Course 4 Solutions
November 2001 Exams

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Question #1
Answer is B

From the Yule-Walker equations:

$$r_1 = f_1 + r_1 f_2$$
$$r_2 = r_1 f_1 + f_2.$$

Substituting the given quantities yields:

$$0.53 = f_1 + 0.53 f_2$$
$$-0.22 = 0.53 f_1 + f_2.$$

The solution is $f_1 = 0.90$ and $f_2 = -0.70$.

The next Yule-Walker equation is:

$$\begin{aligned} r_3 &= f_1 r_2 + f_2 r_1 \\ &= 0.90(-0.22) + -0.70(0.53) \\ &= -0.57. \end{aligned}$$

Question #2
Answer is E

For an interval running from c to d, the uniform density function is $f(x) = g/[n(d-c)]$ where g is the number of observations in the interval and n is the sample size. The contribution to the second raw moment for this interval is:

$$\int_c^d x^2 \frac{g}{n(d-c)} dx = \left. \frac{gx^3}{3n(d-c)} \right|_c^d = \frac{g(d^3 - c^3)}{3n(d-c)}.$$

For this problem, the second raw moment is:

$$\frac{1}{90}\left[\frac{30(25^3 - 0^3)}{3(25-0)} + \frac{32(50^3 - 25^3)}{3(50-25)} + \frac{20(100^3 - 50^3)}{3(100-50)} + \frac{8(200^3 - 100^3)}{3(200-100)}\right] = 3958.33.$$

Question #3
Answer is B

Because the Bayes and Bühlmann results must be identical, this problem can be solved either way. For the Bühlmann approach, $m(l) = v(l) = l$. Then, noting that the prior distribution is a gamma distribution with parameters 50 and 1/500, we have:

$$m = E(l) = 50/500 = 0.1$$

$$v = E(l) = 0.1$$

$$a = Var(l) = 50/500^2 = 0.0002$$

$$k = v/a = 500$$

$$Z = 1500/(1500+500) = 0.75$$

$$\bar{X} = \frac{75+210}{600+900} = 0.19.$$

The credibility estimate is $0.75(0.19)+0.25(0.1) = 0.1675$. For 1100 policies, the expected number of claims is 1100(0.1675) = 184.25.

For the Bayes approach, the posterior density is proportional to (because in a given year the number of claims has a Poisson distribution with parameter l times the number of policies) $\frac{e^{-600l}(600l)^{75}}{75!}\frac{e^{-900l}(900l)^{210}}{210!}\frac{(500l)^{50}e^{-500l}}{l\Gamma(50)} \propto l^{335}e^{-2000l}$ which is a gamma density with parameters 335 and 1/2000. The expected number of claims per policy is 335/2000 = 0.1675 and the expected number of claims in the next year is 184.25.

Question #4
Answer is B

All but B can be seen as true from various items on pages 91-96 of *Survival Analysis*. B is false because if the last observed time is a death time, then the number of deaths is equal to the number at risk (that is, $d = Y$). Thus the survival function will be multiplied by zero and will become zero.

Question #5
Answer is B

$$s^2 = \frac{\sum \hat{e}_i^2}{N-2} = \frac{2394}{6} = 399$$
$$s_{\hat{b}}^2 = \frac{s^2}{\sum x_i^2} = \frac{399}{1.62} = 246.3$$
$$s_{\hat{b}} = 15.69$$
$$t_{.95} = 1.943$$

The confidence interval is
$-35.69 \pm 1.943(15.69) = (-66.2, -5.2)$

Question #6
Answer is E

The q-q plot takes the ordered values and plots the jth point at $j/(n+1)$ on the horizontal axis and at $F(x_j;q)$ on the vertical axis. For small values, the model assigns more probability to being below that value than occurred in the sample. This indicates that the model has a heavier left tail than the data. For large values, the model again assigns more probability to being below that value (and so less probability to being above that value). This indicates that the model has a lighter right tail than the data. Of the five answer choices, only E is consistent with these observations. In addition, note that as you go from 0.4 to 0.6 on the horizontal axis (thus looking at the middle 20% of the data), the q-q plot increases from about 0.3 to 0.4 indicating that the model puts only about 10% of the probability in this range, thus confirming answer E.

