SOCIETY OF ACTUARIES
Exam FET
Financial Economic Theory Exam (Finance/ERM/Investment)

Exam FET

AFTERNOON SESSION

Date: Thursday, November 1, 2007
Time: 1:30 p.m. – 4:45 p.m.

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This afternoon session consists of 10 questions numbered 9 through 18 for a total of 60 points. The points for each question are indicated at the beginning of the question. There are no questions that pertain to the Case Study in the afternoon session.

2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.

3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.

2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.

3. The answer should be confined to the question as set.

4. When you are asked to calculate, show all your work including any applicable formulas.

5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets since they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate morning or afternoon session for Exam FET.

6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d'examen pour la version française.

Printed in the U.S.A.

Financial Economic Theory Formulae Sheet
May 2007

Megginson, Corporate Finance Theory

$$V_j = (S_j + D_j) = \frac{NOI_j}{\rho_k} \text{ for any firm j in class k}$$

$$k_j = \rho_c + \frac{(\rho_c - r)D_j}{S_j}$$

$$G_L = \left[1 - \frac{(1-\tau_c)(1-\tau_{ps})}{(1-\tau_{pd})}\right]D_L$$

$$V_L = V_U + PV\ tax\ shields - PV\ bankruptcy\ costs + PV\ agency\ costs\ of\ outside\ equity$$
$$- PV\ agency\ cost\ of\ outside\ debt$$

$$\rho(t) = \left[\frac{d_j(t) + p_j(t+1) - p_j(t)}{p_j(t)}\right]$$

$$p_j(t) = \left[\frac{d_j(t) + p_j(t+1)}{1 + \rho(t)}\right]$$

$$V(t) = \left[\frac{D(t) + n(t)p(t+1)}{1 + \rho(t)}\right]$$

Hull , Options, Futures and Other Derivatives

$$F_0 = S_0 e^{rT}$$

$$F_0 = (S_0 - I)e^{rT}$$

$$F_0 = S_0 e^{(r-q)T}$$

$$f = (F_0 - K)e^{-rT}$$

$$f = S_0 - Ke^{-rT}$$

$$f = S_0 - I - Ke^{-rT}$$

$$f = S_0 e^{-qT} - Ke^{-rT}$$

$$F_0 = S_0 e^{(r-q)T}$$

$$F_0 = S_0 e^{(r-r_f)T}$$

$$F_0 = S_0 e^{rT}$$

$$F_0 = (S_0 + U)e^{rT}$$

$$F_0 = S_0 e^{(r+u)T}$$

$$F_0 \le (S_0 + U)e^{rT}$$

$$F_0 \le S_0 e^{(r+u)T}$$

$$F_0 = S_0 e^{(r+u-y)T}$$

$$F_0 = S_0 e^{cT}$$

$$F_0 = S_0 e^{(c-y)T}$$

$$F_0 = E(S_T) e^{(r-k)T}$$

$$\Delta z = \varepsilon \sqrt{\Delta t}$$

$$z(T) - z(0) = \sum_{i=1}^{N} \varepsilon_i \sqrt{\Delta t}$$

$$dx = a dt + b dz$$

$$dx = a(x,t)dt + b(x,t)dz$$

$$S_T = S_0 e^{\mu T}$$

$$\frac{ds}{S} = \mu dt + \sigma dz$$

$$\Delta S = \mu S \Delta t + \sigma S \varepsilon \sqrt{\Delta t}$$

$$\frac{\Delta S}{S} \sim \phi(\mu \Delta t, \sigma \sqrt{\Delta t})$$

$$dG = \left(\frac{\partial G}{\partial x} a + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial x^2} b^2\right)dt + \frac{\partial G}{\partial x} b dz$$

$$dS = \mu S dt + \sigma S dz$$

$$dG = (\frac{\partial G}{\partial S}\mu S + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial S^2}\sigma^2 S^2)dt + \frac{\partial G}{\partial S}\sigma S dz$$

$$F = Se^{r(T-t)}$$

$$dF = (\mu - r)F dt + \sigma F dz$$

$$dG = (\mu - \frac{\sigma^2}{2})dt + \sigma dz$$

$$c + Ke^{-rT} = p + S_0 e^{-qT}$$

$$c = S_0 e^{-qT} N(d_1) - Ke^{-rT} N(d_2)$$

$$p = Ke^{-rT} N(-d_2) - S_0 e^{-qT} N(-d_1)$$

$$d_1 = \frac{ln(S_0 / K) + (r - q + \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{ln(S_0 / K) + (r - q - \sigma^2 / 2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

$$dS = (r - q)S dt + \sigma S dz$$

$$p = \frac{e^{(r-q)\Delta t} - d}{u - d}$$

$$\frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial F^2}\sigma^2 F^2 = rf$$

$$H_F = e^{-rT} H_A$$

$$H_F = e^{-(r-q)T} H_A$$

$$H_F = e^{-(r-r_f)T} H_A$$

$$N(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$$

$$\Delta\Pi = \Theta\Delta t + \frac{1}{2}\Gamma\Delta S^2$$

$$\Theta + rS\Delta + \frac{1}{2}\sigma^2 S^2 \Gamma = r\Pi$$

$$\Delta = e^{-qT}\left[N(d_1) - 1\right]$$

$$Q_i(T|M) = N\left(\frac{N^{-1}\left[Q_i(T)\right] - a_i M}{\sqrt{1-a_i^2}}\right)$$

$$Q(T|M) = N\left(\frac{N^{-1}\left[Q(T)\right] - \sqrt{\rho}M}{\sqrt{1-\rho}}\right)$$

$$p(k,T|M) = \frac{N!}{(N-k)!k!} Q(T|M)^k \left[1 - Q(T|M)\right]^{N-k}$$

$$e^{-rT_1}\hat{E}\left[c\frac{S_1}{S_0}\right]$$

$$S_0 e^{-qT_2} M\left(a_1, b_1; \sqrt{T_1/T_2}\right) - K_2 e^{-rT_2} M\left(a_2, b_2; \sqrt{T_1/T_2}\right) - e^{-rT_1} K_1 N\left(a_2\right)$$

$$a_1 = \frac{ln\left(S_0/S^*\right) + \left(r - q + \sigma^2/2\right)T_1}{\sigma\sqrt{T_1}} \qquad a_2 = a_1 - \sigma\sqrt{T_1}$$

$$b_1 = \frac{ln\left(S_0/K_2\right) + \left(r - q + \sigma^2/2\right)T_2}{\sigma\sqrt{T_2}} \qquad b_2 = b_1 - \sigma\sqrt{T_2}$$

$$K_2 e^{-rT_2} M\left(-a_2, b_2; -\sqrt{T_1/T_2}\right) - S_0 e^{-qT_2} M\left(-a_1, b_1; -\sqrt{T_1/T_2}\right) + e^{-rT_1} K_1 N\left(-a_2\right)$$

$$K_2 e^{-rT_2} M\left(-a_2, -b_2; \sqrt{T_1/T_2}\right) - S_0 e^{-qT_2} M\left(-a_1, -b_1; \sqrt{T_1/T_2}\right) - e^{-rT_1} K_1 N\left(-a_2\right)$$

$$S_o e^{-qT_2} M\left(a_1, -b_1; -\sqrt{T_1/T_2}\right) - K_2 e^{-rT_2} M\left(a_2, -b_2; -\sqrt{T_1/T_2}\right) + e^{-rT_1} K_1 N\left(a_2\right)$$

