Fall 2004 Course 4 Solutions

Question #1
Key: C

$E(X \mid q) = 3q, Var(X \mid q) = 3q(1-q)$

$\mu = E(3q) = \int_0^1 3q2qdq = 2q^3\Big|_0^1 = 2$

$v = E[3q(1-q)] = \int_0^1 3q(1-q)2qdq = 2q^3 - 1.5q^4\Big|_0^1 = 0.5$

$a = Var(3q) = E(9q^2) - \mu^2 = \int_0^1 9q^2 2qdq - 2^2 = 4.5q^4\Big|_0^1 - 4 = 4.5 - 4 = 0.5$

$k = v/a = 0.5/0.5 = 1$

$Z = \dfrac{1}{1+1} = 0.5$

The estimate is
0.5(0) + 0.5(2) = 1.

Question #2
Key: D

0.35(14) = 4.9
$\hat{\pi}_{0.35} = 0.1(216) + 0.9(250) = 246.6$

Question #3
Key: D

The problem asks for the confidence interval for β_3, which is $28 \pm 1.96(38.8423)^{1/2}$ or 28 ± 12 or [16, 40].

Question #4
Key: E

At $y_1 = 0.9$ the risk set is $r_1 = 7$ and $s_1 = 1$.
At $y_2 = 1.5$ the risk set is $r_2 = 6$ and $s_2 = 1$.
Then, $S_{10}(1.6) = \dfrac{6}{7}\dfrac{5}{6} = 0.7143$.

Question #5
Key: B

$$\Pr\left(class1 \mid claim = 250\right) = \frac{\Pr(claim = 250 \mid class1)\Pr(class1)}{\Pr(claim = 250 \mid class1)\Pr(class1) + \Pr(claim = 250 \mid class2)\Pr(class2)}$$
$$= \frac{0.5(2/3)}{0.5(2/3) + 0.7(1/3)} = \frac{10}{17}.$$
$$E(claim \mid class1) = 0.5(250) + 0.3(2,500) + 0.2(60,000) = 12,875.$$
$$E(claim \mid class2) = 0.7(250) + 0.2(2,500) + 0.1(60,000) = 6,675.$$
$$E(claim \mid 250) = \frac{10}{17}(12,875) + \frac{7}{17}(6,675) = 10,322.$$

Question #6
Key: D

$$L(p) = f(0.74)f(0.81)f(0.95)$$
$$= (p+1)0.74^p(p+1)0.81^p(p+1)0.95^p$$
$$= (p+1)^3(0.56943)^p$$
$$l(p) = \ln L(p) = 3\ln(p+1) + p\ln(0.56943)$$
$$l'(p) = \frac{3}{p+1} - 0.563119 = 0$$
$$p+1 = \frac{3}{0.563119} = 5.32747$$
$$p = 4.32747.$$

Question #7
Key: E

Homogeneous nonstationary processes have the desirable property that if they are differenced one or more times, eventually one of the resulting series will be stationary.

Question #8
Key: E

The sample mean is 1 and therefore $mq = 1$.
For the smoothed empirical 33rd percentile, (1/3)(5 + 1) = 2 and the second smallest sample item is 0. For the 33rd percentile of the binomial distribution to be 0, the probability at zero must exceed 0.33. So, $(1-q)^m > 0.33$ and then $(1-m^{-1})^m > 0.33$. Trial and error gives $m = 6$ as the smallest value that produces this result.

Question #9
Key: C

Let X be the number of claims.

$E(X \mid I) = 0.9(0) + 0.1(2) = 0.2$

$E(X \mid II) = 0.8(0) + 0.1(1) + 0.1(2) = 0.3$

$E(X \mid III) = 0.7(0) + 0.2(1) + 0.1(2) = 0.4$

$Var(X \mid I) = 0.9(0) + 0.1(4) - 0.2^2 = 0.36$

$Var(X \mid II) = 0.8(0) + 0.1(1) + 0.1(4) - 0.3^2 = 0.41$

$Var(X \mid III) = 0.7(0) + 0.2(1) + 0.1(4) - 0.4^2 = 0.44.$

$\mu = (1/2)(0.2 + 0.3 + 0.4) = 0.3$

$v = (1/3)(0.36 + 0.41 + 0.44) = 0.403333$

$a = (1/3)(0.2^2 + 0.3^2 + 0.4^2) - 0.3^2 = 0.006667$

$k = 0.403333 / 0.006667 = 60.5$

$$Z = \frac{50}{50 + 60.5} = 0.45249.$$

For one insured the estimate is 0.45249(17/50) + 0.54751(0.3) = 0.31810.
For 35 insureds the estimate is 35(0.31810) = 11.13.

