different expectations of future investment performance and to identify and evaluate the factors that influence future equity returns. The material is covered in Siegel Ch. 7 and Derrig and Orr "Equity Risk Premium: Expectations Great and Small".

Solution:

(a)

Considerations in setting an ERP:

How the average was calculated: geometric or arithmetic?

- Arithmetic averages are the same or higher than geometric returns

Investment horizon (short versus long)

- Usually relates to the term or maturity of the risk free instrument that was used to determine the ERP

Length of time of the ERP forecast

- Short-run expectations (forecast for up to 10 years) are different from the long-run expectations (forecast for over 10 years)

Conditional or unconditional forecast

- Unconditional forecasts (based on historical averages) forecasts seem to be higher than the conditional short-term forecasts (conditional on future dividend yield, expected earnings, capital gains, etc.)

US vs. International market data

- Data from the non-US equity markets are different and will produce different estimates than the US data

Data source and period used for the estimates

- Different time periods will produce different estimates
- Smaller sub-periods will show much larger variations in returns
- Different proxies for the market will produce different estimates

Real versus nominal returns

7. continued

(b)

Factors that may cause future returns from equity mutual funds to differ from their historical average:

Economic conditions / stability

- Greater economic stability leads to larger savings into riskier assets such as equities (higher returns)

Transaction Costs

- Lower transaction costs lead to a higher demand for equities (higher returns)

Taxes

- Favorable tax structure for equities increases the demand for stocks (higher returns)
- Firms are taking advantage of the low capital gains tax by reducing dividends and either buying back shares or funding capital expenditures. This generates future capital gains (and higher returns)

Higher equity prices allow for a smaller offering of new shares and cheaper capital expenditures – i.e. potential for more growth and higher returns

Drop in the cost of information, communications technology, and in the cost of the capital investments – firms have more money to buy back shares (higher returns).

Flow of funds from abroad may boost domestic equity prices

Corporate profits relative to national income was at or slightly above the historical average in 2000.

Terrorism (lower returns)

- Security expenditures will reduce productivity
- Level of uncertainty will increase

Age wave (potentially lower returns)

- Baby boomers cashing in their investments – excess supply will lead to lower returns
- The effect can be mitigated by a higher economic growth and by developing world can help purchasing the excess assets

8.

Learning Objective:

3 – a. Define and evaluate credit risk as related to fixed income securities, derivatives and reinsurance ceded

This question asks the candidates to estimate implied credit default probabilities, explain the difference between the implied default probabilities and historical bond default rates and calculate the CDS spread. The material is covered in Hull Ch. 20, and 21.

Solution:

(a)

2-year Bond Price = 101.76
3-year Bond Price = 102.58

Difference:
2-year = 101.76 – 99.27 = 2.49 million
3-year = 102.58 – 98.42 = 4.16 million

If default probability for year y = Q(y), then PV of expected loss on

$$\text{2-year bond} = (\text{year 1 loss}) \times (\text{PV factor for } 0.5) \times Q(1) + (\text{year 2 loss}) \times (\text{PV factor for } 1.5) \times Q(2)$$

For 2-year bond:
Year 1 loss = 3 + 101.34 – 40 = 64.34
Year 2 loss = 3 + 100.46 – 40 = 63.46

So $64.34 \times 0.9753 \times Q(1) + 63.46 \times 0.9277 \times Q(2) = 2.49$

Assuming $Q(1) = Q(2)$, $Q(1) = Q(2) = 2.05\%$

For 3-year bond:
Year 1 loss = 3 + 102.17 – 40 = 65.17
Year 2 loss = 3 + 101.34 – 40 = 64.34
Year 3 loss = 3 + 100.46 – 40 = 63.46

So $65.17 \times 0.9753 \times Q(1) + 63.46 \times 0.9277 \times Q(2) + 63.46 \times 0.8825 \times Q(3) = 4.16$

Substituting $Q(1) = Q(2) = 2.05\%$, $Q(3) = 2.92\%$

8. continued

(b)

Usually, risk neutral default probability implied from bond yields are greater than real-world default probabilities implied from historical bond default rates

This is because:

Corporate bonds are relatively illiquid

Subjective default probability used by traders is higher so they need higher spread