Question #7
Answer is C

The posterior probability of having one of the coins with a 50% probability of heads is proportional to (.5)(.5)(.5)(.5)(4/6) = 0.04167. This is obtained by multiplying the probabilities of making the successive observations 1, 1, 0, and 1 with the 50% coin times the prior probability of 4/6 of selecting this coin. The posterior probability for the 25% coin is proportional to (.25)(.25)(.75)(.25)(1/6) = 0.00195 and the posterior probability for the 75% coin is proportional to (.75)(.75)(.25)(.75)(1/6) = 0.01758. These three numbers total 0.06120. Dividing by this sum gives the actual posterior probabilities of 0.68088, 0.03186, and 0.28726. The expected value for the fifth toss is then (.68088)(.5) + (.03186)(.25) + (.28726)(.75) = 0.56385.

Question #8
Answer is D

From Section 8.6 of *Survival Analysis*, the "times" to be considered are $t_1 = 1, t_2 = 2$ and $t_3 = 3$.
$W(t_1) = 3 + 3e^b = 8.4664; \quad W(t_2) = 3 + 2e^b = 6.6442; \quad W(t_3) = 2 + 2e^b = 5.6442.$

By formula (8.6.2),
$$\hat{H}_0(4) = \frac{1}{8.4664} + \frac{1}{6.6442} + \frac{1}{5.6442} = 0.4458.$$
So $\hat{S}_0(4) = e^{-0.4458} = 0.6403$.

Question #9
Answer is A

This material is on page 496 of *Econometric Models*. Answer A is true because the standard deviation is $1/\sqrt{T} = 1/\sqrt{100} = 0.1$, while B is nonsense, C would be correct in reference to one autocorrelation coefficient, but not all fifteen, if the interpretation of a hypothesis test were corrected (95% is the probability of not rejecting a hypothesis, *given* that the null hypothesis is true), D would be correct if it stated 15 degrees of freedom, and E misinterprets the results of a hypothesis test.

Question #10
Answer is A

Because the exponential distribution is memoryless, the excess over the deductible is also exponential with the same parameter. So subtracting 100 from each observation yields data from an exponential distribution and noting that the maximum likelihood estimate is the sample mean gives the answer of 73.

Working from first principles,
$$L(q) = \frac{f(x_1)f(x_2)f(x_3)f(x_4)f(x_5)}{[1 - F(100)]^5} = \frac{q^{-1}e^{-125/q}q^{-1}e^{-150/q}q^{-1}e^{-165/q}q^{-1}e^{-175/q}q^{-1}e^{-250/q}}{(e^{-100/q})^5}$$
$$= q^{-5}e^{-365/q}.$$

Taking logarithms and then a derivative gives
$l(q) = -5\ln(q) - 365/q,\ l'(q) = -5/q + 365/q^2 = 0.$

The solution is $\hat{q} = 365/5 = 73$.

Question #11
Answer is D

The number of claims for each insured has a binomial distribution with $n = 1$ and q unknown. We have

$m(q) = q, v(q) = q(1-q)$

$m = E(q) = 0.1$, given in item (iv)

$a = Var(q) = E(q^2) - E(q)^2 = E(q^2) - 0.01 = 0.01$, given in item (v)

Therefore, $E(q^2) = 0.02$

$v = E(q - q^2) = 0.1 - 0.02 = 0.08$

$$k = v/a = 8, Z = \frac{10}{10+8} = 5/9.$$

Then the expected number of claims in the next one year is (5/9)(0) + (4/9)(0.1) = 2/45 and the expected number of claims in the next five years is 5(2/45) = 2/9 = 0.22.

Question #12
Answer is A

Using the set-up as in the text, the solution proceeds as follows: Taking one year as the unit of time, we have $t = 3$, X is the time between the issue and the first claim on a policy, and we want to estimate $P(X < 2 | X \le 3)$.