$$\max\left(c, p\right) = c + e^{-q(T_2 - T_1)} \max\left(0, Ke^{-(r-q)(T_2-T_1)} - S_1\right)$$

$$H \le K: c_{di} = S_0 e^{-qT}\left(H/S_0\right)^{2\lambda} N\left(y\right) - Ke^{-rT}\left(H/S_0\right)^{2\lambda-2} N\left(y - \sigma\sqrt{T}\right)$$

$$\lambda = \frac{r - q + \sigma^2/2}{\sigma^2}$$

$$y = \frac{ln\left[H^2/(S_0K)\right]}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}$$

$$c_{do} = c - c_{di}$$

$$H \geq K:\; c_{do} = S_0N(x_1)e^{-qT} - Ke^{-rT}N\left(x_1 - \sigma\sqrt{T}\right) - S_0e^{-qT}\left(H/S_0\right)^{2\lambda}N(y_1) + Ke^{-rT}\left(H/S_0\right)^{2\lambda-2}N\left(y_1 - \sigma\sqrt{T}\right)$$

$$c_{di} = c - c_{do}$$

$$x_1 = \frac{\ln\left(S_0/H\right)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}$$

$$y_1 = \frac{\ln\left(H/S_0\right)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}$$

$$H > K:\; c_{ui} = S_0N(x_1)e^{-qT} - Ke^{-rT}N\left(x_1 - \sigma\sqrt{T}\right) - S_0e^{-qT}\left(H/S_o\right)^{2\lambda}\left[N(-y) - N(-y_1)\right]$$
$$+Ke^{-rT}\left(H/S_0\right)^{2\lambda-2}\left[N\left(-y + \sigma\sqrt{T}\right) - N\left(-y_1 + \sigma\sqrt{T}\right)\right]$$

$$c_{uo} = c - c_{ui}$$

$$H \geq K:\; p_{ui} = -S_0e^{-qT}\left(H/S_0\right)^{2\lambda}N(-y) + Ke^{-rT}\left(H/S_0\right)^{2\lambda-2}N\left(-y + \sigma\sqrt{T}\right)$$

$$p_{uo} = p - p_{ui}$$

$$H \leq K:\; p_{uo} = -S_0N(-x_1)e^{-qT} + Ke^{-rT}N\left(-x_1 + \sigma\sqrt{T}\right) + S_0e^{-qT}\left(H/S_0\right)^{2\lambda}N(-y_1) - Ke^{-rT}\left(H/S_0\right)^{2\lambda-2}N\left(-y_1 + \sigma\sqrt{T}\right)$$

$$p_{ui} = p - p_{uo}$$

$$H < K:\; p_{di} = -S_0N(-x_1)e^{-qT} + Ke^{-rT}N\left(-x_1 + \sigma\sqrt{T}\right) + S_0e^{-qT}\left(H/S_0\right)^{2\lambda}\left[N(y) - N(y_1)\right]$$
$$-Ke^{-rT}\left(H/S_0\right)^{2\lambda-2}\left[N\left(y - \sigma\sqrt{T}\right) - N\left(y_1 - \sigma\sqrt{T}\right)\right]$$

$$p_{do} = p - p_{di}$$

$$c_{ELB} = S_0e^{-qT}N(a_1) - S_0e^{-qT}\frac{\sigma^2}{2(r-q)}N(-a_1) - S_{\min}e^{-rT}\left(N(a_2) - \frac{\sigma^2}{2(r-q)}e^{Y_1}N(-a_3)\right)$$

$$a_1 = \frac{ln\left(S_0/S_{\min}\right) + \left(r - q + \sigma^2/2\right)T}{\sigma\sqrt{T}}$$

$$a_2 = a_1 - \sigma\sqrt{T}$$

$$a_3 = \frac{ln(S_0 / S_{\min}) + (-r + q + \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

$$Y_1 = -\frac{2(r - q - \sigma^2 / 2)\, ln(S_0 / S_{\min})}{\sigma^2}$$

$$p_{ELB} = S_{\max} e^{-rT}\left(N(b_1) - \frac{\sigma^2}{2(r-q)} e^{Y_2} N(-b_3)\right) + S_0 e^{-qT} \frac{\sigma^2}{2(r-q)} N(-b_2) - S_o e^{-qT} N(b_2)$$

$$b_1 = \frac{ln(S_{\max} / S_o) + (-r + q + \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

$$b_2 = b_1 - \sigma\sqrt{T}$$

$$b_3 = \frac{ln(S_{\max} / S_0) + (r - q - \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

$$Y_2 = -\frac{2(r - q - \sigma^2 / 2)\, ln(S_{\max} / S_0)}{\sigma^2}$$

$$\max(0, S_T - S_\tau) + (S_\tau - K)$$

$$r - \frac{1}{2}\left(r - q - \frac{\sigma^2}{6}\right) = \frac{1}{2}\left(r + q + \frac{\sigma^2}{6}\right)$$

$$M_1 = \frac{e^{(r-q)T} - 1}{(r-q)T} S_0$$

$$M_2 = \frac{2e^{(2(r-q)+\sigma^2)T} S_0^2}{(r - q + \sigma^2)(2r - 2q + \sigma^2)T^2} + \frac{2S_0^2}{(r-q)T^2}\left(\frac{1}{2(r-q)+\sigma^2} - \frac{e^{(r-q)T}}{r - q + \sigma^2}\right)$$

$$\sigma^2 = \frac{1}{T}\ln\left(\frac{M_2}{M_1^2}\right)$$

$$V_o e^{-q_V T} N(d_1) - U_0 e^{-q_U T} N(d_2)$$

$$d_1 = \frac{ln(V_o / U_o) + (q_U - q_V + \hat{\sigma}^2 / 2)T}{\hat{\sigma}\sqrt{T}} \qquad d_2 = d_1 - \hat{\sigma}\sqrt{T}$$

$$\hat{\sigma} = \sqrt{\sigma_U^2 + \sigma_V^2 - 2\rho\sigma_U\sigma_V}$$

$$\lambda = \frac{\rho}{\sigma_m}(\mu_m - r)$$

$$d\ln S = \left(\theta(t) - a\ln S\right)dt + \sigma dz$$

Babbel and Fabozzi , Investment Management for Insurers

$$D(t,T) = \frac{1}{e^{s(t,T)\times(T-t)}} = \frac{1}{e^{\phi(T-t)\times(T-t)}} E\left[\frac{1}{e^{\int_t^T r_s ds}}\right]$$

$$r_s^* = r_s + \phi(s-t) + \phi'(s-t)\times(s-t)$$

$$D(t,T) = \frac{1}{e^{s(t,T)\times(T-t)}} = E\left[\frac{1}{e^{\int_t^T (r_s + \phi(T-t))ds}}\right] = E\left[\frac{1}{e^{\int_t^T r_s^* ds}}\right]$$

$$D_S = (D_A - D_L)\frac{A}{S} + D_L$$

$$P(j) \approx P(i)\left[1 - D(i)(j-i) + \frac{1}{2}C(i)(j-i)^2\right]$$

$$D(i) = \frac{-P'(i)}{P(i)} \qquad C(i) = \frac{P''(i)}{P(i)}$$

$$P'(i) \approx \frac{P(i+\Delta t) - P(i-\Delta t)}{2\Delta t}$$

$$P''(i) \approx \frac{P(i+\Delta t) - 2P(i) + P(i-\Delta t)}{(\Delta i)^2}$$

$$D = \frac{\sum tc_t v^{mt+1}}{p} \qquad C = \frac{\sum t(t+\frac{1}{m})c_t v^{mt+2}}{p}$$