Default correlation can't be diversified away

Corporate bonds are highly skewed with limited upside but have unlimited downside

(c)

First, calculate the survival probability $S(t)$:

$$S(1) = 1 - Q(1) = 1 - 2.05\% = 0.9795$$

$$S(2) = S(1) - Q(2) = 0.979502.05\% = 0.9591$$

$$S(3) = S(2) - Q(3) = 0.9591 - 2.92\% = 0.9299$$

Second, calculate the present value of expected payment

Let *s* be the CDS spread

$$\sum_{t=1}^{3} S(t) \times \text{Discount Factor}(t) \times s = (0.9795 \times 0.9512 + 0.9591 \times 0.9048 + 0.9299 \times 0.8607) \times s$$

$$= 2.6s$$

Third, calculate the present value of accrual payment

$$\sum_{t=1}^{3} Q(t) \times 0.5 \times \text{Discount Factor}(t - 0.5) \times s = (0.0205 \times 0.9753 + 0.0205 \times 0.9277 + 0.0292 \times 0.8825) \times 0.5s$$

$$= 0.0324s$$

Fourth, calculate the present value of expected payoff

Binary swap, hence payoff is 1

$$\sum_{t=1}^{3} Q(t) \times \text{Discount Factor}(t - 0.5) \times 1 = 0.0324 \times 2$$

$$= 0.0648$$

Present value of expected payment + Present value of accrual payment = Present value of expected payoff

$$2.6s + 0.0324s = 0.0648$$

$$s = 2.46\%$$

9.

Learning Objectives:

4 – a. Explain how an investment policy and an investment strategy can help manage risk and create value
4 – b. Describe how client's needs and constrains affect the selection of an investment strategy or the construction of a portfolio
4 – c. Describe how liability requirements affect the selection of an investment strategy or the selection of an optimal portfolio

This question asks the candidates to analyze the impact on the key stakeholders of moving pension assets from equities to bonds. The material draws from V-C111-07.

Solution:

(a)

(i)
Plan Participants

Advantages

- Stable pension asset performance with bonds
- Liability matching strategy is a better guarantee that plan sponsor will be able to pay benefits accrued when due

Disadvantages

- Less return from bonds may lead to plan being underfunded
- Less well funded plan may result in freezing or terminating the plan or in elimination of future benefit increases

(ii)
Shareholders

Advantages

- Tax benefit from prefunding
- Converts stock market systematic risk to firm-specific risk
- Benefits from possibility for tax arbitrage
- *Tepper arbitrage*
  Switching pension plan assets to bonds and adjust shareholders portfolio to equities to offset and retain equivalent overall risk
- *Black arbitrage*
  Company buys back own stock and issues bonds while switching plan assets to bonds

9. **(a) (ii) continued**

Disadvantages

- Reduce upside potential of investment in equities
- Lower investment return of plan assets may result in higher contributions

(b)

- Assets and liabilities of the plan remain unchanged whether invested in bonds or equities
- Investment in bonds will lower risk and therefore increase P/E ratio
- Lower risk but also lower earnings which may then lower the share price
- Overall effect on share price will therefore be little to none depending on how the market analyzes the strategy

(c)

Some say that equity hedges the salary and inflation increases in pension plans
Evidence supports a negative correlation between unexpected inflation and equity returns
However, future salary increases should not be included in current pension liability until actually granted
Inflation-indexed fixed income securities (such as TIPs) can be purchased to match benefit when increases are known

10.

Learning Objective:

6 – b. Describe how behavioral finance explains the existence of some market anomalies

This question tests the candidates' understanding of behavioral finance as it relates to consumer insurance purchasing behavior and impacts to investment strategies. The material draws from Schiller "From Efficient markets Theory to Behavioral Finance", Malkiel "The Efficient Market Hypothesis and Its Critics" and Robin and Thaler "Anomalies: Risk Aversion".

Solution:

(a)

Short term momentum is a behavioral finance concept that can be used to explain the apparent serial correlation in stock price returns. Investors are watching what other investors do and the jumping on the bandwagon thus amplifying the general market trend. It could also be due to under-reaction to new information.

