FET Illustrative Solutions
Fall 2007

1.

Learning Objectives:
2d - Apply the concept of economic capital and describe methodologies for allocating capital within a financial organization.

The question builds in difficulty from basic definitions, to understanding how it applies to one line of business, to explaining (but not doing) a calculation. Regulatory (and rating agency) capital estimates have been in place for many years, so part c) asks the candidate why a new basis is necessary. This question is based on 8FE-406-02, Standard & Poor's Insurance Liquidity Model for 2000.

Solution:

(a)

Definition

Sufficient surplus to cover potential losses at a given risk tolerance level, over a specified time horizon

- Potential losses include:
  - adverse mortality
  - adverse persistency
  - those due to interest rate changes

- risk tolerance level can be either
  - specified percentile (e.g. 95th) of the loss distribution across all scenarios
  - N% Conditional Tail Expectation (CTE) - the average of the (100-N) worst scenarios

- time horizon
  - For the Traditional block, should be based on the difference between the age at which the contract matures, and the youngest issue age

1. **continued**

(b)

Uses

- Capital budgeting information - projections of capital needed or released for the block can provide information about potential capital shortfalls in the future
- Economic Capital can uncover potential risks to the acquiring company of the Traditional Block
- Assumptions and methodology may not be the same as that used for LifeCo, because the acquiring company's mix of business and attendant correlation of risks may be different
- Performance evaluation

(c)

Regulatory vs. Economic Capital

- Regulatory capital reflects requirements for the industry as a whole, while Economic Capital reflects the risk profile of a company individually

- Regulatory capital is meant to assess solvency of a company, while Economic Capital is meant to measure the appropriate level of capital required for the risks the company seeks to undertake

(d)

Computation of Economic Capital using a stochastic approach

- Two categories of models can be used for the stochastic approach
  - univariate and multivariate
- Using either of the two models, generate random scenarios and compute the actual capital needed under each scenario
- Generate scenarios under an approach such as risk neutral - average over all scenarios is equal to the market price at the start of the projection
- real world - scenarios are based on arbitrary expectations, which can be formulated based on:
  - historical averages
  - expected future trends
  - average scenario reproducing the initial conditions

- Adjust for covariance between mortality and persistency if using a univarite approach
- Rank the capital needs under each scenario to form the distribution of capital needs

- Economic capital is then based on the level of capital to cover losses at the desired risk tolerance level - either a percentile or a CTE

2.

Learning Objectives:

4a – Describe the issues influencing investment strategies, including liquidity requirements, valuation concerns, cash flow variability, compliance risk, regulatory constraints, taxation impacts, and investment management mandates.

The candidate is asked to explain the ALM risks of one line of business and tune it with investment strategy changes. See the RSA Vol. 22 #3 article, the Course 6-28 note, and 8V-120-03.

Solution:

(a)

- Guaranteed minimum credited rate
- Cash flows are interest sensitive
- Disintermediation risk when interest rates change
- Reinvestment risk upon issue of policies due to high acquisition costs

(b)

- Asset duration of 8.9 is much higher than liability duration of 4.0
- If interest rates increase by 1%, surplus will decrease by around $20 million
- Dollar duration difference between A + L is $ 2.235 million, significantly higher than guideline of $400,000.
- Partial duration exposure shows significant interest rate risk between 7 and 20 years.
- Drop in long-term rates will significantly reduce the economic surplus.

(c)

- Assets invested in commercial mortgages and real estate have high yield and effective duration close to liability duration, but prepayments can lead to uncertain CFs and reinvestment risk
- Real estate has liquidity issues
- Below investment grade bonds and privates also have liquidity risk

(d)

Benefits

- Some of the risks might be offsetting and correlations between products can be helpful
- Duration mismatch can be reduced when the segments are combined
- Economics of scale

2. (d) continued

Drawbacks

- Cannot effectively manage segment-specific risks
- Performance attribution by product is impossible

(e)

- Reduce the effective duration of the non-trad asset portfolio by selling assets with long durations and buying assets with shorter durations
- Hedge guaranteed minimum interest rates by buying interest rate floors
- Reduce duration mismatch by entering into swaps

3.

Learning Objectives:

5a – Explain the rationale for managing risk and for the selection of the appropriate hedging level.

5b – Identify and describe financial and non-financial risks faced.

5c – Assess the overall corporate risk exposure arising from financial and non-financial risks.

This is a synthesis question asking candidates to recommend a strategy for LifeCo's currency exposure. The answer for part (a) is expected to be based on the factors described in 8V-102-00 Currency Hedging Rules for Plan Sponsors that are critical in determining the optimal hedge ratio. In part (b), the candidates are expected to compare the currency hedging strategies discussed in 8V-103-00 Managing Currency Exposures in International Portfolios. In part (c), the candidates are asked to evaluate LifeCo's currency exposure and in part (d) make a recommendation of a strategy for the risk.

Solution:

(a)

1. International allocation
    - The higher the allocation, the higher the hedge ratio

2. Hedging cost
    - The higher the cost, the lower the hedge ratio.
    - Contract trading cost
    - Management fees and custody costs

3. Sponsor risk aversion
    - The higher the risk aversion, the higher the hedge ratio

(b)

1. Full hedging
    - Remove completely currency risk from the portfolio
    - Give up upside potential
        - Allow to focus on managing portfolio in local term.
    - Sell forward contracts with amount equal to currency exposure
    - Need rebalancing when asset value changes.
    - Easy to implement
    - Ignore correlation across currencies

3. **(b) continued**

2. Minimum-variance / regression hedging
    - Combine asset risk and currency risk and manage the portfolio as a total.
    - Find an optimal hedge ratio via volatilities/correlations between assets & currency
    - Consider both asset risk and currency risk in deriving hedge ratio.
    - Easy to implement once the hedge ratio is found.
    - Need accurate forecast of asset and currency volatilities/ correlations

3. Downside/ option-based hedging
    - Hedge from unfavorable currency movements
    - Retain upside potentials
    - 3 ways to implement:
        a) buy put options on individual currency, hedge each separately
        b) buy puts on a basket of currencies, hedge a basket of currency
        c) buy puts on total base currency value of portfolio

(c)

- Euro liability is valued at $100m, 6.7% of total GIC, and 2% of total liability
    - International allocation is small.
- The liability will be matured over next 5 years.
- Euro/US exchange rate has been stable over its short history.