$$P(j) = P(i)\exp\left[-\int_i^j D(s)ds\right]$$

$$D(j) \approx D(i) + \left[D^2(i) - C(i)\right](j-i)$$

$$P(j) \approx P(i)\exp\left[-D(i)(j-i)\right]$$

$$S(j) \approx S(i)\left[1 + C^S(j-i)^2\right]$$

$$D^S(j) \approx -C^S(i)(j-i)$$

$$di_t = \mu(t,i_t)dt + \sigma(t,i_t)dz_t$$

$$i_T = i_0 + \int_0^T \mu(t,i_t)dt + \int_0^T \sigma(t,i_t)dz_t$$

$$\frac{dP_t}{P_t} = \left(\frac{\partial_t P_t}{P_t} - D_t\mu_t + \frac{1}{2}C_t\sigma_t^2\right)dt - D_t\sigma_t dz_t$$

$$\frac{dP_t}{P_t} = \left(i_t - (T-t)\mu_t + \frac{1}{2}(T-t)^2\sigma_t^2\right)dt - (T-t)\sigma_t dz_t$$

$$dD_t = \left(\partial_t D_t + (D_t^2 - C_t)\mu_t + \frac{1}{2}\left[D_t(D_t^2 - C_t) - \partial_t C_t\right]\sigma_t^2\right)dt + (D_t^2 - C_t)\sigma_t dz_t$$

$$P(j) = P(i)\left[1 - D(i)\bullet\Delta i + \frac{1}{2}\Delta i^T C(i)\Delta\right]$$

$$D_k(i) = \frac{-\partial_k P(i)}{P(i)} \qquad C_{kl}(i) = \frac{-\partial_{kl}P(i)}{P(i)}$$

$$\partial_k P(i) \approx \frac{\left[P(i+\Delta i E_k) - P(i-\Delta i E_k)\right]}{\left[2\Delta i\right]}$$

$$\partial_{kl}P(i) \approx \frac{\left[P(i+\Delta i(E_j+E_k)) - P(i-\Delta i(E_l-E_k)) - P(i+\Delta i(E_k-E_l)) + P(i-\Delta i(E_k+E_l))\right]}{\left[2\Delta i\right]^2}$$

$$\frac{\Delta P}{P} = -\sum_{i=1}^{n} D_i \Delta F_i$$

$$D_i = -\frac{1}{P}\frac{\partial P}{\partial F_i}$$

$$\Delta P = A - \sum_{i=1}^{n} D_i X_i + \frac{1}{2}\sum_{i=1}^{n} C_i X_i^2 + Y$$

$$D_i = -\frac{P_i - P_i^{'}}{2\Delta F_i}$$

$$C_i = \frac{P_i + P_i^{"} - 2P}{(\Delta F_i)^2}$$

$$A = \mu - \frac{1}{2}\sum_{i=1}^{n} C_i \sigma_i^2$$

$$\mu = E\Delta P$$

$$\sigma^2 = Var(\Delta P) = \sum_{i=1}^{n}\sum_{j=1}^{n} D_i D_j \sigma_{ij} + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} C_i C_j \sigma_{ij}^2 + s^2$$

$$\mu_3 = E(\Delta P - \mu)^3 = 3\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} D_i D_j C_k \sigma_{ik}\sigma_{jk} + \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} C_i C_j C_k \sigma_{ij}\sigma_{jk}\sigma_{ki}$$

Chew, The New Corporate Finance

$$G_L = \left[1 - \frac{(1-t_c)(1-t_{ps})}{(1-t_{pb})}\right] B_L$$

Trigeorgis, Real Options

$$NPV = \sum_{t=1}^{T} \frac{\alpha_t E(c_t)}{(1+r_1)\ldots(1+r_t)} - I$$

$$E(r_j) = r + \beta_j \left[E(r_m) - r\right]$$

$$Expanded\,(strategic)\,net\,present\,value\,(NPV^*) = \left[Direct\,(passive)\,NPV + strategic\,value\right]$$
$$+ flexibility\,value$$

$$\frac{d\pi_A}{dK_A} = \frac{\partial \pi_A}{\partial K_A} + \frac{\partial \pi_A}{\partial \alpha_B}\frac{d\alpha^* B}{dK_A}$$

Rasmusen, Games and Information, An Introduction to Game Theory

best response: $\pi_i(s_i^*, s_{-i}) \geq \pi_i(s_i^{'}, s_{-i}) \forall s_i^{'} \neq s_i^*$

dominated strategy: $\pi_i(s_i^d, s_{-i}) < \pi_i(s_i, s_{-i}) \forall s_{-i}$

dominant strategy: $\pi_i(s_i^*, s_{-i}) > \pi_i(s_i^{'}, s_{-i}) \forall s_{-i} \forall s_i^{'} \neq s_i^*$

weakly dominated:
$\pi_i(s_i^{'}, s_{-i}) \geq \pi_i(s_i, s_{-i}) \forall s_{-i} \; and \; \pi_i(s_i^{'}, s_{-i}) > \pi_i(s_i, s_{-i}) \; for \; some \; s_{-i}$

Nash equilibrium: $\forall i, \; \pi_i(s_i^*, s_{-i}^*) \geq \pi_i(s_i, s_{-i}^*) \forall s_i^{'}$

pure strategy: $s_i : \omega_i \rightarrow a_i$

mixed strategy: $s_i : \omega_i \rightarrow m(a_i) \;\; where \;\; m \geq 0 \;\; \int_{A_i} m(a_i) da_i = 1$

completely mixed: $m > 0$

minimax strategies: $\min imize_{s_{-i}} \max imize_{s_i} \pi_i(s_i, s_{-i})$

maximin strategies: $\max imize_{s_i} \min imize_{s_{-i}} \pi_i(s_i, s_{-i})$

evolutionarily stable strategy (ESS): $\pi \; (s^*, s^*) > \pi \; (s \;, s^*) \; or$

$$(a) \;\; \pi(s^*, s^*) = \pi(s \,, s^*) \; and \; (b) \;\; \pi(s^*, s \,) > \pi(s \,, s^{'})$$

epsilon equilibrium: $\forall i, \; \pi_i(s_i^*, s_{-i}^*) \geq \pi_i(s_i, s_{-i}^*) - \varepsilon, \forall s_i \in S_i$

FET-100-07

$$PV = \frac{1}{(1 + r_{risk\,free})} \left[(1 - p^D) 100 + p^D RV \right]$$

$$Default\ prob \approx 1 - \frac{1}{\left[1 + \frac{S_{market}}{1 - \frac{RV}{100}} \right]^t}$$

Counterparty Credit Charge (CCC):

$$ccc = (100\% - RR_{CP}) \sum_{t=t_0}^{tN} \sum_{R=Def}^{AAA} prob_{joint} \left\{ CP_{in\,default} RE_{rating=R} \right\} OP_{rating=R}$$

FET-101-07 None

FET-102-07

$$F = \sum_i \max\left(S_{i0}, S_{iT}\right) = \sum_i S_{iT} + \sum_i \max\left(0, S_{i0} - S_{iT}\right)$$

$$F = \max\left(\sum_i S_{i0}, \sum_i S_{iT}\right) = \sum_i S_{iT} + \max\left(0, \sum_i \left(S_{i0} - S_{iT}\right)\right)$$