In order to profit from it:
One can use either momentum strategy or contrarian strategy
Momentum strategy is buying stocks that display positive serial correlation and positive relative strength
Contrarian strategy (smart money strategy) is buying stocks out of favor

(b)

Myopic loss aversion describes the tendency of people to exhibit extreme loss aversion, failing to take into account the "whole" picture and making decisions in an isolated manner. This behavior can be seen in the purchase of relatively expensive insurance covering relatively immaterial risks, when the insurance is sold separately. The customer is willing to buy low deductibles and low limits, when he should be buying high deductibles and deep coverage. Also buy collision damage waivers on rental cars, extended warranties on household appliances, and internal wiring protection.

(c)

(i)

On average, the company will be selling securities when the prices increase and buying securities when the price decreases, in order to maintain the target portfolio mix. This will result in favorable results by emulating "buy low and sell high" strategy if prices stay near the mean of its distribution.

10. (c) continued

(ii)

In the left tail, the force-rebalancing will buy more shares of sub-fund that had been down and will go down further (due to serial correlation). It makes the claims worse than no-rebalancing.

11.

Learning Objectives:

2 – c. Identify limitations of each option pricing technique

2 – d. Describe how option pricing models can be modified or alternatives techniques that can be used to deal with option pricing techniques limitations

This question tests the basic understanding of the major limitations of the Black-Scholes-Merton model, and asks the candidate to use one of the limitations to set up an investment strategy. The material is from Chew V.32 and Hull Ch. 24.

Solution:

(a)

- Mixed jump diffusion model:
- It follows a Poisson process
- The stock price moves stochastically with jumps such that

$$\frac{dS}{S} = (v - q - \Lambda k)\,dt + \sigma dz + dp$$

- Where

Λ = average number of jumps

k = average size of jumps

dp follows a Poisson process

Variance-Gamma

- Is a pure jump model
- G is a gamma process and defines the way at which information arrives
- Up and down stock price movements, with small frequent jumps and occasionally large jumps

Both mixed jump diffusion and Variance Gamma have larger (fatter) tails on both sides than the lognormal distribution

(b)

With Merton's model

- Sell at the money option
- Buy out-of-the money options (underpriced under BSM model)

Under Variance Gamma Model

- U-shaped volatility smile
- Sell short in-the-money calls and puts

12.

Learning Objective:

1 – c. Define and apply the concepts of martingale, market price of risk, and measures in single and multiple state variable contexts

These concepts underlie all interest rate derivative mathematics. The material draws from Hull Ch. 25.

Solution:

(a)

Market price of risk $\lambda = \dfrac{(\mu - r)}{\sigma}$

Therefore, market price or risk for gasoline $= \dfrac{(0.09 - 0.06)}{0.15} = 0.2$

(b)

$\mu - r = \sum \lambda(i)\,\sigma(i)$

Expected return on MPG stock $= r_{stock} - r_f$

$= 0.2 \times 0.15 + 0.1 \times 0.25 - 0.05 \times 0.3$

$= 0.04$

(c)

Martingale is a zero drift stochastic process
Has the form $d\theta = \sigma dZ$
dZ is a Wiener process

(d)

i. Martingale is a zero drift stochastic process
ii. This is a risk neutral world since the market price of risk is = 0

12. continued

(e)

Use σg as the numeraire

$$dg = \left(r + \sigma f \sigma g\right) f \; dt + f \; \sigma f \; dz$$
$$dg = \left(r + \sigma g^2\right) g \; dt + g \; \sigma g \; dz$$

Using Ito's Lemma on $\ln\left(f\right)$ and $\ln\left(g\right)$

$$d \ln\left(f\right) = \left[\left(r + \sigma f \sigma g\right) - \frac{1}{2}\sigma f^2\right] dt - \sigma f \; dz$$
$$d \ln\left(g\right) = \left[\left(r + \sigma g^2\right) - \frac{1}{2} o g^2\right] dt - \sigma g \; dz$$

Subtracting, we get $d \ln\left(f / g\right) = \left(\sigma f - \sigma g\right)^2 \; dt - \left(\sigma f - \sigma g\right) dz$

Apply Ito's Lemma again,

$$d\left(f/g\right) = \left(\sigma f - \sigma g\right) f/g \; dz$$

So, f/g is a martingale.