(d)

- Unhedged given the limited exposure to currency risk.
    - Most likely too costly if hedge
    - Allow more diversification of currency exposure

4.

Learning Objectives:

4a – Describe the issues influencing investment strategies, including liquidity requirements, valuation concerns, cash flow variability, compliance risk, regulatory constraints, taxation impacts, and investment management mandates.

4c – Describe liquidity requirements of an investor and their impact upon portfolio management.

Most companies must monitor and manage their liquidity, or exposure to illiquid assets, to maintain their ratings. This question follows up on a risk noted in the case study by asking the candidate to assess the exposure and recommend corrective action. See the CIA Educational Note, RSA Vol 27#2, and 8FE-406.

Solution:

(a)

- Liquidity risk is referred to as the company's ability to meet financial obligations through cash flow or asset sales at fair market value
- Liquidity crisis is due to expected or unexpected cash demand.

(b)

Management should consider

- Setting liquidity guidelines
- Monitoring position relative to liabilities & take action if guidelines are exceeded.
- How rating agencies measure risks and impact on its ratings
- Cash management and investment department should work together to define short & medium terms needs
- Product design team should explicitly consider liquidity implication of the product features.
- External factors are condition of the economy, condition of the industry.
- Internal factors are cash flow predictability, accessing to financing, credit rating, customer base, balance sheet strength, quality field force

(c)

Asset side

- Public Corp (below Inv. Grade)
- Private Corps
- Commercial Mortgages
- Real Estate

Liability Side

- Allow policyholder to take loan at 7%

4. continued

(d)

Liquidity depends on the type of assets the capital is invested in

(e)

- If possible, for new business, revise policy provisions. Example given: use floating policy loan rates, use market value adjustment for surrender
- Adjust asset portfolio, increase cash equivalent, like treasury, also more allocation to government and public corporate (investment grade)

5.

Learning Objectives:

4c - Describe liquidity requirements of an investor and their impact upon portfolio management.

The candidate is asked to work through a liquidity ratio calculation. Liquidity management has received new attention in the wake of 2007's subprime mortgage meltdown and subsequent liquidity contraction. The material is in 8FE-406, the SOA Specialty Guide on Economic Capital.

Solution:

		Immediate Scenario		Ongoing Scenario	
Asset Class	BV	Factor	Liquid Assets	Factor	Liquid Assets
Government	58.7	100	58.70	100	58.70
Public Corporate (inv. Grade)	469.5	96	450.72	100	469.50
Public Corporate (below inv. Grade)	117.4	0	-	25	29.35
Private Corporate (inv. Grade)	258.2	65	167.83	75	193.65
Private Corporate (below inv. Grade)	140.9	0	-	20	28.18
Agency Pass-throughs	64.6	90	58.14	90	58.14
CMO's	64.6	80	51.68	80	51.68
Cash and short-term	30.0	100	30.00	100	30.00
Commercial Mortgages	135.0	0	-	0	-
Equities	18.8	70	13.16	85	15.98
Real Estate	105.0	0	-	0	-
All other assets	37.5	0	-	0	-
Total	1500.2		830.23		935.18

Liabilities:

Total book value 1500
500 have 6% SC ending during next year
250 have MVA
750 have no withdrawal restrictions

Immediate Scenario

Apply 90% liab risk factor
Apply surrenderability risk factors
Apply covariance factor of 70%

Risk-adjusted liquid liabilities

= 90% * (250 * 50% + 500 * 50% + 750 * 100%) * 70%
= 708.75

5. continued

Ongoing Scenario

Apply 100% liab risk factor
Apply surrenderability risk factors
Apply covariance factor of 70%

Risk-adjusted liquid liabilities

= 100% * (250 * 50% + 500 * 100% + 750 * 100%) * 70%
= 962.5

Result	Immediate Scenario	Ongoing Scenario
Risk-Adjusted Liquid Assets	830.23	935.18
Risk-Adjusted Liquid Liabilities	708.75	962.5
Liquidity Ratio	117%	97%

6.

Learning Objectives:

1e – Evaluate the impact of embedded options on risk/return characteristics of assets and liabilities.

3d – Describe how derivatives, synthetic securities, and financial contracting may be used to manage risk.

This question asks the candidate to synthesize a wide range of syllabus material; see in particular 8V-313-01 "Variable Annuities: No Loss Propositions", Hull Chapter 14, and Hardy Chapters 6 and 12. The candidate is walked through the analysis and hedge design of a guarantee on a variable (equity-linked) annuity contract. Key issues in the analysis are highlighted, including assumptions behind the Black-Scholes-Merton model and consideration of their practical shortcomings.

Solution:

This solution is adjusted to take into consideration that Normal Distribution tables were not available.

(a)

We require

$$\Pr\left[S_2 < 600 \mid S_0 = 450\right]$$

$$= \Pr\left[S_2 / S_0 > 1.3333\right]$$

And

$$S_2 / S_0 \sim \log N\left(2\times(.09),\left(2\times 0.22^2\right)\right)$$

So the probability is

$$\Phi\left(\frac{\ln(1.3333)-0.18}{\sqrt{2}0.22}\right) = \Phi(0.345)$$

(b)

We require the expectation under the P measure.

That is

$$E_0^P\left[(600-S_2)^+ \mid S_0 = 450\right]$$

6. **(b) continued**

We know from Black Scholes that

$$E_0^Q\left[e^{-2r}\left(600-S_2\right)^+ \mid S_0=450\right]=600e^{-2r}\Phi\left(-d_2\right)-S_0\Phi\left(-d1\right)$$

Where

$$d_1=\frac{\ln\left(S_0/K\right)+\left(r+\sigma^2/2\right)2}{\sqrt{2}\sigma}$$

$$d_2=\frac{\ln\left(S_0/K\right)+\left(r-\sigma^2/2\right)2}{\sqrt{2}\sigma}$$

So under Q-measure:

$$E_0^Q\left[\left(600-S_2\right)^+ \mid S_0=450\right]=600\Phi\left(-d_2\right)-S_0e^{2r}\Phi\left(-d_1\right)$$

Now P-measure is exactly the same distribution, same σ, but replacing r by $\mu+\sigma^2/2$, where $\mu=0.09$.

