

**November 2001 - Course 4
Society of Actuaries**

****BEGINNING OF EXAMINATION****

- 1.** For a second-order autoregressive process, you are given:

$$r_1 = 0.53$$

$$r_2 = -0.22$$

Determine r_3 .

- (A) Less than -0.70
- (B) At least -0.70 , but less than -0.30
- (C) At least -0.30 , but less than 0.10
- (D) At least 0.10 , but less than 0.50
- (E) At least 0.50

2. You are given:

Claim Size	Number of Claims
0-25	30
25-50	32
50-100	20
100-200	8

Assume a uniform distribution of claim sizes within each interval.

Estimate the second raw moment of the claim size distribution.

- (A) Less than 3300
- (B) At least 3300, but less than 3500
- (C) At least 3500, but less than 3700
- (D) At least 3700, but less than 3900
- (E) At least 3900

3. You are given:

- (i) The number of claims per auto insured follows a Poisson distribution with mean I .
- (ii) The prior distribution for I has the following probability density function:

$$f(I) = \frac{(500I)^{50} e^{-500I}}{I\Gamma(50)}$$

- (iii) A company observes the following claims experience:

	Year 1	Year 2
Number of claims	75	210
Number of autos insured	600	900

The company expects to insure 1100 autos in Year 3.

Determine the expected number of claims in Year 3.

- (A) 178
- (B) 184
- (C) 193
- (D) 209
- (E) 224

4. Which of the following statements about the Product-Limit estimator is false?
- (A) The Product-Limit estimator is based on the assumption that knowledge of a censoring time for an individual provides no further information about this person's likelihood of survival at a future time had the individual continued in the study.
 - (B) If the largest study time corresponds to a death time, then the Product-Limit estimate of the survival function is undetermined beyond this death time.
 - (C) When there is no censoring or truncation, the Product-Limit estimator reduces to the empirical survival function.
 - (D) Under certain regularity conditions, the Product-Limit estimator is a nonparametric maximum likelihood estimator.
 - (E) The Product-Limit estimator is consistent.

5. You fit the following model to eight observations:

$$Y = a + bX + e$$

You are given:

$$\hat{b} = -35.69$$

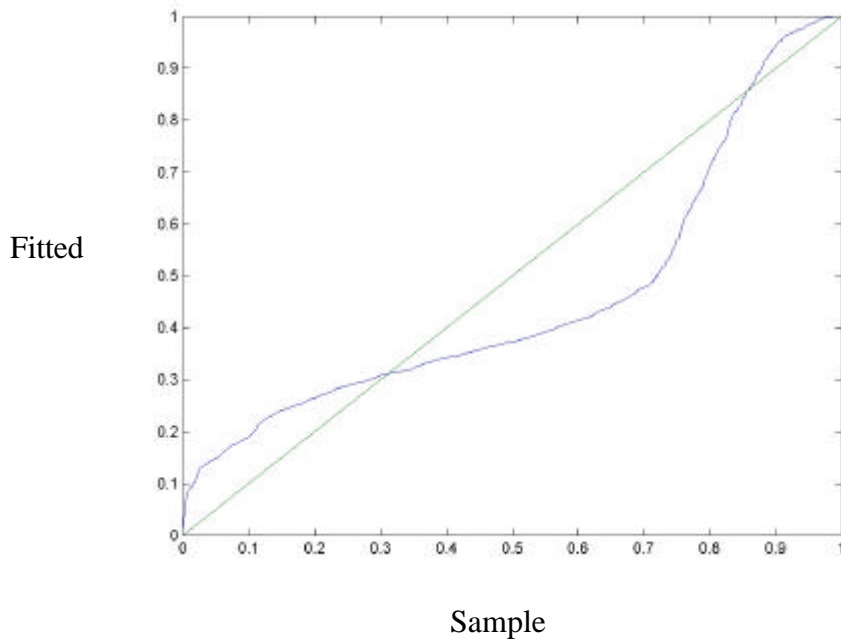
$$\sum (X_i - \bar{X})^2 = 1.62$$

$$\sum (Y_i - \hat{Y}_i)^2 = 2394$$

Determine the symmetric 90-percent confidence interval for b .

- (A) (-74.1, 2.7)
- (B) (-66.2, -5.2)
- (C) (-63.2, -8.2)
- (D) (-61.5, -9.9)
- (E) (-61.0, -10.4)

6. The graph below shows a q - q plot of a fitted distribution compared to a sample.



Which of the following is true?

