# FETE Complete Illustrative Solutions
# Fall 2008

## 1.

**Learning Objectives:**

**2 – c.** Describe the process, methods, and uses of insurance securitization and recommend a structure that is appropriate for a given set of circumstances.

*This is a recall and synthesis question asking candidates to (a) explain issuer and investor motivations for entering into securitization transaction, (b) evaluate and recommend and appropriate securitization structure, and (c) recommend a securitization method to maintain surplus to within a tolerance. See FE-C118-07 Securitization of Life Insurance Assets and Liabilities.*

**Solution:**

**(a)**

**Benefits for LifeCo**

- Reduce agency costs
- Reduce deadweight costs
- Reduce information asymmetries and cost of capital
- Improve capital position
- Facilitate risk management

**Benefits for Investors**

- Create new asset classes
- Allow investors to choose the type of cashflow they would like to invest in
- Reduce information costs by structuring cashflow into tranches
- Portfolio diversification

# 1. continued

**(b)**
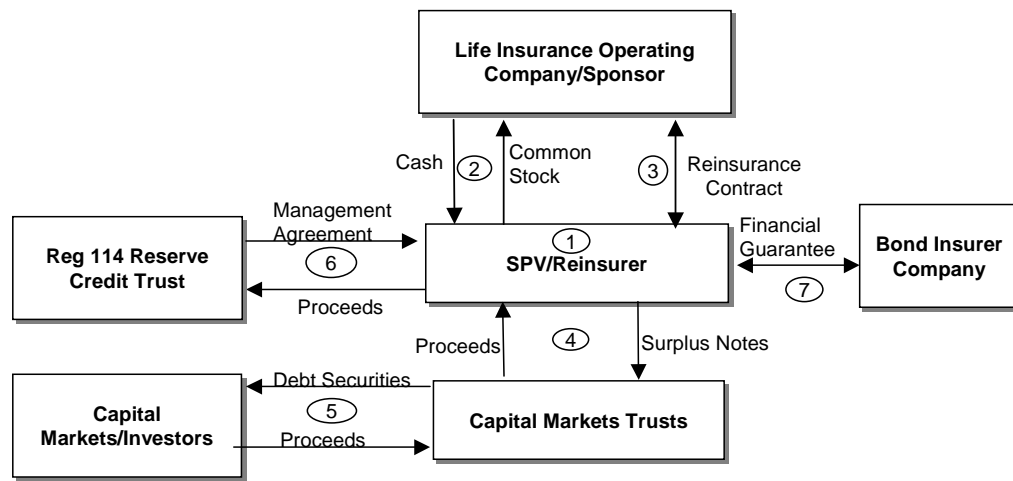


1 – Create a wholly owned SPV (on-shore reinsurer)
2 – In exchange for cash, SPV provides common stock / equity
3 – LifeCo enters into a reinsurance contract with the SPV
4 – SPV issues surplus notes to a capital markets trust
5 – Capital markets trust issues non-recourse debt to investors
6 – Proceeds are invested in reserve credit trust in order to qualify as reinsurance for regulatory purposes
7 – Third party bond insurer can provide financial guarantee to SPV

Reserve build up and then disappear, creating a humped back strain
Cost to LifeCo = rate paid on debt – earned rate in reserve credit trust + cost of guarantor and costs of establishing structure

**(c)**

Trad life block has $300 million in liabilities which means the maximum mortality impact is $3 million. Since the expected dollar values of claims is $10 million, must create a securitization that caps LifeCo's claims payments at $13 million.

Issue a mortality bond or could use an SPV on a non-recourse basis. Investors will not get paid if LifeCo's claims exceed $13 million.

# 2.

**Learning Objectives:**

**3 – d.** Evaluate the impact of embedded options in assets and liabilities.

*This question probes the investment logistics behind an EIA product. See study notes V-C101-07 and FET-101-07*

**Solution:**

**(a)**

External index

- Most contracts based on S&P 500
    - Easily recognizable for customers
    - Call options needed to hedge are readily available
- Does not include growth due to dividends

Index return formula

- Most common
    - Annual ratchet
    - High Watermark
    - Point to Point
    - Point to average

Participation rate

- The costlier the index return formation, the more expensive the option and the lower the participation rate
- Minimum guarantee return the higher the minimum guarantee return, the lower the participation rate because the cost of providing downside protection increases
- When overall interest rates fall, cost of providing downside protection increases

Minimum guarantee rate

# 2. continued

**(b)**

Both are not feasible approaches due to significant differences between the two products

In particular, EIL has
- Unpredictable premium cash flow
  - Each premium received could be treated as a single premium for investment purposes
    - However, administratively difficult and would require changes to the GIC admin system
  - Could bundle premiums during pre-defined period (contribution window) to gain appropriate size
    - Longer windows not as appealing to consumers
    - A larger number of buckets require more administration
- Small premiums
  - May not be able to purchase required options due to minimum sizes
- Account value deductions
  - Not part of GIC admin system
  - Changes the notional amount being hedged during the year

**(c)**

Overall objectives for the ALM function
>    EIL products have additional risks that need to be considered
>    Disintermediation risk
>    Equity participation risk
>    Due to the free look provisions, premiums may have to be returned to
>        policyholder

Frequency of meetings
>    Although meetings need not be more frequent than monthly, reporting of
>        risks should be at least weekly

ALM Guidelines
>    Determine if using exact or modified hedging for the participation risk
>        Exact hedging assumes no interim terminations
>            Company benefits when there are actual decrements
>            But costs more to purchase options
>    Modified duration assumes a certain percentage of decrements
>        Options cost less
>        But company exposed to having a larger in force than assumed
>        Will benefit is decrements larger than assumed

# 3.

## Learning Objectives:

**3 – b.** Evaluate the risk / return characteristics of complex derivatives

**3 – f.** Demonstrate mastery of option pricing techniques and theory

*This question tests the knowledge of convexity adjustment and its application in exotic options. Candidates need to understand the formula and apply it in a proper way. Most points will be awarded if candidates show good understanding but miscalculate a few numbers.*

## Solution:

**(a)**

$L = 100M$

$T = 3$

$y_i = 6\%$

$F_i = 6\%$

$\sigma_{y,i} = 18\%$

$\sigma_{F,i} = 25\%$

$\rho_i = 0.5$

$\tau_i = 1$

Convexity adjustment $= -\dfrac{1}{2} y_i^2 \, \sigma_{y,i}^2 \, t_i \, \dfrac{G_i''(y_i)}{G_i'(y_i)}$

$G(y) = $ price of bond within term 3 years and yield $y$.

$G(y) = \dfrac{6}{1+y} + \dfrac{6}{(1+y)^2} + \dfrac{106}{(1+y)^3}$

$G'(y) = \dfrac{-6}{(1+y)^2} - \dfrac{12}{(1+y)^3} - \dfrac{318}{(1+y)^4}$

$G''(y) = \dfrac{12}{(1+y)^3} + \dfrac{36}{(1+y)^4} + \dfrac{1272}{(1+y)^5}$

$G'(y_i) = G'(0.06) = -267.301$

$G''(y_i) = 989.103$

**3.** **(a) continued**

For a one year period, $t_i = 1$, and the convexity adjustment is

$$-\frac{1}{2}(0.06)^2 (0.18)^2 \frac{989.103}{-267.301}$$
$$= 0.0002158$$

So the convexity adjustment is 2.158 bps per year.

**(b)**

$$t_i = 2$$

The adjustment includes both convexity and timing adjustment

$$= y_i + (\text{convexity adj} + \text{timing adj}) \times (t_i)$$
$$= y_i - \frac{1}{2} y_i^2 \sigma_{y,i}^2 t_i \frac{G_t''(y_i)}{G_t'(y_i)} - \frac{y_i \tau_i F_i \rho_i \sigma_{y,i} \sigma_{F,i} t_i}{1 + (0.06)(1)}$$
$$= 0.06 + 0.0002158 \times 2 - \frac{(0.06)(1)(0.06)(0.5)(0.18)(0.25)(2)}{1 + (0.06)(1)}$$
$$= 6.02788\%$$

**(c)**

The net cashflow received at 2 year point should assume to be
$$(6.02788\% - 6\%) \times 100M = 27,878$$

# 4.

### Learning Objectives:

**2 – d.** Evaluate the alternative options for utilizing capital and recommend the most appropriate use in a given situation.