So under P-measure:

$$d_1^P=\frac{\ln\left(S_0/K\right)+\left(\mu+\sigma^2\right)2}{\sqrt{2}\sigma}=-0.035$$

$$d_2^P=\frac{\ln\left(S_0/K\right)+2\mu}{\sqrt{2}\sigma}=-0.346$$

And

$$E_0^P\left[\left(600-S_2\right)^+ \mid S_0=450\right]=600\Phi\left(0.346\right)-S_0e^{2\mu+\sigma^2}\Phi\left(0.035\right)$$

(c)

It's a put option.
Using the Black Scholes equation above:

$$\text{Price}=E_0^Q\left[e^{-2r}\left(600-S_2\right)^+ \mid S_0=450\right]=600e^{-2r}\Phi\left(-d_2\right)-S_0\Phi\left(-d1\right)$$

6. **(c) continued**

Where

$$d_1 = \frac{\ln\left(S_0 / K\right) + \left(r + \sigma^2 / 2\right)2}{\sqrt{2}\sigma} = -0.4477$$

$$d_2 = \frac{\ln\left(S_0 / K\right) + \left(r - \sigma^2 / 2\right)2}{\sqrt{2}\sigma} = -0.7588$$

$$\left(\Rightarrow \text{Price} = 118.54\right)$$

(d)

- The expected cost under the P measure is the real world average outgo
- B-S gives the discounted expected cost under the risk neutral measure.
- B-S is the cost of replicating the option payoff.
- The risk neutral measure is a risk adjusted measure, gives more weight to losses than P measure.
- Note that the BS is discounted, the expected cost is not.

(e)

The Delta at 0:

$$\text{delta} = -\Phi\left(-d_1\right) = -\Phi\left(-0.4477\right)$$

The stock part at t = 0.5 :

$$\text{stock part} = 0.7697 \quad S_{0.5} = -338.69$$

Where S_t is the asset MV at t.

Bond part at t = 1.0

$$\text{bond part} = 600e^{-r}\Phi\left(-d_2\right)$$

Cashflow at t = .5 :

Hedge brought forward = $-302.76 \times 440/450 + 421.30e^{r/2} = 135.93$
Cost of new hedge = $-338.69 + 469.33 = 130.64$
Cashflow required = -5.29.

6. (e) continued

Cashflow required at $t = 2$:

Hedge brought forward $= -370.52 \times 520/550 + 425.79e^{r/2} = 86.26$
Guarantee cost: 80.00.
Cashflow required = -6.26.

(f)

The total discounted cost is:

$$118.54 + (-5.29)e^{-0.5r} + 7.66e^{-r} + (-13.15)e^{-1.5r} + (-6.26)e^{-2r} = 102.80$$

This is different to the Black Scholes cost because:

1. We are hedging discretely, not continuously as assumed in Black Scholes.
2. Discrete hedging introduces jumps in the stock price that B-S does not anticipate.
3. The 1/2 year interval is large for a hedging strategy, making hedging errors high.
4. B-S assumes GBM. Perhaps the assets are not following GBM.
5. B-S assumes constant volatility; markets exhibit stochastic volatility. Makes B-S less accurate.
6. Perhaps the 22% volatility assumption is not appropriate.

(g)

The traditional approach, also called the actuarial method:

- Use Monte Carlo Simulation
- Assume a real world distribution for S_t
- Generate a large sample of simulated values for the final liability.
- Discount at the risk free rate.
- Apply a risk measure to the loss distribution for the capital requirement.

(h)

- Delta hedging protects against highly adverse outcomes.
- Delta hedging is dynamic, allows for gradual changes in capital requirements.
- Real world approach is more profitable on average.
- Real world approach is easier to understand and explain.

7. **Learning Objectives:**
The candidate will be able to:

a) Define the elements of a game, including information sets, etc. Nash equilibrium, mixed strategies.

b) Explain the prisoners' dilemma and other special cases of a two-person, two-state, single period game.

c) Explain the qualitative implications of repeated games.

This is an application question asking candidates to determine game theory strategies under certain scenario. The best sources for answering this question could be found in Rasmusen's "Games and Information" textbook, pages 11-31.

Solution:

(a)

For Country X outcome is equal to the program saving less the consulting fee.

Table of outcomes:

		President of Country X	
		State-run Insurance	Open Insurance market
World Life	Accept Consulting Assignment	(35, 60-35 =25)	(20, 45-20=25)
	Decline Consulting Assignment	(0, 60-40=20)	(40, 45 -0 = 45)

Payoff = (World Life, Country X)

Assumptions: No other insurers enter the open market; no opportunity for bargaining or discussion, so all decisions are made simultaneously.

(b)

For World Life, there are no dominated or dominating strategies. Depending on what Country X's President decides, World Life could be better off accepting or declining.

For the President of Country X, the open insurance market is a weakly dominant strategy and State-run is a weakly dominated strategy. If the President chooses open-market his outcome is always as good or better than if he chose state-run. The dominance is weak because one strategy is not always better.

7. continued

(c)

There are two Nash equilibriums:

- State-run Insurance/Accept
- Open Insurance Market/Decline

Neither player has an incentive to deviate from their strategy unless the other player deviates.

(d)

Pareto optimality applies to Open Insurance Market/Decline since the outcomes for both players are higher here than in the other Nash equilibrium.

(e)

With no competition World Life's profit in (Open Insurance Market, Decline) is 40.

In the presence of competition, this reduces to 40 x (1-.75) = 10

Probability of competition = Z%

The expected value of the outcome X for (Open Insurance, Decline) under possible competition is:

$$X = 10 \times Z\% + 40 (1-Z\%)$$
$$X = 40 - 30Z\%$$

In order for Accept to dominate Decline as a strategy for World Life, the outcome for (Open Market, Accept) must exceed (open Market, Decline)

$$X \leq 20$$
$$40 - 30Z\% \leq 20$$

The maximum probability of competition that is acceptable to World Life before Accept the Assignment is always a dominant choice is:

$$40-30Z\% = 20$$
$$30Z\% = 20$$
$$Z\% = 66.667\%$$

8.

Learning Objectives:
5g – Calculate effective duration of a portfolio
5h – Contrast modified duration and effective duration measures

This is an application question asking candidates to calculated effective duration of a portfolio and contrast modified duration and effective duration measures. The best sources for answering this question could be found in Ch. 17 and 18 of the Babbel & Fabozzi "Investment Management for Insurers" textbook. Credit was also given for answers based on other reading from the syllabus resources if relevant.