13.

Learning Objective:

4 - f. Assess a portfolio position against portfolio management objectives and recommend a strategy to rebalance the portfolio

This question asks the candidates to assess a portfolio position against a stated objective and rebalance to incorporate new asset classes. The material draws from Litterman Ch. 26 and 28.

Solution

(a)

Private Equity

Unique investment strategy
Returns depend on investment manager's expertise
Standard risk and returns measure are not appropriate
Illiquid
Lack of transparency
Prices are stale
Example of Private Equity: Angel Investments, incubator …

(b)

Hedge Funds

Transparency issues
Reporting frequency not like public markets
No absolute benchmark
No limitation on style changes
Inconsistent weighting
Self reporting bias
Survivorship bias
Backfill bias
Investability
No short sale constraint
Capacity constraint
Illiquidity
No absolute benchmarks or reference
No limit on change in styles for manger
Manager can short sale
Active role in investment

(c)

Hedge fund W add return with the same volatility as the current portfolio
Hedge fund X has too high volatility
Hedge fund Y and Z have too high volatility as well
W is better than Y and Z, and Z better than Y
Invest in W

14.

Learning Objective:

4 – d. Describe and compare specialized financial instruments that can be used in the construction of an asset portfolio supporting financial institutions and pension plan liabilities

This question asks the candidates to calculate the Single Monthly Mortality (SMM), Conditional Prepayment Rate (CPR) and Public Securities Association (PSA) model of a pool of mortgage loans, describe the key factors that impact the prepayment behavior and describe how the prepayment profile of subprime mortgage collateral differs from conforming agency mortgages. The material is from HFIS, Ch. 23 and 26.

Solution:

(a)

$$\begin{aligned}\text{SMM} &= 100\times\left[\frac{\left(\text{scheduled balance}-\text{actual balance}\right)}{\left(\text{scheduled balance}\right)}\right]\\ &= 100\times\left[\frac{\left(250,000,000-245,000,000\right)}{250,000,000}\right]\\ &= 2\%\end{aligned}$$

$$\begin{aligned}\text{CPR} &= 100\times\left[1-\left(1-\left(\frac{\text{SMM}}{100}\right)\right)^{12}\right]\\ &= 100\times\left[1-\left(1-\left(0.02\right)\right)^{12}\right]\\ &= 21.52\%\end{aligned}$$

$$\begin{aligned}\text{PSA} &= 100\times\left[\frac{\text{CPR}}{\left(\min\left(\text{age},30\right)\times 0.2\right)}\right]\\ &= 100\times\left[\frac{0.2152}{\left(30\times 0.2\right)}\right]\\ &= 3.69\end{aligned}$$

14. continued

(b)

The key factors that impact the prepayment behavior of Agency Mortgage Backed Securities:

- Current interest rate level – a decrease in interest rate creates an incentive for homeowners to refinance and this increases payment level; conversely, an increase in interest rates will slow down prepayment levels
- Time of the year – relocations are more frequent during Spring/Summer
- Economic condition – affects level of relocation
- Slow of the yield curve – if the yield curve is steep, homeowners are encouraged to borrow via ARMs and continue to refinance to take advantage of short-term interest rates
- Burnout – refinance opportunity with a loan pool declines over time regardless of whether rates continue to decline
- Seasoning – seasoned mortgage homeowners are more likely to relocate and make additional partial payments

(c)

Prepayment profile of subprime mortgage collateral differs from conforming agency mortgages

- There is generally less refinancing in subprime mortgages even if current market rates drop below current loan rates as subprime mortgage holders may not have sufficient good credit to get a new loan and take advantage of rate opportunities
- More restrictions for subprime mortgage holders because they lack good credit history to pursue alternatives
- Because of these conditions, the prepayment profile of subprime mortgages has better convexity profile than confirming agency mortgages
- Subprime mortgages season quicker and reach a stable state faster in relation to conforming mortgages

15.