*This is a recall, application and synthesis question asking candidates to (a) describe the assumptions underlying real option analysis [recall], (b) contrast real option analysis to net present value method [recall], and (c) recommend whether to proceed with an 'abandonment type' project using a real option decision tree [application and synthesis]. See Copeland, Weston, and Shastri, Chapter 9.*

### Solution:

**(a)**

Company has option to abandon the project. This option allows the firm to give up the project and sell it for what it is worth.

Key assumptions for pricing real options:

- Marketed asset disclaimer adjustment – this allows the use of replication even if there are no twin traded securities
- Real options pricing require the principle of no arbitrage
- Recombining binomial trees will work regardless of the expected cash flow pattern

**(b)**

$$NPV = -I_o + \sum_{t=1}^{n} \frac{E(FCF)}{(1+WACC)^t}$$

Projects with positive NPC should be undertaken.
Managers seem willing to accent projects with negative NPV because they think the flexibility adds back more than enough value.
NPV implicitly assumes no decisions are made in the future.
NPV treats deferral options as mutually exclusive events
Real options analysis uses replicating portfolios
Real options analysis models optimal future decisions
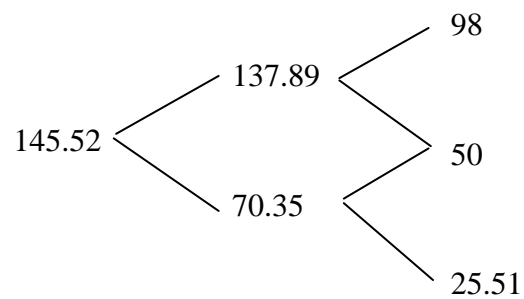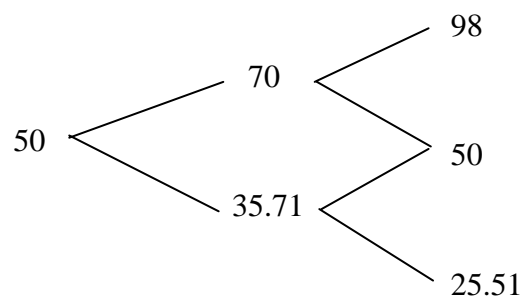
**4.** **continued**

**(c)**

$$WACC = 0.09$$
$$r = 0.0$$
$$u = 1.4$$
$$d = \frac{1}{u} = 0.7143$$
$$p = 0.5$$
$$I_o = 150$$

```
                            98
                70
       50                   50
                35.71
                            25.51
```

```
                            98
                137.89
       145.52               50
                70.35
                            25.51
```
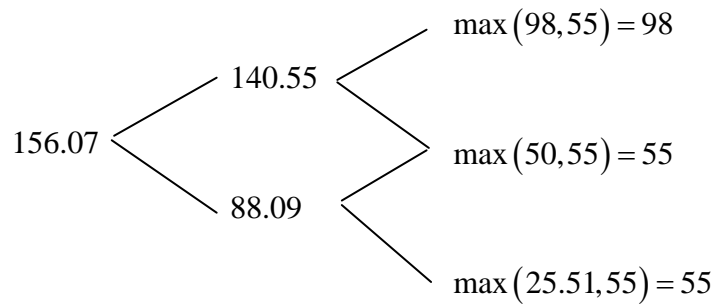
$$* \ \frac{(0.5)(98)+(0.5)(50)}{1.09}+70 = 67.89+70 = 137.89$$

Real option has ability to abandon project at $t = 1$ or $t = 2$ for 55.

**4.** **(c) continued**

$$\text{max}(98,55) = 98$$

140.55

156.07

$$\text{max}(50,55) = 55$$

88.09

$$\text{max}(25.51,55) = 55$$

* $m(98)+1.05B = 98$ $\Rightarrow$ $m = 0.896,\ B = 9.718$ $\Rightarrow$ $m(137.89-70)+B+70 = 140.55$

$m(50)+1.05B = 55$

* $m(50)+1.05B = 55$ $\Rightarrow$ $B = 52.38,\ m = 0$ $\Rightarrow$ $m(70.35-35.71)+B+35.71$

$m(25,51)+1.05B = 55$ $= 52.38+35.71 =$

$m(137.89)+1.05B = 140.55$ $\Rightarrow$ $1.0193 = m+0.0076B$ $\Rightarrow$ $m7.0193-0.0076B$

$m(70.35)+1.05B = 88.09$ $(70.35)(1.0193-0.0076B)+1.05B = 88.09$ $\Rightarrow$ $B = 31.78$

$71.71-0.53466B+1.05B = 88.09$ $\Rightarrow$ $m = 0.7777$

$m(145.52-50)+B+50 = 156.07$

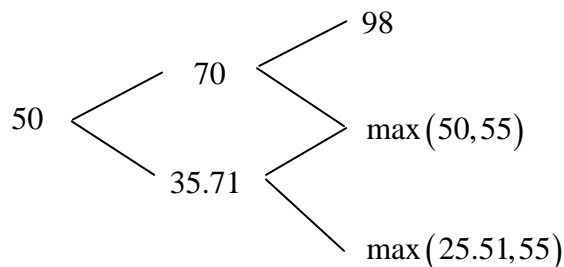* Since NPV is $156.07 > 150$ $(150 = I_o)$, should invest.

**(c)**
**Alternate Solution**

Using a Risk Neutral framework to come to the equivalent solution:
$$\rho_{rn}: \frac{98 \times \rho_{rn} + 50 \times (1-\rho_{rn})}{1.05} = 67.89 \text{ implies } \rho_{rn} = 0.443$$

Value the tree as below with the 55 abandonment option

98

70

50

$$\text{max}(50,55)$$

35.71

$$\text{max}(25.51,55)$$

**4.** **(c) continued**

$$E^+ = \frac{\left(98 \times 0.443 + 55 \times (1 - 0.443)\right)}{1.05} = 70.52$$

$$E^- = \frac{\left(55 \times 0.443 + 55 \times (1 - 0.443)\right)}{1.05} = 52.38$$

$$E = \frac{\left((70 + 70.52) \times 0.443 + (35.71 + 52.38) \times (1 - 0.443)\right)}{1.05} = 106.02$$

NPV with option is $106.02 + 50 - 150 = 6.02$
Therefore, accept project.

# 5.

## Learning Objectives:

**4 – a.** Describe and evaluate equity and interest rate models
**4 – e.** Contrast commonly used equity and interest rate models
**4 – f.** Recommend an equity or interest rate model for a given situation

*This question tests for an understanding of a specific interest rate model. The material references Hull Chapters 7 and 31 and study note FET-105-08.*

## Solution:

**(a)**

MBS cashflows are path dependent because prepayment rates are path dependent. Previous prepayments determine which tranches are still in force in the future.

- Model term structure of interest rates
- Use implied volatilities to calibrate the model
- Model should be arbitrage free
- The model can generate monthly prepayment rates and refinance rates
- Use the model to generate interest rate scenarios
- # of scenarios should be large
- Calculate monthly cash flows under each path at each node
- OAS is the spread that makes the PV of each path's cash flow equal to the market price
- Can use OAS to compare different securities
- The value of the MBS is the average of P of each path's cash flows

**(b)**

$P(t,T)$ is a price at $t$ of a zero-coupon bond with \$1 maturing at time $T$.
$v(t,T,\Omega) = $ volatility of $P(t,T)$

For a bond, volatility goes to 0 at maturity.

