DP-RC,B Complete Illustrative Solutions
November 2008

1.

Learning Objectives:

Determine employer normal cost using the Projected Unit Credit method including appropriate adjustments for the termination benefit and employee contribution

Solution:

Under Projected Unit Credit (Contributory Plan):

$$\mathrm{NC}^{Tot}_{t} = \sum\left[\Delta B_j \; D_y \big/ D_x \; \ddot{a}^{(12)} + \mathrm{TBen}^{j}\left(I_x - I_y\right)\big/ I_x\right]$$
$$\mathrm{NC}^{ER}_{t} = \mathrm{NC}^{Tot}_{t} - \mathrm{C}^{EE}_{t}$$

$$\Delta B_j = b(x) = \mathrm{Sal}_{(x-1)} \times (1.04)^{(y-x-1)} \times 0.02$$
$$\mathrm{PVAB}_t = B(x) \times a^{(12)}_{65} \times (1+i)^{(x-y)}$$
$$B(x) = \mathrm{Sal}_{x-1} \times 0.02 \times (x-w)$$

x = age at valuation date
w = age at plan entry
y = age at retirement

Calculate NC^{ER}_{t} on January 1, 2008:

Member: $x = 35,\ w = 35,\ y = 65$

Determine benefit payable on termination

$$\mathrm{PVAB}_{36} = 75,000 \times 0.02 \times 11 \times 1.0575^{(-29)}$$
$$= 3,261$$

1. continued

$$\begin{aligned}\text{CWI}_{36} &= 0.06 \times 75,000 \times 1.0575 \\ &= 4,759\end{aligned}$$

$$\begin{aligned}\text{ExcessB}_{36} &= \text{CWI}_{36} - 0.5 \times \text{PVAB}_{36} \\ &= 4,759 - 0.5 \times 3,261 \\ &= 3,129\end{aligned}$$

$$\begin{aligned}\text{TBen}_{36} &= \text{PVAB}_{36} + \text{ExcessB}_{36} \\ &= 3,261 + 3,129 \\ &= 6,390\end{aligned}$$

$$\begin{aligned}\text{NC}_{35}^{Tot} &= 75,000 \times 1.04^{29} \times 0.02 \times 0.95 \times 11 \times 1.0575^{(-30)} + 0.05 \times 6,390 \times 1.0575^{(-1)} \\ &= 9,136 + 302 \\ &= 9,438\end{aligned}$$

Determine 2008 Employer Normal Cost

$$\text{C}_{(x)}^{EE} = .06 \times \text{Sal}_{(x-1)} = \text{EE contribution for year at age } x$$

$$\begin{aligned}\text{C}_{(35)}^{EE} &= 0.06 \times 75,000 \\ &= 4,500\end{aligned}$$

$$\begin{aligned}\text{NC}_{35}^{ER} &= \text{NC}_{35}^{Tot} - \text{C}_{35}^{EE} \\ &= 9,438 - 4,500 \\ &= 4,938\end{aligned}$$

2.

Learning Objectives:

Determine the 2008 & 2009 contributions using the Entry Age Normal method reflecting both the normal cost and amortization of the unfunded actuarial liability, if any. Determine gain/loss by source from 2008 to 2009 and illustrate that the unit credit method would produce a lower 2009 contribution.

Solution:

(a)

Calculate 2008 contribution

$$\text{NC}_w = \frac{\left(\text{PVFB}_w\right)}{\text{PVFY}_w}$$

$$\text{PVFB}_t = B\left(y\right) \times \ddot{a}_{58} \times \left(1+i\right)^{(t-y)}$$

$$B\left(y\right) = 100 \times 12 \times \left(y-w\right) \times \left(1 - 0.0025 \times 12 \times \left(65-y\right)\right)$$

$$\text{PVFY}_t = \ddot{a}_{y-x}$$

Calculate NC & AL on January 1, 2008:

$$\text{AL}_t = \text{PVFB}_t - \text{PVFNC}_t$$

Member A: $x = 40,\ w = 28,\ y = 58$

$$\text{PVFB}_w = 100 \times 12 \times 30 \times 12 \times 1.06^{-30} \times \left(1 - 0.0025 \times 12 \times \left(65-58\right)\right)$$
$$= 59,420$$