FET-103-07 None

FET-104-07

$$\sigma_t^2 = \alpha_0 + \alpha_1 (Y_{t-1} - \mu)^2 + \beta \sigma_{t-1}^2$$

$$F_{t^+} = F_{t^-}(1-m) = F_{(t-1)}(1-m)\frac{S_t}{S_{t-1}}$$

$$F_{(t+u)^+} = F_t \frac{S_{t+u}(1-m)^u}{S_t}$$

$$M_t = \left(F_{t^-}\right) m_c = m_c F_{0^-} \frac{s_t (1-m)^{t-1}}{S_0}$$ *errata sheet*

$$C_n = -{}_n p_x^\tau (G - F_n)^+$$

$$C_t = -{}_t p_x^\tau M_t^d + {}_{t-1|1} q_x^d \left(G - F_t\right)^+$$ *note: M should have d superscript*

$$C_t = -{}_t p_x^\tau F_0 - S_t (1-m)^t m_d + {}_{t-1|1} q_x^d \left(G - F_0 - S_t (1-m)^t\right)^+$$ *errata sheet*

$$C_t = {}_{t-1|1} q_x^d \left(G_r - F_t\right)^+ - {}_t p_x^\tau M_t \quad \text{where} \quad n_r < t < n_{r+1}$$

$$C_{n_r} = {}_{n_r - 1|1} q_x^d \left(G_r - F_{n_r^-}\right)^+ + {}_{n_r} p_x^\tau \left(G_r - F_{n_r^-}\right)^+ - {}_{n_r} p_x^\tau M_{n_r}$$

$$\log(1+i_t) \big| \rho_t^y = \mu_{\rho_t^y}^y + \phi_{\rho_t^y}^y \left(\log(1+i_{t-1}) - \mu_{\rho_t^y}^y\right) + \sigma_{\rho_t^y}^y \varepsilon_t$$

$$H_0 = B(0,n) \mathrm{E}_Q \left[F_n (ga_{65}(n) - 1)^+\right]$$

$$H_0 = F_0 \mathrm{E}_Q \left[\left(\frac{ga_{65}^d(0,n)}{B(0,n)} - 1\right)^+\right]$$

$$H_t = F_t \left\{ga_{65}(t)\Phi(d_1(t)) - \Phi(d_2(t))\right\} \text{ where}$$

$$d_1(t) = \frac{\log(ga_{65}(t)) + \sigma_y^2 (n-t)/2}{\sigma_y \sqrt{n-t}} \quad \text{and} \quad d_2(t) = d_1(t) - \sigma_y \sqrt{n-t}$$

FET-105-07 None

FET-106-07

$$dS = \mu S dt + \sigma S dZ$$

$$dr = \mu(r,t) r dt + r\sigma dZ$$

$$\sigma(t,T) = \frac{\sigma\left(\frac{\Delta r(t,T)}{r(t,T)}\right)}{\sqrt{\Delta t}}$$

$$\sigma(t,T) = \frac{\sigma(\Delta r(t,T))}{\sqrt{\Delta t}}$$

$$dr = a(b-r)dt + \sigma\sqrt{r}dZ$$

$$dr = a(b-r)dt + \sigma dZ \text{, (a>0)}$$

$$dr = a_1 + b_1(l-r)dt + r\sigma_1 dZ$$

$$dl = (a_2 + b_2 r + c_2 l)dt + l\sigma_2 dW$$

$$dV = M(t,r)dt + \Omega(t,r)dZ$$

$$M(t,r) = V_t + \mu(t,r)V_r + \frac{1}{2}\sigma(t,r)^2 V_{rr}$$

$$\Omega(t,r) = \sigma(t,r)V_r$$

$$d\Pi = (M_1(t,r) - \Delta M_2(t,r))dt + (\Omega_1(t,r) - \Delta\Omega_2(t,r))dZ$$

$$d\Pi = r\Pi dt$$

$$V_t + (\mu(t,r) - \lambda(t,r)\sigma(t,r))V_r + \frac{1}{2}\sigma(t,r)^2 V_{rr} - rV = 0$$

$$P_i^n(1) = 2\left[\frac{P(n+1)}{P(n)}\right]\frac{\delta^i}{(1+\delta^n)} \qquad \delta = e^{-2r(1)\sigma}$$

$$P_i^n(T) = \frac{1}{2}P_i^n(1)\left\{P_i^{n+1}(T-1) + P_{i+1}^{n+1}(T-1)\right\}$$

$$r_i^n(1) = \ln\frac{P(n)}{P(n+1)} + \ln\left(\frac{1}{2}\left(\delta^{\frac{-n}{2}} + \delta^{\frac{n}{2}}\right)\right) + \left(\frac{n}{2} - i\right)\ln\delta$$

Note: Typo in text $r_i^n(1)1 =$ either way will receive full credit.

$$dr = (f'(0,t) + \sigma^2 t)dt + \sigma dz$$

$$r(n)\sigma^s(n) = \frac{-\frac{1}{2}\ln\left[\delta(n)\delta(n-1)\ldots\delta(1)\right]}{n}$$

$$P_i^n(1) = \left[\frac{P(n+1)}{P(n)}\right]\left[\frac{(1+\delta_{n-1}\delta_{n-2}..\delta_1)..(1+\delta_{n-1})2}{(1+\delta_n..\delta_1).(1+\delta_n)}\right]\delta_n^i$$

$$dr = (f'(0,t) + \sigma^2(t)t + \tfrac{\sigma(t)}{\sigma(t)}\left[r(t) - f(0,t)\right])dt + \sigma(t)dZ$$

$$P_{i\,j}^n(1) = \frac{P(n+1)}{P(n)}\frac{(1+\delta_{n-1}^1..\delta_1^1)(1+\delta_{n-1}^1..\delta_2^1)..(1+\delta_{n+1}^1)2}{(1+\delta_n^1..\delta_1^1).(1+\delta_n^1\delta_{n-1}^1)(1+\delta_n^1)}\times$$

$$\frac{(1+\delta_{n-1}^2..\delta_1^2)(1+\delta_{n-1}^2..\delta_2^2)..(1+\delta_{n-1}^2)2}{(1+\delta_n^2..\delta_1^2)(1+\delta_n^2..\delta_2^2)...(1+\delta_n^2)}(\delta_n^1)^i(\delta_n^2)^j$$

$$dr = \left\{f'(t) + |\sigma(t)|^2 t + \frac{|\sigma(t)|\cos\phi(t)}{|\sigma(t)|\cos\theta(t)}\left[r - f(t)\right]\right\}dt + \sigma(t)dW$$

$$d\ln r = (\theta(t) - \frac{\sigma'(t)}{\sigma(t)}\ln r)dt + \sigma(t)dW$$

$$dr(t) = (\alpha(t) - \beta r(t))dt + \sigma dW(t)$$

where $\alpha(t) = \dfrac{\partial f(0,t)}{\partial T^*} + \beta f(0,t) + \dfrac{\sigma^2}{2\beta}(1 - e^{-2\beta t})$

$$dr = \left[\theta(t) + \mu - ar\right]dt + \sigma_1 dW$$

$$du = -bu dt + \sigma_2 dZ$$

$$dP(t,T^*) = r(t)P(t,T^*)dt + \sigma^P(t,T^*)P(t,T^*)dZ$$

$$df(t,T^*) = \sigma^P(t,T^*)\sigma_{T^*}^P(t,T^*)dt - \sigma_{T^*}^P(t,T^*)dZ$$

$$dP(t,T^*) = r(t)P(t,T^*)dt + \sigma(T^* - t)P(t,T^*)dZ(t,T^*)$$

$$L(t,T^*) = \frac{1}{\Delta}\left(\frac{P(t,T^*)}{P(t,T^*+\Delta)} - 1\right)$$

$$dL(t,T^*) = L(t,T^*)\left[\sum_{j=i}^{N^*} \frac{L(t, j\Delta)\Delta}{1+L(t, j\Delta)\Delta}\Lambda(T^* - j\Delta)\Lambda(T^* - t)dt + \Lambda(T^* - t)dZ\right]$$

$$L(k, j+1) = L(k, j)\exp\left[\left(\sum_{i=j+1}^{k} \frac{L(i, j)\Delta}{1+L(i, j)\Delta}\Lambda_{i-j-1}\Lambda_{k-j-1} - \frac{\Lambda_{k-j-1}^2}{2}\right)\Delta + \Lambda_{k-j-1}\sqrt{\Delta}\tilde{Z}\right]$$

