May 2002 Course 6 Solutions

Question #1

a)   price-weighted index: can be replicated by buying one of every stock in index
- equal to the sum of the prices of all stocks in the index, divided by the number of stocks.
- the divisor is adjusted for stock splits and stock dividends >10% so that the index is unaffected by the event.
- gives more weight to more expensive stocks

market-value weighted index: can be replicated by buying amounts of each stock in proportion to the outstanding market value of each stock.
- stocks with large market value get more weight
- no adjustments need to be made for splits/dividends

equally-weighted index: can be replicated by buying equal dollar amounts of each stock.
- need to constantly re-balance portfolio as stock prices change to maintain equal weight
- can just take average of individual stock returns.

b)

	t_0	t_1	% change
Price-weighted index	(10+9+15)/3=11.3333	(15+10+15)/3 = 13.3333	$\left(\dfrac{13.3333}{11.3333}-1\right)\times100\% = +17.647\%$
Market value weighted	(10x100+9x125+15x200) = 5125	(15x100+10x125 +15x200) = 5750	$\left(\dfrac{5750}{5125}-1\right)\times100\% = +12.195\%$
Equally-weighted			$\left[\left(\dfrac{15}{10}+\dfrac{10}{9}+\dfrac{15}{15}\right)\dfrac{1}{3}-1\right]\times100\% = +20.370\%$

c)   PWI = MVWI

$$\frac{\left(15+10+X\right)}{3\left(11.3333\right)} = \frac{\left(15x100+10x125+Xx200\right)}{5125}$$

5125(25+X)= 3(11.3333)(2750+200X)
128,125+5125X=93500+6800X
34,625 = 1675X
X=20.67

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∴ if the end of period price for stock III was $20.67, the percentage increase for the price-weighted index and the market value-weighted index would be equal.

Question #2

Assume $U = E[r] - .005A\boldsymbol{s}^2$ The utility function.
Calculate the optimal risky portfolio, P. Obtain with the following weights.

$$W_I = \frac{\left(E[r_I] - r_f\right)\boldsymbol{s}_{II}^2 - \left(E[r_{II}] - r_f\right)Cov(r_I, r_{II})}{\left(E[r_I] - r_f\right)\boldsymbol{s}_{II}^2 + \left(E[r_{II}] - r_f\right)\boldsymbol{s}_I^2 - \left(E[r_I] - r_f + E[r_{II}] - r_f\right)Cov(r_I, r_{II})}$$

$r_f = .05$

$E[r_I] = .09 \qquad E[r_{II}] = .12 \qquad \boldsymbol{s}_I^2 = .13^2 \qquad \boldsymbol{s}_{II}^2 = .20^2$

$Cov(r_I, r_{II}) = .0093 = 93\%$

$$W_I = \frac{(.09 - .05).2^2 - (.12 - .05)(.0093)}{(.09 - .05).2^2 + (.12 - .05)(.13)^2 - (.09 - .05 + .12 - .05)(.0093)}$$

$$= \frac{.0016 - .000651}{.0016 + .001183 - .001023} = \frac{.000949}{.00176}$$

$$= .5392$$

$W_{II} = 1 - W_I = .4608$

$E[r_p] = W_I E(r_I) + W_{II} E(r_{II}) = .103824$

$\boldsymbol{s}_p^2 = W_I^2 \boldsymbol{s}_i^2 + W_{II}^2 \boldsymbol{s}_{II}^2 + 2Cov(r_I, r_{II})W_I W_{II}$

$\qquad = .0049134 + .008493 + .00462$

$\qquad = .0180278$

$\boldsymbol{s}_p = .13427$

$U = E(r_c) - .005A\boldsymbol{s}_c^2 \qquad\qquad\qquad E(r_c) = y\left(E(r_p) - r_f\right) + r_f$

$\dfrac{dU}{dy} = 0 \Rightarrow y^* = \dfrac{E[r_p] - r_f}{.01A\boldsymbol{s}_p^2} \qquad$ y* is proportion of
portfolio invested in
optimal risky portfolio P.

i)　　　for investor with A = 4

$$y^* = \frac{10.3824 - 5}{.01(4)(13.427)^2} = \frac{5.3824}{7.21137} = 74.64\%$$

optimal portfolio　　74.64% in P　　40.25% in stock I
　　　　　　　　　　　　　　　　　34.39% in stock II
　　　　　　　　　　25.36% in r_f

