Society of Actuaries Course 8V Fall 2003
****BEGINNING OF EXAMINATION****
MORNING SESSION

Questions 1-6 pertain to the Case Study.

1. *(4 points)* LifeCo is reviewing its Operational Guidelines for the Use of Derivatives for consistency with The Group of Thirty recommendations.

(a) Identify and describe the various derivatives risks as outlined in the Group of Thirty recommendations.

(b) Assess LifeCo's management of these risks associated with the derivatives used to hedge its liabilities.

Questions 1-6 pertain to the Case Study.

2. *(10 points)* LifeCo's ALM committee is concerned about the impact of the following capital market events on the company's value:

(1) Low equity market return
(2) Higher than expected credit loss
(3) Low interest rate environment

(a) Describe the minimum guarantees offered in the Variable Annuity product and how the guarantees can be priced using the
  (i) actuarial approach, and
  (ii) capital market approach.

(b) Assess the potential impact of the capital market events described above on the Variable Annuity product.

(c) Recommend risk management strategies to mitigate the risks associated with the capital market events described above on the Variable Annuity product.

(d) Describe the characteristics of the Payout Annuity product cash out flows.

(e) Assess the potential impact of the capital market events described above on the Payout Annuity product.

(f) Recommend investment strategies to mitigate the impact of the capital market events described above on the Payout Annuity product through
  (i) asset allocation, and
  (ii) use of derivative products.

Justify your recommendations.

Questions 1-6 pertain to the Case Study.

3. *(5 points)* LifeCo's ALM report for December 2000 indicates a significant mismatch in the Non-Traditional Life Products segment.

(a) *(1 Point)* Compare the use of dollar duration to effective duration as a risk metric for this product line.

(b) *(4 Points)* Describe various approaches to ALM as outlined by Glacy and Vilms and assess how each approach might be applied to the non-traditional life segment in order to mitigate the mismatch problem.

Questions 1-6 pertain to the Case Study.

4. *(8 points)* LifeCo is considering how to evaluate the performance of a new GIC product. You are given the following pricing assumptions for LifeCo's new GIC product:

Term:	3 years
Maturity Value:	123% of the initial deposit
Lapse:	None
Policyholder Options:	None
Premium Size:	$1,000
Required Return on Capital:	8% per annum
Required Capital Factor for GICs:	3.6% of liabilities
Asset Return:	8.5%
Tax Rate:	35%
Risk-Free Zero-Coupon Rate Curve:	1-year: 5.70%
	2-year: 6.08%
	3-year: 6.21%
GIC Competitors:	
Required Return on Capital:	12%
Required Capital Factor for GICs:	5% of liabilities
Asset Return:	8%

(a) Compare the "entity-specific" and "fair value" systems.

(b) Calculate the tax-adjusted liability value at issue for LifeCo's new GIC product using the Cost of Capital method.

(c) Calculate the liability value at issue and the required spread of LifeCo's new GIC product using the Total Return method as presented by Ho, Scheitlin and Tam.

(d) Describe the arguments for and against reflecting an entity's credit standing in the fair value of its liabilities.

(e) Assess how the Cost of Capital method reflects credit standing and how this might impact the valuation of LifeCo's liabilities.

Questions 1-6 pertain to the Case Study.

5. *(3 points)* LifeCo is concerned about the mismatch of its assets and liabilities at the total company level.

(a) Calculate the modified duration of surplus excluding separate accounts.

(b) Estimate the change in interest rates that would reduce total company surplus to zero.

(c) Calculate the modified duration of liabilities needed to immunize surplus.

(d) Criticize the manner in which the interest rate risk exposure is quantified in (b) and (c).

6. *(5 points)* LifeCo's ALM committee manages the investment needs and risk exposures of several business segments with very different characteristics.

(a) Explain how the target asset duration should be changed to address any imbalance between benefit duration and renewal premium duration for each of LifeCo's product lines.

(b) Describe the constraints on asset sales that must be considered when rebalancing asset segments.

7. *(3 points)*

(a) Explain the two basic properties of utility functions as described by Gerber and Pafumi.

(b) Explain the principle of equivalent utility.

8. *(4 points)* You are given the following information for zero-coupon bonds:

Maturity	Interest Rate
1-year	4%
2-year	5%
3-year	6%

(a) Construct two sample portfolios of equal value, each using zero-coupon bonds at all three maturities, such that the portfolio durations are the same.