Where $\sigma_j^2 j = \sum_{i=1}^{j} \Lambda_{j-i}^2$

caplet $C_k = L\delta_k P(t_{k+1})\left[F_k N(d_1) - R_x N(d_2)\right]$

$$\text{where } d_1 = \frac{\ln\left[\frac{F_k}{R_X}\right] + \sigma_k^2 \frac{t_k}{2}}{\sigma_k \sqrt{t_k}} \qquad d_2 = d_1 - \sigma_k \sqrt{t_k}$$

$$\text{swaption} = \sum_{i=1}^{mn} \frac{L}{m} P(t_i)\left[R_F N(d_1) - R_X N(d_2)\right] = L * A\left[R_F N(d_1) - R_X N(d_2)\right]$$

where $A = \frac{1}{m}\sum_{i=1}^{mn} P(t_i) \quad 1 \le i \le mn$

$$P(k+1, j) = P(k, j)\exp\left[\left(r(k) - \frac{\sigma^2(j-k)}{2}\right)\Delta + \sigma(j-k)\sqrt{\Delta}Z(j-k)\right]$$

$$\sigma^*(T^* - t) = (a + b(T^* - t))\exp(-c(T^* - t)) + d$$

$$L(k, j+1) = L(k, j)\exp\left[\left(\sum_{i=j+1}^{k} \frac{L(i, j)\Delta}{1+L(i, j)\Delta}\Lambda_{i-j-1}\Lambda_{k-j-1} - \frac{\Lambda_{k-j-1}^2}{2}\right)\Delta + \Lambda_{k-j-1}\sqrt{\Delta}Z\right]$$

$$P(T^*, i; T) = \frac{P(T^* + T)}{P(T^*)} \cdot 2 \cdot \frac{\prod_{t=T}^{T+T^*-1} h(t)}{\prod_{t=1}^{T^*-1} h(t)} \delta^{Ti} \quad \text{where } h(t) = \frac{1}{1+\delta^t}$$

FET-107-07 None

FET-108-07

$$V(E) = V(F) - V(D) = V(F) - D_{DF} + P(V(F), D) = C(V(F), D)$$

$$V^*(F) = S + D(1+\frac{m}{n})$$

$$V_R(E) = -C + V_R(F) - D + P\{V_R(F), D\} = -C + V_R - D + P_R$$
$$V_N(E) = V_N(F) - D + P\{V_N(F), D\} = V_N - D + P_N$$

$$\textit{face value} + \textit{principal forgiven} - \textit{default put assu}\min g\,\textit{reinvestment} = D - (P_N - P_R - NPV) - P_R$$
$$= (D - P_N - B - NPV) + B = \textit{value of regular debt} + \textit{saving in bankruptcy} \cos t$$

FET-109-07

$$RBC = \frac{1}{2}\left[C_0 + C_{4a} + \left[(C_1 + C_{3a})^2 + C_2^2 + C_{3b}^2 + C_{4b}^2\right]^{\frac{1}{2}}\right]$$

FET-110-07

market value of surplus = market value of assets – market of liabilities
= PV (assets cash flows) – PV (liabilities cash flows)
= PV of net cash flows

Total return = income + realized returns + unrealized returns

FET-111-07 None

FET-112-07 None

FET-113-07

$$\sigma_Y^2 = \sum_{i=1}^{n} \sigma_{x_i}^2 = \sum_{i=1}^{n} \omega_i^2 \sigma_i^2$$
$$MCaR = k\sigma_r = k\sqrt{\sum_{i=1}^{n} \omega_i^2 \sigma_i^2} = \sqrt{\sum_{i=1}^{n} k^2 \omega_i^2 \sigma_i^2} = \sqrt{\sum_{i=1}^{n} DCaR_i^2}$$

$$Total\ CaR = \sqrt{\sum_{i=1}^{n} CaR_i^2 + \sum_{i=1}^{n}\sum_{i \neq j} CaR_i CaR_j \rho_{ij}}$$

FET-114-07

$$NPV = (1-d)V\{S^+\} - (C - \mu) - (1+m)V\{S^-\}$$
$$= \mu - (dV\{S^+\} + mV\{S^-\})$$

$$V\{S^+\} = \frac{\sigma(n(z) + zN(z))}{(1+r)}$$

$$V\{S^-\} = \frac{\sigma(n(z) - zN(-z))}{(1+r)}$$

FET-115-07 None

FET-116-07 None

FET-117-07 None

FET-118-07

$$\left[N + kLe^{-sT}\right]e^{sT} = L + \left(kLe^{-sT}\right)e^{rT}$$

$$N = Le^{-sT}\left[1 - k + ke^{(r-s)T}\right]$$

$$V(R) = \sum_{t=1}^{T} Le^{-sT} = L\frac{d(1-d^T)}{1-d} = Le^{-sT}\frac{e^{sT}-1}{e^s-1} \quad \text{where } d = e^{-s}$$

$$D(R) = -\frac{1}{V(R)}\frac{dV(R)}{ds} \qquad d(R) = \frac{e^s}{e^s-1} - \frac{T}{e^{sT}-1}$$

$$D(A) = \frac{(1+k)V(R)D(R)}{V(A)}$$

$$V(FL) = Le^{-sT}\frac{p}{e^s-p}\frac{e^{ns}-p^n}{e^{ns}} = Le^{-sT}\sum_{t=1}^{n} p^t e^{-st}$$

$$V(FP) = N\frac{p}{e^s-p}\frac{e^{ns}-p^n}{e^{ns}}$$

$$V(FR) = (N - Le^{-sT})\frac{p}{e^s-p}\frac{e^{ns}-p^n}{e^{ns}}$$

$$D(A) = \frac{(1+k)V(R)D(R)}{V(A)} - \frac{V(FR)D(FR)}{V(A)}$$

$$D(FL) = \frac{e^s}{e^s-p} - \frac{np^n}{e^{ns}-p^n} + T$$

$$D(N_f) = \frac{e^s}{e^s-p} - \frac{np^n}{e^{ns}-p^n}$$

$$D(N_v) = \frac{e^s}{e^s-p} - \frac{np^n}{e^{ns}-p^n} + T\left[1 + \frac{(1-b)ke^{aT+(b-1)sT}}{1-k+ke^{aT+(b-1)sT}}\right]$$

$$D(FP)=\frac{e^s}{e^s-p}-\frac{np^n}{e^{ns}-p^n}+vT\left[1+\frac{(1-b)\,ke^{aT+(b-1)sT}}{1-k+ke^{aT+(b-1)sT}}\right]$$