Solution:

(a)

Assets – Non-callable Government Treasuries, use either one
- Payments made by the government to holders of its obligations do not change when yield curve changes

Assets – Callable Corporate Bonds, use effective duration
- Call options introduced => cash flows related to levels or historic interest rates movements

Assets – Mortgage Backed Securities, use effective duration
- Prepayment options will alter expected cash flows when levels or historical interest rates change

Liabilities, use effective duration
- Are replete with many options and provisions that make the cash flows patterns sensitive to interest rates movements

(b)

Asset portfolio duration = W(1) * D(1) + W(2) * D(2) + W(3) * D(3)

W(i) - market value of asset (i) / market value of portfolio, D(i) = effective duration of asset (i), for non-callable treasuries D(i) can also be modified duration

Asset Portfolio duration = 10/70 * 0.69 + 40/70 * 2.92 + 20/70 * 6.57
= 3.64

8. continued

(c)

Conditions for Immunization:

$$D(A) = L / A * D(L)$$

And

$C(A) > L/A * C(L)$
$D(A)$ = Effective duration of assets
$D(L)$ = Effective duration of liabilities
$C(A)$ = Convexity of assets
$C(L)$ = Convexity of liabilities
A = Total Asset Market Value, Total Liability Market Value

Both conditions have to be satisfied

$3.64 <> 50 / 70 * 2.4$ => condition 1 not met

$45 < 50/70 * 65$ => condition 2 not met => not immunized

9.

Learning Objectives:
5h – Contrast modified duration and effective duration measures

This question asks candidates to apply a formula on the formula sheet to the data for a particular company. It asks the candidates to analyze from the results whether the goal of the company is satisfied by using the duration method and to describe the limitations of using this method to study interest rate sensitivity. In summary, this question asks candidates to calculate the duration of liabilities which would satisfy the company's target, and then comment on the faults of the method itself. The source for this question is Ch. 17of the Babbel & Fabozzi "Investment Management for Insurers" textbook.

Solution:

(a)

$$\begin{aligned} D_S &= (D_A - D_L) \times \left(\frac{A}{S}\right) + D_L \\ &= (18 - D_L) \times \left(\frac{6{,}500{,}000}{600{,}000}\right) + D_L \\ &= 180 - 9D_L \end{aligned}$$

For D_S to be between 5 and 10, we need $5 < 180 - 9D_L < 10$

or $18.9 < D_L < 19.4$

(b)

$$\begin{aligned} D_S &= (D_A - D_L) \times \left(\frac{A}{S}\right) + D_L \\ &= (17 - D_L) \times \left(\frac{6{,}500{,}000}{1{,}000{,}000}\right) + D_L \\ &= 110.5 - 5.5D_L \end{aligned}$$

For D_S to be between 5 and 10, we need $5 < 110.5 - 5.5D_L < 10$

or $18.3 < D_L < 19.2$

(c)

Limitations of using this approach:

- Cash flows are not fixed, as implied in the formula.
- 2 years is too short-term so not credible enough.
- Formula is only a simplification which should be used with other measures, such as convexity.
- Will only work for small changes in interest rate.

9. **(c) continued**

- Factors that do not directly affect the magnitude of durations are ignored.
- As the credit quality increases or if options are introduced, traditional duration will differ more versus effective duration, increasing the probability of error if a poor duration measure is chosen.
- Effective duration measures are based on stochastic interest rate models.
- Should consider a variety of methods to assess interest rate sensitivity, not a single method.

10.

Learning Objectives:

1a – Define the cash flow characteristics of complex derivatives including exotic options, credit derivatives, interest rate derivatives, swaps, and other non-traditional derivatives

Counterparty credit risk is very much in the news and on risk management committee agendas. This application question asks the candidate to mitigate an emerging credit exposure. See Hull chapter 21 and the "JP Morgan Guide to Credit Derivatives".

Solution:

(a)

- Under a CDS Peckham would pay a regular premium to a 3rd party.
- The fee is based on a notional amount based on Peckham's exposure to loss if Falkirk defaults
- In the event of a 'credit event' – i.e., full or partial failure of Falkirk, the 3rd party would make payment.

(b)

Advantages of CDS

- No need for Falkirk to be aware
- Isolates the credit risk
- Avoids adverse effect on relationship
- Recapture may require raising of capital

Disadvantages of CDS

- CDS may be very expensive given Falkirk's situation
- Exposure to 3rd party counterparty risk

(c)

- Reference Entity
  - Falkirk Re
- Notional amount (Notional principal)
  - Basis for payment
  - This could be coinsured reserve amount
- Credit event (default, insolvency, nonpayment)
- CDS spread
- Effective Date
- Contract Termination Date

10. continued

(d)

- Dynamic CDS has variable notional amount
- In this situation, one could use economic reserve, statutory reserves

(e)

$$\text{Spread}\times\left(\text{E}\left[\text{Prem}\right] + \text{E}\left[\text{Accrual}\right]\right)=\text{E}\left[\text{Payoff}\right]$$

$$\text{E}\left[\text{Payoff}\right] = .25\times\left(1-.75\right)\times 1.423\times\exp\left(-.03\right) + .75\times.1\times\left(1-.75\right)\times 1.391\times\exp\left(-.09\right)$$
$$+ .75\times.9\times.05\times\left(1-.75\right)\times 1.384\times\exp\left(-.15\right) = 120.1792$$

$$\text{E}\left[\text{Prem}\right] = .75\times 1.423\times\exp\left(-.06\right) + .75\times.9\times 1.391\times\exp\left(-.12\right) + .75\times.9\times.95\times 1.384$$
$$\times\exp\left(-.18\right) = 2,578.55$$

$$\text{E}\left[\text{Accrual}\right] = .05\times.25\times 1.423\times\exp\left(-.03\right)\times.5 + .05\times.75\times.1\times 1.391\times\exp\left(-.09\right)\times.5$$
$$+ .05\times.75\times.9\times.05\times 1.384\times\exp\left(-.15\right)\times.5 = 240.36$$

$$\text{Spread} = \left(2818.9\right)/\left(2578.55 + 240.36\right) = 4.26\%$$

11.

Learning Objectives:
1d – Define option adjusted spread analysis and its limitations

This straightforward question is based on the Fabozzi "Handbook of Mortgage Backed Securities", chapter 23. Different models for assessing the value of an MBS are examined and then applied to a hypothetical CMO tranche structure.