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$$E[r_c] = .05 + .7464(.103824 - .05) = 9.017\%$$
$$s_c = .7464(.13427) = 10.02\%$$

ii)   for investor with A = 2

$$y^* = \frac{10.3824 - 5}{.01(2)(13.427)^2} = 149.28\%$$

   Assuming that this investor can borrow at the risk free rate, the optimal portfolio is 149.28% in P and borrowing 49.28% @ $r_f = .05$

$$E[r_c] = .05 + 1.4928(.103824 - .05) = 13.035\%$$
$$s_c = 1.4928(.13427)$$
$$= .2004$$

   invest   1.4928(.5392) = 80.49%   in stock I
             1.4928(.4608) = 68.79%   in stock II
   borrow   49.28 @ $r_f = 5\%$

   If investor cannot borrow at risk-free rate, then he will invest such that his utility function is maximized with the following weights.

$$W_I = \frac{E[r_I] - E[r_{II}] + .01A\left(s_{II}^2 - Cov(r_I, r_{II})\right)}{.01A\left(s_I^2 + s_{II}^2 - 2Cov(r_I, r_{II})\right)}$$
$$W_{II} = 1 - W_I$$
$$= \frac{9 - 12 + .01(2)\left(20^2 - .93\right)}{.01(2)\left(20^2 + 13^2 - .93(2)\right)}$$
$$= \frac{4.9814}{11.3428} = .4392$$
$$W_{II} = 56.08\%$$

Invest 43.92% in stock I
       56.08% in stock II

$$E(r_p) = 10.6824 \qquad\qquad U = 10.274$$
$$s_p^2 = .0204 \Rightarrow s_p = 14.29\%$$

Question #3

(i)   For the continuous binomial option pricing model

$$u = e^{s\sqrt{h}} \quad \text{where} \quad h = \text{time in years}$$
$$= 3/12 \ = \tfrac{1}{4}$$
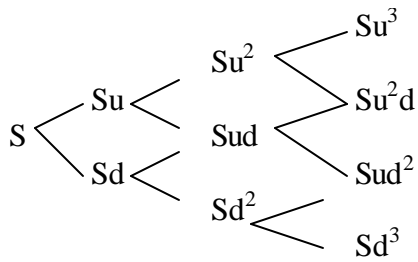
$$u = e^{0.3\sqrt{0.25}} = 1.16183$$
$$d = u^{-1} = 0.86071$$
$$p = \text{probability of an up-move}$$
$$= \frac{e^{rh} - d}{u - d}$$
$$= \frac{e^{0.05(0.25)} - 0.86071}{1.16183 - 0.86071}$$
$$= 0.50435$$
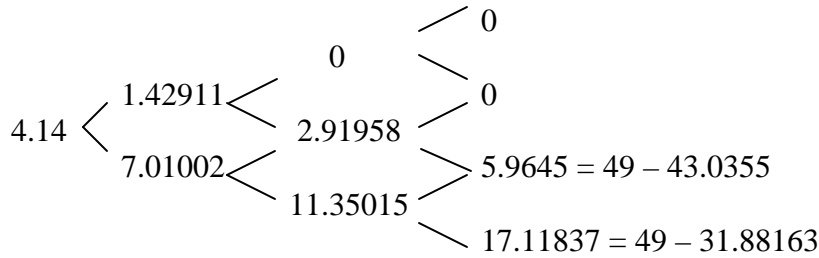
$$q = 1 - p = 1 - 0.50435 = 0.49565$$

Calculate the stock price tree
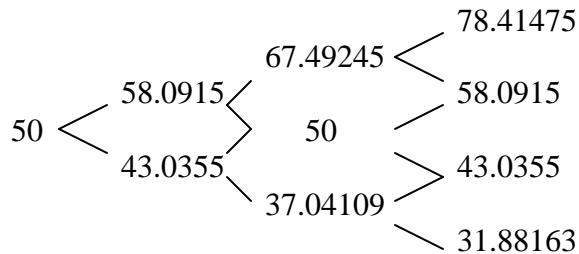
European put payoffs at time 3 and values- only exercised at maturity

Payoff= Max (Strike – stock, 0)
      = Max (49 – stock, 0)

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European put


```
                                                    0
                                    0
                    1.42911
4.14                                                0
                                    2.91958
             7.01002                                5.9645 = 49 – 43.0355
                                    11.35015
                                                    17.11837 = 49 – 31.88163
```