(b) Approximate the impact on the value of the two portfolios of a 1% increase in zero-coupon interest rates at every maturity using the portfolios' modified duration.

(c) Approximate the impact on the value of the two portfolios of a 1% increase in the 1-year zero-coupon rate and no change in the 2 and 3-year zero-coupon rates using the portfolios' modified duration.

(d) Evaluate the validity of the approximations in (b) and (c).

9. *(4 points)* You maintain a model that determines the 1-month 95% confidence value at risk (VAR) of your company's guaranteed minimum accumulation benefit (GMAB) portfolio.

(a) Describe the key limitations of the VAR methodology.

(b) Describe methods that can be used to test and address these limitations.

10. *(5 points)* A large Canadian life insurer, CanCo, is about to purchase a mid-sized U.S. insurer, TargetCo. Assume:

(1) TargetCo will continue all of its existing lines of business.
(2) TargetCo will reinvest profits in its U.S. operations.
(3) TargetCo's Corporate functions will be performed in CanCo's Canadian office.
(4) TargetCo's Sales and Marketing and other lines of business functions will continue to operate in the U.S.

The post-acquisition strategic plan assumes:

	CanCo	**TargetCo**
Pro forma Cash Flow Summary		
Operational Expenses – Sales and Marketing	CAD 200	USD 50
Operational Expenses – Corporate and Other	CAD 300	CAD 75
Policy Benefits and Other Expenses	CAD 100	USD 20
Premium and Investment Income	CAD 700	USD 150
Balance Sheet (at date of acquisition)		
Assets	CAD 11,000	USD 2,500
Liabilities	CAD 9,000	USD 2,000
Surplus	CAD 2,000	USD 500

Exchange Rate
1 USD = 1.5 CAD volatility = 4%

Risk-Free Interest Rate
Canada: 4%
US: 3%

(a) Describe the financial and economic impacts that foreign exchange rate risk could have
  (i) in general, on a multinational company, and
  (ii) specifically, on CanCo and TargetCo following this acquisition.

10. Continued

(b) Determine the impact of an immediate 10% depreciation in the U.S. dollar on
(i) the consolidated surplus of CanCo and TargetCo in Canadian dollars, and
(ii) the net cash flow of TargetCo itself.

(c) Describe the external concerns associated with the purchase that may affect current and potential CanCo shareholders.

(d) Outline how options and other financial instruments could be used to manage the foreign exchange exposure of TargetCo's net cash flow position and compare TargetCo's risk exposure under each approach.

(e) Calculate the cost of a European style currency option that would protect TargetCo's net cash flow position in U.S. dollars for one year. Assume that the currency value follows a geometric Brownian motion process and all cash flows are end-of-year.

11. (*6 points*) You are given the following information.

Current index value = 800
Annual volatility of index = 20%
No dividends
Price of 1 month at-the-money European call index option = 25.025
One-month time steps for all calculations
Risk-free rate is the same for all periods

(a) Calculate the risk-free rate implied in the call price using a binomial tree.

(b) Estimate an upper and lower bound on the price of a 2-month American lookback call.

(c) Calculate the price of a 2-month American lookback call using a binomial tree.

12. *(3 points)* ABC Life has formulated an ALM strategy whereby the long-term liability cash flows of its non-participating life block are backed with equities. ABC Life does not feel that duration is an appropriate metric to measure the interest rate sensitivity of this asset class. Stochastic modeling techniques are used to measure the likelihood that the equities will be sufficient to meet the long-term cash flows beyond 20 years.

Critique the use of duration for equities in the ALM strategy for ABC Life.

****END OF EXAMINATION****
MORNING SESSION

****BEGINNING OF EXAMINATION****
AFTERNOON SESSION

13. *(3 points)* Describe the key features of derivatives risk management aimed at preventing derivatives mishaps in an insurance company setting.

14. *(4 points)* ABC Life is investigating hedging the currency exposure of its international operations.

(a) Describe the critical factors for identifying the optimal degree of currency exposure to be hedged.

(b) For each of the critical factors indicate whether the optimal hedge ratio of the currency exposure increases or decreases as the factor increases.

(c) Explain the two economic theories relating to exchange rates.

(d) Assess the impact on the hedging strategy for the currency risk assuming the economic theories are:
(i) True
(ii) False

15. *(4 points)* Given the following information.