Solution:

(a)

(i) *Nominal Spread*:

Find Treasury security with maturity equal to average life of MBS for benchmarking.

The nominal Spread is the cashflow yield on MBS – yield on Treasury benchmark.

(ii) *Zero Volatility Spread*:

The Zero Volatility Spread is the constant spread that will make the price of a security equal to the present value of its cash flows when added to the yield at each point on the spot rate Treasury curve where a cash flow is received

Option Adjusted Spread:

Assume we have an arbitrage free interest rate model. We simulate N interest rate paths and determine the cash flows for each path using a dynamic prepayment model.

The OAS is the fixed spread over the simulated interest paths that makes the average PV cash flows for all paths equal to the MV.

(b)

The OAS is the best spread measure. The main reasons are:

- It is a stochastic measure, reflects range of interest rate paths
- It assumes dynamic prepayments, where the others assume deterministic prepayments. Prepayments are key to value and risk balance.
- OAS measures compensation after adjusting for prepayment risk.
- Other measures do not adjust for option value.

11. continued

(c)

The cheaper tranches have high OAS (value) , low option cost (risk). Duration measures interest rate risk, which may be an issue.

A has fairly low OAS, which measures value. Option cost and duration measure prepayment and interest rate risk. The only possible advantage of A is low duration.

B has higher value (OAS) than A, with lower option risk. It looks more attractive, but also has higher duration.

C has even higher value (OAS), only slightly higher option risk, but considerably higher duration.

D has only slightly higher value but much greater option risk than **C**. It looks less attractive unless the lower duration is needed.

12.

Learning Objectives:
6b – Explain the prisoners' dilemma and other special cases of a two-person, two-state, single period game.

*This is a synthesis question asking candidates to demonstrate mastery of the Cournot-Nash equilibrium and Stackelberg equilibrium, and understanding of their similarity and difference. The best source for answering this question could be found on Pages 87-90 of Rasmusen's "**Game and Information**" textbook.*

Solution:

(a)

Step 1:
Find reaction functions (best-response functions) for the players (text, page 88)

- Idea: for Blue, set the first partial derivative of P_{blue} with respect to x equal to zero

Calculation:
From $P_{blue} = x/(x+y) - s\,x$, $dP_{blue}/dx = (1\,(x+y) - x\,1)/(x+y)^2 - s = y/(x+y)^2 - s$.
Let $dP_{blue}/dx = 0$, i.e., $y/(x+y)^2 - s = 0$.
Thus the reaction function for Blue is

$$\mathbf{y = s\,(x+y)^2} \qquad (1)$$

Similarly, for Red, set the first partial derivative of P_{red} w.r.t. y equal to zero.
And find that the reaction function for Red is

$$\mathbf{x = t\,(x+y)^2} \qquad (2)$$

Step 2:
Derive equilibrium allocations by solving the reaction system (page 88)

- Adding *(1)* to *(2)*, we have $x+y = (s+t)\,(x+y)^2$.
- Since $x+y > 0$, it follows that $x+y = 1/(s+t)$.
- Plugging it back to *(1)* and *(2)* we solve

$$\mathbf{y = s/(s+t)^2} \quad \text{and} \quad \mathbf{x = t/(s+t)^2}. \qquad (3)$$

12. (a) continued

Step 3:
Evaluate the equilibrium profits

$\mathbf{P_{blue}} = x/(x+y) - s\,x = t/(s+t) - s\,t/(s+t)^2 = \mathbf{t^2/(s+t)^2}$, *(4)*

$\mathbf{P_{red}} = y/(x+y) - t\,y = s/(s+t) - t\,s/(s+t)^2 = \mathbf{s^2/(s+t)^2}$. *(5)*

(b)

Idea 1:
Blue forecasts that Red will move along Red's reaction curve (page 90), which is $x = t\,(x+y)^2$ from *(2)*.

Idea 2:
Blue moves to maximize its own profit subject to *(2)*.
Calculation:

- Re-write *(2)* as $x+y = (x/t)^{0.5}$ and plug it into $P_{blue} = x/(x+y) - s\,x$
- It follows that $P_{blue} = (t\,x)^{0.5} - s\,x$
- From calculus or quadratic function, P_{blue} obtains its maximum at $\mathbf{x = t/(4s^2)}$.
- Plugging $x = t/(4s^2)$ back to *(2)*, we solve $\mathbf{y} = 1/(2s) - t/(4s^2) = \mathbf{(2s-t)/(4s^2)}$, which is positive since $2s > t$.
- Plugging the allocations x and y to the profit functions, we have

$\mathbf{P_{blue}} = (t\,x)^{0.5} - s\,x = t/(2s) - s\,t/(4s^2) = \mathbf{s\,t/(4s^2)}$, *(6)*

$\mathbf{P_{red}} = y/(x+y) - t\,y = (2s-t)/(2s) - t\,(2s-t)/(4s^2) = \mathbf{(2s-t)^2/(4s^2)}$. *(7)*

(c)

Need to verify that P_{blue} at equilibrium in (b), as the leader, is at least as great as in (a).
That is, from *(6)* and *(4)*, $s\,t/(4s^2) >= t^2/(s+t)^2$.
Since $s, t > 0$, it amounts to see that $\mathbf{(s+t)^2 >= 4st}$, which holds since $(s-t)^2 >= 0$.

Notes:

It's not true in general that in (b) Blue, as the leader, derives greater profit than Red, as the follower. We can compare P_{blue} with P_{red} using *(6)* and *(7)*. Namely, $P_{blue} - P_{red} = s\,t/(4s^2) - (2s-t)^2/(4s^2) = (4s-t)(t-s)/(4s^2)$. Since $4s-t > 0$, $\mathbf{P_{blue} > P_{red}}$ **if and only if t > s, that is, if Red has higher unit cost than Blue**. Similarly, compare the sum of *(4)* and *(5)* to that of *(6)* and *(7)*, we can see that the **total** profit in (a) is greater than in (b) if and only if $t > s$, too.

13.

Learning Objectives:

2d) - Apply the concept of economic capital and describe methodologies for allocating capital within a financial organization.

The question continues Question 1's exploration of capital management. Allocating capital, whether regulatory, rating agency, or economic, is not a mathematical exercise. Rather, it requires business judgment and influence to successfully implement a system – in any kind of company. The material is from 8FE-208, "Allocation of Risk Capital in Financial Institution", 8FE-325 "Capital Allocation in Financial Firms", and Chew chapter 31 "Theory of Risk Capital in Financial Firms".

