DP-RC,B Illustrative Solutions
Fall 2007

1.

Solution:

(a)

Pensions projected to age 65:

$$A: 2\% \times 50000 \times (20+10) \times 1.04^{(60-50-1)} \times 85\% = 36,294$$

$$B: 2\% \times 30000 \times (5+30) \times 1.04^{(60-30-1)} \times 85\% = 55,668$$

$$PFVB_a = ben \times 1.06^{-(60-50)} \times 12 = 243,200$$

$$AL_a = \frac{svc}{totSvc} \times PVFB_a = 162,133$$

$$PVFB_b = ben \times 1.06^{-(60-30)} \times 12 = 116,308$$

$$AL_b = \frac{svc}{totSvc} \times PVFB_b = 16,615$$

$$AL = 162,133 + 16,615 = 178,749$$

$$NC_a = \frac{PVFB_a}{totSvc\ a} = 8,107$$

$$NC_b = \frac{PVFB_b}{totSvc\ b} = 3,323$$

$$UL = AL - 150000 - 28,749$$

$$\text{Amortization} = \frac{UL}{\ddot{a}_{\overline{3}|}} = 10,146$$

Where $\ddot{a}_{\overline{3}|} = 2.833$

Total cost = NC_a + NC_b + amortization = 21,576

(b)

Actual fund value at 1/1/2008 $= (150000 + 21576) \times 1.04 = 178,439$

Expected fund value $= (150000 + 21576) \times 1.06 = 181,871$

Investment loss: expected – actual = 3,432

Salary gain:

Expected AL $= (AL_t + NC_t) \times 1.06 = 201,589$

1. **(b) continued**

Actual AL:

$$A: 2\% \times 50000 \times (20+10) \times 1.04^{(60-50-1)} \times .85 = 34,899$$

$$B: 2\% \times 30000 \times (5+30) \times 1.04^{(60-31-1)} \times .85 = 53,527$$

$$PVFB_a = ben \times 1.06^{-(60-51)} \times 12 = 247,876$$

$$AL_a = \frac{svc}{totSvc} \times PVFB_a = 173,514$$

$$PVFB_b = ben \times 1.06^{-(60-31)} \times 12 = 118,545$$

$$AL_b = \frac{svc}{totSvc} \times PVFB_b = 20,322$$

$$AL = 173514 + 20322 = 193,836$$

Salary gain = expected AL – actual AL = 7,753

Change of method gain

With new method AL = F

PVFB = 366,421

PVFB – F = 187,982

$$PVFS = 50000 \times \ddot{a}_{\overline{9}|j} + 30000 \times \ddot{a}_{\overline{29}|j} = 1,092,346$$

$\ddot{a}_{\overline{9}|j}$ where $j = \frac{1.06}{1.04} = 8.350$

$\ddot{a}_{\overline{29}|j}$ where $j = \frac{1.06}{1.04} = 22.495$

$$U_t = \frac{(PVFB\text{-}F)}{PVFS} = 17.2\%$$

$$NC = U_t \times (50000 + 30000) = 13,767$$

Since there AL = F there are no gains and losses to amortize.
Total 2008 contribution = 13,767

2.

Solution:

(a)

Calculate employer cost EAN

$$\text{PVFNC}_{\text{w}} + \text{PVFContEE}_{\text{W}} = \text{PVFB}_{\text{W}}$$

$$\text{NC}_{\text{W}} = \frac{(\text{PVFB} - \text{PVFContEE})}{\text{PVFY}_{\text{W}}}$$