Question #10
Key: A

For the given intervals, based on the model probabilities, the expected counts are 4.8, 3.3, 8.4, 7.8, 2.7, 1.5, and 1.5. To get the totals above 5, group the first two intervals and the last three. The table is

Interval	Observed	Expected	Chi-square
0—500	3	8.1	3.21
500—2498	8	8.4	0.02
2498—4876	9	7.8	0.18
4876—infinity	10	5.7	3.24
Total	30	30	6.65

Question #11
Key: C

$$r^2_{YX_3 \bullet X_2} = \frac{R^2 - r^2_{YX_2}}{1 - r^2_{YX_2}}$$

$$(-0.4)^2 = \frac{R^2 - (0.4)^2}{1 - (0.4)^2}$$

$$R^2 = 0.16(0.84) + 0.16 = 0.2944.$$

Question #12
Key: E

Let $\hat{H} = \hat{H}(t)$ and $\hat{v} = V\hat{a}r(\hat{H}(t))$. The confidence interval is $\hat{H}U$ where $U = \exp(\pm z_{\alpha/2}\sqrt{\hat{v}} / \hat{H})$. Multiplying the two bounds gives $0.7(0.357) = \hat{H}^2$ for $\hat{H} = 0.49990$. Then,
$\hat{S} = \exp(-0.49990) = 0.60659.$

Question #13
Key: C

$$0.575 = \Pr(N = 0) = \int_0^k \Pr(N = 0 \mid \theta)\pi(\theta)d\theta$$

$$= \int_0^k e^{-\theta} \frac{e^{-\theta}}{1 - e^{-k}} d\theta = -\frac{e^{-2\theta}}{2(1 - e^{-k})}\Bigg|_0^k = -\frac{e^{-2k}}{2(1 - e^{-k})} + \frac{1}{2(1 - e^{-k})}$$

$$= \frac{1 - e^{-2k}}{2(1 - e^{-k})} = \frac{1 + e^{-k}}{2}$$

$$e^{-k} = 2(0.575) - 1 = 0.15$$

$$k = 1.90.$$

Question #14
Key: C

The sample -1 moment is $\frac{1}{6}\left(\frac{1}{15}+\frac{1}{45}+\frac{1}{140}+\frac{1}{250}+\frac{1}{560}+\frac{1}{1340}\right)=0.017094$. The sample -2 moment is $\frac{1}{6}\left(\frac{1}{15^2}+\frac{1}{45^2}+\frac{1}{140^2}+\frac{1}{250^2}+\frac{1}{560^2}+\frac{1}{1340^2}\right)=0.00083484$.
Then the equations are

$$0.017094=\frac{1}{\theta(\tau-1)},$$

$$0.00083484=\frac{2}{\theta^2(\tau-1)(\tau-2)}.$$

Divide the square of the first equation by the second equation to obtain
$0.35001=\frac{\tau-2}{2(\tau-1)}$ which is solved for $\tau=4.33356$. From the first equation,

$$\theta=\frac{1}{3.33356(0.017094)}=17.55.$$

Question #15
Key: E

This is an MA(2) model. For this model, $\rho_2=\frac{-\theta_2}{1+\theta_1^2+\theta_2^2}$. Substituting the given value of ρ_2 and $\theta_2=0.7-\theta_1$ gives

$$-0.155=\frac{-(0.7-\theta_1)}{1+\theta_1^2+(0.7-\theta_1)^2}=\frac{-0.7+\theta_1}{1+\theta_1^2+0.49-1.4\theta_1+\theta_1^2}$$

$$-0.155(1.49-1.4\theta_1+2\theta_1^2)=-0.7+\theta_1$$

$$-0.23095+0.217\theta_1-0.31\theta_1^2=-0.7+\theta_1$$

$$0=0.31\theta_1^2+0.783\theta_1-0.46905$$

$$\theta_1=0.5 \text{ or } -3.$$

Only the first solution (0.5) is acceptable.