$\Rightarrow$ Set $\quad t = T$

$$v(t,T,\Omega) = v(T,T,\Omega) = \frac{\sigma}{\lambda}\left(e^{-\lambda(T-T)} - 1\right) = \frac{\sigma}{\lambda}(1-1) = 0$$

$\Rightarrow t = 0 \qquad v(t,T,\Omega) = 0$

## 5.     continued

**(c)**

$$dF(t,T) = v(t,T,\Omega_t)\, v_T(t,T,\Omega_t)\, dt - v_T(t,T,\Omega_t)\, d_t$$

$$v_T(t,T,\Omega_t) = \frac{2v(t,T,\Omega)}{2T} = \frac{2\left(\dfrac{\sigma}{\lambda}e^{-\lambda(T-t)}\right)}{2T} = -\sigma e^{-\lambda(T-t)}$$

$$dF(t,T) = -\frac{\sigma^2}{\lambda}\left(e^{-2\lambda(T-t)} - e^{-\lambda(T-t)}\right)dt + \sigma e^{-\lambda(T-t)}dz$$

**(d)**

HJM was instantaneous fwd rate which is
- Not observable
- HJM is hard to calibrate
- LMM uses fwd rate
    Trades are more used to it
- LIBOR MM can easily be calibrated to swap rates

# 6.

**Learning Objectives:**

**1 – d.** Define and compare risk metrics used to quantify economic capital and describe their limitations

**1 – f.** Identify regulatory capital requirements and describe how they affect decisions

*This question explores VaR's shortcomings and possible enhancements to address these to set an appropriate level of capital. It brings together material covered in 8E-115-07, Hardy chapter 9, FET-138-07, and the AAA monograph.*

**Solution:**

**(a)**

- Factor based RBC is a rigid method to calculate capital
- It may be a good industry made model for ensuring solvency, but doesn't adequately capture the risks in a particular company
- Takes items from the balance sheet, applies a multiple to it, and that's it
- Takes into consideration:
    - C1: Asset risk
    - C2: Insurance / pricing risk
    - C3: Interest rate risk
    - C4: Operational risks
    - C0: Others

- The RBC ratio is given by $\text{RBC} = \dfrac{\text{Total Adjusted Capital-C1}}{\text{C2+C3+C4}}$
    - It's easy for insurance companies to find ways to arbitrage on these rules through accounting tricks and regulatory loopholes
    - It's simple and not computationally intensive – just need to multiply a bunch of numbers and add them up

- VaR at $\alpha$ percentile is the maximum loss for a particular portfolio over a specific time horizon that we're $\alpha$ % confident that the losses won't be great
    - It requires simulation to calculate a distribution of losses, and funding the convex quantile
    - It's one number that summarizes the worst loss – should really be reported as a confidence interval instead
    - It's very dependent on the underlying model, data, time horizon and methodology
    - It's a better way to report capital because it allows the insurance company freedom to capture the underlying economics and business of their portfolios
    - More complicated than using factor-based method

# 6.  continued

**(b)**

- VaR isn't a coherent risk measure. It's not bounded below by the average loss, and it's not subadditive.
- VaR doesn't capture all forms of risk e.g. it ignores operational risk and liquidity risk
- It may give senior management a false sense of security
- It's very dependent on the model, data, time horizon and methodology. Creative actuaries can massage their calculations to produce a "desirable" VaR
- It doesn't focus on the right tail of loss the distribution (only uses 1 number). This tail may contain severe losses
- Uses simulation, which can be very computationally intense and subject to much programming errors

- VaR is usually reported as just a point estimate
- It's seductive but dangerous, i.e., numbers can be misleading

**(c)**

- VaR should be reported on a confidence level instead of a single number. Could use Hardy's formula

  $\beta\%$ confidence is interval for $\text{VaR} = \left( L_{(N\alpha - A)}, L_{(N\alpha + A)} \right)$ where $L_{(N\alpha)}$ is the

  quartile (VaR) measure, and $A = \Phi^{-1}\left( \dfrac{1+\beta}{2} \right) \sqrt{N\alpha(1-\alpha)}$

  $L_{(x)}$ is the $x^{th}$ biggest loss

- VaR calculation should be peer reviewed by other actuaries to ensure correctness
  - There shouldn't be any pressure from senior management to massage VaR numbers. There should be a clear independent structure so that actuaries won't have conflicts of interest
- Should also incorporate explicit measures for other risks like credit risk and operational risks
- For credit risk, can use options – approach, ICMU or Credit Metrics method
- No widely accepted model for operational risk. Need to use a lot of professional judgment and historical data
- Use a reasonable time horizon when reporting VaR. A VaR is still acceptable for short horizons but is useless for long term, e.g. 1 year or more
- Should use a more robust measure like the conditional tail expectation (CTE). It's the average of all losses, given that it's greater than the loss reported by VaR
  - It's a coherency risk measure, and takes less iterations than VaR to converge
  - It captures all the losses in the right tail

# 7.

**Learning Objectives:**

        **4 – a.** Critique the following modeling methods
        **4 – d.** Describe and evaluate equity and interest rate models
        **4 – e.** Contrast commonly used equity and interest rate models
        **4 – f.** Recommend an equity or interest rate model for a given situation
        **4 – g.** Describe issues and best practices in the estimation or calibration of
            financial models

*The question compares different models' tail behavior. It is based on the Hardy, Freeland, and Till paper.*

**Solution:**

**(a)**

**(i)**

The ILN model assumes a normal distribution with mean $\mu$ and variance $\sigma^2$. Since volatility is constant, volatility is not stochastic, there is no clustering and there is no association between high volatility and low returns.

**(ii)**

The GARCH(1, 1) model is $Y_t = \mu + \sigma_t^2 z_t$ (where $z_t$ are *i.i.d.*, and $z_t \sim N(0,1)$)

$$\sigma_t^2 = \alpha_0 + \alpha_1 \left( Y_{t-1} - \mu \right)^2 + \beta_1 \sigma_{t-1}^2$$

Volatility is stochastic when $t > 1$ but deterministic at $t = 1$.
Since volatility at each point is dependent on the prior volatility, clustering is captured, although mean reversion causes it to fade gradually.
There is no association between high volatility and low returns.

**(iii)**

The RSLN(2) model is $Y_t \mid \rho_t = \mu_{\rho t} + \sigma_{\rho t} z_t$

$$\rho_t \mid \rho_{t-1} = \begin{cases} 1 & \text{w.p. } p_{\rho t-1}, 1 \\ 2 & \text{w.p. } p_{\rho t-1}, 2 = \left( 1 - p_{\rho t-1}, 1 \right) \end{cases}$$

Stochastic volatility is capture because volatility moves between the two regimes.
Periods of clustering occur since the probability of leaving a regime is less than
        the probability of remaining.
There is an association between high volatility and low returns because the two
        regimes can be chosen appropriately to model this.

**7.** **(a) continued**

**(iv)**

The AAA SLV model is $Y_t = \dfrac{\mu_t}{12} + \left(\dfrac{\sigma_t}{12}\right) z_{y,t}$ where $\mu_t = A + B\sigma_t + C\sigma_t^2$

$$\log \sigma_t = v_t = (1-\theta) v_{t-1} + \theta \log \tau + \sigma_v z_{v,t}$$

Volatility is stochastic because it is a separate process with its own source of randomness

Clustering occurs because the volatility function depends on the previous value

High volatility can be associated with low returns through the mechanism of the correlation structure

**(b)**

The RSLN model is recommended

The $q - q$ fit on the 45 degree line is good for both models, but GARCH has 2 values in the left tail that poor fits

The area of the poor fit is the most critical in modeling VA liabilities

# 8.

**Learning Objectives:**

**2 – f. Recommend an optimal capital structure and how to implement it for a given business or strategy and to justify the recommendation.**

*This question tests the candidate's ability to apply the theoretical concepts and formulas for a firm's weighted average cost of capital, taxes, and net income in a real-world scenario. It further challenges the candidate's understanding of personal versus corporate tax rates and the difference in net income required to sustain a higher marginal tax rate. Credit is given for correctly applying the appropriate formulas from the formula sheet and for justifying a recommendation based on the results of the analysis. See Financial Theory and Corporate Policy, Chapter 15.*

**Solution:**

**(a)**

**Before Incorporating**

Using the security market line equation

$$k_s = r_f + \beta_u \left( E\left[ r_m - r_f \right] \right) = 0.04 + 0.7\left( 0.08 - 0.04 \right) = 0.068$$

Since unlevered firm, $WACC = k_s = 0.068$

**After Incorporating**

$$k_s = r_f + \beta_L \left( E\left[ r_m - r_f \right] \right) = 0.04 + 1.3\left( 0.08 - 0.04 \right) = 0.092$$

$$WACC = \left( 1 - r \right) k_b \left[ \frac{B}{B+S} \right] + k_s \left[ \frac{S}{B+S} \right]$$

$$WACC = \left( 1 - 0.4 \right) \times \left( 0.04 \right) \times \left[ 0.5 \right] + 0.092\left[ 0.5 \right] = \left( 0.116 \right) \times \left[ 0.5 \right] = 0.058$$

**(b)**

Let $EBIT_{BEF}$ and $EBIT_{AFT}$ be the earnings before interest and taxes before and after HillTop incorporates, respectively.