	POOL A	POOL B
Originator	GNMA	FNMA
Type	30-year fixed	50% 30-year fixed 50% balloon
Weighted Average Coupon	8.0%	9.0%
Weighted Average Maturity	25.6 years	19.2 years
Maturity Distribution	27 years 80% 20 years 20%	25 years 20% 22 years 20% 19 years 20% 16 years 20% 14 years 20%
% Refinance Mortgagors % Home Purchase Mortgagors	20% 80%	80% 20%

Economic Environment: Strong housing market			
Current Yield Curve			
1-year Treasury	1.5%		
10-year Treasury	3.5%		
30-year Treasury	5.5%		
Rates for New Issues of 30-Year Fixed Mortgages for Calendar Year 2003 through November			
January	8.6%	July	7.5%
February	8.4%	August	7.0%
March	8.2%	September	6.5%
April	8.0%	October	6.0%
May	7.9%	November	6.5%
June	7.6%		

(a) Describe the four primary structural components of the Goldman Sachs mortgage backed securities prepayment model including
   (i) primary variables that drive change in these components, and
   (ii) expected impact of these variables on total prepayments.

(b) Compare and contrast the impact on prepayments in the two mortgage pools for each of the variables described in (a).

16. *(4 points)* You are building a model for forecasting equity real estate values in portfolio asset projections, given economic scenario inputs.

(a) List and justify the items you will need in order to build a robust model for equity real estate values.

(b) Describe a method for determining the optimal level of equity real estate in your company's various portfolios.

17. *(8 points)* You are the Chief Risk Officer of Pusillanimous Re that provides annual catastrophe reinsurance to the marketplace. You invest the asset portfolio in cash and three zero-coupon bonds, each with 1-year time to maturity. You are given the following information:

Reinsurance exposure: Excess cost layer between 100 million and 350 million of 1 billion total earthquake exposure.

	Cash	Bond A	Bond B	Bond C
Market Value (millions)	2.0	2.2	2.0	1.8
Face Value (millions)	2.0	2.4	2.4	2.4
Expected default loss (%)	0	5	not given	15
Expected recovery rate (%)	100	not given	30	45
Historical default rate (%)	0	1	2	4

You are concerned about the default risk of your asset portfolio and the magnitude of your liability exposure.

(a) Describe how you would calculate the 1-year default probability of Bond B using
(i) bond market prices, and
(ii) equity prices of the bond issuer.

(b) Calculate the 1-year expected default loss of Bond B implied by bond market prices.

(c) Explain the reasons for the differences typically observed between the default probabilities derived using historical default data and those derived from the bond market prices.

(d) Compare and contrast the use of add-up credit default swaps (CDS) and first-to-default CDS for management of the default risk in the asset portfolio.

(e) Describe how a first-to-default basket CDS on the asset portfolio can be valued using a Gaussian copula approach.

(f) Explain how CAT bonds work and how they can be used to manage the option structure of your liability risk exposure.

18. (*4 points*) The financial results at your insurance company over the past year have been dismal. Your new CFO has asked you to determine the reasons behind this. She has recently read about transfer pricing for banks, and is interested in applying this concept to your company.

Total returns for your company:

	Duration	Credit Rating	Return
Assets	10	A	5.70%
Liabilities	7	AAA (claims paying)	6.20%

Total return of generic assets over the past year:

	Credit Rating		
Duration	A	AA	AAA
5	6.00%	6.50%	7.50%
6	5.90%	6.40%	7.40%
7	5.80%	6.30%	7.30%
8	5.70%	6.20%	7.20%
9	5.60%	6.10%	7.10%
10	5.50%	6.00%	7.00%

Your CFO wants to see the returns attributed to the following four sources:

1) Liability performance
2) Asset performance due to interest rate risk
3) Asset performance due to credit risk
4) Asset performance due to selection of individual securities

(a) Construct the appropriate benchmark portfolios from the generic assets provided.

(b) Attribute your company's performance to the four sources described above, using the benchmarks constructed in (a).

(c) Describe the considerations in benchmark selection when applying transfer pricing to an insurance company.

19. *(5 points)* The current value of the assets and liabilities are respectively A_0 and L_0. The liability in five years is designated L_5. Senior management is concerned that the asset portfolio could drop in value below the liability value in five years.