Learning Objectives:

4 – c. Describe how liability requirements affect the selection of an investment strategy or the selection of an optimal portfolio

4 – d. Describe and compare specialized financial instruments that can be used in the construction of an asset portfolio supporting

This question asks the candidates to price and assess the attractiveness of longevity bonds as a hedge and a diversifying investment. The material is from "Living with Mortality: Longevity Bonds and Other Mortality-Linked Securities"

Solution:

(a)

Bond Payments:

t	$S(t)$	Payment	LIBOR
0	1	0	5%
1	0.96	960,000	5%
2	0.9168	916,800	5%
3	0.87096	870,960	5%

Discount rate = 1 + .05 - .003 = 1.047

Price of the bond $\frac{960,000}{(1.047)}+\frac{916,800}{(1.047)^2}+\frac{870,960}{(1.047)^3}=2,512,096$

(b)

Benefits

- Payments will increase if more survivors
- This is similar to pension liabilities
- High quality issue

Risks

- General population survivorship may not match pension population survivorship
- This would eliminate hedge value
- Credit risk (small but present)
- Some market risk due to changing LIBOR

15. **continued**

(c)

(i)

- Low correlation with other assets
- Potential for stable returns
- Hedge other risks or liabilities
- Take position on mortality

(ii)

- Hedge risk in social programs
- Market participant
- Create market for this asset
- Create liquidity

16.

Learning Objectives:

3 – f. Recommend a credit risk management strategy for a given situation
4 – g. Evaluate complex situations associated with the presence of embedded options, hedging strategies, accounting considerations, taxation and capital requirements under a range of economic environments
4 – i. Recommend an investment strategy for a given situation, including portfolio policy and objectives, asset selection criteria, capital market expectations, and risk management strategy

This question explores fixed income hedging of corporate spreads and the underlying Treasury yield curve. It draws on material in Babbel & Fabozzi Ch. 8 and 19, and Crouhy Ch. 12.

Solution:

(a)

$$\#\text{of contracts} = \frac{\text{DVBP Portfolio}}{\text{DVBP Futures} \times \text{Conversion Factor}}$$
$$= \frac{100,000}{80.5 \times 1.20}$$
$$= 1,035$$

Short 1,035 contracts.

(b)

After 10 bps upward shift,

Change in Portfolio Value $= -\text{DVBP} \times 10 = -\$1,000,000$
Portfolio Value decreases by \$1,000,000

Changes in Futures Position $= 1,035 \times 80.5 \times 1.20 \times 10 = \$999,810$
Futures position increases by \$999,810

Change in value of Hedged Portfolio $= -\$1,000,000 + \$999,810 = -\$190$

16. continued

(c)

The risk related to the portfolio is widening credit spreads; which implies a reduction in the value of the bonds.

To mitigate the risk, can purchase

- Credit spread options
- Credit default swaps
- Credit spread forwards
- Credit linked notes
- Total return swaps

Use Credit Linked Notes

Buying put options on sector index

Shorting a basket of stocks in the sector

(d)

Treasuries

Advantages

- Liquid, default free

Disadvantages

- Can not hedge spread risk, on balance sheet, some issues may be in short supply (expensive to borrow)

Interest Rate Swaps

Advantages

- Off balance sheet, no supply problems, good to hedge spread risk

Disadvantages

- Less liquid than futures, liquidated through counterparty (rather than traded), counterparty risk

Interest rate caps / floors

17.

Learning Objective:

4 – g. Evaluate complex situations associated with the presence of embedded options, hedging strategies, accounting considerations, taxations and capital requirements under a range of economic environments

This question asks the candidates to demonstrate an understanding of guaranteed benefits, costs associated with each and sensitivity to market conditions. The material is from Hardy Ch.8.

Solution:

(a)

GMMB

- Straight forward put option on segregated fund
- Assume guarantee is fixed but adjust for contact allowing resets
- Black-Scholes standard formula as a starting point except need to define 'm' as annual management charge and adjust formula
- Derived price doesn't reflect lapses or mortality

GMDB

- Liability is the same as the GMMB expect the time to payment is unknown (contingent upon death)
- Then term is a random variable
- Black-Scholes standard formula as starting point and adjust for random variable
- Cost of hedge portfolio can be viewed as two parts – split between a risk free portfolio and a risk portfolio