Then we have:

**Before Incorporating**

$k_s = p$ for an unlevered firm

$$V_u = \frac{E\left[ EBIT_{BEF} \right] \times \left( 1 - t_{BEF} \right)}{\rho} = \frac{E\left[ EBIT_{BEF} \right] \times \left( 1 - 0.28 \right)}{0.068}$$

**8.** **(b) continued**

**After Incorporating**

$$V_L = \frac{E[EBIT_{AFT}] \times (1 - t_{AFT})}{WACC} = \frac{E[EBIT_{AFT}] \times (1 - 0.4)}{0.058}$$

To support a 10% increase in the value of the firm, we need:

$$V_L = 1.1 V_u$$

$$\frac{E[EBIT_{AFT}] \times (1 - 0.4)}{0.058} = \frac{1.1 \times E[EBIT_{BEF}] \times (1 - 0.28)}{0.068}$$

Solving for $E[EBIT_{AFT}]$, $\quad E[EBIT_{AFT}] = \frac{1.1 \times E[EBIT_{BEF}] \times (1 - 0.28) \times (0.058)}{(0.068) \times (1 - 0.4)}$

$$E[EBIT_{AFT}] = 1.12588 \times E[EBIT_{BEF}]$$

**(c)**

**Reasons to Incorporate**
- All responsibilities no longer on one person
- Reduces WACC
- Only a modest increase in EBIT is needed to keep same firm value
- Adam is no longer personally liable as a sole owner
- Good for sustaining the business in the long-term, i.e., business can continue after he retires
- As a corporation, HillTop will have the potential to issue additional debt or equity

**Reasons Not to Incorporate**
- Potential for mismanagement
- Potential problems involving business and family
- Don't know the experience level of the three potential owners
- Adam's loss of decision-making power
- Higher tax rate
- Additional accounting and administration, formal procedures, etc.
- Increased dependence on market return (since $\beta_L > \beta_U$) leads to greater volatility in earnings

**(d)**

Do not incorporate

Too many unknowns and risks for Adam to take his only business

Adam's loss of ownership decision-making not compensated

Don't have information on nature of business, Adam's intentions to his nephew, his retirement, his business, estate planning issues, etc.

# 9.

### Learning Objectives:

**5 – c.** Explain information asymmetry and how it can affect financial markets, especially insurance markets.

**5 – d.** Define principal – agency theory and explain how it affects capital structure, portfolio management, and risk management.

*This is a recall and application question asking candidates to explain topics related to signaling firm quality with debt. The expected answer was based on the content of information asymmetry in Chapter 12 of Copeland's textbook. Credit was also given for answers that express the understanding of information asymmetry, separating equilibrium, and their application to the debt issue.*

### Solution:

**(a)**
- Information asymmetry occurs when one group of participants has better or more timely information than other groups.
- Firm's managers have better knowledge about the firm's prospects than its investors.

**(b)**
- Increase debt
  - Signal that management has confidence in future earnings levels

- Increase dividends
  - Signal if larger cash flows are expected

- Share repurchase
  - Signal that stock is undervalued

- Split stocks
  - Signal management's confidence in earnings expectation

**(c)**

$$V_0 \text{ of good firms } = \frac{250}{(1+10\%)} = 227.27$$

$$V_0 \text{ of bad firms } = \frac{150}{(1+10\%)} = 136.36$$

# 9. (c) continued

**For good firms, manager's compensation**
$$= 0.1 \times 227.27 \times (1 + 10\%) + 0.15 \times 250 \text{ if } D > 150$$
$$= 62.5$$

$$= 0.1 \times 136.36 \times (1 + 10\%) + 0.15 \times 250 \text{ if } D \leq 150$$
$$= 52.5$$

Since marginal payoff from telling the truth is greater, the good firms will always tell the truth.

**For bad firms, manager's compensation**
$$= 0.1 \times 227.27 \times (1 + 10\%) + 0.15 \times 150 - C \text{ if } D > 150$$
$$= 47.5 - C$$

$$= 0.1 \times 136.36 \times (1 + 10\%) + 0.15 \times 150 \text{ if } D \leq 150$$
$$= 37.5$$

To provide incentive to managers in bad firms to signal correctly, $47.5 - C < 37.5$
$\Rightarrow C > 10$ to ensure managers in bad firms signal correctly.

# 10.

### Learning Objectives:

**3 – f.** Demonstrate mastery of option pricing techniques and theory for equity and interest rate derivatives

*This question tests basic understanding of the Black-Scholes-Merton model, setting up a currency hedge, and the concept of hedge ratio. The topic is covered in Hull Chapter 14.*

### Solution:

**(a)**

$$S_0 = 120, \quad K = 117, \quad T = \frac{180}{365}, \quad \sigma = 10\%, \quad r_j = 0.4988\%, \quad r_u = 4.97\%$$

By $B-S$ Formula, $p = ke^{-r_j T} N(-d_2) - S_0 e^{r_n T} N(-d_1)$

Where

$$d_1 = \frac{\ln\left(\frac{S_0}{k}\right) + \left(r_j - r_u + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}, \quad d_2 - d_1 - \sigma\sqrt{T}$$

$$\Rightarrow d_1 = \frac{\ln\left(\frac{120}{117}\right) + \left(0.4988\% - 4.97\% + \frac{0.1^2}{2}\right)\left(\frac{180}{365}\right)}{0.1\sqrt{\frac{180}{365}}}$$

$$= 0.08165$$

$$d_2 = 0.08165 - 0.1\sqrt{\frac{180}{365}} = 0.0114$$

$$N(-d_1) = N(-0.08165) = 1 - N(0.08165) = 0.4675$$
$$N(-d_2) = N(-0.0114) = 1 - N(0.0114) = 0.4954$$

$$\therefore p = 117e^{-0.4988\% \times \frac{180}{365}} \times 0.4954 - 120e^{-4.97\%\left(\frac{180}{365}\right)} \times 0.4675$$

$$= 3.0777$$

$$\therefore \text{Option Premium} = 3.0777 \times 10,000$$
$$= 30777$$

**10.** continued

**(b)**

$$dx = Y \times dQ(t) + Y r_u Q dt + r_j \left[ x_0 - YQ \right] dt$$

**(c)**

$$\text{Portfolio} = Y \times 120 + (30,777 - 120Y)$$

By $B-S$, $y = -e^{r_n T} \times N(-d_1)$

$$= -e^{-\frac{180}{365} \times 4.97\%} \times 0.4675 \qquad \left[ \text{From part } (a) \right]$$

$$= -0.4562$$

$\therefore$ has to short $0.4652 \times 10,000 = 4,562$ USD

**(d)**

From Part (b)

$$dx = (-4562 \times 0.127) + \left( -4562 \times 0.0497 \times \frac{120}{365} \right) + \frac{0.004988 \times \left[ 30,777 + 4562.120 \right]}{365} = 645.73$$

$$x_0 = 30,777$$

$$x_1 = x_0 + dx$$

$$= 30,131.27$$

# 11.

**Learning Objectives:**

**4 – g.** Describe issues and best practices in the estimation or calibration of financial models

*This question on model risk is particularly timely given the financial crisis, and pushes the candidate to think critically about the Black-Scholes-Merton model. Business awareness and judgment is where actuaries distinguish themselves relative to more purely quantitative colleagues. See study note V-C101-07*

**Solution:**

**(a)**

**Fundamental** – *This is what was received*
- Postulates and data together with a means of drawing dynamic inferences from them