- (A) The tails of the fitted distribution are too thick on the left and on the right, and the fitted distribution has less probability around the median than the sample.
- (B) The tails of the fitted distribution are too thick on the left and on the right, and the fitted distribution has more probability around the median than the sample.
- (C) The tails of the fitted distribution are too thin on the left and on the right, and the fitted distribution has less probability around the median than the sample.
- (D) The tails of the fitted distribution are too thin on the left and on the right, and the fitted distribution has more probability around the median than the sample.
- (E) The tail of the fitted distribution is too thick on the left, too thin on the right, and the fitted distribution has less probability around the median than the sample.

7. You are given the following information about six coins:

Coin	Probability of Heads
1 – 4	0.50
5	0.25
6	0.75

A coin is selected at random and then flipped repeatedly. X_i denotes the outcome of the i th flip, where “1” indicates heads and “0” indicates tails. The following sequence is obtained:

$$S = \{X_1, X_2, X_3, X_4\} = \{1, 1, 0, 1\}$$

Determine $E(X_5|S)$ using Bayesian analysis.

- (A) 0.52
- (B) 0.54
- (C) 0.56
- (D) 0.59
- (E) 0.63

8. To study the effect of smoke alarms on the size of fire insurance claims, a proportional hazards model was used on a random sample of six claims. A single covariate Z was used with $Z = 0$ indicating the absence and $Z = 1$ indicating the presence of a smoke alarm. The sizes of the claims (in standard units) were:

Without a smoke alarm: 2, 5, 7

With a smoke alarm: 1, 3, 6

The maximum likelihood estimate of the parameter \mathbf{b} was 0.6.

Using Breslow's estimate of the baseline cumulative hazard rate, estimate the probability that a claim from a property without a smoke alarm will exceed 4 units.

- (A) 0.44
- (B) 0.50
- (C) 0.57
- (D) 0.64
- (E) 0.67

9. Based on 100 observations of a time series, you determine the following sample autocorrelation coefficients:

k	1	2	3	4	5	6	7	8	9	10
\hat{r}_k	0.06	0.02	0.02	-0.16	-0.14	-0.01	0.03	-0.01	0.01	-0.04

k	11	12	13	14	15
\hat{r}_k	0.03	0.10	-0.17	0.02	0.01

You also determine that the Box-Pierce Q statistic, where $Q = T \sum_{k=1}^{15} \hat{r}_k^2$, is 9.38.

You must decide if the time series has been generated by a white noise process.

Which of the following is true?

- (A) If the time series has been generated by a white noise process, then the sample autocorrelation coefficients are distributed approximately according to a normal distribution with mean zero and standard deviation 0.1.
- (B) Because the absolute values of three of the fifteen sample autocorrelation coefficients exceed 0.1, the probability is 80% that the time series is not generated by a white noise process.
- (C) Because none of the absolute values of the fifteen sample autocorrelation coefficients exceeds 0.2, the probability is 95% that all of the true autocorrelation coefficients are simultaneously zero.
- (D) The Q statistic is approximately chi-square distributed with 85 degrees of freedom.
- (E) Because the Q statistic does not exceed its critical value at the 0.05 level of significance, the probability is 95% that the true autocorrelation coefficients are all zero.

- 10.** You observe the following five ground-up claims from a data set that is truncated from below at 100:

125 150 165 175 250

You fit a ground-up exponential distribution using maximum likelihood estimation.

Determine the mean of the fitted distribution.

- (A) 73
- (B) 100
- (C) 125
- (D) 156
- (E) 173

- 11.** An insurer writes a large book of home warranty policies. You are given the following information regarding claims filed by insureds against these policies:
- (i) A maximum of one claim may be filed per year.
 - (ii) The probability of a claim varies by insured, and the claims experience for each insured is independent of every other insured.
 - (iii) The probability of a claim for each insured remains constant over time.
 - (iv) The overall probability of a claim being filed by a randomly selected insured in a year is 0.10.
 - (v) The variance of the individual insured claim probabilities is 0.01.

An insured selected at random is found to have filed 0 claims over the past 10 years.

Determine the Bühlmann credibility estimate for the expected number of claims the selected insured will file over the next 5 years.