Value of any node earlier than time 3

$$V = [V_H P + V_L (l - p)]e^{-rh}$$ Value of the European put $= 4.14$

(ii)   American put- can be optimal to exercise at any time if intrinsic value is greater than value, then exercise (substitute intrinsic value for value)


```
                                                            0
                                            0
                            1.42911
            4.29                                            0
                                            2.91958
                    7.30800                                 5.9645
                            Max (49-37.04109=
                            11.95891,11.35015)
                            =11.95891                        17.11837
```
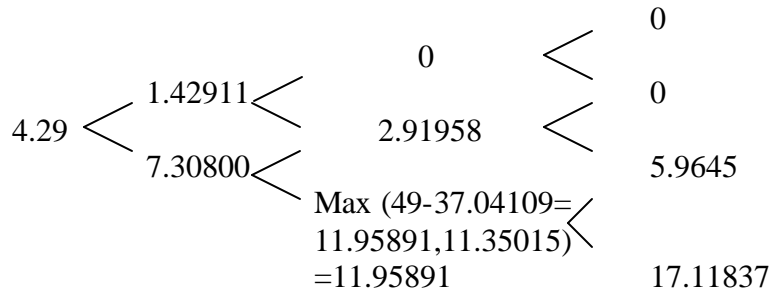
Value of the American put $= 4.29$

(iii)   European call

Payoff = Max (stock - strike, 0)   only exercised at maturity
       = Max (stock – 49, 0)


```
                            29.41475 = 78.41475 - 49
              19.10127
    11.73064                9.0915
6.95            4.52834
    2.25550                0
        0
                            0
                    0
```
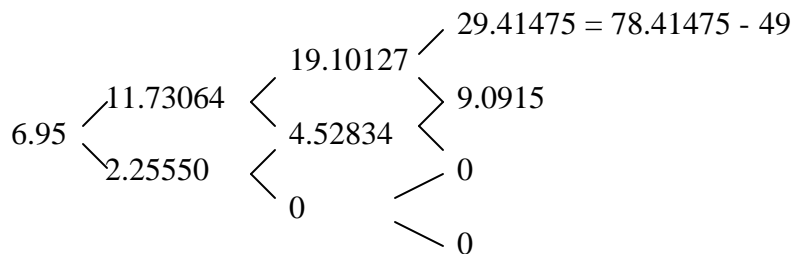
Value of the European call $= 6.95$

Value of any node earlier than time 3

$$V = [V_H P + V_L (1 - p)]e^{-rh}$$

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Question #4

Loyalty
- The trustee should act to the best interest of the participants of the fund, avoid any contradicting interest.
- Should not manage fund to the interest of the plan sponsors.

Care and Diversify
- The trustee should manage the fund with care, taking into account economic outlook and participants interest.
- Should diversify the fund accordingly to reduce the risk of the pensioners unless prove diversification is not prudent.

Impartiality
- The trustee should not perform anything to the interest of a particular group in the pension fund.
- Should treat every participant the same and do anything to the interest to all participants equally.

Delegate
- Should delegate fund management or investment according to any suitable parties, to help manage the fund.
- But should bear in mind the responsibility and liability even delegate to the other parties.

Follow Statutory Constraints
- Should follow all regulation constraints.
- Any accounting issues, valuation method
- To comply to any tax rules and investment guidelines

Make the Property Productive
- Should manage the fund such that the investments are productive, or should maintain comfortable surplus for the pensioners.
- Should set-up objective and try to carry it out.

Re Co-Trustee
- Make sure all co-trustees have the same objective and same management mechanisms.
- All co-trustees follow the guidelines and manage or invest the fund in the same manner.

Act According to the Trust Agreement
- Should make sure is acting according to the terms written in the trust
  agreement.
- Should not carry out anything that is in contradiction to the trust
  agreement.
- Should manage trust in a prudent manner otherwise not stated in the trust
  agreement.

Purpose of funding
- finding purpose affect which method to use
- comply with purpose will produce better results

Plan sponsor's financial objective
- financial objectives should be taken into account, stable
  contribution or flexible contribution?
- Stabilize business profit or retain comfortable surplus?

Plan sponsor's business or industry
- each industry has different needs, so need to consider that and
  use the appropriate method

Type of benefits
- is the benefit (with employer contribution) DC or DB will
  affect the funding method used.
- % of salary or level for all?

Method of accrual of benefit
- Does past services take into account or is any benefits related
- All these affect funding method used

Regulatory constraints
- Should comply with any statutory rules
- Accounting or reporting or tax rules should be followed.

Funding method that require lowest cost may be more flexible to plan sponsors

Should avoid method that generates negative NC or negative UAL

Should avoid fluctuations to funded liability ratio.

May want to tap into lump sum benefit since those are usually discounted in a higher rate.