**Phenomenological**
- Model which helps visualize a process which cannot directly be observed

**Statistical**
- Uses regression analysis to find the best-fit solution

**(b)**
- Inapplicable model
- Incorrect model
- Correct model, incorrect solution
- Correct model, inappropriate use
- Unstable, unreliable data
- Software / Hardware bugs
- Changing conditions in real world render model useless
- Wrong assumptions concerning dynamics and relationships between variables
- Variables modeled poorly

**(c)**
- Only let those who take pride in their work work on model development
- View development as an interdisciplinary endeavor – make sure everyone knows their role and role of others
- Roll out testing slowly
- Create a good user interface
- Test complex models with simple cases first
- Test model at boundaries
- Do not ignore small discrepancies

# 12.

**Learning Objectives:**

**5 – g.** Explain the qualitative implications of repeated games

*This question asks the candidate to compare and contrast game strategies when the game is played non-repeatedly, versus finitely repeated, versus infinitely repeated. The material is covered in Eric Rasmussen chapters 1 and 5.*

**Solution:**

**(a)**

Players: NorthSouth and PropRed
Action: Maximum effort and minimum effort

Payoffs:

$$\frac{100}{2} - 25 = 25 \quad \text{If both use maximum effort}$$

$$\frac{100}{2} - 10 = 40 \quad \text{If both use minimum effort}$$

$$90 - 25 = 65 \quad \text{If it is the only one use maximum effort}$$

$$10 - 10 = 0 \quad \text{If it is the only one use minimum effort}$$

$$
\begin{array}{ccccc}
 & & \text{PropRed} & & \\
 & & \text{Max} & & \text{Min} \\
\text{Max} & (25, 25) & \leftarrow & (65, 0) \\
\text{NorthSouth} & \uparrow & & \uparrow \\
\text{Min} & (0, 65) & \leftarrow & (40, 40)
\end{array}
$$

**(b)**

If not repeat. See matrix above. Max effort is a dominant strategy (strong form) for NS. No matter what effort PropRed use, NS is better off using max effort. Smiliarly, PropRed has max effort as a dominant strategy.

We can see if NS chose minimum effort, PR chose max, NS will switch to max → Not equilibrium.

If NS chose min, PR also min, PR will switch to max → Not equilibrium.

Based on symmetric structure, (max, max) is the only equilibrium.

# 12. continued

**(c)**

If finitely repeated. We start from last game (max, max) is the only Nash equilibrium. (Proof, see (b). No player has the incentive to switch from max no matter what strategy the other chose.) Same logic applies to every previous game. So always max is a Nash equilibrium.

**(d)**

A subgame Nash perfect eq is still an equilibrium in the entire game and also Nash equilibrium in every subgame.

**(e)**

Let NS deviate for Tit-Tat once. NS will max and pay off is (65, 0). From now on, the pay off will be (0, 65), (65, 0) alternate If the discount factor is close to 1. Future payoff have almost the same impact as current pay off. PR is better off not punishing NS once and accept (65, 0) once and keep playing min to get (40, 40) later.

So Tit-for-Tat is not Nash subgame perfect.

**(f)**

Deviation from Grim has to hurt the player.

No deviation: $PV = 40 + \dfrac{40}{1+r} + \dfrac{40}{(1+r)^2} + ... = 40 + \dfrac{40}{r}$

Deviate: $PV = 65 + \dfrac{25}{1+r} + \dfrac{25}{(1+r)^2} + ... = 65 + \dfrac{25}{r}$

No Dev $\geq$ Dev $\Rightarrow 40 + \dfrac{40}{r} \geq 65 + \dfrac{25}{r}$

$$r \leq \dfrac{15}{25} = 60\%$$

# 13.

**Learning Objectives:**

**2 – a.** Describe the steps necessary to obtain funds for a given project or firm from any specified source, and be able to recommend a specific approach to raising capital in a given situation.

*This is a recall and application question asking candidates to (a) determine the value of the firm's debt and equity given the implementation of 2 different projects, (b) given a specific project and financing method, calculate the impact to existing debt and equity, and (c) describe convertible debt and calculate the conversion option point and value of this new debt. See Doherty, Chapter 13.*

**Solution:**

**(a)**

### Project A

| | | | |
|---|---|---|---|
| **(i)** | Net Present Value | (PV cash flows – capital cost) | $(240 - 200) = 40$ |
| **(ii)** | Value of the Firm | Low = (Existing Op low + Proj A cs's) | $(100 + 240) = 340$ |
| | | High = (Existing Op low + Proj A cf's) | $(200 + 240) = 440$ |
| | Value of the Firm | | $0.5 \times (340 + 440) = 390$ |
| **(iii)** | Value of Old Debt | | $0.5 \times (100 + 100) = 100$ |
| **(iv)** | Value of New Debt | | $0.5 \times (200 + 200) = 200$ |
| **(v)** | Value of Equity | Low = (low VF – debt) | $(340 - 300) = 40$ |
| | | High = (high VF – debt) | $(440 - 300) = 140$ |
| | Equity | | $0.5 \times (40 + 140) = 90$ |

### Project B

| | | | |
|---|---|---|---|
| **(i)** | New Present Value | Prob high = prob low = 0.5 | 0.5 |
| | | (low PV cash flows – capital cost) | $(40 - 200) = -160$ |
| | | (high PV cash fhighs – capital cost) | $(310 - 200) = 110$ |
| | | E(NPV) | $0.5 \times (-160 + 100) = -25$ |

## 13. (a) (Project B) continued

| | | | |
|---|---|---|---|
| **(ii)** | Value of the Firm | (1) = (E Op low + Proj B cf's low – BR) | $(100 + 40 - 100) = 40$ |
| | | (2) = (E Op high + Proj B cf's low – BR) | $(200 + 40 - 100) = 140$ |
| | | (3) = (E Op low + Proj B cf's high) | $(100 + 310) = 410$ |
| | | (4) = (E Op high + Proj B cf's high) | $(200 + 310) = 510$ |
| | | E(value of firm) | $0.25 \times (40 + 140 + 410 + 510)$ $= 275$ |
| **(iii)** | Value of Old Debt | (1) since firm value $< 100$ | 40 |
| | | (2, 3, 4) since firm value $> 100$ | 100 |
| | | Old debt value | $0.25 \times (40 + 100 + 100 + 100)$ $= 85$ |
| **(iv)** | Value of New Debt | | $\text{Min}\left( \max\left( \begin{array}{l} 0, \text{value of firm} - \\ \text{old debt} \end{array} \right), 200 \right)$ |
| | | (1) | $\text{Min}(\max(0, 40 - 100), 200) = 0$ |
| | | (2) | $\text{Min}(\max(0, 140 - 100), 200) = 40$ |
| | | (3, 4) | $\text{Min}(\max(0, 410 - 100), 200)$ $= 200$ |
| | | New debt value | $0.25 \times (0 + 40 + 200 + 200) = 110$ |
| **(v)** | Value of Equity | | $\max(\text{value of firm} - \text{total debt}, 0)$ |
| | | (1) | $\max(40 - 300, 0) = 0$ |
| | | (2) | $\max(140 - 300, 0) = 0$ |
| | | (3) | $\max(410 - 300, 0) = 110$ |
| | | (4) | $\max(510 - 300, 0) = 210$ |
| | | Equity | $0.25 \times (0 + 0 + 110 + 210) = 80$ |

### (b)

**(i)**

Bond holders willing to pay = 110 (proj B (a)(iv))

**(ii)**

Capital cost = 100

Additional capital from shareholders $(200 - 110) = 90$

# 13. continued

**(c)**

Use a convertible

**(i)**

Features

Normal bond with option to convert to fixed number of shares

Convert if bond value less than diluted share value

Has bondholder value – lowers interest on debt

Makes risky projects less attractive to shareholders

**(ii)**

Value of firm for conversion

Shares after conversion $\quad (100 + 200) = 300$

Convert if $\qquad \frac{2}{3} \times (\text{value of Equity}) > 200$

$\qquad\qquad$ Or $(\text{value of Equity}) > 300$

$\qquad\qquad (\text{value of firm} - \text{senior debt}) > 300$

$\qquad\qquad$ value of firm $> 400$

**(iii)**

Value of new debt

| | | |
|---|---|---|
| Low | value of firm $= 340 < 400$ | 200 |
| High | value of firm $= 440 > 400$ | |
| Converted Share value | $\frac{2}{3} \times (\text{value of firm} - \text{senior debt})$ | $\frac{2}{3} \times (440 - 100) = 226.67$ |
| Value of new debt | $0.5 \times (200 + 226.67) = 213.33$ | |