GMAB

- Benefit is a series of puts and calls
- Starting guarantee is starting fund value
- At time t (a reset point), if guarantee is greater than the current fund value, no change in guarantee
- If fund value is higher, then reset the guarantee to that level
- Material indicates the difference between fund value and guarantee (fund > guarantee) is paid into the fund so the next period they are equal
- Derive hedge costs by using standard Black-Scholes formula with adjustments to reflect the nature of complicated benefit

17. (a) continued

Key hedging challenges that apply to all three guarantees

- Bias that dynamic hedging is not practical because assumptions are overly simplified. Uncertainty of models / parameters is very large
- Dynamic hedging using the B-S formula approach will produce a hedge portfolio that is very likely to meet the nature of the benefits
- Issues such as transaction costs are not considered in the analysis but can be adjusted for – assume they are small for bonds but overall as a percentage of the change in value of the stock portfolio
- Discrete hedging error with certain maturity date – hedging error is introduced when continual trading is not used
- Model error
- Adjust for lapses and mortality

(b)

Fund alpha = B / (S × annuity factor) (multiple monthly factor by 12 to get annual equivalent since monthly deduction)

$$\text{Fund X alpha} = \frac{12 \times 1.7}{90 \times 52.0} = 0.0044$$
$$\text{Total MER(\%)} = 2.5 + 0.2 + 0.44 = 3.14$$

$$\text{Fund Y alpha} = \frac{12 \times 1.3}{100 \times 48.0} = 0.0033$$
$$\text{Total MER(\%)} = 2.5 + 0.2 + 0.33 = 3.03$$

$$\text{Fund Z alpha} = \frac{12 \times 1.0}{80 \times 51.0} = 0.0029$$
$$\text{Total MER(\%)} = 2.5 + 0.2 + 0.29 = 2.99$$

Fund Z would be the best to support the guarantee and meet the objective of the marketing department. The cost of the segregated fund to the policyholder will be 2.99%.

18.

Learning Objective:

4 – i. Evaluate the need to revise an investor's investment policy and recommend revisions over a lifetime

This question asks the candidates to create an investment policy statement for a client, amend it for a real life situation and then discuss solutions. The material draws from Magnin Ch.11, Babbel & Fabozzi, Ch. 26 and HFIS Ch. 48.

Solution:

(a)

- Liquidity: must provide some in case of unexpected situations, however not much is needed due to current salary
- Risk tolerance: current has 100% in equities, high risk tolerance. Plans to use proceeds of business sale to buy retirement income so not counting on investment to fund retirement, therefore high risk tolerance
- Time horizon: plans to retire in 20 years, long time horizon allows for more risk taking
- Objectives: enjoying 10% return

100% in equities is risk, however, doesn't rely on this income for the future

(b)

- Liquidity: must increase liquidity due to possible cost. Equities are fairly liquid, however, $1M to liquidate once may be too much, may start investing in liquid GIC.
- Risk tolerance: decrease significantly, must ensure sudden drops in value of equities may deplete portfolio when decide to liquidate
- Time horizon: may need cushion soon. Can no longer focus in long-term return
- Objectives: may need safe and steady return

(c)

(i)

Conservative

- Portion of portfolio switches to coupons payable bonds that cover monthly care of mother
- Can invest majority in fixed income investment and leave some in equity for future appreciation and possible future long-term care cost inflation

18. **(c) continued**

(ii)

Moderate

- Match future payments to long-term care exactly with coupons payable bonds and leave the rest in equity

19.

Learning Objective:

3 – c. Describe, contrast and assess credit risk measurement techniques and models

The material is from Cruohy, Ch. 10.

Solution:

(a)

Computations for Bond A

Compute value at time 1 if rates increase

$$= \frac{(1-\text{default rate})\times 100 + (\text{default rate})\times(1-\text{Loss given Default})\times 100}{(1+\text{risk free rate up})}$$

$$= \frac{\left[(1-0.25)\times 100 + 0.25\times(1-0.4)\times 100\right]}{1+0.12}$$

$$= 80.357$$

Compute value at time 1 if rates decrease

$$= \frac{(1-\text{default rate})\times 100 + (\text{default rate})\times(1-\text{Loss given Default})\times 100}{(1+\text{risk free rate down})}$$