Question #5

a.)   - Solvency test only
        - Based on formula that ignores many subtle but important issues
        - Cannot and does not reflect all strengths and weaknesses of company, such as:
                            - Strong underwriting
                            - Good contracts
                            - Customer loyalty
        - Does not differentiate within a given class of investment vehicles.
        - Companies can manipulate the RBC formula

b.) $C1 = 0.003(2,000,000) + 0.04(5,000,000) + 0.12(3,000,000)$
              $= 566,000$

        $C2 = 150,000,000 \times 0.001 = 150,000$

        $C3 = 4,000,000 \times 0.01 = 40,000$

        $C4 = 500,000 \times 0.02 = 10,000$

        $$RBC = 10,000 + \sqrt{150,000^2 + (566,000 + 40,000)^2}$$
              $= 634,288$

        Before reinsurance, RBC ratio = Available Capital / RBC

        $= 800,000 / 634,288$

        $= 126\%$

        Changes to RBC after reinsurance:

        $\Delta C1 = -8,000,000(6.0\% - 0.5\%)(50\%) = -220,000$
        $C1 = 566,000 - 220,000 = 346,000$

        $\Delta (C2) = -100,000,000(0.1\%)(50\%) = -50,000$
        $C2 = 150,000 - 50,000 = 100,000$

        $\Delta (C3) = -3,000,000(1.0\%)(50\%) = -15,000$
        $C3 = 40,000 - 15,000 = 25,000$

        $\Delta (C4) = 0$
        $C4 = 10,000 - 0 = 10,000$

RBC = 10,000 + SQRT(100,000^2 + (346,000 + 25,000)^2))
      = 394,291

Delta(Available capital) = 4,000,000(50%)(3%) = 120,000

Available capital = 800,000 + 120,000 = 920,000

After reinsurance, RBC Ratio = 920,000 / 394,241
= 233%

c.)   Upgrade all assets to minimum BB
C1 = 2,000,000(0.3%) + 8,000,000(4.0%)
    = 326,000

RBC = 10,000 + SQRT(150,000^2 + (326,000 + 40,000)^2))
      = 405,545

RBC Ratio = Available Capital / RBC

= 800,000 / 405,545

= 197%

d.)   I recommend the reinsurance option because it raises the RBC ratio more than upgrading assets does.

Question #6

- Markowitz model requires $(n^2-n)/2$ estimates of covariance and $2n$ mean /variances assumptions.
- Allows one analyst to study one sector and another to study another each relative to index.

a).   single-index model:

$$R_i = \partial_i + b_m + R_m + e_i$$

R_i refers to excess return of security i

R_m refers to excess return of market

b_i will be the same with ß of CAPM $\left(b = \dfrac{\mathrm{cov}(r_i - r_m)}{s_m^2} \right)$

e_i will be the firm-specific factor

Advantages:
1. Using single index model only requires estimation of different e_i and cov(r_i, r_M) for each security i, then estimate R_m, s_m^2 for the market portfolio. It will reduce calculation greater than CAPM.
2. Single-index model assumes specific security i's excess return only related to excess market return and form-specific factor e_i.

b). The values of Markowitz efficient frontier.

	Z(r)	s_r
Stock I	6% + 12% = 18%	40%
Stock II	6% + 5% = 11%	25%
Stock III	6% + 12% = 18%	4%
Market	6% + 8% = 14%	25%

$$\text{cov}(r_I, r_{II}) = \boldsymbol{b}_1 \boldsymbol{b}_2 \cdot \boldsymbol{s}_m^2 = 1.25 \times 0.75 \times 20^2 = 375$$
$$\text{cov}(r_I, r_{III}) = \boldsymbol{b}_1 \boldsymbol{b}_3 \cdot \boldsymbol{s}_m^2 = 1.25 \times 1.75 \times 20^2 = 875$$
$$\text{cov}(r_{II}, r_{III}) = \boldsymbol{b}_2 \boldsymbol{b}_3 \cdot \boldsymbol{s}_m^2 = 0.75 \times 1.75 \times 20^2 = 525$$

∴ covariance – variance matrix is:

$$\begin{vmatrix} 1,600 & 375 & 875 \\ 375 & 625 & 525 \\ 875 & 525 & 1,600 \end{vmatrix}$$

c).

$$Z_p = \frac{1}{3} \times 18\% + \frac{1}{3} \times 11\% + \frac{1}{3} \times 18\% = 15.67\%$$

$$\boldsymbol{s}_p^2 = \left(\frac{1}{3}\right)^2 \times 1,600 + \left(\frac{1}{3}\right)^2 \times 625 + \left(\frac{1}{3}\right)^2 \times 1,600 + 2 \times \frac{1}{3} \times \frac{1}{3} \times 375 + 2 \times \frac{1}{3} \times \frac{1}{3} \times 525 + 2 \times \frac{1}{3} \times \frac{1}{3} \times 875$$

$$= 819$$

$$\Rightarrow \boldsymbol{s}_p = 28.62\%$$

Question #7

According to the CIA guidance note the risk management practice issues are:
1. Rebalancing Practices
2. Liquidity Management Practices
3. Asset Options
4. Embedded liability options
5. Use of derivatives
6. Equity Investments
7. Long Duration Liabilities
8. Tax/Legislative Implications

I will now elaborate on each of these.