# 14.

**Learning Objectives:**

**1 – d.** Define and compare risk metrics used to quantify economic capital and describe their limitations

**1 – e.** Apply the concept of economic capital and describe methodologies for allocation capital within a financial organization

*This question probes the practical issues of allocating capital with diversification benefits among very different business units, which was explored in note FET-11-07.*

**Solution:**

**(a)**

**(i)**

Equally $CaR(A) = 150$

$CaR(B) = 300$

$CaR(C) = 400$

$CaR(A) + CaR(B) + CaR(C) = 850$

Using correlation matrix

$$\left[ CaR(A+B+C) \right]^2 = \begin{bmatrix} 150, & 300, & 400 \end{bmatrix} \begin{bmatrix} 1 & -0.3 & 0.7 \\ -0.3 & 1 & 0.2 \\ 0.7 & 0.2 & 1 \end{bmatrix} \begin{bmatrix} 150 \\ 300 \\ 400 \end{bmatrix}$$

$$= \begin{bmatrix} 340 & 335 & 565 \end{bmatrix} \begin{bmatrix} 150 \\ 300 \\ 400 \end{bmatrix}$$

$$= 5.1 \times 10^4 + 10.05 \times 10^4 + 22.6 \times 10^4$$

$$= 37.75 \times 10^4$$

$CaR(A+B+C) = 614$

$\text{Factor} = \dfrac{614}{850} = 72\%$

$CaR'(A) = 150 \times \dfrac{614}{850} = 108.43$

$CaR'(B) = 300 \times \dfrac{614}{850} = 216.85$

$CaR'(C) = 400 \times \dfrac{614}{850} = 289.13$

# 14. (a) continued

**(ii)**
Marginal

$$CaR'(A) = 614 - 475 = 139$$
$$CaR'(B) = 614 - 400 = 214$$
$$CaR'(C) = 614 - 300 = 314$$

**(iii)**
Internal beta

$$\beta_A = \frac{(oVCA, \text{Total Return})}{Var(\text{Total Return})}$$
$$= \frac{140}{191.2}$$
$$= 0.73$$

$$\beta_B = \frac{160}{191.2} = 0.84$$

$$\beta_C = \frac{100}{191.2} = 0.52$$

$$W(\beta_A + \beta_B + \beta_C) = 1 \quad \Rightarrow \quad \text{ensure } 100\% \text{ allocation}$$

$$W = \frac{1}{0.73 + 0.84 + 0.52} = \frac{1}{2.09} = 0.48$$

$$CaR'(A) = 614 \times W\beta_A = 614 \times 0.73 \times 0.48 = 215$$
$$CaR'(B) = 614 \times W\beta_B = 246$$
$$CaR'(C) = 614 \times W\beta_C = 153$$
$$\Rightarrow \quad \text{Total} = 614$$

# 14. continued

**(b)**
- Return on CaR
  - This may reject a project, which has a positive economic profit or EVA, but has a lower return on CaR than target.
  - This problem can be resolved by using the Economic Value added or Residual Value measure, such that $EVA = Return - k \times CaR$
    - Any project, which has the positive value after charging an economic cost, will add the value to shareholders
- Business unit level measure of $CaR$ = Internal Beta.
  - Diversification benefit is beyond control of each BU
    - Recommend to use BU's individual CaR without allocating diversification benefit to BU for their performance measurement

# 15.

### Learning Objectives:

**3 – a.** Define the cash flow characteristics of complex derivatives including exotic options, interest rate derivatives, swaps, and other non-traditional derivatives.

**3 – c.** Identify embedded options in assets and liabilities

**3 – d.** Evaluate the impact of embedded options on risk/return characteristics of assets and liabilities

**3 – f.** Demonstrate mastery of option pricing techniques and theory for equity and interest rate derivatives

**3 – g.** Identify limitations of each option pricing technique

*The question is based on Chapter 12 of Hardy (GAOs). It requires candidates to derive equations that are stated in the reading (easy derivation, but unseen), and to adapt the formula for slight variations in the option.*

### Solution:

**(a)**

The fund at $t$ is $F_t = \dfrac{1000 S_t}{S_0 (1-m)^{12t}}$, where $m = 0.005$ and $t$ is in years.

The annuity payable per year is $X$, say, where $X = \max\left(\dfrac{F_n g, F_n}{a_{65}(n)}\right)$

The cost of the annuity is $X a_{65}(n)$

The available funds from the policyholder's account are $F_n$, so the option payoff is

$$
\begin{aligned}
P &= \left( X \, a_{65}(n) - F_n \right)^+ \\
&= \left( \max\left[ \dfrac{F_n g, F_n}{a_{65}(n)} \right] a_{65}(n) - F_n \right)^+ \\
&= \left( \max\left[ F_n g a_{65}(n), F_n \right] - F_n \right)^+ \\
&= F_n \max\left[ g a_{65}(n) - 1, 0 \right] \\
&= 1000 \dfrac{S_n}{S_0} (0.995)^{12n} \max\left[ g a_{65}(n) - 1, 0 \right]
\end{aligned}
$$

## 15. continued

**(b)**

- Interest Rate Risk
    - Low interest rates will increase the value of $a_{65}(n)$. The option will be in the money when $ga_{65}(n) > 1$, that is when $a_{65}(n) > \dfrac{1}{g}$.
- Longevity Risk
    - Increased longevity will lead to higher values for $a_{65}(n)$, same problems as interest rate.
- Stock price risk
    - Once the option is in the money, the value increases proportionately with $S_t$.
- Lapse risk
    - If lapses are different to expected, leads to over or under hedging.

- Also causes problems with reducing MC income.
- MC may not cover expenses.

**(c)**

- Interest rate risk is hedgeable if all cashflows are well-defined. In this case there is a lot of uncertainty in the cash flows, and all payments are in units of underlying fund.
- The liabilities are very long term, and there are unlikely to be long enough bonds to hedge with.
- Pricing is difficult, delta hedging of complex interest rate options is very, very difficult
- Difficulty predicting lapses leads to over or under hedging.
- Longevity risk very difficult to offset – little capital market capacity
- Using available models, the gamma risk is very large; the hedge position tends to involve very high values long in the annuities and short in bonds
- No commonly accepted joint model (in $q$-measure or $P$-measure) for stocks and bonds
- Static hedge using swaptions requires certain cashflows; uncertainty in cashflows introduces significant basis risk.
- Static hedge introduces counterparty risk.

# 15. continued

**(d)**

The annuity part of the option has a minimum value of $F_n$. Hence it is never optimal under the original annuity not to annuitize. Therefore, the value does not change. Symbolically, we lose the $(\ )^+$ in the equations under (a), but end up with the same payoff:

$$P = \left( X\ a_{65}(n) - F_n \right)$$
$$= \left( \max\left( F_n g, \frac{F_n}{a_{65}}(n) \right) a_{65}(n) - F_n \right)$$
$$= \left( \max\left( F_n g a_{65}(n), F_n \right) - F_n \right)$$
$$= F_n \max\left( g a_{65}(n) - 1, 0 \right)$$
$$= 1000 \frac{S_n}{S_0} (0.995)^{12n} \max\left( g a_{65}(n) - 1, 0 \right)$$

**(e)**

Now the annuity is based not on $F_n$ but on $H = \max\left( F_n, 1000(1.06)^n \right)$. Then

$$P^* = \left( X\ a_{65}(n) - F_n \right)^+$$
$$= \left( \max\left( H\ g, \frac{H}{a_{65}(n)} \right) a_{65}(n) - F_n \right)$$
$$= \left( \max\left( H\ g\ a_{65}(n), H \right) - F_n \right)$$
$$= H \max\left( g\ a_{65}(n), 1 \right) - F_n$$

**(f)**

- Clearly, pricing and hedging are even more complex
- We have introduced a floor function for the option value. Now the units of payoff are in terms of the greater of a fixed amount $\left( 1000(1.06)^{12n} \right)$ and the fund value. For this risk, we would expect to take a long position in bonds maturing at $n$. However, for the GAO, we are short in bonds. This will increase the hedging error risk.