- (A) 0.04
- (B) 0.08
- (C) 0.17
- (D) 0.22
- (E) 0.25

12. A study of the time to first claim includes only policies issued during 1996 through 1998 on which claims occurred by the end of 1999.

The table below summarizes the information about the 50 policies included in the study:

Number of Policies			
Year of Issue	Time to First Claim		
	1 year	2 years	3 years
1996	5	9	13
1997	6	10	
1998	7		

Use the Product-Limit estimator to estimate the conditional probability that the first claim on a policy occurs less than 2 years after issue given that the claim occurs no later than 3 years after issue.

- (A) Less than 0.20
- (B) At least 0.20, but less than 0.25
- (C) At least 0.25, but less than 0.30
- (D) At least 0.30, but less than 0.35
- (E) At least 0.35

13. You fit the following model to four observations:

$$Y_i = \mathbf{b}_1 + \mathbf{b}_2 X_{2i} + \mathbf{b}_3 X_{3i} + \mathbf{e}_i, \quad i = 1, 2, 3, 4$$

You are given:

i	X_{2i}	X_{3i}
1	-3	-1
2	-1	3
3	1	-3
4	3	1

The least squares estimator of \mathbf{b}_3 is expressed as $\hat{\mathbf{b}}_3 = \sum_{i=1}^4 w_i Y_i$.

Determine (w_1, w_2, w_3, w_4) .

- (A) $(-0.15, -0.05, 0.05, 0.15)$
- (B) $(-0.05, 0.15, -0.15, 0.05)$
- (C) $(-0.05, 0.05, -0.15, 0.15)$
- (D) $(-0.3, -0.1, 0.1, 0.3)$
- (E) $(-0.1, 0.3, -0.3, 0.1)$

14. For a group of insureds, you are given:

- (i) The amount of a claim is uniformly distributed but will not exceed a certain unknown limit q .
- (ii) The prior distribution of q is $p(q) = \frac{500}{q^2}$, $q > 500$.
- (iii) Two independent claims of 400 and 600 are observed.

Determine the probability that the next claim will exceed 550.

- (A) 0.19
- (B) 0.22
- (C) 0.25
- (D) 0.28
- (E) 0.31

15. You are given the following information about a general liability book of business comprised of 2500 insureds:

- (i) $X_i = \sum_{j=1}^{N_i} Y_{ij}$ is a random variable representing the annual loss of the i^{th} insured.
- (ii) $N_1, N_2, \dots, N_{2500}$ are independent and identically distributed random variables following a negative binomial distribution with parameters $r = 2$ and $\mathbf{b} = 0.2$.
- (iii) $Y_{i1}, Y_{i2}, \dots, Y_{iN_i}$ are independent and identically distributed random variables following a Pareto distribution with $\mathbf{a} = 3.0$ and $\mathbf{q} = 1000$.
- (iv) The full credibility standard is to be within 5% of the expected aggregate losses 90% of the time.

Using classical credibility theory, determine the partial credibility of the annual loss experience for this book of business.

- (A) 0.34
- (B) 0.42
- (C) 0.47
- (D) 0.50
- (E) 0.53

16. Which of the following statements about moving-average models is false?

- (A) Both simple (unweighted) moving-average models and exponentially weighted moving-average (EWMA) models can be used to forecast future values of a time series.
- (B) EWMA models always give greater weight to more recent observations of the time series.
- (C) Forecasts using EWMA models represent true averages because the weights applied to the observations sum to one.
- (D) Moving-average forecasts are adaptive because they automatically adjust themselves to the most recently available data.
- (E) With an EWMA model, the differences between adjacent forecasted values of a time series from a fixed starting point increase as the number of steps ahead increases.

17. To estimate $E[X]$, you have simulated X_1, X_2, X_3, X_4 and X_5 with the following results:

1 2 3 4 5

You want the standard deviation of the estimator of $E[X]$ to be less than 0.05.

Estimate the total number of simulations needed.