Rebalancing Practices
Must rebalance due to time, drift, and non-parallel interest rate shifts
Low risk factors for rebalancing
- Position reported frequently and process to rebalance in place
- Computer optimizing software used to rebalance
- Assets are liquid or can use derivatives
- History of quickly eliminating unwanted exposures

Liquidity Management Practices Low Risk Factors
1. MIS in place to understand different cash outflow scenarios of inforce
2. Don't rely on writing new business for liquidity
3. Sufficient liquid assets to meet adverse cash outflow demands
4. Disciplined investing in illiquid assets
5. Derivative expertise if use derivatives

Asset Options Low Risk Factors
1. No options can be exercised against the company
2. Potential options are accurately modeled

Embedded Liability Options Low Risk Factors
1. No discretionary withdrawls are allowed
2. Discretionary withdrawls are market value adjusted if they are permitted
3. Embedded liability options are accurately modeled
4. Policyholders can't annuitize at book value when they want to
5. Policyholders can't selectively use rate quotes
6. Ensure that surrender charges are reasonably high to prevent early surrenders and lessen exposure to company.

<u>Use of Derivatives Low Risk Factors</u>
Derivatives are good if you don't have liquid assets but they are risky because they are complex and highly leveraged.

1. Limits on purchasing derivatives without senior management approval
2. Significant expertise in company
3. Derivatives are accurately modeled in ALM process

<u>Equity Investments</u>
Risky because
       a. assets don't have guarantees; liabilities do
       b. expected return and volatility are difficult to measure
       c. they are often uncorrelated with interest rate volatility

Low risk factors for equity investments
       a. no equities are used
       b. if equities are used, they are segmented and used to back very long duration liabilities
       c. use very little junk bonds

<u>Long Duration Liabilities</u>
Often it is hard or impossible to find assets that have maturities greater than 30 years

Options to manage risk
    a. discount liability ash flows past 30 years to duration 30 and manage as a time 30 cash flow
    b. manage in total rate of return segment
    c. use derivatives to leverage asset sensitivity to match liability sensitivity

Low risk factors for long duration liabilities
    a. No liabilities issued that have maturities greater than 30 years
    b. Procedures in place to handle unique risk

<u>Tax/Legislative Liabilities</u>
Do not plan to manage interest rate using techniques that are bad for taxes or not permitted by regulators.

<u>Analysis and Recommendations</u>
Target market is professionals under 30, which means they will have long duration liabilities (ie, 73 yrs). I recommend that they reconsider this strategy because it ill be tough to find assets to match the liabilities.

Policy surrenders should be not permitted or market value adjusted. If they pay the Max [book value, market value], they may put themselves at more risk.

The fact that liabilities are supported by fixed income securities is good because they are less risky. If 50% is moved to equities considerable interest rate risk may arise. I would not recommend such a big shift to equities.

Right now, they are measuring interest rate risk exposure annually. Interest rate risk exposure should be monitored on a day to day basis so that actions can be taken to correct bad exposures quickly. If the position is only reported once a year, you do not have a good handle on your exposure to interest rate risk. As such, I recommend they implement a process that measures exposure on a day to day basis.

Recommend liabilities being backed by assets should only represent statutory margins/required surplus.
Limitations of duration mismatch:
    -do not measure exposure to non-parallel yield curve shift
    -do not always capture the effects of options
A wide range of scenarios covering changes in level, terms structure, sector spread and default rates should be chosen when doing scenario testing.

Question #8

I. Market or Interest Rate risk
- biggest risk faced by an investor
- magnitude of price response to changes to interest rates
- magnitude depends on: maturity, coupons, embedded options

Duration is a summary measure of price sensitivity to yields

II. Reinvestment Risk
- risk that interest rates will decline and reinvestment of coupons will be at lower than current rates.
- risk larger for longer holding periods
- opposite risk to market risk with respect to yields
- when reinvestment and market risk offset each other over a time horizon it is referred to as immurization.

III. Call or Timing Risk
- magnitude depends upon the parameters of the call and current market conditions

IV. Credit or Default Risk
- risk that promised payments are not made.
- rating agencies include: Moody's and Standard & Poor's
- institutional lenders also do credit ratings
- investors will be concerned with any changes in credit ratings because this will impact yield spreads over treasuries.