# 16.

## Learning Objectives:

**1 – e.** Apply the concept of economic capital and describe methodologies for allocating capital within a financial organization

*This question is about understanding ways of calculating and allocating capital. It is based primarily on the paper "Capital Allocation by Percentile Layer."*

## Solution:

**(a)**

$$VaR(99\%) = \$1,000 \quad \because 1\% \text{ chance, loss} > 91000.$$
$$= \text{Amt of capital needed}$$

**(b)**

$$CTE(98\%) = \frac{\$1,000 \times 1.0\% + \$1400 \times 0.8\% - \$1,800 \times 0.2\%}{2\%}$$
$$= \$1,240$$
$$= \text{Amt of capital needed}$$

**(c)**

Entire \$1,000 allocated to Life $\because$ only Life responsible for penetrating loss at $VaR(99\%)$.

**(d)**

| Layer of Capital | Amount | Share % | | | | |
|---|---|---|---|---|---|---|
| | | Health (400) | Health (800) | Life (1000) | H&L (400 + 1000) | H&L (800 + 1000) |
| $[0, 400]$ | 400 | $\frac{39.2\%}{51\%}$ | $\frac{9.8\%}{51\%}$ | $\frac{1.0\%}{51\%}$ | $\frac{0.8\%}{51\%}$ | $\frac{0.2\%}{51\%}$ |
| $[400, 800]$ | 400 | | $\frac{9.8\%}{11.8\%}$ | $\frac{1.0\%}{11.8\%}$ | $\frac{0.8\%}{11.8\%}$ | $\frac{0.2\%}{11.8\%}$ |
| $[800, 1000]$ | 200 | | | $\frac{1.0\%}{2.0\%}$ | $\frac{0.8\%}{2.0\%}$ | $\frac{0.2\%}{2.0\%}$ |
| Total allocated capital to events | | 307.45 | 409.07 | 141.74 | 113.39 | 28.35 |

**16.** **(d) continued**

$$\therefore \text{to health} = 307.45 \times \frac{400}{400} + 409.07 \times \frac{800}{800} + 141.74 \times \frac{0}{1000}$$

$$+ 113.39 \times \frac{400}{1400} + 28.30 \times \frac{800}{1800}$$

$$= \$761.52$$

$$\therefore \text{to life} = 1000 - 761.52 = \$238.48$$

**(e)**

Total cap reg/mt at $CTE(98\%) = \$1,240$ as $m(b)$.

$$E\left(L_L \middle| TL > VaR(98\%)\right) = \$1000 \times \frac{1\%}{2\%} + \$1000 \times \frac{0.8\%}{2\%} + \$1000 \times \frac{0.2\%}{2\%} = \$1000$$

$$E\left(L_H \middle| TL > VaR(98\%)\right) = \$0 \times \frac{1\%}{2\%} + \$400 \times \frac{0.8\%}{2\%} + \$800 \times \frac{0.2\%}{2\%} = \$240$$

**(f)**

I recommend percentile layer method
∵ Capital B is held primarily for losses under the capital requirement (95%
     change to meet the losses)
Percentile layer method correctly allocates more capital to health which is the
     major loss contributor below the capital requirement

# 17.

### Learning Objectives:

**2 – e.** Describe the process, methods and effects of potential acquisition of a business including its effects on capital structure and price/earnings multiples.

*This is a recall, application and synthesis question asking candidates to (a) determine the appraisal value [application], (b) list advantages and disadvantages of financing the deal through debt and equities [recall], and (c) recommend the best way to finance the deal by comparing the earnings per share for the combined company [synthesis]. See Toole and Herget, Chapters 2 & 4, and Copeland, Weston, Shastri, Chapter 18.*

### Solution:

**(a)**

| Distributable Earnings (DE) | | | | |
|---|---|---|---|---|
| Yr | netcF – | $\Delta$StatRes. – | $\Delta$reg. capital | = |
| 2008 | 7,480 | 5000 | 384 | 2,096 |
| 2009 | 8,500 | 5500 | 458 | 2,592 |
| 2010 | 9,720 | 5900 | 598 | 3,222 |
| 2011 | 11,070 | 6400 | 680 | 3,990 |
| 2012 | 13,300 | 6700 | 770 | 5,830 |

$\text{netCF} = \text{Prem} + 1\text{nv inc} - \text{Death \& Surrender benefit} - \text{exp} - \text{comm} - \text{taxes}$

$$\text{Appraisal value} = \sum_{1}^{5} v_{wAcc}^{t} \, DE_{t}$$

$wAcc = 0.3 \times 5\% + 0.7 \times 10\% = 8.5\%$

$\text{Appraisal value} = 13.412 \text{ million}$

**(b)**

Advantage of debt
- Lower cost
- Lock in fixed cost for several years
- Sends positive signal

Disadvantages of debt
- Not sufficient taxable income to allow tax deduction
- High cost
- Restrict future financing amount
- Adds financial burden

## 17.  (b) continued

Advantage of equity
- No need to repay principal
- Rating agencies view favorably
- Claim limited to residual amount
- Dividends receive preferential tax treatment

Disadvantage of equity
- Stack dividends not tax deductible to payer
- Dilutive to earnings $\downarrow$ P/E ratio
- Greater risk to capital provider

**(c)**

Finance by debt

Goodwill annuitization $\dfrac{2m}{4} = 0.5m$

After tax interest expense     = 7% (appraisal valuation)
                                             = 1.08m

Net income of combined company $= 10m + 2m + 1m - 0.5m - 1.08m$

(net inc of great south and net inc of Apple Life + after cnt synergies – after tax goodwill annuitization – after tax int exp)

$$= 11.42m$$

$$\text{EPS} = \frac{11.42}{1} = 11.42$$

Finance by equity

**Net Income of Company**

| | | |
|---|---|---|
| – | Net. Inc. of Great South Life | 10 $m$ |
| + | Net. Inc. of Apple Life | 2 $m$ |
| + | After tax cut synergies | 1 $m$ |
| – | After tax deal cut | 1 $m$ |
| – | After tax goodwill annuitization | 0.5 $m$ |
| | | 11.5 $m$ |

$$EPS = \frac{11.5}{1.2} = 9.58$$

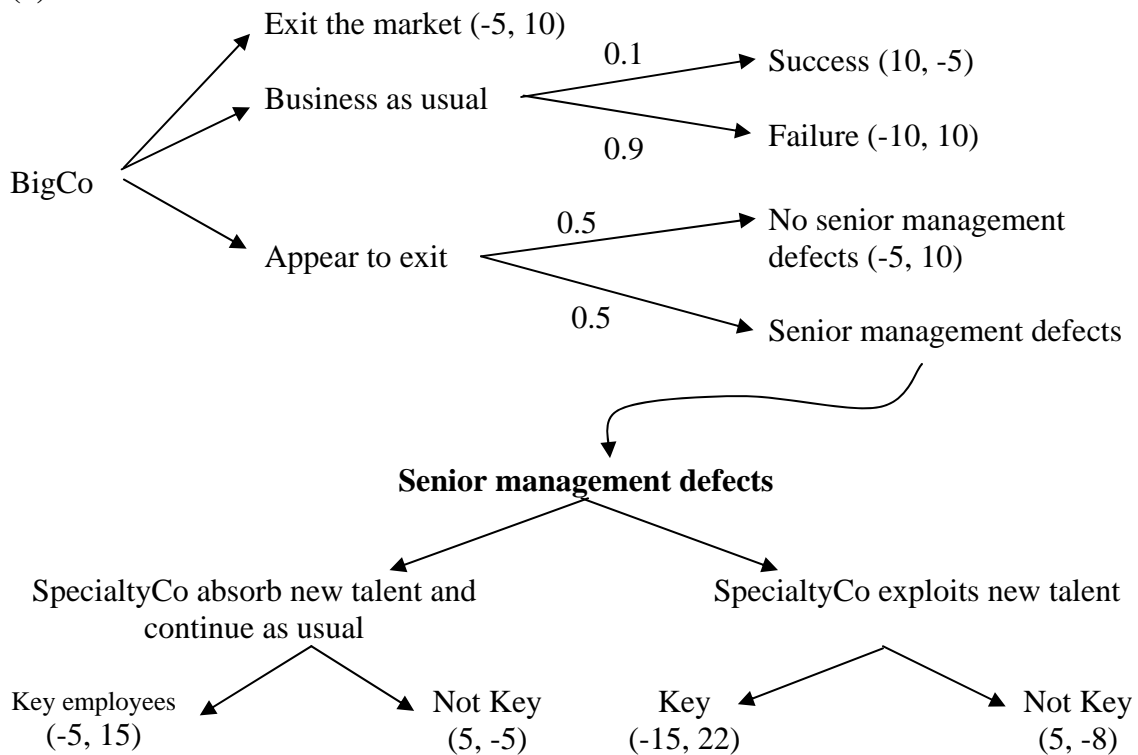Finance the deal through debt generate higher EPS