Question #16
Key: A

For each simulation, estimate the LER and then calculate the squared difference from the estimate, 0.125.

Simulation	First claim	Second claim	Third claim	LER	Squared difference
1	600	600	1500	0.111111	0.000193
2	1500	300	1500	0.090909	0.001162
3	1500	300	600	0.125000	0.000000
4	600	600	300	0.200000	0.005625
5	600	300	1500	0.125000	0.000000
6	600	600	1500	0.111111	0.000193
7	1500	1500	1500	0.066667	0.003403
8	1500	300	1500	0.090909	0.001162
9	300	600	300	0.250000	0.015625
10	600	600	600	0.166667	0.001736

The last column has an average of 0.002910 which is the bootstrap estimate.

Question #17
Key: B

The subscripts denote the three companies.

$$x_{I1} = \frac{50,000}{100} = 500, \quad x_{I2} = \frac{50,000}{200} = 250, \quad x_{II1} = \frac{150,000}{500} = 300$$

$$x_{II2} = \frac{150,000}{300} = 500, \quad x_{III1} = \frac{150,000}{50} = 3,000, \quad x_{III2} = \frac{150,000}{150} = 1,000$$

$$\bar{x}_I = \frac{100,000}{300} = 333.33, \quad \bar{x}_{II} = \frac{300,000}{800} = 375, \quad \bar{x}_{III} = \frac{300,000}{200} = 1,500, \quad \bar{x} = \frac{700,000}{1,300} = 538.46$$

$$\hat{v} = \frac{\begin{array}{l}100(500-333.33)^2 + 200(250-333.33)^2 + 500(300-375)^2 + 300(500-375)^2 \\ +50(3,000-1,500)^2 + 150(1,000-1,500)^2\end{array}}{(2-1)+(2-1)+(2-1)}$$

$$= 53,888,888.89$$

$$\hat{a} = \frac{300(333.33-538.46)^2 + 800(375-538.46)^2 + 200(1,500-538.46)^2 - 53,888,888.89(3-1)}{1,300 - \frac{300^2+800^2+200^2}{1,300}}$$

$$= 157,035.60$$

$$k = \frac{53,888,888.89}{157,035.60} = 343.1635, Z = \frac{200}{200+343.1635} = 0.3682$$

Question #18
Key: D

Let α_j be the parameter for region j. The likelihood function is $L = \left(\prod_{i=1}^{n} \frac{\alpha_1}{x_i^{\alpha_1+1}}\right)\left(\prod_{i=1}^{m} \frac{\alpha_2}{y_i^{\alpha_2+1}}\right)$. The expected values satisfy $\frac{\alpha_2}{\alpha_2 - 1} = 1.5\frac{\alpha_1}{\alpha_1 - 1}$ and so $\alpha_2 = \frac{3\alpha_1}{2+\alpha_1}$. Substituting this in the likelihood function and taking logs produces

$$l(\alpha_1) = \ln L(\alpha_1) = n\ln\alpha_1 - (\alpha_1 + 1)\sum_{i=1}^{n}\ln x_i + m\ln\left(\frac{3\alpha_1}{2+\alpha_1}\right) - \frac{2+4\alpha_1}{2+\alpha_1}\sum_{i=1}^{m}\ln y_i$$

$$l'(\alpha_1) = \frac{n}{\alpha_1} - \sum_{i=1}^{n}\ln x_i + \frac{2m}{\alpha_1(2+\alpha_1)} - \frac{6}{(2+\alpha_1)^2}\sum_{i=1}^{m}\ln y_i = 0.$$

Question #19
Key: A

The unrestricted model has ESS = 4,053 + 2,087 = 6,140 and the restricted model has ESS = 10,374. The unrestricted model has 37 observations and 4 parameters while the restricted model has 37 observations and 8 parameters. The test statistic has 8 – 4 = 4 numerator degrees of freedom and 37 – 8 = 29 denominator degrees of freedom. The test statistic is

$$F = \frac{\frac{10{,}374 - 6{,}140}{4}}{\frac{6{,}140}{29}} = 5.$$

The critical value is about 2.7 and so the value is statistically significant.