V. Yield Curve or Maturity Risk
- risk that the shape of the yield curve changes in a manner different than anticipated
- hedges will no longer hold

VI. Inflation or Purchasing Power Risk
- bonds with fixed cash flows will be at risk due to unanticipated inflation
- the value of cash flows in real terms is a function of the rate of inflation

VII. Marketability or Liquidity Risk
- represents the ease at which an issue can be sold at or near its true value
- a common measure is the bid-ask spread
- not a major concern of the intent to hold to maturity
- spreads will widen during recession and narrow during expansion

VIII. Exchange Rate or Currency Risk
- risk when non-dollar denominational issues are held
- 2 risks: currency fluctuation & changing foreign interest rates

IX. Volatility Risk
- value of interest rate options increase when interest rate volatility increases

X. Legal or Political Risk
- expropriation
- change in tax status from tax free to taxable
- decision by a regulatory body that an instrument is unsuitable
- tax free vs. tax spread changing because of different tax rates

XI. Event Risk
- the ability of a borrower to repair may be jeopardized due to:
- natural or industrial accident
- corporate restructuring or takeover
- may be firm specific, or related to event risk of other firms or may be systematic (CAPM)

XII. Sector Risk
- yield spreads across different sectors respond differently to different events:
- premium vs. discount
- corporate vs. mortgage-booked, etc.

XIII. Basis Risk
- all risks other than market risk

Question #9

In order to cash flow match, each liability cash flow must be matched with sufficient assets. We first need to match the longest liabilities and work back.

5th year: Assets > liabilities, 5 year assets must be sold

$$\text{sale of 5 year bond} = \frac{A_5 - L_5}{1 + couponrate} = \frac{2200 - 1980}{1 + 10\%} = 200 \quad \therefore \text{ sell \$200 of par amount of}$$

5 year bond

Asset before sale	194	254	41	200	2200
Assets sold	-20	-20	-20	-20	-20
Adjusted Assets	174	234	21	180	1980

4th year: Assets = liabilities, the cash flows are matched

3rd year: Liabilities > Assets, 3 year assets must be purchased

$$\text{purchase of 3 year bond} = \frac{L_3 - A_3}{1 + couponrate} = \frac{445 - 21}{1 + 6\%} = 400 \quad \therefore \text{ purchase \$400 of amount}$$

of 3 year bond

Assets before purchase	174	234	21	180	1980
Asset purchased	+24	+24	+424	+0	+0
Adjusted assets	198	258	445	180	1980

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2nd year: Assets > Liabilities, 2 year assets must be sold

sale of 2 year bond $= \dfrac{A_2 - L_2}{1 + couponrate} = \dfrac{258 - 69}{1 + 5\%} = 180$ $\therefore$ sell \$180 of par amount of

2 year bond

Assets before sale	198	258	445	180	1980
Assets sold	-9	-189	0	0	0
Adjusted assets	189	69	445	180	1980

1st year: Liabilities > Assets, 1 year asset must be purchased but there are no 1 year assets. Can purchase \$21 (210-189) of 90-day T-bills or mix T-bills and 2 year bond for better match.

Question #10

For this "collared floater", the corporations payments will be:

If $L \geq 12\%$, 12.5%
If $7\% \leq L \leq 12\%$, $L + .5\%$
If $L \leq 7\%$, 7.5%

We want to pay instead a fixed rate of X%. Thus, we need:

If $L \geq 12\%$, $12.5\% - X\%$
If $7\% \leq L \leq 12\%$, $L + .5\% - X\%$
If $L \leq 7\%$, $7.5\% - X\%$
Let's try the middle one first:

This is a swap. We want to receive L +.5% and pay X%. This will be an off-market swap. Since we want to receive L +.5%, we need to pay for the extra .5% for 5 years.
I'll assume we'll pay for it by paying an extra .5% on the swap as well, so that we'll pay 8.75% + .5% = 9.25%. This then must be our X.

So now, if for any L, we receive L + .5% - 9.25%.
So if L≥12%, we want to get 17.5% - 9.25%, so we don't need the extra L – 12%. So we'll sell a cap at 12%. Similarly, we need to buy a floor at 7%. Then our total cash flow should equal what we desired:

If L≥12%:

     receive   L + .5% -9.25%   on swap

     pay   <u>L – 12%</u>   on cap

                       .5% - 9.25% + 12% = 12.5% - 9.25%

       receive 3.25%, which is 12.5% - 9.25%, as hoped.

If 7% ≤ L ≤ 12%, receive L + .5% - 9.25% on swap, as hoped
       neither cap nor floor pays

If L ≤ 7%,

     receive   L + .5% - 9.25%   on swap

     receive   <u>7% - L</u>   on floor

                      5% - 9.25% + 7% = 7.5% - 9.25%

      = -1.75%, as hoped

What will this cost up-front?

- The swap is free, since both fixed and variable rates were adjusted.
- The cap will bring us income of .65%(10) = .065 million.
- The floor will cost us .9%(10) = .09 million, for a total upfront cost of .09 - .065 = .025 million.