$$\text{PVFB}_t = B(y) \times \ddot{a}_{65(5)} \times (1+i)^{(t-y)}$$

$$B(y) = 1200 \times (y - w)$$

$$\text{PVFContEE}_t = 1000\ddot{a}_{y-x}$$

$$\text{PVFY}_t = \ddot{a}_{y-x}$$

$$\text{NC}_t = \text{NC}_w$$

$$\ddot{a}_{65(5)} = \ddot{a}_5 + {}_5P_{65}\ddot{a}_{70}(1+i)^{-5}$$

$$= 4.43 + 0.93 \times 9.6 \times 1.065^{-5}$$

$$= 10.95$$

Calculate NC & AL on January 1, 2007

$$\text{AL}_t = \text{PVFB}_t - \text{PVFContEE}_t - \text{PVFNC}_t$$

Member A: x = 35, w = 30, y = 65

$$\text{PVFB}_w = 1200 \times 35 \times 10.95 \times 1.065^{-35}$$

$$= 50,749$$

$$\text{PVFY}_w = \ddot{a}_{35} = 14.580$$

$$\text{PVFContEE}_w = 1000 \times 14.580$$

$$= 14,580$$

$$\text{NC}_w = \frac{50,749 - 14,580}{14.58}$$

$$= 2,481$$

$$\text{PVFYt} = \ddot{a}_{30} = 13.910$$

$$\text{AL} = 50,749 \times 1.065^5 - (2,481 + 1,000) \times 13.910$$

$$= 21,110$$

2. **(a) continued**

Member B: x = 45, w = 35, y = 65

$$\text{PVFB}_w = 1200 \times 30 \times 10.95 \times 1.065^{-30}$$
$$= 59,598$$

$$\text{PVFY}_w = \ddot{a}_{30} = 13.910$$

$$\text{PVFYContEE}_w = 1000 \times 13.91$$
$$= 13,910$$

$$\text{NC}_w = \frac{59,598 - 13,910}{13.91}$$
$$= 3,285$$

$$\text{PVFY}t = \ddot{a}_{20} = 11.730$$
$$\text{AL} = 59,598 \times 1.065^{10} - (3,285 + 1,000) \times 11.730$$
$$= 61,610$$

Determine Employer Contribution:

$$\text{UAL} = 21,110 + 61,610 = 82,720$$
$$\text{Amortization over 15 years} = \frac{82,720}{10.01}$$
$$= 8,264$$

$$\text{Contribution} = \text{NC}_A + \text{NC}_B + \text{Amortization Payment}$$

$$= 2,481 + 3,285 + 8,264$$
$$\text{Contribution} = 14,030$$

(b)

Calculate Employer cost under ILP (Contributory Plan):

$$\text{PVFNC}_a + \text{PVFContEE}_a = \text{PVFB}_a$$
$$\text{NC}_a = \frac{(\text{PVFB}_a - \text{PVFContEE}_a)}{\text{PVFY}_a}$$
$$\text{PVFB}_t = B(y) \times \ddot{a}_{65(5)} \times (1+i)^{(x-y)}$$
$$B(y) = 1200 \times (y - w)$$
$$\text{PVFContEE}_t = 1000\ddot{a}_{y-x}$$

2. (b) continued

$$\text{PVFY}_t = \ddot{a}_{y-x}$$
$$\text{NC}_t = \text{NC}_a$$
$$\ddot{a}_{65(5)} = \ddot{a}_5 + {}_5P_{65}\ddot{a}_{70}\left(1+i\right)^{-5}$$
$$= 4.43 + 0.93 \times 9.6 \times 1.065^{-5}$$
$$= 10.95$$

Calculate NC & AL on January 1, 2007:

$$\text{AL}_t = \text{PVFB}_t - \text{PVFContEE}_t - \text{PVFNC}_t$$

Member A: x = a = 35, w = 30, y = 65

$$\text{PVFB}_a = 1200 \times 35 \times 10.95 \times 1.065^{-30}$$
$$= 69,531$$

$$\text{PVFY}_a = \ddot{a}_{30} = 13.91$$

$$\text{PVFContEE}_a = 1000 \times 13.910$$
$$= 13,910$$

$$\text{NC}_w = \frac{69,531 - 13,910}{13.91}$$
$$= 3,999$$

AL = 0

Member B: x = a = 45, w = 35, y = 65

$$\text{PVFB}_a = 1200 \times 30 \times 10.95 \times 1.065^{-20}$$
$$= 111,873$$

$$\text{PVFY}_a = \ddot{a}_{20} = 11.73$$

$$\text{PVFContEE}_a = 1000 \times 11.730$$
$$= 11,730$$

$$\text{NC}_w = \frac{\left(111,873 - 11,730\right)}{11.730}$$
$$= 8,537$$

AL = 0

2. **(b) continued**

Determine Employer Contribution:

UAL = 0

Amortization Pmt = 0

$$\text{Contribution} = \text{NC}_A + \text{NC}_B$$
$$= 3,999 + 8,537 = 12,536$$

$$\text{Contribution} = 12,536$$

(c)

Under Frozen Initial Liability (Contributory Plan):

$$\text{NC}_t = \frac{\sum \text{PVFB}t - \sum \text{PVFContEE}t - \text{UAL}t - \text{F}t}{\sum \text{PVFY}t \times nt}$$

UAL_t is determined under EAN method

Calculate NC & AL on January 1, 2007:

$$\sum \text{PVFB} = \text{PVFB}_A + \text{PVFB}_B$$
$$= 69,531 + 111,874$$
$$= 181,405$$

$$\sum \text{PVFY} = \text{PVFY}_A + \text{PVFY}_B$$
$$= 13.910 + 11.73$$
$$= 25.64$$

$$\sum \text{PVFContEE} = \text{PVContEE}_A + \text{PVFContEE}_B$$
$$= 1000 \times (13.910 + 11.73)$$
$$= 25,640$$

$$\text{UAL} = \text{UAL}_A + \text{UAL}_B = 82,720$$

$$\text{U}_{07} = \frac{181,405 - 25,640 - 82,720 - 0}{25.64}$$
$$= 2,849$$

$$\text{FIL NC } 2,849 \times 2 = 5,698$$

2. **(c) continued**

Determine Employer Contribution:

$$\text{Amortization over 15 years} = \frac{82,720}{10.01} = 8,264$$

Contribution = FIL NC + Amortization Pmt

$$\text{Contribution} = 5,698 + 8264 = 13,692$$

3.

Solution:

(a)

Retirement Pension at retirement under normal form

Retirement Age = (1/1/2007 – 1/1/1952) = 55 years of age
Credited Service = (1/1/2007 – 1/1/1990) = 17 years of service

$$B_{65} = \text{monthly benefit} \times \text{service} = 50 \times 12 \times 17$$

$$B_{65} = \$10,200$$

$$\text{ERF} = (65 - 55) \times 3\% = 30\%$$

$$B_{55} = \text{DB Annual Early Retirement Benefit at Age } 55 = 10,200 \times (1 - 30\%)$$

$$B_{55} = \$7,140$$

Actuarial Equivalence Calculation

Actuarial present values of the early retirement pension under the normal form is equal to the Level Income Option pension

Present Value of both company and government pension payable under the Normal form:

$$\text{PVFB} = B_{55}\ \ddot{a}_{55}^{(12)} + G_{65}\ v^{10}\ {}_{10}p_{55}\ \ddot{a}_{65}^{(12)}$$

$$= 7,140 \times 13.6 + 3,500 \times \left(\frac{1}{1.06}\right)^{10} \times 0.93 \times 11.8$$

$$= 97,104 + 21,447$$

$$= \$118,551$$

Determine level benefit to be paid for Nancy's lifetime:

$$\text{PVFB} = X * \ddot{a}_{55}^{(12)}$$

$$\$118,551 = X * 13.6$$

$$X = \$8,717$$

OR (alternate equation for 12 pts):