No. Of Policies	T_i	X_i	R_i	d_i	Y_i	$P(X < x_i \mid X \le 3)$
5	0	1	2			
6	1	1	2			
7	2	1	2	18	18	0
9	0	2	1			
10	1	2	1	19	30	$\left(14/27\right) \times \left(11/30\right) = 0.1901$
13	0	3	0	13	27	$\left(14/27\right) = 0.5185$

The answer is the middle number in the last column, namely 0.1901.

Alternatively, perhaps all that one remembers is that for right-truncated data the Kaplan-Meier estimate can be used provided we work with, in this case, the variable $R = 3 - X$. Then the observations become left truncated. The probability we seek is

$\Pr(X < 2 \mid X \leq 3) = \Pr(3 - R < 2 \mid 3 - R \leq 3) = \Pr(R > 1 \mid R \geq 0)$ and because R cannot be negative, this reduces to $\Pr(R > 1)$.

The six entries in the original table can be identified as follows:

Number of entries	Left truncation point	Value of R
5	0	2
9	0	1
13	0	0
6	1	2
10	1	1
7	2	2

The left truncation point is three minus the right truncation point. For example the entries in the second row of the original table could have X values of 1 or 2, but no higher. So they have a right truncation point 2 which for R is a left truncation point of 3 - 2 = 1. We then observe that the risk set at time 0 is 27 (the observations with a left truncation point at 0) and of them, there were 13 deaths. The Kaplan-Meier estimate of surviving past time 0 is then (14/27). At time 1 the risk set has 30 members (the 43 who were left truncated at 0 or 1 less the 13 who died at time 0) of which 19 died (had an R value of 1). The Kaplan-Meier estimate of surviving past time 1 is (14/27)(11/30) = 0.1901.

Question #13
Answer is B

From the matrix formulas for multiple regression,

$$X = \begin{bmatrix} 1 & -3 & -1 \\ 1 & -1 & 3 \\ 1 & 1 & -3 \\ 1 & 3 & 1 \end{bmatrix}, \quad X'X = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 20 \end{bmatrix}, \quad \left(X'X\right)^{-1} = \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & 1/20 & 0 \\ 0 & 0 & 1/20 \end{bmatrix} \text{ and then}$$

$$\hat{b} = (X'X)^{-1}X'Y = \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & 1/20 & 0 \\ 0 & 0 & 1/20 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ -3 & -1 & 1 & 3 \\ -1 & 3 & -3 & 1 \end{bmatrix} Y$$

$$= \begin{bmatrix} 1/4 & 1/4 & 1/4 & 1/4 \\ -3/20 & -1/20 & 1/20 & 3/20 \\ -1/20 & 3/20 & -3/20 & 1/20 \end{bmatrix} Y.$$

The coefficients of $\hat{b}_3$ are found in the third row of the matrix.

Alternatively, one may use formula (4.5) on page 86 of *Econometric Models*.

Question #14
Answer is E

The model distribution is $f(x\,|\,q) = 1/q,\ 0 < x < q$. Then the posterior distribution is proportional to

$$p(q\,|400{,}600) \propto \frac{1}{q}\frac{1}{q}\frac{500}{q^2} \propto q^{-4},\ q > 600.$$

It is important to note the range. Being a product, the posterior density function is non-zero only when all three terms are non-zero. Because one of the observations was equal to 600, the value of the parameter must be greater than 600 in order for the density function at 600 to be positive. Or, by general reasoning, posterior probability can only be assigned to possible values. Having observed the value 600 we know that parameter values less than or equal to 600 are not possible.

The constant is obtained from $\int_{600}^{\infty} q^{-4}\,dq = \frac{1}{3(600)^3}$ and thus the exact posterior density is

$p(q\,|400{,}600) = 3(600)^3 q^{-4},\ q > 600.$ The posterior probability of an observation exceeding 550 is

$$\Pr(X_3 > 550|400{,}600) = \int_{600}^{\infty} \Pr(X_3 > 550|\,q) p(q\,|400{,}600)\,dq$$

$$= \int_{600}^{\infty} \frac{q-550}{q} 3(600)^3 q^{-4}\,dq = 0.3125$$

where the first term in the integrand is the probability of exceeding 550 from the uniform distribution.