Question #20
Key: D

Let $K_y(x)$ be the contribution at x of the data point at y. It is

$$K_y(x) = \begin{cases} 0, & x < y - 1.4 \\ \frac{x - y + 1.4}{2.8}, & y - 1.4 \le x \le y + 1.4 \\ 1, & x > y + 1.4. \end{cases}$$

For the particular points,

$K_2(4) = 1, K_{3.3}(4) = \frac{4 - 3.3 + 1.4}{2.8} = 0.75, K_4(4) = 0.5, K_{4.7}(4) = 0.25$. The kernel estimate is the weighted average $\frac{1}{8}(1) + \frac{2}{8}(0.75) + \frac{2}{8}(0.5) + \frac{3}{8}(0.25) = 0.53125.$

Question #21
Key: B

The mean is mq and the variance is $mq(1-q)$. The mean is 34,574 and so the full credibility standard requires the confidence interval to be ±345.74 which must be 1.96 standard deviations. Thus,

$$345.74 = 1.96\sqrt{mq(1-q)} = 1.96\sqrt{34{,}574}\sqrt{1-q}$$

$$1-q = 0.9, \quad q = 0.1.$$

Question #22
Key: A

Only the Kolmogorov-Smirnov test statistic tends toward zero as the sample size goes to infinity. As a consequence, the critical value for the K-S statistic has the square root of the sample size in the denominator. For the Anderson-Darling and the Chi-square goodness-of-fit test statistics, the sample size appears in the numerator of the test statistics themselves. The Schwarz Bayesian Criterion involves an adjustment to the likelihood function, which does not go to zero as the sample size goes to infinity.

Question #23
Key: A

The adjustment is to divide the regression equation by the square root of the multiplier in the variance. The new equation is

$$\frac{Y_i}{\sqrt{X_i}} = \frac{\beta X_i}{\sqrt{X_i}} + \frac{\varepsilon_i}{\sqrt{X_i}}$$

$$Y_i' = \beta X_i' + \varepsilon_i'.$$

The least squares solution is

$$\hat{\beta} = \frac{\sum X_i' Y_i'}{\sum X_i'^2} = \frac{\sum \sqrt{X_i}\frac{Y_i}{\sqrt{X_i}}}{\sum X_i} = \frac{\sum Y_i}{\sum X_i} = \frac{30.1}{12.5} = 2.408.$$

Question #24
Key: E

The sample average is (14 + 33 + 72 + 94 + 120 + 135 + 150 + 150)/8 = 96. The model average is $E(X\wedge 150)=\int_0^{150} x\frac{1}{\theta}dx+\int_{150}^{\theta}150\frac{1}{\theta}dx=\frac{150^2}{2\theta}+150\frac{\theta-150}{\theta}=150-\frac{11,250}{\theta}$. The equation to solve is $150-\frac{11,250}{\theta}=96,\quad \frac{11,250}{\theta}=54,\quad \theta=\frac{11,250}{54}=208.3.$

Question #25
Key: C

$$E(N\,|\,1)=5, E(N\,|\,2)=8(0.55)=4.4, \mu=0.5(5)+0.5(4.4)=4.7$$
$$Var(N\,|\,1)=5, Var(N\,|\,2)=8(0.55)(0.45)=1.98, v=0.5(5)+0.5(1.98)=3.49$$
$$a=0.5(5)^2+0.5(4.4)^2-4.7^2=0.09, k=3.49/0.09=38.7778$$
$$Z=\frac{3}{3+38.7778}=0.0718, 4.6019=0.0718\frac{7+r}{3}+0.9282(4.7)$$

The solution is r = 3.