Solution:

(a)

1. **Top-down approach**
   Capital at risk (CaR) and performance objectives are assigned by senior management to each risk-taking unit

2. **Creation of an internal market for capital at risk (CaR)**
   Amount of CaR allocated to each unit and its price (i.e., the RAROC objective each unit assumes) is determined by matching aggregate CaR demand coming from risk-taking units and aggregate CaR supply provided by corporate headquarters

3. **Allocate CaR through negotiation**
   CaR and risk adjusted performance (RAP) target levels derived through an iterative process where senior management's judgments lead to initial proposals, which are then negotiated with the business units

(b)

Allocated CaR

- The greater the influence of individual units on CaR assignments *(as is the case for internal market for capital*), such units should be evaluated on allocated CaR
- Since EJC Trust's business units have *little influence* on risk capital assigned to them, the use of allocated CaR is **not** appropriate in this case.

Actual Amount of Utilized CaR

- When CaR is allocated in a *highly centralized, top-down fashion*, units should not be blamed for not utilizing an amount of capital (or excess capital unused) that may be disproportionate to the risk-taking opportunities they face.
- Hence, utilized CaR is a more appropriate measure of risk capital for EJC Trust.

13. continued

(c)

- It's inappropriate because of the inconsistency between the basis for profits being monthly versus the basis for CaR being daily. Using month-end daily CaR doesn't convert something that's daily to monthly. A measure of overall utilized CaR is required to address this problem.

(d)

- Total CaR is generally less than the sum of individual business unit CaRs
- Any RAROC measure based on "diversified" CaR is influenced by the correlation structure among existing business returns. Good for management decisions, bad for measuring individual unit performance and determining bonuses.
- Use of "undiversified" CaR is appropriate when evaluating individual business units
- Use of "diversified" CaR is appropriate when making acquisition or divestiture decisions, or when reviewing units' contributions to the firm's overall risk profile in order to define strategic plans

(e)

Table 1 ($ Millions)

Business Unit	Stand Alone Risk Capital	Profit
Unit A	200	40
Unit B	100	15
Unit C	200	50

Table 2: Correlation Among Business Units

Business Unit	Unit A	Unit B
Unit B	0.00	
Unit C	0.00	0.30

Set up and complete correlation matrix

Business Unit	Unit A	Unit B	Unit C
Unit A	1.00	0.00	0.00
Unit B	0.00	1.00	0.30
Unit C	0.00	0.30	1.00

13. (e) continued

Set up and complete the table of squared CaRs
(or cross products of CaRs)

Business Unit	Unit A	Unit B	Unit C
Unit A	40,000	20,000	40,000
Unit B	20,000	10,000	20,000
Unit C	40,000	20,000	40,000

Total CaR = Sum of Cross product: 102,000
Square Root: 319.4

CaR Adjustment Factor = (Total CaR) / (Total Stand-Alone CaR) = 319.4 / 500 = 63.9%

Business Unit	Stand-Alone Risk Capital	×	Adjustment Factor	=	Diversified CaR
Unit A	200		63.9%		127.7
Unit B	100		63.9%		63.9
Unit C	200		63.9%		127.7
	500				319.4

14.

Learning Objectives:

2b) Describe the steps necessary to obtain funds for a given project or firm from any specified source, and be able to recommend a specific approach to raising capital in a given situation.

The candidate must explain alternatives for raising capital and how they affect a company's capital structure. The material is based on Chew Chapter 18, Megginson Chapter 9, and Doherty Chapter 13.

Solution:

(a)

- Issuing common stock
  - is generally perceived negatively and reduces stock price.
  - can cause earnings per share dilution, which is believed to reduce stock prices.
  - increases the supply in the market, which can be expected to reduce stock prices.
  - can reduce stock prices due to negative information effects:
    - Management could believe that the stock price is overvalued
    - Stock issuance reduces leverage which is perceived negatively

- Issuing non-callable debt
  - is generally perceived positively by the stock market, and can increase stock price.
  - signals management confidence in ability to make debt payments.
  - increases leverage which is perceived positively.

(b)

Investment banks can provide:

- Liquidity and payment intermediation – taking deposits and providing commercial loans
- Maturity intermediation – issuing short term liabilities and purchasing longer term assets
- Denomination intermediation
- Information intermediation
- Diversification intermediation

Investment banks can also:

- Advise on raising capital (issuance type, features, amount)
- Underwrite the offering
- Provide stabilization activity and Green Shoe option

14. continued

(c)
(i)

The junior debt is non-convertible

Project A

Probability	PV Earnings	Old Debt	New Debt	Bankruptcy	Equity
0.6	2200	500	800	0	900
0.4	1700	500	800	0	400
Expected Value	2000	500	800	0	700

Project B

Probability	PV Earnings	Old Debt	New Debt	Bankruptcy	Equity
0.42	2500	500	800	0	1200
0.18	1400	500	800	0	100
0.28	2000	500	800	0	700
0.12	900	500	300	100	0
Expected Value	1970	500	740	12	718

Choose Project B since expected equity value is greater.

(ii)

The junior debt is convertible to 10 million shares at the option of the debtholder.

New debtholders will convert to equity if their value after conversion is greater than 800 million.

So will convert when $\frac{1}{2}x(\text{value of firm - value of old debt}) > 800,$

or when value of firm > 2100

Project A

Probability	PV Earnings	Old Debt	New Debt	Bankruptcy	Equity
0.6	2200	500	850	0	850
0.4	1700	500	800	0	400
Expected Value	2000	500	830	0	670

14. (c)(ii) continued

Project B					
Probability	PV Earnings	Old Debt	New Debt	Bankruptcy	Equity
0.42	2500	500	1000	0	1000
0.18	1400	500	800	0	100
0.28	2000	500	800	0	700
0.12	900	500	300	100	0
Expected Value	1970	500	824	12	634

Choose Project A since expected equity value is greater.

(d)

- Convertible bonds help to align the interests of shareholder and bondholders, bondholders can participate on the upside.
- Convertible bonds reduce bankruptcy risk, since less risky projects are selected.
- Convertible bonds reduce the under-investment problem.

15.

Learning Objectives:

2e) Recommend an optimal capital structure and how to implement it for a given business or strategy and to justify the recommendation.