$$D(FR)=\frac{V(FP)D(FP)-V(FL)D(FL)}{V(FP)-V(FL)}=\frac{p}{e^s-p}\frac{e^{ns}-p^n}{e^{ns}}+T\left[\frac{vNM-Le^{-sT}}{N-Le^{-sT}}\right]$$

$$\textit{where}\qquad M=1+\frac{(1-b)\,ke^{aT+(b-1)sT}}{1-k+ke^{aT+(b-1)sT}}$$

$$\frac{1}{N}\frac{dN}{ds}=-vTM$$

$$\frac{1}{N_c}\frac{dN_c}{ds}=-wTM$$

$$\frac{1}{N}\frac{dN_{rel}}{ds}=\frac{1}{N}\left[\frac{dN}{ds}-\frac{dN_c}{s}\right]$$

$$\frac{1}{V(FR)}\frac{dV(FR)}{dp}=\left[\frac{e^s}{p(e^s-p)}-\frac{np^n}{p(e^{ns}-p^n)}\right]$$

$$Dp(FR)=q(w-v)TM\left[\frac{e^s}{e^s-p}-\frac{np^n}{e^{ns}-p^n}\right]$$

$$D_{tot}(FR)=D_s(FR)+D_p(FR)=\left[\frac{p}{e^s-p}\frac{e^{ns}-p^n}{e^{ns}}\right]+T\left[\frac{vNM-Le^{-sT}}{N-Le^{-sT}}\right]+q(w-v)TM\left[\frac{e^s}{e^s-p}-\frac{np^n}{e^{ns}-p^n}\right]$$

FET-119-07

$$P^*-P=-P\,D(i)\,d(i)$$

FET-120-07

$$\text{maximize } U(P)=ExpRet(P)-\frac{\left[ExpRisk(P)\right]^2}{rt}-\frac{\left[ExpTE(P)\right]^2}{tet}$$

FET-121-07 None

FET-122-07

$$spending_t = smoothing\ rate\left[spending_{t-1}(1+\inf lation_{t-1})\right]+$$
$$(1-smoothing\ rate)(spending\ rate * beginning\ market\ value_{t-1})$$

FET-123-07

$$\min\left(E\left[DK\left(\alpha+\frac{1}{\alpha}(P-\beta)^2\right)\right]\right)$$

$$\beta=\frac{E[DP]}{E[D]}$$

$$\alpha=\frac{1}{E[D]}\sqrt{E\left[DP^2\right]E[D]-E[DP]^2}$$

$$D_t=D_0\exp\left\{\frac{1}{2\sigma^2}\left(\mu^2-\left(r+\frac{\sigma^2}{2}\right)^2\right)t+\frac{1}{\sigma^2}\left(r-\mu-\frac{\sigma^2}{2}\right)X_t\right\}$$

$$D_t=D_0\left(\frac{S_t}{S_0}\right)^{-\alpha}\exp\left\{-r(1-\alpha)t+\frac{1}{2}\sigma^2\alpha(\alpha-1)t\right\}$$

$$S_t=S_0e^{X_t}, X_t\sim N\left(\left(r-\frac{\sigma^2}{2}\right)(T-t),\sigma^2(T-t)\right)$$

FET-124-07

$$R_{S(L)}=\left(\frac{A_0}{L_0}R_A\right)-R_L$$

$$R_A=R_f+\beta_A r_Q+\alpha$$

$$\sigma_A^2=\beta_A\sigma_Q^2+\omega_A^2$$

$$\max(U_S)=R_S-\lambda\sigma_S^2$$

$$\max(U_S)=\left(\frac{A_0}{L_0}-1\right)R_F+\beta_S\mu_Q-\lambda_\beta\beta_S^2\sigma_Q^2+\left(\frac{A_0}{L_0}\alpha_A-\alpha_L\right)-\lambda_\omega\left[\left(\frac{A_0}{L_0}\right)^2\omega_A^2-2\frac{A_0}{L_0}\omega_A\omega_L+\omega_L^2\right]$$

$$P_{TIPS}=\frac{F}{(1+r)^T}$$

$$P_{EQUITY} = \sum_{t=0}^{\infty} \frac{Dvd_0 \left(1+g_r\right)^t}{\left(1+r\right)^t}$$

FET-125-07 None

FET-126-07 None

FET-127-07 None

FET-128-07 None

FET-129-07 None

FET-130-07 None

FET-131-07 None

FET-132-07

$$\sigma_j^2 w_j^2 + \sum_{\substack{i=1 \\ i \neq j}}^{n} \sigma_j \sigma_i w_i w_j \rho_{ij}$$

$$\sum_{i=1}^{n} \sigma_i^2 w_i^2 + \sum_{j=1}^{n} \sum_{\substack{i=1 \\ i \neq j}}^{n} \sigma_j \sigma_i w_i w_j \rho_{ij}$$

FET-133-07 None

FET-134-07 None

FET-135-07 None

FET-136-07 None

FET-137-07 None

FET-138-07

$$c = \int_{W^*}^{\infty} f\left(w\right) dw$$

$$VAR = W_0 \times \alpha \sigma \sqrt{\Delta t}$$

$$se\left(\hat{q}\right) = \sqrt{\frac{c\left(1-c\right)}{T\ f\left(q\right)^2}}$$

FET-139-07 None

$$dr_t = a\left[b - r_t\right]dt + \sigma dW_t$$

$$\frac{dP(t,T)}{P(t,T)} = r_t dt - \sigma_P(T-t)\,dW_t$$

$$\sigma_P(t,T) = \frac{\sigma}{a}\left(1 - e^{-a(T-t)}\right)$$

$$P(t,T) = G(T-t)\ \exp\left(-H(T-t)r_t\right)$$

$$H(T-t) = \frac{1 - e^{-a(T-t)}}{a}$$

$$G(T-t) = \exp\left[\left(\frac{\sigma^2}{2a^2} - b\right)(T-t) + \left(b - \frac{\sigma^2}{a^2}\right)H(T-t) + \frac{\sigma^2}{4a^2}H\left(2(T-t)\right)\right]$$

$$\frac{dA_t}{A_t} = \mu dt + \sigma_A\left[\rho dW_t + \sqrt{1-\rho^2}\ dZ_t\right]$$

$$B_T = \max\left[0, \delta\left(\frac{L_0}{A_0}(A_T - A_0) - \left(L_T^* - L_0\right)\right)\right] = \delta\alpha\max\left[0, A_T - \frac{L_T^*}{\alpha}\right]$$

$$L_T^* = L_0 e^{r^* T}$$

$$L_T = L_T^* + B_T = \delta\alpha A_T + (1-\delta)L_T^*$$

$$E_T = \max\left[0, A_T - L_T^*\right] - \delta\alpha\max\left[0, A_T - \frac{L_T^*}{\alpha}\right]$$

$$E_t = C_E\left(A_t, L_T^*\right) - \delta\alpha C_E\left(A_t, \frac{L_T^*}{\alpha}\right)$$

Note: study note has a typo; full marks for either.