Question #26
Key: A

These observations are truncated at 500. The contribution to the likelihood function is $\frac{f(x)}{1-F(500)}=\frac{\theta^{-1}e^{-x/\theta}}{e^{-500/\theta}}$. Then the likelihood function is

$$L(\theta)=\frac{\theta^{-1}e^{-600/\theta}\theta^{-1}e^{-700/\theta}\theta^{-1}e^{-900/\theta}}{\left(e^{-500/\theta}\right)^3}=\theta^{-3}e^{-700/\theta}$$
$$l(\theta)=\ln L(\theta)=-3\ln\theta-700\theta^{-1}$$
$$l'(\theta)=-3\theta^{-1}+700\theta^{-2}=0$$
$$\theta=700/3=233.33.$$

Question #27
Key: B

In 1992, $D_t = 1$ and the equation is
$E(\ln Y_t) = (\beta_1 - \beta_4 \ln X_{2t_0} - \beta_5 \ln X_{3t_0}) + (\beta_2 + \beta_4)\ln X_{2t} + (\beta_3 + \beta_5)\ln X_{3t}$.
Elasticity is the percent change in Y due to a 1% change in X. Let x be a given value of X_{2t}. Then the expected value of $\ln Y_t$ is $s + 0.53\ln x$ where s represents the rest of the equation and 0.53 is the estimated value of $\beta_2 + \beta_4$. With a 1% increase in x, the new value is $s + 0.53\ln(1.01x)$ which is the old value plus $0.53\ln(1.01) = 0.0052737$. Exponentiating indicates that the new Y value will be $e^{0.0052737} = 1.0052876$ times the old value. This is a 0.53% increase, and so the elasticity is 0.53. Most texts note that the Taylor series approximation indicates that the coefficient itself is a reasonable estimate of the elasticity. So going directly to $\beta_2 + \beta_4 = 0.53$ is a reasonable way to proceed to the answer.

Question #28
Key: A

For group A let the hazard rate function be the baseline function, $h_0(x)$. For group B let the hazard rate function be $h_0(x)e^{\beta}$. Then the partial likelihood function is
$L = \frac{1}{1}\frac{e^{\beta}}{1+e^{\beta}}\frac{e^{\beta}}{1+2e^{\beta}}\frac{1}{2+2e^{\beta}} = \frac{e^{2\beta}}{2(1+2e^{\beta})(1+e^{\beta})^2}$. Taking logarithms and differentiating leads to

$$l = 2\beta - \ln(2) - \ln(1+2e^{\beta}) - 2\ln(1+e^{\beta})$$

$$l' = 2 - \frac{2e^{\beta}}{1+2e^{\beta}} - \frac{2e^{\beta}}{1+e^{\beta}} = 0$$

$$2(1+2e^{\beta})(1+e^{\beta}) = 2e^{\beta}(1+e^{\beta}) + 2e^{\beta}(1+2e^{\beta})$$

$$1 + 3e^{\beta} + 2e^{2\beta} = e^{\beta} + e^{2\beta} + e^{\beta} + 2e^{2\beta}$$

$$0 = e^{2\beta} - e^{\beta} - 1$$

$$e^{\beta} = \frac{1 \pm \sqrt{1+4}}{2} = 1.618$$

where only the positive root can be used. Because the value is greater than 1, group B has a higher hazard rate function than group A. Therefore, its cumulative hazard rate must also be higher.

Question #29
Key: E

For a compound Poisson distribution, *S*, the mean is $E(S \mid \lambda, \mu, \sigma) = \lambda E(X) = \lambda e^{\mu + 0.5\sigma^2}$ and the variance is $Var(S \mid \lambda, \mu, \sigma) = \lambda E(X^2) = \lambda e^{2\mu + 2\sigma^2}$. Then,