$$B_{55}\ \ddot{a}_{55}^{(12)} = X\left(\ddot{a}_{55}^{(12)} - v^{10}\ {}_{10}p_{55}\ \ddot{a}_{65}^{(12)}\right) + \left(X - G_{65}\right) v^{10}\ {}_{10}p_{55}\ \ddot{a}_{65}^{(12)}$$

$$X = \frac{B_{55(\text{normal form})} + G_{65}\ v^{10}\ {}_{10}p_{55}\ {}_{65}\ddot{a}_{65}^{(12)}}{\ddot{a}_{55}^{(12)}}$$

$$X = 7,140 + 3,500 \times \left(\frac{1}{1.06}\right)^{10} \times 0.93 \times \frac{11.8}{13.6} = 8,717$$

3. **(a) continued**

Monthly pension from age 55 – 65 from the pension plan $= \frac{8,717}{12} = \$726$

Monthly pension after 65 from the pension plan

$= \frac{8,717 - 3,500}{12} = \$434.75$

(b)

Additional liability of modified cash refund is equal to the present value of the death benefit payable if Nancy were to die before pension payments = contributions with interest.

Present Value of death benefit at Age 55:

Number of monthly pension payments before death benefit is zero

$= \frac{9,240}{7,140} \times 12 = 16$ months

$\text{PVDB}_{55} = q_{55}\, v\,(\text{EEcont} - \text{1st year of pension payments}) +$

$p_{55}\, q_{56}\, v^2\,(\text{EECont} - \text{2 years of pension payments})$

$$\text{PVDB}_{55} = (1 - 0.99)\left(\frac{1}{1.06}\right)(9,240 - 7,140) + 0.99 \times 0.02\left(\frac{1}{1.06}\right)^2 (9,240 - \min(9,240, 14,280))$$

$\text{PVDB}_{55} = 19.81$

$$X = \frac{B_{55\,(\text{normal form})} \times \ddot{a}_{55}^{12} - \text{PVDB}^{55}}{\ddot{a}_{55}^{12}}$$

$$= \frac{(7,140 \times 13.6 - 19.81)}{13.6}$$

$$= \frac{7,139}{12}$$

$= \$595$

3. **continued**

(c)

Pension at Retirement under normal form from part a = $7,140

Pension at Retirement under optional form

$$\text{Pension under normal form} * \frac{\ddot{a}_{55}^{(12)}}{\ddot{a}_{55:55\,(60\%)}^{(12)}}$$

$$\begin{aligned}\ddot{a}_{55:55\,(60\%)}^{(12)} &= \ddot{a}_{55}^{(12)} + .60 * \left(\ddot{a}_{55\,(spouse)}^{(12)} - \ddot{a}_{55:55}^{(12)} \right) \\ &= 13.6 + .60 * (13.1 - 12.5) \\ &= 13.96\end{aligned}$$

Nancy's annual pension at January 1, 2007 under JS60% form

$$\begin{aligned}&= \frac{7{,}140 \times 13.6}{13.96} \\ &= \frac{\$6{,}956}{12} \\ &= \$580\end{aligned}$$

4.

Solution:

(a)

$$\begin{aligned}\text{AL}_{07} &= \text{Annual Pension}\left(\text{A}\right) * \ddot{a}74\left(\text{indexed}\right) + \text{Annual Pension}\left(\text{B}\right) * \ddot{a}75\left(\text{indexed}\right)\\ &= \$30,000 * 9.1 + \$40,000 * 8.7\\ &= \$274,000 + \$348,000\\ &= \$621,000\end{aligned}$$

$$\begin{aligned}\frac{\text{Surplus}_{07}}{\left(\text{Unfunded Actuarial Liability}\right)_{07}} &= \text{Assets}_{07} - \text{AL}_{07}\\ &= \$500,000 - \$621,000\\ &= \left(\$121,000\right)\end{aligned}$$

(b)

$$\begin{aligned}\text{2007 Benefit Payments} &= \$30,000 + \$40,000\\ &= \$70,000\end{aligned}$$