$$C_E\left(A_t, L_T^*\right) = A_t N(d_1) - P(t,T) L_T^* N(d_2)$$

$$C_E\left(A_t, \frac{L_T^*}{\alpha}\right) = A_t N(d_3) - P(t,T)\frac{L_T^*}{\alpha} N(d_4)$$

$$d_1 = \frac{\ln A_t / P(t,T) L_T^* + \bar{\sigma}(t,T)^2 (T-t)/2}{\bar{\sigma}(t,T)\sqrt{(T-t)}} = d_2 + \bar{\sigma}(t,T)\sqrt{(T-t)}$$

$$d_3 = \frac{\ln \alpha A_t / P(t,T) L_T^* + \bar{\sigma}(t,T)^2 (T-t)/2}{\bar{\sigma}(t,T)\sqrt{(T-t)}} = d_4 + \bar{\sigma}(t,T)\sqrt{(T-t)}$$

$$\bar{\sigma}(t,T)^2 = \frac{1}{T-t}\int_t^T \left[\left(\rho\sigma_A + \sigma_P(u,T)\right)^2 + \left(1-\rho^2\right)\sigma_A{}^2\right] du$$

$$E_t = A_t\left[N(d_1) - \delta\alpha N(d_3)\right] - P(t,T) L_T^*\left[N(d_2) - \delta N(d_4)\right]$$

$$L_t = L_T^* P(t,T) - P_E\left(A_t, L_T^*\right) + \delta\alpha C_E\left(A_t, \frac{L_T^*}{\alpha}\right)$$

$$P_E\left(A_t, L_T^*\right) = -A_t N(-d_1) + P(t,T) L_T^* N(-d_2)$$

$$L_t = A_t\left[N(-d_1) + \delta\alpha N(d_3)\right] + P(t,T) L_T^*\left[N(d_2) - \delta N(d_4)\right]$$

$$(1-\alpha) A_0 = C_E\left(A_0, L_T^*\right) - \delta\alpha C_E\left(A_0, \frac{L_T^*}{\alpha}\right)$$

$$\delta = \frac{C_E\left(A_0, L_T^*\right) - (1-\alpha) A_0}{\alpha C_E\left(A_0, \frac{L_T^*}{\alpha}\right)}$$

$$\eta_P(t,T) = H(T-t)$$

$$\eta_A(t,T) = -\frac{\rho\sigma_A}{\sigma}$$

$$\eta_L(t,T) = \eta_P(t,T) - \frac{A_t}{L_t}\left[\eta_P(t,T) - \eta_A\right]\left[N(-d_1) + \delta\alpha N(d_3)\right]$$

$$D_L = \frac{\ln\left(1 - a\eta_L(0,T)\right)}{a}$$

$$\eta_A(t,T) = \frac{E_t}{A_t}\eta_E(t,T) + \frac{L_t}{A_t}\eta_L(t,T)$$

$$\eta_E(t,T)=\eta_P(t,T)-\frac{A_t}{E_t}\left[\eta_P(t,T)-\eta_A\right]\left[N(d_1)-\delta\alpha N(d_3)\right]$$

$$\hat{P}(t,T,\hat{r}_t)=G(T-t)\exp\left(-H(T-t)\hat{r}_t\right)$$

$$\hat{A}_t(\hat{r}_t)=A_t\exp\left(-\eta_A(\hat{r}_t-r_t)\right)$$

AAA Monograph: Fair Valuation of Insurance Liabilities Principles and Methods

$$r_L=r_A-e\left(\frac{r_E}{1-t}-r_A\right)$$

$$MVM_t=L_{t-1}\left(r_f-r_L\right)$$

RBC Phase 2 – AAA Paper C3 Phase 2 Report (pages 1-18) None

RSA Vol. 22 #3 "Strategic Investment Policy Formulation and Implementation" None

CIA Guidance Notes: An Overview of an Investment Policy Statement in an A/L Management Context None

CIA Educational Note: Liquidity Risk Management None

RSA Vol. 27 #2 "Liquidity Modeling and Management" None

****BEGINNING OF EXAMINATION****
EXAM FET
FINANCIAL ECONOMIC THEORY EXAM (FINANCE/ERM/INVESTMENT)
Afternoon Session

9. *(5 points)* Languid Life has decided to focus on surplus duration to determine the appropriateness of the interest rate sensitivity of its cash flows. The current target for the surplus duration is between 5 and 10 years. You are given the following information:

YEAR 1

Current assets	6,000,000
Current liabilities	5,400,000
Duration of assets	18 years

YEAR 2

Current assets	6,500,000
Current liabilities	5,500,000
Duration of assets	17 years

(a) Calculate the range for the liability duration that satisfies the surplus target in Year 1.

(b) Calculate the range for the liability duration that satisfies the surplus target in Year 2.

(c) Describe the limitations of Languid Life's approach.

10. *(8 points)* You are the CFO of Peckham Life Insurance Company. Recently you uncovered evidence that Falkirk Re, a major reinsurance partner of yours, may be in financial distress and could potentially default on its obligations in the near future. Peckham Life has one coinsurance arrangement in force with Falkirk Re for a block of UL policies. Peckham Life has a unilateral right of recapture in three years on the business ceded to Falkirk Re.

(a) *(1 point)* Explain how a Credit Default Swap would provide Peckham with the protection it is seeking.

(b) *(1 point)* Describe the advantages and disadvantages of using a Credit Default Swap compared with immediate recapture of the business.

(c) *(1 point)* Define the significant contract terms to be negotiated in order to enter into a credit default swap to mitigate the exposure to Falkirk Re.

(d) *(1 point)* Explain how a dynamic credit swap might be used in this situation.

Your corporate department has provided the following exposure estimates for the coinsurance treaty with Falkirk Re.

End of Year	Coinsured Reserve ($Millions)
2007	$1,423
2008	$1,391
2009	$1,384

You have in turn provided these estimates to Friendly Bank and instructed them to provide a price quote for a three year dynamic credit default swap assuming that:

- The above listed exposure amounts are the fixed payoff amounts upon which both premium and default payments are to be made.
- Premium payments made by Peckham are in arrears (EOY).
- Default payments are assumed to be made at the middle of each year.
- Falkirk Re is assumed to have default probabilities of 25%, 10% and 5% respectively in each of the coming three years.
- Recovery rate of 75% is assumed.
- The risk free LIBOR rate is 6%.

Friendly Bank has quoted a credit default spread of 500 basis points.

(e) *(4 points)* Determine whether the price quoted by Friendly Bank is high or low. Show your work.

11. *(4 points)* You work as an analyst for a fixed income money manager specializing in Mortgage Backed Securities (MBS).

(a) Describe the following

(i) Nominal Spread

(ii) Zero Volatility Spread

(iii) Option Adjusted Spread

(b) Recommend with reasons the best spread measure in (a) for measuring the relative value of MBS.

(c) You are given the following analysis for four tranches of a CMO:

Tranche	OAS (bps)	Option Cost (bps)	Effective Duration
A	39	12	1.2
B	50	10	3.5
C	60	11	8.5
D	65	62	5.4

Assess the relative advantages and disadvantages of each tranche.

12. *(5 points)* A particular insurance product is offered by only two insurance companies: Company Blue and Company Red. In a non-cooperative way, each company may allocate a certain amount of effort for the purpose of gaining a market share proportional to the amount of effort exerted.

The cost per unit of effort is s for Company Blue and t for Company Red, where s and t are positive numbers such that $t < 2s$.

Denote by x and y the amount of effort from Company Blue and Company Red, respectively. If both x and y are zero, the profits to both firms are zero. Otherwise, the profits are given as below:

$$P_{Blue}(x, y) = x/(x+y) - sx$$
$$P_{Red}(x, y) = y/(x+y) - ty$$

(a) Calculate the Cournot-Nash equilibrium allocations, x and y, and the equilibrium profits, P_{Blue} and P_{Red}, in terms of s and t, **assuming that both companies are to move simultaneously**.