$$= \frac{\left[(1-0.25)\times 100 + 0.25\times(1-0.4)\times 100\right]}{1+0.09}$$

$$= 82.56881$$

Compute the expected payoff at time 1 if rates increase

$$= \text{Value at time } 1\times\left((1-\text{default rate}) + \text{default rate}\times(1-\text{loss given default})\right)$$

$$= 80.357\times\left((1-0.25)+(0.25)\times(1-0.4)\right)$$

$$= 72.3213$$

Compute the expected payoff at time 1 if rates decrease

$$= \text{Value at time } 1\times\left((1-\text{default rate}) + \text{default rate}\times(1-\text{loss given default})\right)$$

$$= 82.56881\times\left((1-0.25)+(0.25)\times(1-0.4)\right)$$

$$= 74.31193$$

19. (a) continued

Compute the value at time 0

$$= \frac{\left[\left(\text{Probability of increase}\right) \times \text{Payoff if increase} + \left(\text{Probability of decrease}\right) \times \text{Payoff if decrease}\right]}{\left(1 + \text{current risk free rate}\right)}$$

$$= \frac{\left(0.75 \times 72.3213 + 0.25 \times 74.31193\right)}{1 + 0.10}$$

$$= 66.19914$$

(b)

Compute value at time 1 if rate increase

$$= \frac{\left(1 - \text{default rate}\right) \times 100 + \left(\text{default rate}\right) \times \left(1 - \text{Loss given default}\right) \times 100}{\left(1 + \text{risk free rate up}\right)}$$

$$= \frac{\left[\left(1 - 0.05\right) \times 100 + 0.05 \times \left(1 - 0.9\right) \times 100\right]}{1 + 0.12}$$

$$= 85.26786$$

Compute value at time 1 if rates decrease

$$= \frac{\left(1 - \text{default rate}\right) \times 100 + \left(\text{default rate}\right) \times \left(1 - \text{Loss given default}\right) \times 100}{\left(1 + \text{risk free rate down}\right)}$$

$$= \frac{\left[\left(1 - 0.05\right) \times 100 + 0.05 \times \left(1 - 0.9\right) \times 100\right]}{1 + 0.09}$$

$$= 87.61468$$

Compute the expected payoff at time 1 if rate increase

$$= \text{Value at time } 1 \times \left(\left(1 - \text{default rate}\right) + \text{default rate} \times \left(1 - \text{loss given default}\right)\right)$$

$$= 85.26786 \times \left(\left(1 - 0.5\right) + \left(0.5\right) \times \left(1 - 0.9\right)\right)$$

$$= 81.4308$$

Compute the expected payoff at time 1 if rates decrease

$$= \text{Value at time } 1 \times \left(\left(1 - \text{default rate}\right) + \text{default rate} \times \left(1 - \text{loss given default}\right)\right)$$

$$= 87.61468 \times \left(\left(1 - 0.5\right) + \left(0.5\right) \times \left(1 - 0.9\right)\right)$$

$$= 83.67202$$

19. **(b) continued**

Compute the value at time 0

$$= \frac{\left[\left(\text{Probability of increase}\right) \times \text{Payoff if increase} + \left(\text{Probability of decrease}\right) \times \text{Payoff if decrease}\right]}{\left(1 + \text{current risk free rate}\right)}$$

$$= \frac{\left(0.75 \times 81.4308 + 0.25 \times 83.67202\right)}{1 + 0.10}$$

$$= 74.53737$$

(c)

Analysis

Market Value of Bond A is greater than its modeled value
80 > 66.19914

Market Value of Bond B is less than its modeled value
70 < 74.53737

Conclusion: Selling Bond A and purchasing Bond B is effective.

20.

Learning Objective:

2 – e. Explain how numerical methods can be used to effectively model complex assets or liabilities

This question tests a basic understanding of the implicit finite difference method and the explicit finite difference method and how to use representative scenarios approach and why the technique is superior to Monte Carlo simulation for evaluating capital required for a derivative security. The material is from Hull Ch. 17 and Longley-Cook and Kehrberg, "Efficient Stochastic Modeling".