$$E(S) = E[E(S \mid \lambda, \mu, \sigma)] = \int_0^1 \int_0^1 \int_0^1 \lambda e^{\mu + 0.5\sigma^2} 2\sigma d\lambda d\mu d\sigma$$
$$= \int_0^1 \int_0^1 e^{\mu + 0.5\sigma^2} \sigma d\mu d\sigma = \int_0^1 (e-1) e^{0.5\sigma^2} \sigma d\sigma$$
$$= (e-1)(e^{0.5} - 1) = 1.114686$$
$$v = E[Var(S \mid \lambda, \mu, \sigma)] = \int_0^1 \int_0^1 \int_0^1 \lambda e^{2\mu + 2\sigma^2} 2\sigma d\lambda d\mu d\sigma$$
$$= \int_0^1 \int_0^1 e^{2\mu + 2\sigma^2} \sigma d\mu d\sigma = \int_0^1 0.5(e^2 - 1) e^{2\sigma^2} \sigma d\sigma$$
$$= 0.5(e^2 - 1)0.25(e^2 - 1) = 0.125(e^2 - 1)^2 = 5.1025$$
$$a = Var[E(S \mid \lambda, \mu, \sigma)] = \int_0^1 \int_0^1 \int_0^1 \lambda^2 e^{2\mu + \sigma^2} 2\sigma d\lambda d\mu d\sigma - E(S)^2$$
$$= \int_0^1 \int_0^1 \frac{2}{3} e^{2\mu + \sigma^2} \sigma d\mu d\sigma - E(S)^2 = \int_0^1 \frac{1}{3}(e^2 - 1) e^{\sigma^2} \sigma d\sigma - E(S)^2$$
$$= \frac{1}{3}(e^2 - 1)\frac{1}{2}(e-1) - E(S)^2 = (e^2 - 1)(e-1)/6 - E(S)^2 = 0.587175$$
$$k = \frac{5.1025}{0.587175} = 8.69.$$

Question #30
Key: D

The equations to solve are $0.4 = e^{-(\theta/1.82)^\tau}$, $0.8 = e^{-(\theta/12.66)^\tau}$. Taking logs yields $0.91629 = (\theta/1.82)^\tau$, $0.22314 = (\theta/12.66)^\tau$. Taking the ratio of the first equation to the second equation gives $4.10635 = (12.66/1.82)^\tau = 6.95604^\tau$. Taking logs again, $1.41253 = 1.93961\tau$ and then $\tau = 0.72825$. Returning to the first (logged) equation, $0.91629 = (\theta/1.82)^\tau$, $0.88688 = \theta/1.82$, $\theta = 1.614$.

Question #31
Key: B

The number of degrees of freedom for the Box-Pierce test is $K - p - q$.

Question #32
Key: C

There are $n/2$ observations of $N = 0$ (given $N = 0$ or 1) and $n/2$ observations of $N = 1$ (given $N = 0$ or 1). The likelihood function is

$$L = \left(\frac{e^{-\lambda}}{e^{-\lambda}+\lambda e^{-\lambda}}\right)^{n/2}\left(\frac{\lambda e^{-\lambda}}{e^{-\lambda}+\lambda e^{-\lambda}}\right)^{n/2} = \frac{\lambda^{n/2}e^{-n\lambda}}{(e^{-\lambda}+\lambda e^{-\lambda})^n} = \frac{\lambda^{n/2}}{(1+\lambda)^n}.$$

Taking logs, differentiating and solving provides the answer.

$$l = \ln L = (n/2)\ln\lambda - n\ln(1+\lambda)$$

$$l' = \frac{n}{2\lambda} - \frac{n}{1+\lambda} = 0$$

$$n(1+\lambda) - n2\lambda = 0$$

$$1-\lambda = 0, \quad \lambda = 1.$$

Question #33
Key: D

The posterior density function is proportional to the product of the likelihood function and prior density. That is, $\pi(q\,|\,1,0) \propto f(1\,|\,q)f(0\,|\,q)\pi(q) \propto q(1-q)q^3 = q^4 - q^5$. To get the exact posterior density, integrate this function over its range:

$$\int_{0.6}^{0.8} q^4 - q^5 dq = \left.\frac{q^5}{5} - \frac{q^6}{6}\right|_{0.6}^{0.8} = 0.014069 \text{ and so } \pi(q\,|\,1,0) = \frac{q^4-q^5}{0.014069}.$$ Then,

$$\Pr(0.7 < q < 0.8\,|\,1,0) = \int_{0.7}^{0.8}\frac{q^4-q^5}{0.014069}dq = 0.5572.$$