This question uses the Modigliani and Miller capital structure propositions to explore how capital structure and tax status affects one's perspective on an acquisition (or even on lines of business within a firm). See Megginson Chapter 7.

Solution:

(a)

- All physical assets owned by corporations
- 2 securities – risky equity / risk free debt
- Can borrow / lend at risk free rate
- Homogeneous expectations of future profits
- No growth so all cash flows are perpetuities
- No market friction, costs, or taxes

(b)
(i)

$$K = \rho + \frac{(\rho - r)D}{S}$$

$$K_{GH} = .1 + \frac{(.1 - .05)(.5)}{.5} = .15$$

$$K_{DW} = .1 + \frac{(.1 - .05)(0.0)}{1} = .10$$

$K_{GH} > K_{DW} \Rightarrow$ GenHouse better off

(ii)

$$V_L = V_U + \text{tax shield} = \frac{\text{NOI}(1-t)}{\rho}$$

Before purchase:

$$\text{Value } DW = \frac{1,000,000(1-.25)}{.1} = 7,500,000$$

$$\text{Stock } DW = \frac{7,500,000}{500,000} = 15$$

$$\text{Value } GH = \frac{1,000,000(1-.25)}{.1} + 5,000,000(.25) = 8,750,000$$

15. **(b)(ii) continued**

$$\text{Stock } GH = \frac{8,750,000 - 5,000,000}{250,000} = 15$$

After purchase:

$$\text{Value } DW = \frac{(1,000,000 + 500,000)(1 - .25)}{.1} = 11,250,000$$

$$\text{Stock}_{DW} = \frac{11,250,000}{500,000 + 250,000} = 15$$

GH issues another 1,875,000 debt to purchase.

$$\text{Value } GH = \frac{(1,000,000 + 500,000)(1 - .25)}{.1} + (5,000,000 + 1,875,000)(.25)$$

$$= 12,968,750$$

$$\text{Stock}_{GH} = \frac{(12,968,750 - 6,875,000)}{250,000 + 125,000} = 16.25$$

$\text{Stock}_{GH} > \text{Stock}_{DW}$ due to tax shield. GenHouse better off.

16.

Learning Objectives:

3c - Demonstrate how to apply funding and portfolio management strategies to control equity and interest rate risk, including key rate risks. Explain the concepts of immunization including modern refinements and practical limitations.

This is an application question with recall and synthesis asking the candidates to demonstrate their knowledge of duration, VaR, and stress testing with respect to analyzing the risk of fixed income derivatives. The reference for the question is Babel & Fabozzi, Investment Management for Insurers, Chapter 22, "Portfolio Risk Management". The key insight from the reading is that fixed income derivatives have asymmetric price distributions and require different risk measures than other simpler securities. Credit was also given for points from other readings.

Solution:

(a)

- Interest rate level
- Rates of benchmark securities
- Spreads over government rates
- Volatility of interest rates
- Exchange rates

(b)

- Derivatives have embedded options
- Options have asymmetric price distributions
- Duration does not adequately capture this asymmetry
- Price change may not be a linear function of interest rate level change
- Changes in rates are not instantaneous – time needs to be incorporated
- Dollar change may be more appropriate because many derivatives positions start with low or zero value
- Security specific risk not reflected in duration
- Exposure changes as rates change
- Duration assumes parallel rate shift

(c)

$\lambda = \mu_3 / \sigma^3 = -82,000 / 3,000^{3/2} = -0.5$

$\mathrm{K}(\lambda) = 2.69$

$\sigma = 3000^{1/2} = 54.77$

VaR = 2.69 x 54.77 = 147.34 (can subtract mean of 25)

16. continued

(d)

VaR = 2.33 x 54.77 = 127.62 (can subtract mean of 25)

(e)

- Use the gamma
- Gamma is better because it captures the skewness in the distribution
- The normal assumes that the skewness is zero
- VaR using the normal is too low

(f)

Stress testing

- can measure portfolio's changes to market conditions over longer time period
- can examine more extreme events
- can examine more persistent events
- can examine path-dependent events
- does not rely on a specific form of the value response curve such as linear or quadratic
- derivatives will not change in a linear or quadratic form
- can appeal to intuition by showing the situations under which a loss can occur
- required or recommended by various oversight agencies and auditors
- management can be more involved
- allows judgment
- less systematic than VaR
- somewhat ad hoc
- no probability assigned to scenario

VaR

- measures instantaneous changes to prices
- produces a single value summarizing the risk
- required or recommended by various oversight agencies and auditors
- not coherent risk measure
- has a probability associated with it
- ignores extreme tail events
- dependent on method and assumptions used
- should use a confidence interval with it

SHOULD USE BOTH TOGETHER AS A COMPREHENSIVE MEASUREMENT OF RISK

17.

Learning Objectives:

3b - Demonstrate how to apply funding and portfolio management strategies to control equity and interest rate risk, including key rate risks. Explain the concepts of immunization including modern refinements and practical limitations.

5g - Calculate effective duration and effective key-rate durations of a portfolio.

This is an application question asking candidates to calculate key rate duration and effective duration of a portfolio and compare the two for measuring the interest rate risk. The best sources for answering this question could be found in 8V-115: Key Rate Durations: Measures of Interest Rate Risks, Thomas S. Y. Ho. Credit was also given for points from other readings.

Solution:

(a) Let P′ be the price of the bond after each key rate is shocked.
Let P be the price of the original bond.

$$\begin{aligned} \mathrm{P} &= 3e^{(-4.75\%)(0.5)} + 3e^{(-4.95\%)(1)} + 3e^{(-5\%)(1.5)} + 103e^{(-5.05\%)(2)} \\ &= 2.9296 + 2.8551 + 2.7832 + 93.1051 \\ &= 101.6730 \end{aligned}$$

We shock each key rate by 0.1% each time.