- (A) Less than 150
- (B) At least 150, but less than 400
- (C) At least 400, but less than 650
- (D) At least 650, but less than 900
- (E) At least 900

18. You are given the following information about a book of business comprised of 100 insureds:

- (i) $X_i = \sum_{j=1}^{N_i} Y_{ij}$ is a random variable representing the annual loss of the i^{th} insured.
- (ii) N_1, N_2, \dots, N_{100} are independent random variables distributed according to a negative binomial distribution with parameters r (unknown) and $\mathbf{b} = 0.2$.
- (iii) Unknown parameter r has an exponential distribution with mean 2.
- (iv) $Y_{i1}, Y_{i2}, \dots, Y_{iN_i}$ are independent random variables distributed according to a Pareto distribution with $\mathbf{a} = 3.0$ and $\mathbf{q} = 1000$.

Determine the Bühlmann credibility factor, Z , for the book of business.

- (A) 0.000
- (B) 0.045
- (C) 0.500
- (D) 0.826
- (E) 0.905

19-20. Use the following information for questions 19 and 20.

For a mortality study of insurance applicants in two countries, you are given:

(i)

	Country A			Country B		
t_i	d_i	Y_i	q_i	d_i	Y_i	q_i
1	20	200	0.05	15	100	0.10
2	54	180	0.10	20	85	0.10
3	14	126	0.15	20	65	0.10
4	22	112	0.20	10	45	0.10

- (ii) Y_i is the number at risk over the period (t_{i-1}, t_i) . Deaths during the period (t_{i-1}, t_i) are assumed to occur at t_i .
- (iii) q_i is the reference hazard rate over the period (t_{i-1}, t_i) . Within a country, q_i is the same for all study participants.
- (iv) $S^T(t)$ is the Product-Limit estimate of $S(t)$ based on the data for all study participants.
- (v) $S^B(t)$ is the Product-Limit estimate of $S(t)$ based on the data for study participants in Country B.

19. Determine $|S^T(4) - S^B(4)|$.

- (A) 0.06
- (B) 0.07
- (C) 0.08
- (D) 0.09
- (E) 0.10

19-20. (Repeated for convenience) Use the following information for questions 19 and 20.

For a mortality study of insurance applicants in two countries, you are given:

(i)

	Country A			Country B		
t_i	d_i	Y_i	q_i	d_i	Y_i	q_i
1	20	200	0.05	15	100	0.10
2	54	180	0.10	20	85	0.10
3	14	126	0.15	20	65	0.10
4	22	112	0.20	10	45	0.10

- (ii) Y_i is the number at risk over the period (t_{i-1}, t_i) . Deaths during the period (t_{i-1}, t_i) are assumed to occur at t_i .
- (iii) q_i is the reference hazard rate over the period (t_{i-1}, t_i) . Within a country, q_i is the same for all study participants.
- (iv) $S^T(t)$ is the Product-Limit estimate of $S(t)$ based on the data for all study participants.
- (v) $S^B(t)$ is the Product-Limit estimate of $S(t)$ based on the data for study participants in Country B.

20. Calculate $\hat{A}(4)$, the estimated cumulative excess mortality at time 4 under the additive mortality model, based on the data for all study participants.

- (A) 0.21
- (B) 0.31
- (C) 0.36
- (D) 0.41
- (E) 0.52

21. Three models have been fit to 20 observations:

Model I: $Y = \mathbf{b}_1 + \mathbf{b}_2 X_2 + \mathbf{b}_3 X_3 + \mathbf{e}$

Model II: $Y = \mathbf{b}_1 + \mathbf{b}_2(X_2 + X_3) + \mathbf{e}$

Model III: $Y - X_3 = \mathbf{b}_1 + \mathbf{b}_2(X_2 - X_3) + \mathbf{e}$

You are given:

Model	<i>ESS</i>
I	484
II	925
III	982

Calculate the value of the F statistic used to test the hypothesis $H_0: \mathbf{b}_2 + \mathbf{b}_3 = 1$.

- (A) Less than 15
- (B) At least 15, but less than 16
- (C) At least 16, but less than 17
- (D) At least 17, but less than 18
- (E) At least 18

22. You fit an exponential distribution to the following data:

1000 1400 5300 7400 7600

Determine the coefficient of variation of the maximum likelihood estimate of the mean, $\hat{\mu}$.

- (A) 0.33
- (B) 0.45
- (C) 0.70
- (D) 1.00
- (E) 1.21

- 23.** You are given the following information on claim frequency of automobile accidents for individual drivers:

	Business Use		Pleasure Use	
	Expected Claims	Claim Variance	Expected Claims	Claim Variance
Rural	1.0	0.5	1.5	0.8
Urban	2.0	1.0	2.5	1.0
Total	1.8	1.06	2.3	1.12

You are also given:

- (i) Each driver's claims experience is independent of every other driver's.
- (ii) There are an equal number of business and pleasure use drivers.