$$\text{PVFY}_w = \ddot{a}_{30} = 14.59$$

$$\text{NC}_w = \frac{\left(59,420\right)}{14.59}$$
$$= 4,073$$

$$\text{PVFY}_t = \ddot{a}_{18}$$

$$AL = 59,420 \times 1.06^{12} - 4,073 \times 11.48$$
$$= 72,806$$

Determine 2008 Contribution:

$$\text{UAL} = \text{AL} - \text{Assets}$$
$$= 72,806 - 70,000$$
$$= 2,806$$

2. **(a) continued**

$$\text{Amortization of UAL over 15 years} = \frac{2,806}{7.8} = 360$$

Contribution = NC + Amortization Payment
Contribution = 4,073 + 360
= 4,433

(b)

Calculate the change in Unfunded AL by source at January 1, 2009

Investment Loss = Expected Assets – Actual Assets
Expected Assets
(Assets January 1, 2008 + NC) × 1.06 = 78,517

Actual Assets
(Assets January 1, 2008 + NC) × 1.02 = 75,554

Investment return loss = 78,517 – 75,554 = 2,963

Loss on Contributions
= (expected contributions – actual contributions) × *I*
= (4,433 – 4,073) × 1.06
= 382

Loss on Benefit Increase

$$\text{Expected AL} = (72,806 + 4,073) \times 1.06$$
$$= 81,492$$

$$\text{Actual AL} = (72,806 + 4,073) \times 1.1 \times 1.06$$
$$= 89,641$$

Loss on Benefit Increase = 8,149

Total Loss = 11,494

2. continued

(c)

Determine 2009 contribution

$$NC_w = \frac{(PVFB_w)}{PVFY_w} \text{ OR } NC_w = NC_w(\text{old ben}) \times 1.1$$

$$PVFB_w = AL_w \times 1.1$$
$$= 59,420 \times 1.1$$
$$= 65,362$$

$$NC_w = \frac{(PVFB_w)}{PVFY_w}$$
$$= \frac{65,362}{14.59}$$
$$= 4,480$$

$$\text{Amortization of UAL} = \frac{14,088}{7.8} = 1,806$$

Total Contribution, January 1, 2009 = 1,806 + 4,480
= 6,286

(d)

Company can lower their 2009 contribution by using unit credit method

$$PVFB_t = B(y) \times \ddot{a}_{65} \times (1+i)^{(t-y)}$$
$$B(y) = 110 \times 12 \times (y-w) \times (1 - 0.0025 \times 12 \times (65-y))$$

AL = PVFB × service to date / total service
= 139,412 × 13 / 30
= 60, 412

UAL = AL – assets
= 60,412 – 75,554
= - 15,142
= Surplus no amortization payments

Contribution = NC = AL / past service
$$= \frac{60,412}{13}$$
$$= 4,647$$

3.

Learning Objectives:

Determine the normal cost as at January 1, 2008 and 2009 using the Attained Age Normal method. Using the UAL and the fund assets as at January 1, 2009, determine the accrued liability at this date.

Solution:

(a)

Normal cost at January 1, 2008

$$\mathrm{NC}_t = \frac{\left(\sum \mathrm{PVFB}_t - \mathrm{AL}_t\right)}{\sum \mathrm{PVFS}_t \times \sum S_t}$$

$$\begin{aligned}\mathrm{AL}_t &= F_t + \mathrm{UAL}_t \\ &= 60,000 + 10,000 \\ &= \$70,000\end{aligned}$$

$$\sum \mathrm{PVFB} = \begin{bmatrix} 0.01\times 50,000 \times 1.04^{(60-50)} \times 20 \times \ddot{a}_{60}^{(12)} \times v^{10} + \\ 0.01\times 50,000 \times 1.04^{(60-40)} \times 30 \times \ddot{a}_{60}^{(12)} \times v^{20} \end{bmatrix}$$

$$\begin{aligned}\sum \mathrm{PFVB} &= \begin{bmatrix} 0.01\times 50,000 \times 1.4802 \times 20 \times 12.0 \times 0.5584 + \\ 0.01\times 50,000 \times 2.1911 \times 30 \times 12.0 \times 0.3118 \end{bmatrix} \\ &= \$222,164\end{aligned}$$

$$\begin{aligned}\sum \mathrm{PVFS} &= 50,000 \times 1.04 \times \ddot{a}_{\overline{10}|j} + 50,000 \times 1.04 \times \ddot{a}_{\overline{20}|j}, \qquad \text{where } j = \frac{1.06}{1.04} - 1 \\ &= \$1,351,093\end{aligned}$$

$$\begin{aligned}\mathrm{NC} &= \frac{\left(222,164 - 70,000\right)}{1,351,093} \times \left(100,000 \times 1.04\right) \\ &= \$11,713\end{aligned}$$