Question #15
Answer is C

$$E(N) = rb = 0.40$$
$$Var(N) = rb\,(1+b) = 0.48$$
$$E(Y) = q/(a-1) = 500$$
$$Var(Y) = q^2 a \Big/ \left[(a-1)^2 (a-2)\right] = 750{,}000$$

Therefore,

$$E(X) = 0.40(500) = 200$$
$$Var(X) = 0.40(750{,}000) + 0.48(500)^2 = 420{,}000$$

The full credibility standard is $n = \left(\frac{1.645}{0.05}\right)^2 \frac{420{,}000}{200^2} = 11{,}365$ and then

$Z = \sqrt{2500/11{,}365} = 0.47.$

Question #16
Answer is E

In an EWMA model, all forecasted values from a fixed starting point are the same. Therefore, the differences between all forecasted values are zero. The other four statements are true based on material from pages 476-477 of *Econometric Models*.

Question #17
Answer is E

The sample variance is $s^2 = \dfrac{(1-3)^2+(2-3)^2+(3-3)^2+(4-3)^2+(5-3)^2}{4} = 2.5$. The estimator of $E[X]$ is the sample mean and the variance of the sample mean is the variance divided by the sample size, estimated here as $2.5/n$. Setting the standard deviation of the estimator equal to 0.05 gives the equation $\sqrt{2.5/n} = 0.05$ which yields $n = 1000$.

Question #18
Answer is E

$$m(r) = E(X|r) = E(N)E(Y) = r\beta\theta/(\alpha-1) = 100r$$

$$v(r) = Var(X|r) = Var(N)E(Y)^2 + E(N)Var(Y)$$

$$= r\beta(1+\beta)\theta^2/(\alpha-1)^2 + r\beta\alpha\theta^2/[(\alpha-1)^2(\alpha-2)] = 210{,}000r.$$

$$v = E(210{,}000r) = 210{,}000(2) = 420{,}000$$

$$a = Var(100r) = (100)^2(4) = 40{,}000$$

$$k = v/a = 10.5$$

$$Z = 100/(100+10.5) = 0.905.$$

Question #19
Answer is B

Using all participants, $S^{\mathrm{T}}(4) = \left(1-\dfrac{35}{300}\right)\left(1-\dfrac{74}{265}\right)\left(1-\dfrac{34}{191}\right)\left(1-\dfrac{32}{157}\right) = 0.41667.$

Using only Country B, $S^{\mathrm{B}}(4) = \left(1-\dfrac{15}{100}\right)\left(1-\dfrac{20}{85}\right)\left(1-\dfrac{20}{65}\right)\left(1-\dfrac{10}{45}\right) = 0.35.$

The difference is, $S^{\mathrm{T}}(4) - S^{\mathrm{B}}(4) = 0.41667 - 0.35 = 0.0667 = 0.07.$

Question #20
Answer is B

From page 168 of *Survival Analysis*,

$$\Theta(4) = \frac{0.05(200)+0.10(100)}{300} + \frac{0.10(180)+0.10(85)}{265} + \frac{0.15(126)+0.10(65)}{191}$$
$$+\frac{0.20(112)+0.10(45)}{157} = 0.47099.$$

Then,

$$\hat{A}(4) = \frac{35}{300} + \frac{74}{265} + \frac{34}{191} + \frac{32}{157} - 0.47099 = 0.30675.$$

Question #21
Answer is D

The unrestricted model is Model I with *ESS* = 484. To obtain the restricted model, substitute $1-b_2$ for b_3 to yield Model III with *ESS* = 982. Then,

$F = \dfrac{(982-484)/1}{484/17} = 17.49$. The 17 in the denominator is the sample size of 20 less the 3 parameters in the unrestricted model. The 1 in the numerator is the 3 parameters in the unrestricted model less the 2 parameters in the restricted model.