Determine the Bühlmann credibility factor for a single driver.

- (A) 0.05
- (B) 0.09
- (C) 0.17
- (D) 0.19
- (E) 0.27

24. When estimating a time-series model based on T observations, which of the following is false?
- (A) Assuming normally distributed errors and setting aside the problem of determining past unobservable process values, the maximum likelihood estimators are the same as the least-squares estimators.
 - (B) The Yule-Walker equations are sufficient to provide the initial guesses for the parameter values.
 - (C) If the model has been specified correctly, the residuals $\hat{\epsilon}_t$ constitute a white noise process.
 - (D) If the model has been specified correctly, the residual autocorrelations \hat{r}_k for large displacements are themselves uncorrelated, normally distributed random variables with mean 0 and variance $1/T$.
 - (E) Several residuals \hat{r}_k much larger than $2/\sqrt{T}$ indicate that the model should be respecified.

25. You are investigating insurance fraud that manifests itself through claimants who file claims with respect to auto accidents with which they were not involved. Your evidence consists of a distribution of the observed number of claimants per accident and a standard distribution for accidents on which fraud is known to be absent. The two distributions are summarized below:

Number of Claimants per Accident	Standard Probability	Observed Number of Accidents
1	0.25	235
2	.35	335
3	.24	250
4	.11	111
5	.04	47
6+	.01	22
Total	1.00	1000

Determine the result of a chi-square test of the null hypothesis that there is no fraud in the observed accidents.

- (A) Reject at the 0.005 significance level.
- (B) Reject at the 0.010 significance level, but not at the 0.005 level.
- (C) Reject at the 0.025 significance level, but not at the 0.010 level.
- (D) Reject at the 0.050 significance level, but not at the 0.025 level.
- (E) Do not reject at the 0.050 significance level.

26. You are given the following data on large business policyholders:

- (i) Losses for each employee of a given policyholder are independent and have a common mean and variance.
- (ii) The overall average loss per employee for all policyholders is 20.
- (iii) The variance of the hypothetical means is 40.
- (iv) The expected value of the process variance is 8000.
- (v) The following experience is observed for a randomly selected policyholder:

Year	Average Loss per Employee	Number of Employees
1	15	800
2	10	600
3	5	400

Determine the Bühlmann-Straub credibility premium per employee for this policyholder.

- (A) Less than 10.5
- (B) At least 10.5, but less than 11.5
- (C) At least 11.5, but less than 12.5
- (D) At least 12.5, but less than 13.5
- (E) At least 13.5

27. You are given the following information about a group of 10 claims:

Claim Size Interval	Number of Claims in Interval	Number of Claims Censored in Interval
(0-15,000]	1	2
(15,000-30,000]	1	2
(30,000-45,000]	4	0

Assume that claim sizes and censorship points are uniformly distributed within each interval.

Estimate, using the life table methodology, the probability that a claim exceeds 30,000.

- (A) 0.67
- (B) 0.70
- (C) 0.74
- (D) 0.77
- (E) 0.80

28. You fit the model $Y = \mathbf{a} + \mathbf{b}X + \mathbf{e}$.

The error variance is proportional to $X^{-1/2}$.

Which of the following models corrects for this form of heteroscedasticity?

(A) $YX^{1/4} = \mathbf{a}X^{1/4} + \mathbf{b}X^{5/4} + \mathbf{e}^*$

(B) $YX^{1/4} = \mathbf{a} + \mathbf{b}X^{5/4} + \mathbf{e}^*$

(C) $YX^{1/2} = \mathbf{a}X^{1/2} + \mathbf{b}X^{3/2} + \mathbf{e}^*$

(D) $YX^{-1/4} = \mathbf{a}X^{-1/4} + \mathbf{b}X^{3/4} + \mathbf{e}^*$

(E) $YX^{-1/2} = \mathbf{a}X^{-1/2} + \mathbf{b}X^{1/2} + \mathbf{e}^*$

- 29.** In order to simplify an actuarial analysis Actuary A uses an aggregate distribution $S = X_1 + \dots + X_N$, where N has a Poisson distribution with mean 10 and $X_i = 15$ for all i .