One method to model the asset returns would be using a multivariate normal model and modified Cholesky decomposition with the following assumptions:

$$m = [0.12, 0.06, 0.085] = [\text{equities, fixed income, real estate}]$$

$$\text{covariance matrix} = \begin{bmatrix} 0.04 & 0.003 & 0.014 \\ 0.003 & 0.0025 & -0.0015 \\ 0.014 & -0.0015 & 0.01 \end{bmatrix}$$

(a) Evaluate potential problems in using this method to model the asset returns.

(b) Explain and evaluate the use of an option-based portfolio hedging strategy for a portfolio of assets, A_0, and liabilities, L_0, where $A_0 > L_0$, using
(i) a put, and
(ii) a call.

(c) Demonstrate that the strategies in (b) for the put and call options are equivalent, assuming that cash flows are reinvested.

(d) Explain and evaluate the use of dynamic hedging by replicating the put option using a risky portfolio and a duration-matched Treasury portfolio.

20. *(6 points)* A financial institution has sold short three European call options on a stock with the following characteristics:

Stock price:	100
Strike price:	110
Volatility:	20%
Expiry:	2 years
Contract size:	1,000 shares
Dividends:	none
Risk-free rate:	5%

(a) (4 Points) Describe and evaluate strategies the financial institution can use to manage the risk of this portfolio.

(b) (1 Point) Calculate delta, theta and gamma for the portfolio.

(c) (1 Point) Verify the calculations of delta, theta and gamma by showing that the results for the portfolio satisfy the Black-Scholes-Merton differential equation.

21. *(6 points)* A model of the type GARCH(1,1) is being used to set volatility for integer values of strike price for an option pricing model that is based on a lognormal distribution. You have the following information on the model for a specific underlying asset:

- A weight of 10% is given to a variance of 4% that is reached asymptotically as the strike price increases toward infinity.
- The model assumes returns are a constant 20%.
- A weight of 80% is given to the variance determined for the prior strike price.
- The variance is 9% for a strike price of 0.

(a) Calculate the change in volatility for each of strike prices of 1 and 2 assuming the variance for a strike price of 0 decreases from 9% to 8%.

(b) Assess whether the model with the given parameters is more appropriate for an equity option or a currency option. Support your answer by interpreting the resulting implications for the volatility smile.

(c) Describe how to determine a maximum likelihood estimate for the above model, assuming the distribution of returns conditioned on the variance is normal.

22. *(7 points)* You would like to calibrate a new Monte Carlo pricing model by comparing the model price to the Black-Scholes price of a European call option on a non-dividend paying stock. The European call option and the underlying stock have the following characteristics:

Current stock value	=	100
Expected growth rate of the stock price	=	10%
Volatility	=	25%
Exercise price	=	95
Time to maturity	=	6 months
Risk-free interest rate	=	6%
Black-Scholes price of the option	=	11.37

Assume that the percentage change in the stock price is normally distributed.

(a) Calculate the control variate technique adjustment using stratified sampling on 3 intervals.

(b) Describe additional techniques that can be used to improve the efficiency of the Monte Carlo method.

(c) Describe how the Monte Carlo method can be adapted to calculate the price of American options.

23. *(6 points)* You are given the following information about an interest rate collar with quarterly resets. Assume quarterly compounding unless otherwise indicated.

Notional amount	=	100,000,000
Strike interest rate for cap	=	7.0%
Strike interest rate for floor	=	4.5%
Time to expiration	=	2 years

Time t	Forward 3-month interest rate starting at time t	Zero-coupon interest rate to time t (continuous compounding)
9 months	5.0%	4.49%
12 months	5.2%	4.64%
15 months	5.3%	4.77%

(a) Compare the collar to a portfolio of bond options with equivalent payoffs.

(b) Calculate, using Black's model, the price of the floorlet that prevents the interest rate from being lower than the strike price for 3 months starting in one year. Assume that the volatility of the underlying three-month rate is 20%.

(c) Assess the suitability of Black's model for valuing options embedded in a mortgage backed security.

24. *(3 points)* You have been asked to review the trading costs of the following stock:

Time	Price	Number of shares traded
(Previous close is 44 1/8)		
1:40 pm	44 1/8	1000
1:42	44 1/4	2000
1:45	44	500
1:46	43 7/8	5000
1:50	43 3/4	2000
1:51	43 7/8	400
1:54	44 1/4	6000

(a) Explain why the measurement of trading costs is important.

(b) Calculate the price impact of trading for the above stock using the "money flow" system as adapted by Birinyi.

*****END OF EXAMINATION*****