Solution:

(a)

Concept Using Implicit Finite Difference Method
Calculate American-style put on dividend-paying stock

Build a grid of time (i) by stock price (j)
Time is $i\Delta t$ and stock price is $j\Delta s$ at node (i,j)
Estimate value at each point node
At maturity time T, put value is $\max\left[K - j\Delta s, 0\right]$
When S = 0, put value is K
Define put value along 3 edges of grid, where S = 0, S = Smax and t=T

Solve formula – quadratic interpolation between nearby points

Iterate back to get f0,j

(b)

Assumptions to Apply Explicit Finite Difference Model

- Reduces no. of equations and computation time of implicit method
- Assume slope and curvature (convexity) – 1st and 2nd derivatives
- Of the price function are constant
- From time period to time period
- That is $\frac{\partial f}{\partial s}$ and $\frac{\partial 2\, f}{\partial s 2}$ at (i, j) same as at $(i+1, j)$

20. continued

(c)

How to Use Representative Scenarios Approach

- Create scenarios
- Choose the Representative Scenarios (RS)
- Choose the one furthest from it's RS as the new one
- Assign probability to each RS
- Run the Cash Flow Model
- Analyze the results

Why Superior to Monte Carlo (MC)?

- For MC, cash flow project very time consuming – 1+ hrs per scenario
- Best to implement strategies quickly
- Need to preserve dist. shape to assess tail metrics e.g. capital
- Low discrepancy (quasi-MC) converge to mean and Std. Dev.

21.

Learning Objectives:

1 – f. Recommend an equity or interest rate model for a given situation
2 – a. Demonstrate mastery of option pricing techniques and theory for equity, interest rate and credit derivatives

The candidate is asked to describe a pricing methodology for interest rate derivatives and assess interest rate interest rate models for that methodology. This question illustrates how financial models integrate with pricing methodologies. This question draws mainly on V-C125-07. Other references complement the main one.

Solution:

(a)

Select benchmark securities whose characteristics are similar to the characteristics of the valued security. Make sure there is enough liquidity.
Calibrate of the model

- Select appropriate yield curve
- Use Blacks Model to estimate the volatility that best replicate benchmark prices
- Value the derivatives using the calibrated model

(b)

Only arbitrage free interest rate models are appropriate for pricing interest rate derivatives. They fit the current term structure exactly. Equilibrium models need risk premium adjustments to be able to price interest rate derivatives.

- An arbitrage free model
- An equilibrium model
- An equilibrium model
- An arbitrage free model

Only the first and fourth are appropriate to price interest rate derivatives

22.

Learning Objectives:

6 – a. Explain how behavioral characteristics of individuals or firms affect the investment or capital management process
6 – c. Identify and apply concepts of behavioral finance with respect to policy holder

This question asks the candidates to discuss the key variables of modeling of policy holder behavior. The material is from "Modeling Surrender and Lapse Rates with Economic Variables", V-C116-07, and HFIS Ch.26.

Solution:

Surrender rates

Insurance policies

- Financial crisis – in terms of financial crisis, lapses are expected to increase
- Age of policy – older policies are less likely to lapse
- Surrender charge – lapses can be expected to increase after the surrender charges wear off
- Unemployment – lapses increase as unemployment increases because the people will need the money
- Economic Growth – as the economy grows, lapses are expected to decrease
- Spread between market rate and credited rate – the greater the spread, the more likely it is for lapses to increase
- Seasonal Factors – small impact
- Inflation and Foreign Exchange Rate – small impact

Mortgage Prepayment Rates

- Relocations – prepayments from people moving
    - Economic considerations – housing cycle, loan rates
    - Noneconomic considerations – age of loan
    - Affordability of "moving up"

- Assumptions – existing FHA/VA loan is assumed
    - Mortgage is not prepaid
    - Attractive to home purchaser if rate on this mortgage is lower than those available in market today
    - Not common today

22. continued

- Curtailments – partial prepayments
  - Has significant impact on term of loan
  - Increases total prepayments

- Refinancing – borrower prepays and refinances into a lower rate
  - Ready, Willing and Able
  - Ready – waiting to time market
  - Willing – would refinance if beneficial to borrower
  - Able – credit score allows them to refinance
  - Burnout
  - Incentive – need greater incentive to make others refinance after they chose not to at the last opportunity
  - Seasonal affects