Actuary A's work is criticized because the actual severity distribution is given by

$$\Pr(Y_i = 1) = \Pr(Y_i = 2) = 0.5, \text{ for all } i,$$

where the Y_i 's are independent.

Actuary A counters this criticism by claiming that the correlation coefficient between S and $S^* = Y_1 + \dots + Y_N$ is high.

Calculate the correlation coefficient between S and S^* .

- (A) 0.75
- (B) 0.80
- (C) 0.85
- (D) 0.90
- (E) 0.95

30. You are making credibility estimates for regional rating factors. You observe that the Bühlmann-Straub nonparametric empirical Bayes method can be applied, with rating factor playing the role of pure premium.

X_{ij} denotes the rating factor for region i and year j , where $i = 1, 2, 3$ and $j = 1, 2, 3, 4$. Corresponding to each rating factor is the number of reported claims, m_{ij} , measuring exposure.

You are given:

i	$m_i = \sum_{j=1}^4 m_{ij}$	$\bar{X}_i = \frac{1}{m_i} \sum_{j=1}^4 m_{ij} X_{ij}$	$\hat{v}_i = \frac{1}{3} \sum_{j=1}^4 m_{ij} (X_{ij} - \bar{X}_i)^2$	$m_i (\bar{X}_i - \bar{X})^2$
1	50	1.406	0.536	0.887
2	300	1.298	0.125	0.191
3	150	1.178	0.172	1.348

Determine the credibility estimate of the rating factor for region 1 using the method that preserves $\sum_{i=1}^3 m_i \bar{X}_i$.

- (A) 1.31
- (B) 1.33
- (C) 1.35
- (D) 1.37
- (E) 1.39

31. A study of short-term disability claims produced the following information:

- (i) The study period began January 1, 1999 and ended December 31, 2000.
- (ii) A random sample was taken of 100 individuals who began short-term disability claims sometime during the study period.
- (iii) The following results were observed:

t_i	d_i	c_i
2	30	10
3	20	10
5	10	10
8	5	3
12	2	0

where:

d_i = number of claimants who returned to work after spending t_i months on disability

c_i = number of claimants who were still on disability after t_i months as of December 31, 2000

You use the one-sample log-rank test to test whether these 100 individuals come from an exponential survival model with a hazard rate of 0.24.

Determine the value of the chi-square test statistic.

- (A) 2.1
- (B) 2.5
- (C) 3.4
- (D) 4.2
- (E) 5.3

32. You are given:

$$y_t = 0.5y_{t-1} + 2.0 + \mathbf{e}_t$$

$$y_T = 6.0$$

Calculate $\hat{y}_T(3)$, the three-period forecast.

- (A) 4.00
- (B) 4.25
- (C) 4.50
- (D) 4.75
- (E) 5.00

33. You are given:

- (i) Claim amounts follow a shifted exponential distribution with probability density function:

$$f(x) = \frac{1}{q} e^{-(x-d)/q}, \quad d < x < \infty$$

- (ii) A random sample of claim amounts X_1, X_2, \dots, X_{10} :

5 5 5 6 8 9 11 12 16 23

- (iii) $\sum X_i = 100$ and $\sum X_i^2 = 1306$

Estimate d using the method of moments.

- (A) 3.0
(B) 3.5
(C) 4.0
(D) 4.5
(E) 5.0

34. You are given:

- (i) The annual number of claims for each policyholder follows a Poisson distribution with mean q .
- (ii) The distribution of q across all policyholders has probability density function:

$$f(q) = q e^{-q}, q > 0$$

- (iii) $\int_0^{\infty} q e^{-nq} dq = \frac{1}{n^2}$

A randomly selected policyholder is known to have had at least one claim last year.

Determine the posterior probability that this same policyholder will have at least one claim this year.

- (A) 0.70
- (B) 0.75
- (C) 0.78
- (D) 0.81
- (E) 0.86

35. You observe N independent observations from a process whose true model is:

$$Y_i = \mathbf{a} + \mathbf{b} X_i + \mathbf{e}_i$$

You are given:

(i) $Z_i = X_i^2$, for $i = 1, 2, \dots, N$

(ii)
$$b^* = \frac{\sum(Z_i - \bar{Z})(Y_i - \bar{Y})}{\sum(Z_i - \bar{Z})(X_i - \bar{X})}$$

Which of the following is true?