(b)

Accrued Liability and Normal Cost at January 1, 2009:

$$\mathrm{UAL}_{t+1} = \left(\mathrm{UAL}_t + \mathrm{NC}_t\right) \times \left(1 + i\right) - \mathrm{C} - \mathrm{I}_c$$

$$\begin{aligned}\mathrm{UAL}_{2009} &= \left(10,000 + 11,713\right) \times 1.06 \\ &= \$23,016\end{aligned}$$

$$\begin{aligned}F_{t+1} &= 60,000 \times 1.00 + 0 \\ &= \$60,000\end{aligned}$$

$$\begin{aligned}\mathrm{AL} &= 60,000 + 23,016 \\ &= \$83,016\end{aligned}$$

3. **(b) continued**

$$\sum \text{PVFB} = \begin{bmatrix} 0.01\times 50,000\times 1.04^{(60-51)}\times 20\times \ddot{a}_{60}^{(12)}\times v^{9} + \\ 0.01\times 50,000\times 1.04^{(60-41)}\times 30\times \ddot{a}_{60}^{(12)}\times v^{19} \end{bmatrix}$$
$$= \$226,436$$

$$\sum \text{PVFS} = 50,000\times 1.04\times \ddot{a}_{\overline{9}|j} + 50,000\times 1.04\times \ddot{a}_{\overline{19}|j}$$
$$= \$1,271,075$$

$$\text{NC} = \frac{\left(226,436-83,016\right)}{1,271,075}\times\left(100,000\times 1.04\right)$$
$$= \$11,735$$

4.

Learning Objectives:

Using the Aggregate Method and the information provided, determine the 2008 normal cost given the change in salary scale assumption.

Solution:

Aggregate Cost Method

$$TNC \times \ddot{a} = \sum pvB - F$$

$$B_x = 2\% \times S_{r-1} \times (x - e)$$

Present value of future benefit for all participants at January 1, 2008:

Employee A =

$$B_{60} = \left(2\% \times \left(25{,}000 \times (1.02)^{(59-30)}\right)\right) \times (60 - 28)$$

$$B_{60} = \$28{,}413$$

$$pvB = B_{60} \times \ddot{a}_{60}^{(12)} \times v^{(60-30)}$$

$$pvB = 28{,}413 \times 12.2 \times (1.06)^{-(60-30)}$$

$$pvB = 60{,}353$$

Employee B =

$$B_{60} = \left(2\% \times \left(40{,}000 \times (1.02)^{(59-50)}\right)\right) \times (60 - 35)$$

$$B_{60} = \$23{,}902$$

$$pvB = B_{60} \times \ddot{a}_{60}^{(12)} \times v^{(60-50)}$$

$$pvB = 23{,}902 \times 12.2 \times (1.06)^{-(60-50)}$$

$$pvB = 162{,}830$$

$$\sum pvB = 60{,}353 + 162{,}830$$

$$\sum pvB = \$223{,}183$$

Average Annuity

$$\ddot{a} = \sum pv \frac{(\text{future salaries})}{\text{total salaries}}$$

$$\ddot{a} = \frac{792{,}000}{65{,}000}$$

$$\ddot{a} = 12.2$$

4. continued

Find F

$$TNC \times \ddot{a} = \sum pvB - F$$
$$15,000 \times 12.2 = 223,183 - F$$
F = 40,183 at January 1, 2008

Salary Scale increases to 3%

Present value of future benefit for all participants at January 1, 2008 with 3% salary scale

Employee A =

$$B_{60} = \left(2\% \times \left(25,000 \times (1.03)^{(59-30)}\right)\right) \times (60-28)$$
$$B_{60} = \$37,705$$
$$pvB = B_{60} \times \ddot{a}_{60}^{(12)} \times v^{(60-30)}$$
$$pvB = 37,705 \times 12.2 \times (1.06)^{-(60-30)}$$
$$pvB = 80,090$$