Then, we'll pay a fixed-rate of 9.25% each year, on 10 million, is .925 million a year. Actually, since this is semi-annual, we'll pay ½(.925) = .4625 million every 6 months.

Solve for i in
$$.025 + \frac{.4625}{\left(1+\frac{i}{2}\right)} + \frac{.4625}{\left(1+\frac{i}{2}\right)^2} + ... + \frac{.4625}{\left(1+\frac{i}{2}\right)^9} + \frac{10.4625}{\left(1+\frac{i}{2}\right)^{10}} = 10$$

to calculate interest cost.

Credit risk is an issue for the swap we entered into and the floor that we bought. Thus, if Libor rises above 8.75%, we have to worry about the swap paying. If Libor drops below 7%, we have to worry about the floor paying.

Question #11

(a)   Define a floating-rate security and describe its features

- a security whose coupon rate resets at predetermined dates
- coupon rate is based on a defined reference rate
- formula: coupon rate = reference rate +/- quoted margin
- quoted margin is the adjustment the issuer agrees to make to the reference rate
- Common reference rates include: Libor, T-Bills, or Prime rate
- Others include COFI
- There may be limits on the coupon rate including a cap or a floor, or both, which is a collar

(b)   Describe the yield spread measures used to evaluate floating-rate securities

There are 4 margins commonly used:

1. Spread for Life also known as Simple Margin
- Accounts for the amortization of discount/premium

2. Adjusted Simple Margin also called Effective Margin
- An adjustment to spread for life
- Adjustment accounts for the one-time cost of carry when a floater is purchased with borrowed funds

3. Adjusted Total Margin
- Adds an additional adjustment to adjusted simple margin
- This adjustment is the interest earned on the difference between the floater's par value and the carry-adjusted price

4. Discount Margin
- Method that employs discounted cash flows
- Procedure for calculating:
- Project the floater's cash flows assuming the reference rate does not change
- Select a margin
- Discount the cash flows from the first step using the current reference rate and the selected margin
- Compare the above present value to the security's price. If equal, the discount margin is the selected margin; if not equal, select a different margin and repeat until the present value equals the price

**(c)   Describe the factors affecting the price of floating-rate securities**

There are 3 main factors:

1. The time until the next coupon reset date
- The longer the time until the next reset, the more a floater behaves like a fixed-rate security

2. Change in the margin required by the market
- After issue, the margin demanded by the market may increase/decrease and the price of the floater will decrease/increase respectively
- The required margin will depend on:
- Margin available in competitive markets
- Credit quality of the issuer
- Presence of embedded options
- Liquidity of the issue

3. If a cap or floor exists, whether or not it is reached
- If there is a cap, once the coupon rate rises above the cap, then the rate is limited and the floater will trade at a discount
- The floater will trade more and more like a fixed-rate security, the further above the cap the market rate goes.

Question #12

(a) -The reinsurance contract is similar to a put option on the life insurance business.
-The reinsurance would eliminate the loss on the portfolio just like a protective put strategy does to a portfolio of stocks.
-The reinsurance premium is payable at the beginning of the year just like the premium for a one-year option.

(b) Answering the value of the life insurance business is 100, then

bank account

$$100 <^{110}_{110} \qquad 100 <^{12}_{80} \qquad R <^{0}_{20}$$
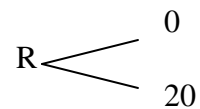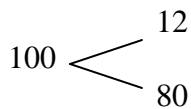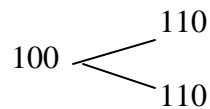
where R is the reinsurance premium

$$\text{Probability of up-move} = q = \frac{1.10 - 0.80}{1.20 - 0.80} = \frac{0.30}{.40} = 0.75$$

$$\text{then } R = \frac{1}{1.10}\{0.75(0) + 0.25(20.00)\} = 4.55$$

Therefore the reinsurance premium is 4.55 or 4.55% of the reinsured business

Replicating portfolio: $D - 100\,\Delta = R$

$$1.10D - 120\,\Delta = 0$$
$$1.10D - 80\,\Delta = 20$$

$1.10D = 120\,\Delta$

$D = \dfrac{120\Delta}{1.10}$ $\qquad \Rightarrow \qquad$ $1.10\left(\dfrac{120\Delta}{1.10}\right) - 80\Delta = 20$
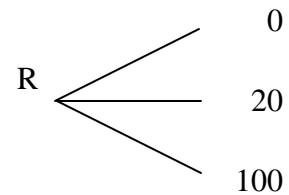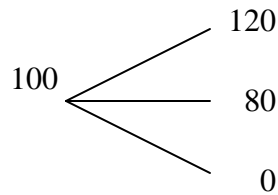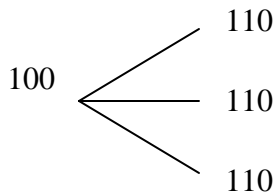
$\qquad\qquad \Rightarrow \qquad 120\,\Delta - 80\,\Delta = 20$