Question #22
Answer is B

For an exponential distribution the maximum likelihood estimate of the mean is the sample mean. We have

$$E(\bar{X}) = E(X) = q,\ Var(\bar{X}) = Var(X)/n = q^2/n.$$
$$cv = SD(\bar{X})/E(\bar{X}) = [q/\sqrt{n}]/q = 1/\sqrt{n} = 1/\sqrt{5} = 0.447.$$

If the above facts are not known, the loglikelihood function can be used:

$$L(q) = q^{-n}e^{-\Sigma x_j/q},\quad l(q) = -n\ln q - n\bar{X}/q,\quad l'(q) = -nq^{-1} + n\bar{X}q^{-2} = 0 \Rightarrow \hat{q} = \bar{X}.$$
$$l''(q) = nq^{-2} - 2n\bar{X}q^{-3},\quad I(q) = E[-nq^{-2} + 2n\bar{X}q^{-3}] = nq^{-2}.$$

Then, $Var(\hat{q}) = q^2/n.$

Question #23
Answer is D

Because the total expected claims for business use is 1.8, it must be that 20% of business users are rural and 80% are urban. Thus the unconditional probabilities of being business-rural and business-urban are 0.1 and 0.4 respectively. Similarly the probabilities of being pleasure-rural and pleasure-urban are also 0.1 and 0.4 respectively. Then,

$$m = 0.1(1.0) + 0.4(2.0) + 0.1(1.5) + 0.4(2.5) = 2.05$$

$$v = 0.1(0.5) + 0.4(1.0) + 0.1(0.8) + 0.4(1.0) = 0.93$$

$$a = 0.1(1.0^2) + 0.4(2.0^2) + 0.1(1.5^2) + 0.4(2.5^2) - 2.05^2 = 0.2225$$

$$k = v / a = 4.18$$

$$Z = 1/(1 + 4.18) = 0.193.$$

Question #24
Answer is C

(A) True – page 551 of *Econometric Models*
(B) True – page 553
(C) False – On page 555 the text says "Then, if the model has been specified correctly, the residuals $\hat{e}_t$ should resemble a white noise process." Answer C is almost the same, however the word "constitute" is used in place of "resemble." This makes the answer false because the residuals are only approximately white noise. That is because they are calculated from the estimated parameter values, which in turn are calculated from the observed values. This causes the residuals to be slightly correlated and thus not white noise. The error terms, which are calculated from the true parameter values do constitute a white noise process, so another way to make statement C true is to replace "residuals" with "errors" and remove the "hat" from e_t.
(D) True – page 555
(E) True – page 556

Question #25
Answer is A

No. claims	Hypothesized	Observed	Chi-square
1	250	235	$15^2/250 = 0.90$
2	350	335	$15^2/350 = 0.64$
3	240	250	$10^2/240 = 0.42$
4	110	111	$1^2/110 = 0.01$
5	40	47	$7^2/40 = 1.23$
6+	10	22	$12^2/10 = 14.40$

The last column sums to the test statistic of 17.60 with 5 degrees of freedom (there were no estimated parameters), so from the table reject at the 0.005 significance level.

Question #26
Answer is C

In part (ii) you are given that $m = 20$. In part (iii) you are given that $a = 40$. In part (iv) you are given that $v = 8{,}000$. Therefore, $k = v/a = 200$. Then,

$$\bar{X} = \frac{800(15) + 600(10) + 400(5)}{1800} = \frac{100}{9}$$

$$Z = \frac{1800}{1800 + 200} = 0.9$$

$$P_c = 0.9(100/9) + 0.1(20) = 12.$$

Question #27
Answer is C

$$\Pr(X > 30{,}000) = S(30{,}000) = \left(1 - \frac{1}{10 - 2/2}\right)\left(1 - \frac{1}{7 - 2/2}\right) = 20/27 = 0.741.$$

Question #28
Answer is A

To correct for heteroscedasticity, divide the model by something proportional to the standard deviation of the error. In this case, divide by $\sqrt{X^{-1/2}} = X^{-1/4}$ which is equivalent to multiplying by $X^{1/4}$. Doing so produces the model in answer A.