Employee B =

$$B_{60} = \left(2\% \times \left(40,000 \times (1.03)^{(59-50)}\right)\right) \times (60-35)$$
$$B_{60} = \$26,095$$
$$pvB = B_{60} \times a_{60}^{(12)} \times v^{(60-50)}$$
$$pvB = 26,095 \times 12.2 \times (1.06)^{-(60-50)}$$
$$pvB = 177,770$$

$$\sum pvB = 80,090 + 177,770$$
$$\sum pvB = \$257,860$$

4. continued

Average Annuity

$$\ddot{a} = \sum pv \frac{(\text{future salaries})}{\text{total salaries}}$$

$$\ddot{a} = \frac{863,000}{65,000}$$

$$\ddot{a} = 13.3$$

$$TNC \times \ddot{a} = \sum pvB - F$$

$$TNC = \frac{(257,860 - 40,183)}{13.3}$$

$$TNC = 16,367$$

5.

Learning Objectives:

Determine the pension under the normal form of pension and then determine the pension under each optional form of pension. To receive full marks, a candidate must include the actuarial equivalence formula and a description of the benefit payable upon death of the member and for the level income option the amount payable both before and after age 65.

Solution:

$$B(Y) = 2\% \times S_{y-1} \times 30 + 1.0\% \times S_{y-1} \times 1$$

$$B(60) = 2\% \times 110,000 \times 30 + 1.0\% \times 110,000 \times 1$$

$$B(60) = 67,100$$

Annual Pension Payable at age 60

Life Only Form

$$B_{20}(58) = \left(1 - [60 - 58] \times 0.05\right) \times 67,100$$

$$B(58) = 60,390 = \text{Annual Pension Payable at age 58}$$

Life Only Form - Upon death of the member, pension will cease

Life Guaranteed for Ten Years Form

$$B_{G10}(58) \times \ddot{a}^{(12)}_{58(10)} = B_{40}(58) \times \ddot{a}^{(12)}_{58}$$

$$\ddot{a}^{(12)}_{58(10)} = \ddot{a}^{(12)}_{\overline{10}|} + {}_{10}P_{58}\ \ddot{a}^{(12)}_{68}(1+i)^{-10}$$

$$= 7.4 + 0.92 \times 9.6 \times 1.065^{-10}$$

$$= 12.1$$

$$B(58) = \frac{(60,390 \times 11.9)}{12.1}$$

$$B(58) = 59,392 = \text{Annual Pension Payable at age 58}$$

Life Guaranteed for 10 years -

Upon death of the member in the first ten years after retirement, pension will continue to the spouse / beneficiary for the balance of the guarantee period

Upon death of the member after ten years after retirement, pension will cease

5. continued

Joint and Survivor 60% Form

$$B_{J\&S60}(58)\times \ddot{a}_{J\&S60}^{(12)} = B_{60}(58)\times \ddot{a}_{58}^{(12)}$$

$$\ddot{a}_{J\&S60}^{(12)} = \ddot{a}_{x}^{(12)} + 0.6\left(\ddot{a}_{y}^{(12)} - \ddot{a}_{x:y}^{(12)}\right)$$

$$= 11.9 + 0.6\times(12.5 - 10.8)$$

$$= 12.9$$

x = age of member

y = age of spouse

$$B_{J\&S60}(58) = \frac{(60,390\times 11.9)}{12.9}$$

$B_{J\&S60}(58) = 55,709$ = Annual Pension Payable at age 58

Joint and Survivor 60% - Upon death of the member, the spouse, if alive, will receive an annual pension of 33, 425

Level Income Form

$$B_{60}(58)\times \ddot{a}_{58}^{(12)} = P\times \ddot{a}_{58}^{(12)} + \text{Gov Ben}\times 12\times \ddot{a}_{\overline{58:7}|}^{(12)} - \text{Solve for } P.$$

$$\ddot{a}_{\overline{58:7}|} = \ddot{a}_{58} - {}_{7}P_{58}\ \ddot{a}_{65}(1+i)^{-7}$$

$$= 11.9 - 0.95\times 10.4\times 1.065^{-7}$$

$$= 5.5$$

$$P = \frac{(60,390\times 11.9 - 1,300\times 12\times 5.5)}{11.9}$$

$P = 53{,}180$

Pension payable before age 65 = 68,780

Pension payable after age 65 = 53,180

Upon death of the member, pension will cease