# 18.

**Learning Objectives:**

**5 – c.** Define the elements of a game, including information sets, etc., Nash equilibrium, mixed strategies.

*This question covers basic game theory from Rasmusen chapters 1 – 3, 7.1 and 9.1.*

**Solution:**

**(a)**



**(b)**

BigCo knows no key defections are possible
Consider (business as usual):
Payoff is $0.1(10) + 0.9(-10) = -8$

Consider (exit the market): payoff $= -5$

Consider (appear to exit)
If no management defect $\rightarrow$ payoff $= -5$
If no management defect $\rightarrow$ payoff $= 5$, since BigCo knows no key defections
$\rightarrow$ payoff $= 0.5(5) + 0.5(-5) = 0$

$\rightarrow$ Dominant strategy is (appear to exit)

**(c)**

Let $p$ = prob. that recruited staff is key.
We want payoff (exploit) > payoff (business as usual)
$$p(22)+(1-p)(-8) > p(15)+(1-p)(-5)$$
$$30p-8 > 20p-5$$
$$10p > 3$$
$$p > \frac{3}{10}$$

**(d)**

Moral hazard with hidden action
Both parties know ability initially but effort decided only by key manager
Contract offered, agent decides on effort, nature makes a move
Complete but uncertain information

Adverse selection
Key manager knows effort before contract is offered
Nature moves to determine manager ability, contract offered
Other party doesn't know manager's ability

**(e)**

BigCo's situation is moral hazard
Since they know which manager are key and which are not before contract is
    offered

# 19.

**Learning Objectives:**

**1 – c.** Evaluate the various profitability measures including IRR, NPV, and ROE, etc.

*This question deals with evaluating projects under uncertainty, differentiating between market pricing of projects and pricing using WACC, and calculating required risk capital for projects. See Copeland, Chapter 9, and Chew, Chapter 31.*

**Solution:**

**(a)**

There are at least four ways to derive a market value for Project 2. Any one of these would earn full marks for (a):

**(i)**
Solve using state prices, where S1 is state price for $1 payoff in state 1; S2 is the state price for $1 payoff in state 2; and S3 is the state price for $1 payoff in state 3:

$$40 \times S1 = 10, \text{ so } S1 = 0.25$$

$$40 \times S1 + 50 \times S2 = 25, \text{ so } S2 = 0.30$$

$$40 \times S1 + 40 \times S2 + 40 \times S3 = \frac{40}{1 + rf} = \frac{40}{1.1}, \text{ so } S3 = 0.359$$

Market value of Project 3 is:

$$30 \times S2 + 34 \times S3 = 30 \times 0.30 + 34 \times 0.359 = 21.21$$

**(ii)**
Solve by converting realistic probabilities to risk neutral probabilities, and then discounting at the risk-free rate:

$$\frac{P1 \times 40}{1.1} = 10, \text{ so } P1 = 0.275$$

$$\frac{\left[ (P1 \times 40) + (P2 \times 50) \right]}{1.1} = 25, \text{ so } P2 = 0.330$$

$$P3 = 1 - 0.275 - 0.330 = 0.395$$

# 19. (a) continued

Market value of Project 3 is:

$$\frac{\left[(P2\times30)+(P3\times34)\right]}{1.1}=21.21$$

**(iii)**
Solve by replication:
$$40\times Q1+40\times Q2+1.1\times M=0$$

$$0\times Q1+50\times Q2+1.1\times M=30$$

$$0\times Q1+0\times Q2+1.1\times M=34$$

$$M=\frac{34}{1.1}=30.91$$

$$Q2=\frac{(30-34)}{50}=-0.08$$
$$Q1=\frac{(-34-40(-0.08))}{40}=-0.77$$

Market value of Project 3 is:

$$10\times Q1+25\times Q2+M=21.21$$

**(iv)**
Solve by WACC:

Assume all projects are funded:

| State | Probability | Payoff |
|-------|-------------|--------|
| 1 | 1/3 | 80 |
| 2 | 1/3 | 80 |
| 3 | 1/3 | 34 |

Expected payoff is $\dfrac{80}{3}+\dfrac{80}{3}+\dfrac{34}{3}=64.66$

**19.** (a) continued

Present value of all three is: $\dfrac{64.66}{1.15} = 56.23$

Market value of Project 3 is: $56.23 - 10 - 25 = 21.23$

**(b)**

Project 1:

$$V_0 = \left(\frac{1}{3}\right) \times \frac{40}{1.15} = 11.59$$

This is greater than $10.00 cost, so fund Project 1 because
$$NPV = 11.59 - 10.00 = 1.59$$

Project 2:

$$V_0 = 11.59 + \left(\frac{1}{3}\right) \times \frac{50}{1.15} = 26.08$$

This is greater than the $25.00 cost, so fund Project 2 because
$$NPV = 26.08 - 25.00 = 1.08$$

Project 3:

$$V_0 = \left(\frac{1}{3}\right) \times \frac{30}{1.15} + \left(\frac{1}{3}\right) \times \frac{34}{1.15} = 18.55$$

This is less than $21.21 cost, so do not fund Project 3 because
$$NPV = 18.55 - 21.21 = -2.66$$

**(c)**

There are no customer liabilities, so net assets equal gross assets.
To make the net assets default free, you would need to buy an $11.00 payoff in
state 2 and an $11.00 payoff in state 3.

Risk capital $= (\$11.00 \times S2) + (\$11.00 \times S3) = (\$11.00 \times 0.300) + (\$11.00 \times 0.359) = \$7.25$

## 20.

**Learning Objectives:**

**1 – a.** Explain the various definitions of capital, including regulatory, rating agency and other risk-based capital requirements, the context in which they are appropriate, and how they affect decisions.

*The question challenges the student to know the original Basel Accord and its flaws. The student must be able to apply the calculation methods in the Accord. The solution is drawn from Crouhy, Galai, and Mark, Chapter 2, pages 53-61 and 68-70.*

**Solution:**

**(a)**

$$RWA = \sum \text{on BS value} \times wt + \sum \text{off BS credit equivalent} \times \text{counterpay } wt \text{ RWA}$$

On BS: $RWA = 15(0.2) + 20(0.5) + 50(1) + 10 + 20$

$$= 93mn$$

Off BS:  Credit equivalent
Banker's acceptance   $= 5 \times 100\% = 5$
Revoluj loan   $= 25 \times 0 = 0$
Call   $= \text{replacement} + \text{notional} \times \text{add-on factor}$
$$= 0.5 + 10 \times 0.1$$
$$= 1.5$$

$$RWA = 93 + 5 \times 50\% + 0 + 1.5 \times 50\%$$
$$= 96.26mn$$

Min capital $= 8\% \times RWA = 8\%(96.25) = 7.7mn$

**(b)**

**(i)**

- No cap required for less than 1 year loan
- Assume all corporation are same risky
- Doesn't recognize netting for derivatives
- Too low charge for mortgages
- Portfolio diversification not recognized
- Only credit risk charge, not other risks

# 20. **(b) continued**

**(ii)**
- BIS98 includes market risk; BIS 2000 include operational, legal, liquidity risk
- BIS 2000
  - at least the same cap req
  - Focus on active international bank
  - Promote global competitive equality
  - Cover more risk types