Let K(1) be the key rate duration for the first key rate at t=0.25.
Shocking the key rate at t=0.25 by 0.1% means that, by linear interpolation, we also shock the rate at t=0.5 by x, where x is:

$$\frac{0.1\%}{0.25-1} = \frac{x}{0.5-1}$$
$$\therefore x = 0.066667\%$$

$$\begin{aligned} \mathrm{P'} &= 3e^{-(4.75\%+0.06667\%)(0.5)} + 3e^{(-4.95\%)(1)} + 3e^{(-5\%)(1.5)} + 103e^{(-5.05\%)(2)} \\ &= 2.9286 + 2.8551 + 2.7832 + 93.1051 \\ &= 101.6720 \end{aligned}$$

$$\mathrm{K}(1) = -\frac{\Delta \mathrm{P}}{\mathrm{P}} \cdot \frac{1}{\Delta \mathrm{r}} = -\frac{(101.6720 - 101.6730)}{101.6730} \cdot \frac{1}{0.1\%} = 0.009835453$$

Let K(2) be the key rate duration for the second key rate at t=1.
Shocking the key rate at t=1 by 0.1% means that, by linear interpolation, we also shock the rate at t=0.5 by x, where x is:

17. **(a) continued**

$$\frac{0.1\%}{1-0.25} = \frac{x}{0.5-0.25},$$
$$\therefore x = 0.033333\%$$

and we also shock the rate at t=1.5 by *y*, where *y* is:

$$\frac{0.1\%}{1-2} = \frac{y}{1.5-2}$$
$$\therefore y = 0.05\%$$

$$\begin{aligned} P' &= 3e^{-(4.75\%+0.03333\%)(0.5)} + 3e^{-(4.95\%+0.1\%)(1)} + 3e^{-(5\%+0.05\%)(1.5)} + 103e^{(-5.05\%)(2)} \\ &= 2.9291 + 2.8523 + 2.7811 + 93.1051 \\ &= 101.6676 \end{aligned}$$

$$K(2) = -\frac{\Delta P}{P}\cdot\frac{1}{\Delta r} = -\frac{(101.6676-101.6730)}{101.6730}\cdot\frac{1}{0.1\%} = 0.053111446$$

Let K(3) be the key rate duration for the third key rate at t=2.
Shocking the key rate at t=2 by 0.1% means that, by linear interpolation, we also shock the rate at t=1.5 by *x*, where *x* is:

$$\frac{0.1\%}{2-1} = \frac{x}{1.5-1}$$
$$\therefore x = 0.05\%$$

$$\begin{aligned} P' &= 3e^{(-4.75\%)(0.5)} + 3e^{(-4.95\%)(1)} + 3e^{-(5\%+0.05\%)(1.5)} + 103e^{-(5.05\%+0.1\%)(2)} \\ &= 2.9296 + 2.8551 + 2.7811 + 92.9191 \\ &= 101.4849 \end{aligned}$$

$$K(3) = -\frac{\Delta P}{P}\cdot\frac{1}{\Delta r} = -\frac{(101.4849-101.6730)}{101.6730}\cdot\frac{1}{0.1\%} = 1.8500$$

All other key rate durations are zero for t>2, since there are no cashflows.

(b) Effective duration is the sum of the key rate durations.

Effective duration = 0.0098+0.0531+1.8500=1.91

17. continued

(c) Effective duration is often inadequate in measuring a security's interest rate risk exposure.

Spot curve rarely moves in a parallel fashion, which makes effective duration not precise enough.

Effective duration is the total risk exposure, key rate durations are the component parts of effective duration

Key rate durations can identify the price sensitivity of a security to each segment of the yield curve.

Key rate durations recognize that the yield curve movement is driven by multiple market factors.

Easy to use key rate durations to create a replicating portfolio of a bond with embedded options using zero-coupon bonds.

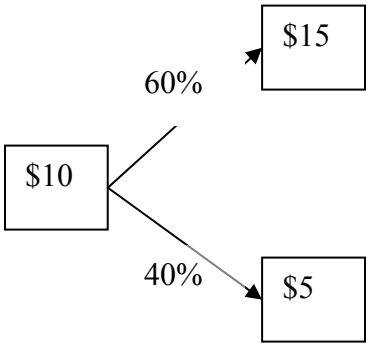
18.

Learning Objectives:
5i - Apply the elements of risk assessment, reduction, and transfer to new product/project proposals based on a cost/benefit analysis.

This is an application question asking candidates to demonstrate mastery of options-based approach of contingent-claims analysis in capital budgeting and understanding the shortcoming of traditional DCF approach in capital budgeting. The best source for answering this question could be found in chapter 5 of the Trigeorgis' "Real Options" textbook.

Solution:

(a)



10 = 1/(1+k)*(.6*15 + .4*5) = 11/(1+k)
k = 10%

(b)

Use k from comparable security (GECH) so k=10% from a.

NPV(GenE) = -51 + 1/1.1*(.6*75+.4*25) = -1

Since NPV < 0, shouldn't invest

(c)

DCF doesn't give a proper discount rate to value projects. Rate is constant.
DCF also doesn't include the value of real options embedded in the project.

18. continued

(d)

If GenE delayed for one year:

$I_1 = I_0 * (1 + r_f) = 51 * 1.03 = 52.53$

$E_1^+ = \max\{75 - 52.53, 0\} = 22.47$

$E_1^- = \max\{25 - 52.53, 0\} = 0$

Company will only invest when cash flow is \$75M

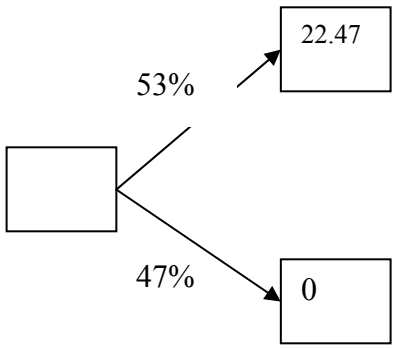
Risk-neutral prob can be calculated:

$$u = 1.5$$

$$d = 0.5$$

$$p_u = \frac{(1+r)-d}{u-d} = \frac{1.03-0.5}{1.5-0.5} = 53\%$$

$$p_d = 1 - p_u = 47\%$$



$$\frac{22.47 * 0.53 + 0 * 0.47}{1.03} = 11.56$$

This is the Expanded NPV.

Expanded NPV = PV + option premium

Option Premium = 11.56 – (-1) = 12.56

18. **continued**

(e)

$$S = 10, \quad S^+ = 15, \quad S^- = 5$$

$$E^+ = 22.47, \quad E^- = 0$$

$$E^+ = NS^+ - (1+r)B, \quad E^- = NS^- - (1+r)B$$

$$22.47 = 15N - 1.03B$$

$$0 = 5N - 1.03B$$

$$22.47 = 10N$$

$$N = 2.247$$

$$B = \frac{5N}{1.03} = 10.91$$

Buy 2.247M shares in GECH, borrow \$10.91M in Treasuries.