(b) Calculate the Stackelberg equilibrium allocations, x and y, and the equilibrium profits, P_{Blue} and P_{Red}, in terms of s and t, **assuming that Company Blue is to move first**.

(c) Verify that Company Blue benefits from its status as the Stackelberg leader in (b).

13. *(7 points)* EJC Trust is an international financial services company composed of three business units operating in profitable banking and reinsurance markets in Europe and Asia. The company is concerned with its current capital position in light of the need to fund future business growth.

You are reviewing the company's capital allocation process, including the measurement of risk capital and the evaluation of business unit risk-adjusted performance.

Capital is currently allocated using a centralized, top-down approach. Business units have little influence on risk capital assigned to them.

Return on capital is currently determined by comparing monthly profits to month-end daily risk capital. The average daily capital at risk ("CaR") times the number of trading days in a given month is being considered as an alternative risk measure for monthly CaR.

You are given the following information about the company:

Table 1 ($ Millions)

Business Unit	Stand-Alone CaR	Profit
Unit A	200	40
Unit B	100	15
Unit C	200	50

Table 2: Correlation Among Business Units

Business Unit	Unit A	Unit B
Unit B	0.00	
Unit C	0.00	0.30

(a) Describe the top-down approach and two alternatives for capital allocation.

(b) Recommend either allocated CaR or utilized CaR as the more appropriate measure of risk capital for EJC Trust. Justify your recommendation.

(c) Assess the appropriateness of the current return on capital measure that the company is using and compare it to the alternative measure of monthly CaR.

(d) Evaluate the appropriateness of allocating any diversification benefits across the business units in proportion to their stand-alone risk capital.

(e) Calculate the diversified CaR based on the approach described in (d).

14. *(7 points)* Fugitive Life Insurance Company is interested in raising capital to support its entry into a new insurance market. It is considering hiring an investment bank to help raise the needed capital. Fugitive's current capital structure is as follows:

Equity	500 million
Senior Debt	500 million
Number of outstanding shares	10 million

Management has made the following forecast for its current business:

Present value of Earnings	Probability
1,200 million	60%
700 million	40%

The company has just raised 800 million of new junior debt and is now considering the following projects:

Project A:	PV Earnings	Probability
	1,000 million	100%

Project B:	PV Earnings	Probability
	1,300 million	70%
	200 million	30%

Assume the new project's earnings are independent of the earnings for current business. Bankruptcy occurs when the PV of total future earnings is insufficient to pay the face amounts of junior and senior debt. Bankruptcy transaction costs are 100 million. Management will select the project that maximizes expected shareholder wealth. There are no taxes.

(a) Contrast the effect on share price of issuing common stock or non-callable debt.

(b) Describe the services that investment banks can provide Fugitive.

(c) Recommend the project that management should pursue under each of the following scenarios:

(i) the junior debt is non-convertible

(ii) the junior debt is convertible to 10 million shares at the option of the debtholder

(d) Explain how convertible debt overcomes some of the problems associated with financing the new project with non-convertible debt.

15. *(5 points)* DustSucker Inc is a company that produces specialty vacuum cleaners. The company is now up for sale and several larger companies are planning to make bids to purchase DustSucker Inc.

GenHouse is a large holding company which wishes to acquire DustSucker with a combination of 50% debt and 50% equity.

DirtWizard is a privately owned company which wishes to acquire DustSucker solely through equity.

You are given the following information about the firms:

All outstanding debt is at 5% annual interest.

	DustSucker	GenHouse	DirtWizard
Expected Return	10%	10%	10%
Pre-tax Operating Income	500,000	1,000,000	1,000,000
Debt Outstanding	0	5,000,000	0
Number of Shares Outstanding	800,000	250,000	500,000

(a) (*1 point*) List the assumptions underlying the Modigliani and Miller (M&M) Capital Structure Model.

(b) (*4 points*) Determine which firm's shareholders would benefit more from the purchase of DustSucker under each of the following, using the assumptions from (a):

(i) no corporate income tax

(ii) the corporate income tax rate is 25%

16. *(7 points)* You are performing a risk analysis on a portfolio that includes fixed income derivatives. You are analyzing duration, 1-year VaR, and stress testing.

(a) Describe the principal risk factors for fixed income derivatives.

(b) Describe why duration may not be an adequate measure of the price change of fixed income derivatives.

You are estimating the probability distribution of the portfolio's 1-year price change using a Gamma distribution. The first three central moments are:

μ_1	25
μ_2	3,000
μ_3	–82,000

The table below gives the 99% quantile, $\kappa(\lambda)$, of the Gamma distribution as a function of the coefficient of skewness, λ, for a unit standard deviation, where $\lambda = \mu_3 / \sigma^3$.

λ	$\kappa(\lambda)$
–2.83	3.99
–2.00	3.61
–1.00	3.03
–0.67	2.80
–0.50	2.69
0.00	2.33
0.50	1.96
0.67	1.83
1.00	1.59
2.00	0.99
2.83	0.71

(c) Calculate the VaR at the 99% level assuming the Gamma distribution.

(d) Calculate the VaR at the 99% level assuming the Normal distribution.

(e) Recommend which VaR measure should be used.

(f) Appraise stress testing vs. VaR as a measure of portfolio risk.

17. *(6 points)* You are analyzing the interest rate risk of a non-callable 2-year government bond using the following data:

t	Cash flow (t)	Spot Rate (t)
0.5	$3.00	4.75%
1.0	$3.00	4.95%
1.5	$3.00	5.00%
2.0	$103.00	5.05%

Spot rates are continuously compounded.

(a) *(4 points)* Calculate the key rate durations for all key rates $(t = 0.25, 1, 2, 3, 5, 7, 10, 15, 20, 25, 30)$

(b) *(1 point)* Calculate the effective duration for the bond.

(c) *(1 point)* Compare key rate durations with effective duration for measuring interest rate risk.

18. *(6 points)* Assume today is January 1, 2008. Your company is evaluating a new venture, project GenE. Your company has until January 1, 2009 to commit capital to project GenE. The publicly traded security GECH has the same risk characteristics as project GenE. Currently, your company uses the traditional discounted-cash-flow ("DCF") approach to appraise capital investment projects.

You are given the following information.

Risk free rate: 3% per annum

Investment cost of project GenE: 51 million

Projected cash flows from project GenE:

Probability	Cash Flow one year after investment
60%	75 million
40%	25 million

Price of security GECH on January 1, 2008 is $10 per share

Each year the price of GECH has a 60% probability of increasing by 50% and a 40% probability of decreasing by 50%.

(a) Determine the expected rate of return for an investment in GECH.

(b) Determine whether your company should invest in project GenE using the traditional DCF approach with a 1-year horizon to appraise the capital investment of the project.

(c) Describe the shortcomings of the traditional DCF approach to capital budgeting.

You have learned from the Society of Actuaries exam syllabi that the Contingent-Claims Analysis ("CCA") approach in capital budgeting can help to overcome the shortcomings described in (c).

(d) Calculate the expanded net present value ("NPV") and the option premium for project GenE using the option-based CCA approach.

A third party wishes to replicate the payoff of project GenE in one year using a hedge portfolio constructed of shares of GECH and 1-year risk free borrowing.

(e) Calculate the number of shares of GECH and 1-year risk free borrowing required for the hedge portfolio.

****END OF EXAMINATION****