Question #34
Key: B

The likelihood function, its logarithm, derivative and solution are

$$L(\beta) = \prod_{i=1}^{5} p(y_i) = \prod_{i=1}^{5}\frac{e^{-\beta x_i}(\beta x_i)^{y_i}}{y_i!}$$

$$l(\beta) = \ln L(\beta) = \sum_{i=1}^{5}[-\beta x_i + y_i\ln(\beta) + y_i\ln(x_i) - \ln(y_i!)]$$

$$l'(\beta) = \sum_{i=1}^{5}(-x_i + y_i/\beta) = -163 + 9/\beta = 0, \quad \hat{\beta} = 9/163.$$

To approximate the variance,

$$l''(\beta) = -\sum_{i=1}^{5} y_i/\beta^2, \quad E\left(\sum_{i=1}^{5} y_i/\beta^2\right) = \sum_{i=1}^{5} x_i\beta/\beta^2 = 163/\beta.$$ The variance is the reciprocal.

Substituting the estimate gives $\hat{\beta}/163 = 9/163^2 = 0.00033874$. The standard error is the square root, 0.0184.

Question #35
Key: B

Replace "increases" with "decreases" to make this statement true.

Question #36
Key: B

The cumulative hazard function for the exponential distribution is $H(x) = x/\theta$. The maximum likelihood estimate of θ is the sample mean, which equals (1227/15) = 81.8.
Therefore $\hat{H}_2(75) = (75/81.8) = 0.917$.
To calculate $\hat{H}_1(75)$ use the following table.

j	1	2	3	4	5	6
y_j	11	22	36	51	69	92
s_j	1	3	1	1	3	2
r_j	15	14	11	10	9	6

Therefore,

$$\hat{H}_1(75) = \frac{1}{15} + \frac{3}{14} + \frac{1}{11} + \frac{1}{10} + \frac{3}{9} = 0.805.$$

Thus, $\hat{H}_2(75) - \hat{H}_1(75) = 0.917 - 0.805 = 0.112$.

Question #37
Key: A

The sample mean is $\frac{0(2000)+1(600)+2(300)+3(80)+4(20)}{3000} = 0.5066667 = \hat{\mu} = \hat{v}$ and the sample variance is
$\frac{2000(0-\hat{\mu})^2 + 600(1-\hat{\mu})^2 + 300(2-\hat{\mu})^2 + 80(3-\hat{\mu})^2 + 20(4-\hat{\mu})^2}{2999} = 0.6901856$. Then,

$\hat{a} = 0.6901856 - 0.5066667 = 0.1835189$, $k = \frac{0.5066667}{0.1835189} = 2.760842$ and

$Z = \frac{1}{1+2.760842} = 0.2659$.

Question #38
Key: E

The cdf is $F(x)=\int_0^x 4(1+t)^{-5}\,dt=-(1+t)^{-4}\Big|_0^x=1-\dfrac{1}{(1+x)^4}$.

Observation (x)	$F(x)$	compare to:	Maximum difference
0.1	0.317	0, 0.2	0.317
0.2	0.518	0.2, 0.4	0.318
0.5	0.802	0.4, 0.6	0.402
0.7	0.880	0.6, 0.8	0.280
1.3	0.964	0.8, 1.0	0.164

K-S statistic is 0.402.

Question #39
Key: B

When forecasting, assume that all future values of the error term are zero.
$y_{T+1}=0.9y_T+1+\varepsilon_{T+1}-0.4\varepsilon_T$, $\hat{y}_{T+1}=0.9(8)+1+0-0.4(0.5)=8$
$y_{T+2}=0.9y_{T+1}+1+\varepsilon_{T+2}-0.4\varepsilon_{T+1}$, $\hat{y}_{T+2}=0.9(8)+1+0-0.4(0)=8.2$.

Question #40
Key: D

This follows from the formula $MSE(\hat{\theta})=Var(\hat{\theta})+[bias(\hat{\theta})]^2$. If the bias is zero, then the mean-squared error is equal to the variance.