Question #29
Answer is E

For Actuary A,

$$E(S) = E(N)E(X) = 10(1.5) = 15$$

$$Var\left(S\right) = E(N)Var(X) + Var(N)E(X)^2 = 10(0) + 10(1.5)^2 = 22.5.$$

The true values are,

$$E(S^*) = E(N)E(Y) = 10(1.5) = 15$$

$$Var\left(S^*\right) = E(N)Var(Y) + Var(N)E(Y)^2 = 10(0.25) + 10(1.5)^2 = 25.$$

Also,

$$E(SS^*) = E\{E[1.5N(Y_1 + \cdots + Y_N) \mid N]\} = E[1.5N(1.5N)] = E(2.25N^2)$$
$$= 2.25[Var(N) + E(N)^2] = 2.25(10 + 100) = 247.5.$$

Then the correlation is $r = \dfrac{247.5 - 15(15)}{\sqrt{22.5(25)}} = 0.95.$

Question #30
Answer is C

The formulas are from Section 5.5 of *Loss Models*.

$$\hat{v} = \frac{3(0.536 + 0.125 + 0.172)}{3+3+3} = 0.27767.$$

$$\hat{a} = \frac{0.887 + 0.191 + 1.348 - 2(0.27767)}{500 - \frac{1}{500}(50^2 + 300^2 + 150^2)} = 0.00693.$$

Then,

$$k = 0.27767/0.00693 = 40.07,\ Z_1 = \frac{50}{50 + 40.07} = 0.55512,\ Z_2 = \frac{300}{300 + 40.07} = 0.88217,$$

$$Z_3 = \frac{150}{150 + 40.07} = 0.78918.$$

The credibility weighted mean is,

$$\hat{m} = \frac{0.55512(1.406) + 0.88217(1.298) + 0.78918(1.178)}{0.55512 + 0.88217 + 0.78918} = 1.28239.$$

The credibility premium for state 1 is

$$P_c = 0.55512(1.406) + 0.44488(1.28239) = 1.351.$$

Question #31
Answer is D

For the one-sample log-rank test, the test statistic is $\left[O(t) - E(t)\right]^2 / E(t)$.

$O(12)$ is the observed number of events at or prior to time 12 = 67

The expected number of events at or prior to time 12 is $E(12) = \sum H_0(T_j) - H_0(L_j)$. From the exponential model, $H_0(t) = 0.24t$, and so $E(12)$ is 0.24 times the total observation time for all subjects. From the table, there were 40 subjects observed for 2 months, 30 for 3, 20 for 5, 8 for 8, and 2 for 12 months for a total of 358 months. Thus, $E(12) = 0.24(358) = 85.92$. The test statistic is

$$\frac{(67 - 85.92)^2}{85.92} = 4.17.$$

Question #32
Answer is B

$\hat{y}_T(1) = 0.5(6) + 2.0 = 5$
$\hat{y}_T(2) = 0.5(5) + 2.0 = 4.5$
$\hat{y}_T(3) = 0.5(4.5) + 2.0 = 4.25.$

Question #33
Answer is D

$$E(X) = \int_d^\infty \frac{x}{q} e^{-(x-d)/q} dx = \int_0^\infty \frac{y+d}{q} e^{-y/q} dx = q + d$$
$$E(X^2) = \int_d^\infty \frac{x^2}{q} e^{-(x-d)/q} dx = \int_0^\infty \frac{y^2 + 2yd + d^2}{q} e^{-y/q} dx = 2q^2 + 2qd + d^2.$$

Both derivations use the substitution $y = x - d$ and then recognize that the various integrals are requesting moments from an ordinary exponential distribution. The method of moments solves the two equations
$q + d = 10$
$2q^2 + 2qd + d^2 = 130.6$
producing $\hat{d} = 4.468.$

It is faster to do the problem if it is noted that $X = Y + d$ where Y has an ordinary exponential distribution. Then $E(X) = E(Y) + d = q + d$ and $Var(X) = Var(Y) = q^2.$

Question #34
Answer is D

The posterior density is proportional to the product of the probability of the observed value and the prior density. Thus, $p(q \mid N > 0) \propto \Pr(N > 0 \mid q) p(q) = (1 - e^{-q}) q e^{-q}$.