$\qquad\qquad \Rightarrow \qquad 40\,\Delta = 20$

$\qquad\qquad \Rightarrow \qquad \Delta = \tfrac{1}{2} \qquad\qquad \therefore D = \dfrac{120\left(\dfrac{1}{2}\right)}{1.10} = 54.55$

Therefore the replicating portfolio is 54.55 in the deposit-taking business and a short position of 50 in the life insurance business

(c)

$100 = 110y_1 + 110y_2 + 110y_3 \qquad\qquad \Rightarrow \qquad y_1 + y_2 + y_3 = \dfrac{1}{1.1}$

$100 = 120y_1 + 80y_2$

let $y_1 = s$

then $\quad s + y_2 + y_3 = \dfrac{1}{1.1} \qquad\qquad \Rightarrow \qquad y_2 = \dfrac{1}{1.1} - s - y_3$

so $\quad 100 = 120s + 80\left(\dfrac{1}{1.1} - s - y_3\right) = 120s + 72.73 - 80s - 80y_3$

$$100 = 120s + 72.73 - 80s - 80y_3$$

$$80y_3 = 40s - 27.27$$

$$y_3 = 0.5s - 0.340875$$

$$y_2 = \frac{1}{1.1} - s - 0.5s + 0.340875 = 1.25 - 1.5s$$

We need to find y_1, y_2, y_3 in order to find $R = 0y_1 + 20y_2 + 100y_3$

$$y = [s \ 1.25\text{-}1.55 \ 0.5s\text{-}0.340875]$$

s>0	1.25-1.5s>0	1.25-1.5s<1	0.5s-0.340875>0
s<1	1.25>1.5s	1.25<1+1.5s	0.5s>0.340875
	$s < \dfrac{1.25}{1.5} = \dfrac{5}{6}$	0.25<1.5s	s>0.68175
		$s > \dfrac{0.25}{1.5} = \dfrac{1}{6}$	

$$\therefore 0.68175 < s < \frac{5}{6}$$

but $e_1 = y_1 = s_{max} \quad -\dfrac{1}{12} = \dfrac{5}{0} - \dfrac{1}{12} = \dfrac{9}{12} = \dfrac{3}{4}$

$$\Rightarrow y = [0.75 \quad 0.125 \quad 0.034125]$$

$$R = 20(0.125) + 100(0.034125) = 2.5 + 3.4125 = 5.9125$$

The new reinsurance premium is 5.9125 or 5.9125% of the reinsurance business

(d)   If the guaranteed rate is 20% then $q = \dfrac{1.20 - 0.80}{1.20 - 0.80} = 1$

That means the risk-neutral probability of a down-move is zero. But the real probability is greater than 0. Therefore the 2 probability measures are not equivalent and the model would not be arbitrage free.

Using the guaranteed rate of 20%, the reinsurance premium would be 0 but there would be a non-zero probability of a positive pay-off from the reinsurance contract.

Question #13

(a)   Must define the following:
                target objective of the pension
                liability characteristics
                risk tolerance (relate to surplus requirement, risk of business, etc.)
                liquidity
                asset mix
                availability of asset
                regulation constraints and taxation
                asset liability matching
                procedure and authority
                diversification of investment
                how to review (appraise) of investment performance (can compare with
                benchmark, peer or liability cost, etc)

(b)   3 possible vehicles
        1. Segregate fund (fund by insurance company or trust)
            pension fund own fixed income security
            must be great size for diversity

        2. Unit of pool fund (by insurance company or combination of many small pension fund
            trust)
            advantage: diversity, exposure to many asset, low cost, expertise available

        3. Fixed-income fund or insurance product
            group annuity (lock return when purchase)
            single premium group annuity

            internally managed fund

Question #14

$$S = 25, q = 6\%, X = 30, t = \frac{60}{360} = \frac{1}{6}, c = 1, p = 6$$

$$c = Se^{-qt}N(d_1) - Xe^{-rt}N(d_2)$$

$$p = Xe^{-rt}N(-d_2) - Se^{-qt}N(-d_1)$$

$$c + Xe^{-rt} = p + Se^{-qt}$$

$$1 + 30e^{-rt} = 6 + 25e^{-.06/6}$$

$$30e^{-rt} = 5 + 25e^{-.01}$$

$$e^{-rt} = .991708$$

Current market price of 150-day T-bill of 1,000 face amount

$\quad = e^{-rt} \cdot \quad$ (price of futures contract on 90-day T-bill delivery in 60 days)
$\quad = .991708 \times 984$
$\quad = 975.84$