Exp $\text{AL}_{08}^{(1)}$ (given no plan changes and expected mortality and indexation)

$$\begin{aligned}&= \left(\text{AL}_{07} - \text{2007 Benefit Payments}\right) * 1.06\\ &= \left(\$621,000 - \$70,000\right) * 1.06\\ &= \$584,060\end{aligned}$$

$\text{AL}_{08}^{(2)}$ (given no plan changes and no mortality and expected indexation)

$$\begin{aligned}&= \$30,000 * 1.02 * 8.7 + \$40,000 * 1.02 * 8.3\\ &= \$266,220 + \$338,640\\ &= \$604,860\end{aligned}$$

$\text{AL}_{08}^{(3)}$ (given no plan changes and no mortality and actual indexation)

$$\begin{aligned}&= \$30,000 * 1.02 * 8.7 + \$40,000 * 1.02 * 8.3\\ &= \$266,220 + \$338,640\\ &= \$604,860\end{aligned}$$

$\text{AL}_{08}^{(4)}$ (actual actuarial liability, given change in plan)

4. **(b) continued**

Actual indexation (old plan):
@ 1.1.2006 – 3.00%
@1.1.2007 – 1.00%
@1.1.2008 – 2.00%

Cumulative effect $= 1.03 * 1.01 * 1.02 = 1.0611$

Deemed indexation (new plan):
@1.1.2006 – 3.00%
@1.1.2007 – 0.00%
@1.1.2008 – 5.00%

Cumulative effect $1.03 * 1.0 * 1.05 = 1.0815$

Ratio of new/old: $\dfrac{1.0815}{1.0611}$

=1.0192

$$\begin{aligned} AL_{08}^{(4)} &= AL_{08}^{(3)} * 1.0192 \\ &= \$604,860 * 1.0192 \\ &= \$616,473 \end{aligned}$$

$$\begin{aligned} \text{Mortality Loss} &= AL_{08}^{(2)} - AL_{08}^{(1)} \\ &= \$604,860 - \$584,060 \\ &= \$20,800 \end{aligned}$$

$$\begin{aligned} \text{Indexation Loss} &= AL_{08}^{(3)} - AL_{08}^{(2)} \\ &= \$604,860 - \$604,860 \\ &= \$0 \end{aligned}$$

$$\begin{aligned} \text{Amendment Loss} &= AL_{08}^{(4)} - AL_{08}^{(3)} \\ &= \$616,473 - \$604,860 \\ &= \$11,613 \end{aligned}$$

$$\begin{aligned} \text{Fund at 1.1.08} &= (\$500,000 - \text{2007 Benefit Payments}) * 1.10 \\ &= \$473,000 \end{aligned}$$

Expected Fund at 1.1.08 = ($500,000 – 2007 Benefit Payments) * 1.06
=$455,800

4. **(b) continued**

Investment Gain = Fund at 1.1.08 – Expected Fund at 1.1.08
= $473,000 - $455,800
= $17,200

$$\frac{\text{Surplus}_{08}}{\left(\text{Unfunded Actuarial Liability}\right)_{08}} = \text{Fund at } 1.1.08 - \text{AL}_{08}^{(4)}$$
$$= \$473,000 - \$616,473$$
$$= \left(\$143,473\right)$$

Unfunded Actuarial Liability $_{07}$	=	($121,000)
Interest on UAL	=	($7,260)
Mortality Loss	=	($20,800)
Amendment Loss	=	($11,613)
Investment Gain	=	($17,200)
Unfunded Actuarial Liability $_{08}$	=	($143,473)

5.