- (A) b^* is a nonlinear estimator of \mathbf{b} .
- (B) b^* is a heteroscedasticity-consistent estimator (HCE) of \mathbf{b} .
- (C) b^* is a linear biased estimator of \mathbf{b} .
- (D) b^* is a linear unbiased estimator of \mathbf{b} , but not the best linear unbiased estimator (BLUE) of \mathbf{b} .
- (E) b^* is the best linear unbiased estimator (BLUE) of \mathbf{b} .

36. For an insurance policy, you are given:

- (i) The policy limit is 1,000,000 per loss, with no deductible.
- (ii) Expected aggregate losses are 2,000,000 annually.
- (iii) The number of losses exceeding 500,000 follows a Poisson distribution.
- (iv) The claim severity distribution has

$$\Pr(\text{Loss} > 500,000) = 0.0106$$

$$E[\min(\text{Loss}; 500,000)] = 20,133$$

$$E[\min(\text{Loss}; 1,000,000)] = 23,759$$

Determine the probability that no losses will exceed 500,000 during 5 years.

- (A) 0.01
- (B) 0.02
- (C) 0.03
- (D) 0.04
- (E) 0.05

37. A survival study gave (1.63, 2.55) as the 95% linear confidence interval for the cumulative hazard function $H(t_0)$.

Calculate the 95% log-transformed confidence interval for $H(t_0)$.

- (A) (0.49, 0.94)
- (B) (0.84, 3.34)
- (C) (1.58, 2.60)
- (D) (1.68, 2.50)
- (E) (1.68, 2.60)

38. You are given:

- (i) Claim size, X , has mean m and variance 500.
- (ii) The random variable m has a mean of 1000 and variance of 50.
- (iii) The following three claims were observed: 750, 1075, 2000

Calculate the expected size of the next claim using Bühlmann credibility.

- (A) 1025
- (B) 1063
- (C) 1115
- (D) 1181
- (E) 1266

39. For an ARMA(1,1) model, you are given:

$$y_t = 0.8y_{t-1} + 3 + \mathbf{e}_t - 0.3\mathbf{e}_{t-1}$$

Calculate r_1 .

- (A) -0.6
- (B) -0.3
- (C) 0.0
- (D) 0.3
- (E) 0.6

40. Losses come from a mixture of an exponential distribution with mean 100 with probability p and an exponential distribution with mean 10,000 with probability $1-p$.

Losses of 100 and 2000 are observed.

Determine the likelihood function of p .

- (A) $\left(\frac{pe^{-1}}{100} \cdot \frac{(1-p)e^{-0.01}}{10,000}\right) \cdot \left(\frac{pe^{-20}}{100} \cdot \frac{(1-p)e^{-0.2}}{10,000}\right)$
- (B) $\left(\frac{pe^{-1}}{100} \cdot \frac{(1-p)e^{-0.01}}{10,000}\right) + \left(\frac{pe^{-20}}{100} \cdot \frac{(1-p)e^{-0.2}}{10,000}\right)$
- (C) $\left(\frac{pe^{-1}}{100} + \frac{(1-p)e^{-0.01}}{10,000}\right) \cdot \left(\frac{pe^{-20}}{100} + \frac{(1-p)e^{-0.2}}{10,000}\right)$
- (D) $\left(\frac{pe^{-1}}{100} + \frac{(1-p)e^{-0.01}}{10,000}\right) + \left(\frac{pe^{-20}}{100} + \frac{(1-p)e^{-0.2}}{10,000}\right)$
- (E) $p \cdot \left(\frac{e^{-1}}{100} + \frac{e^{-0.01}}{10,000}\right) + (1-p) \cdot \left(\frac{e^{-20}}{100} + \frac{e^{-0.2}}{10,000}\right)$

****END OF EXAMINATION****

Final November 2001 Course 4 Exam Answer Key

Exam

<i>Test Item</i>	<i>Key</i>
1	B
2	E
3	B
4	B
5	B
6	E
7	C
8	D
9	A
10	A
11	D
12	A
13	B
14	E
15	C
16	E
17	E
18	E
19	B
20	B
21	D
22	B
23	D
24	C
25	A
26	C
27	C
28	A
29	E
30	C
31	D
32	B
33	D
34	D
35	D
36	A
37	E
38	B
39	E
40	C