The constant of proportionality is obtained from $\int_0^\infty q e^{-q} - q e^{-2q} dq = \frac{1}{1^2} - \frac{1}{2^2} = 0.75.$

The posterior density is $p(q \mid N > 0) = (4/3)(q e^{-q} - q e^{-2q})$.

Then,
$$\Pr(N_2 > 0 \mid N_1 > 0) = \int_0^\infty \Pr(N_2 > 0 \mid q) p(q \mid N_1 > 0) dq = \int_0^\infty (1 - e^{-q})(4/3)(q e^{-q} - q e^{-2q}) dq$$
$$= \frac{4}{3} \int_0^\infty q e^{-q} - 2q e^{-2q} + q e^{-3q} dq = \frac{4}{3}\left(\frac{1}{1^2} - \frac{2}{2^2} + \frac{1}{3^2}\right) = 0.8148.$$

Question #35
Answer is D

Using deviations form $(z_i = Z_i - \bar{Z},\, x_i = X_i - \bar{X},\, y_i = Y_i - \bar{Y})$, we have

$b^* = \sum w_i y_i,\ w_i = \dfrac{z_i}{\Sigma z_j x_j}$ and so b^* is a linear estimator, making (A) false. To check for bias,

$E(b^*) = \sum w_i E(y_i) = \sum w_i \boldsymbol{b} x_i = \boldsymbol{b} \sum \dfrac{z_i}{\Sigma z_j x_j} x_i = \boldsymbol{b}$, making (C) false. We know that the

ordinary least squares estimator, $b = \dfrac{\Sigma x_i y_i}{\Sigma x_i^2}$ is BLUE, so b^* cannot be BLUE, making (E) false and (D) true. Finally, HCE estimators are concerned with estimating variances, so (B) must be false.

Question #36
Answer is A

Let S be the annual aggregate losses, N the number of losses, and X the distribution of an individual loss, limited to 1,000,000. Then $E(S) = E(N)E(X)$ or 2,000,000 = $E(N)$(23,759) and so $E(N)$ = 84.1786, Because the probability of a loss exceeding 500,000 is 0.0106, the number of losses in excess of 500,000 will have a Poisson distribution with mean (0.0106)(84.1786) = 0.89229 per year. That means the number of such losses in 5 years has a Poisson distribution with mean 5(0.89229) = 4.46145. The probability of no such losses is $e^{-4.46145} = 0.01155$.

Question #37
Answer is E

The interval is centered at 2.09 and the plus/minus term is 0.46 which must equal $1.96\hat{s}$ and so $\hat{s} = 0.2347$. For the log-transformed interval we need $f = e^{1.96(0.2347)/2.09} = 1.2462$. The lower limit is 2.09/1.2462 = 1.68 and the upper limit is 2.09(1.2462) = 2.60.

Question #38
Answer is B

From item (ii), $m = 1000$ and a = 50. From item (i), v = 500. Therefore, $k = v/a$ = 10 and Z = 3/(3+10) = 3/13. Also, $\bar{X} = (750 + 1075 + 2000)/3 = 1275$. Then $P_c = (3/13)(1275) + (10/13)(1000) = 1063.46$.

Question #39
Answer is E

Using formula (17.58) on page 536 of *Econometric Models*:

$$r_1 = \frac{(1-f_1 q_1)(f_1 - q_1)}{1+q_1^2 - 2f_1 q_1}$$
$$r_1 = \frac{[1-0.8(0.3)](0.8-0.3)}{1+0.3^2-2(0.8)(0.3)} = 0.623.$$

Question #40
Answer is C

$$f(x) = p\frac{1}{100}e^{-x/100} + (1-p)\frac{1}{10{,}000}e^{-x/10{,}000}$$
$$L(100,200) = f(100)f(2000)$$
$$= \left(\frac{pe^{-1}}{100} + \frac{(1-p)e^{-0.01}}{10{,}000}\right)\left(\frac{pe^{-20}}{100} + \frac{(1-p)e^{-0.2}}{10{,}000}\right)$$