Solution:

(a)

Calculate Normal Cost as at January 1, 2007

$$\text{I/A NC} = \frac{\sum(\text{PVFB-F})}{\text{PVFS}\times\text{S}}$$

F = AL = \$0

Member A

$$\text{PVFB} = 2\%\times 70{,}000\times(1.04)^{14}\times 30\times \ddot{a}^{(12)}_{65}\times v^{15} = \$333{,}826$$

$$\text{PVFS} = \$70{,}000\times \ddot{a}_{\overline{15}|j} \quad \text{where } j = \frac{1.06}{1.04}-1$$
$$= \$70{,}000\times 13.1720$$
$$= \$922{,}040$$

$$\text{I/A NC} = \frac{\$333{,}826-\$0}{\$922{,}040}\times\$70{,}000$$
$$= 36.205\%\times\$70{,}000$$
$$= \$25{,}344$$

Member B

$$\text{PVFB} = 2\%\times 50{,}000\times(1.04)^{24}\times 35\times \ddot{a}^{(12)}_{65}\times v^{25} = \$229{,}940$$

$$\text{PVFS} = \$50{,}000\times \ddot{a}_{\overline{25}|j} \quad \text{where } j = \frac{1.06}{1.04}-1$$
$$= \$50{,}000\times 20.0798 = \$1{,}003{,}990$$

$$\text{I/A NC} = \frac{\$229{,}940-\$0}{\$1{,}003{,}990}\times\$50{,}000$$
$$= 22.903\%\times 50000$$
$$= \$11{,}451$$

$$\text{NC} = \$25{,}344+\$11{,}451$$
$$= \$36{,}795$$

5. continued

(b)

Calculate Normal Cost as at January 1, 2008

$$\begin{aligned} F = AL &= \left(0 + NC_0\right) \times 1.01 \\ &= \$36,795 \times 1.01 = \$37,163 \\ UAL &= AL - F = 0 \end{aligned}$$

Member A

$$\begin{aligned} F &= \$25,344 \times 1.01 \\ &= \$25,597 \end{aligned}$$

$$\begin{aligned} PVFB &= 2\% \times 72,800 \times \left(1.04\right)^{13} \times 30 \times \ddot{a}_{65}^{(12)} \times v^{14} \\ &= \$353,856 \end{aligned}$$

$$\begin{aligned} PVFS &= \$72,800 \times \ddot{a}_{\overline{14}|j} \quad \text{where } j = \frac{1.06}{1.04} - 1 \\ &= \$72,800 \times 12.4061 = \$903,164 \end{aligned}$$

$$\begin{aligned} \text{I/A NC} &= \frac{\left(\$353,856 - \$25,597\right)}{\$903,164} \times \$72,800 \\ &= 36.345\% \times \$72,800 \\ &= \$26,459 \end{aligned}$$

Member B

$$\begin{aligned} F &= \$11,541 \times 1.01 \\ &= \$11,566 \end{aligned}$$

$$\begin{aligned} PVFB &= 2\% \times 52,000 \times \left(1.04\right)^{23} \times 35 \times \ddot{a}_{65}^{(12)} \times v^{24} \\ &= \$243,736 \end{aligned}$$

$$\begin{aligned} PVFS &= \$52,000 \times \ddot{a}_{\overline{24}|j} \quad \text{where } j = \frac{1.06}{1.04} - 1 \\ &= \$52,000 * 19.4467 \\ &= \$1,011,228 \end{aligned}$$

5. **(b) continued**

$$\text{I/A NC} = \frac{(\$234,736 - \$11,566)}{\$1,011,228} \times \$52,000$$
$$= 22.959\% \times \$52,000$$
$$= \$11,939$$

$$\text{NC} = \$26,459 + \$11,939 = \$38,398$$

(c)

Changes in NC:

$$\text{Expected NC} = \$36,795 \times 1.04 = 38,267$$

Change due to fund return:

$$\text{Change in I/A NC} = \frac{(\$25,344 \times .05)}{\$903,164} \times \$72,800 + \frac{(\$11,451 \times .05)}{\$1,011,228} \times \$52,000$$
$$= 102 + 29$$
$$= \$131$$

Alternatively, actual NC = 38,398

Change due to Fund return = 38,398 – 38,267 = 131