May, 2001 Society of Actuaries

****BEGINNING OF EXAMINATION****

1. You are given the following information about an invertible ARMA time-series model:

$$r_1 = -0.4$$
$$r_k = 0,\ k = 2, 3, 4, \ldots$$

Determine q_1.

(A) 0.2

(B) 0.3

(C) 0.4

(D) 0.5

(E) 0.6

2. You are given:

(i) Annual claim counts follow a Poisson distribution with mean l.

(ii) The parameter l has a prior distribution with probability density function:

$$f(l) = \frac{1}{3}e^{-l/3}, \quad l > 0$$

Two claims were observed during the first year.

Determine the variance of the posterior distribution of l.

(A) 9/16

(B) 27/16

(C) 9/4

(D) 16/3

(E) 27/4

3-4. *Use the following information for questions* 3 *and* 4.

You are given the following times of first claim for five randomly selected auto insurance policies observed from time $t = 0$:

1 2 3 4 5

3. Calculate the kurtosis of this sample.

(A) 0.0

(B) 0.5

(C) 1.7

(D) 3.4

(E) 6.8

3-4. *(Repeated for convenience) Use the following information for questions* 3 *and* 4.

You are given the following times of first claim for five randomly selected auto insurance policies observed from time $t = 0$:

1 2 3 4 5

4. You are later told that one of the five times given is actually the time of policy lapse, but you are not told which one.

The smallest Product-Limit estimate of $S(4)$, the probability that the first claim occurs after time 4, would result if which of the given times arose from the lapsed policy?

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

5. A professor ran an experiment in three sections of a psychology course to show that the more digits in a number, the more difficult it is to remember. The following variables were used in a multiple regression:

X_2	=	number of digits in the number
X_3	=	1 if student was in section 1, 0 otherwise
X_4	=	1 if student was in section 2, 0 otherwise
Y	=	percentage of students correctly remembering the number

You are given:

(i) A total of 42 students participated in the study.

(ii) The regression equation $Y = b_1 + b_2X_2 + b_3X_2^2 + b_4X_3 + b_5X_4 + e$ was fit to the data and resulted in $R^2 = 0.940$.

(iii) A second regression equation $Y = g_1 + g_2X_2 + g_3X_2^2 + e^*$ was fit to the data and resulted in $R^2 = 0.915$.

Determine the value of the F statistic used to test whether class section is a significant variable.

(A) 5.4

(B) 7.3

(C) 7.7

(D) 7.9

(E) 8.3

6. You are given:

(i) The full credibility standard is 100 expected claims.

(ii) The square-root rule is used for partial credibility.

You approximate the partial credibility formula with a Bühlmann credibility formula by selecting a Bühlmann k value that matches the partial credibility formula when 25 claims are expected.

Determine the credibility factor for the Bühlmann credibility formula when 100 claims are expected.

(A) 0.44

(B) 0.50

(C) 0.80

(D) 0.95

(E) 1.00

7. You are given a sample of losses from an exponential distribution. However, if a loss is 1000 or greater, it is reported as 1000. The summarized sample is:

Reported Loss	Number	Total Amount
Less than 1000	62	28,140
1000	38	38,000
Total	100	66,140

Determine the maximum likelihood estimate of q, the mean of the exponential distribution.

(A) Less than 650

(B) At least 650, but less than 850

(C) At least 850, but less than 1050

(D) At least 1050, but less than 1250

(E) At least 1250

8. For a mortality study conducted on three individuals observed from time zero, you are given:

(i) The reference hazard rate, q, for each individual is constant over time.

(ii)

Person	Time of Death	q
1	7	0.25
2	11	0.15
3	14	0.50

Determine the estimate of $B(14)$, the cumulative relative excess mortality at time 14.

(A) Less than 1.0

(B) At least 1.0, but less than 2.0

(C) At least 2.0, but less than 3.0

(D) At least 3.0, but less than 4.0

(E) At least 4.0

9. A Dickey-Fuller unit root test was performed on 100 observations of each of three price series by estimating the unrestricted regression

$$Y_t - Y_{t-1} = a + bt + (r-1)Y_{t-1},$$

and then the restricted regression

$$Y_t - Y_{t-1} = a\,.$$

You are given:

(i)

Price	Error Sum of Squares (ESS)	
Series	Unrestricted	Restricted
I	3233.8	3552.2
II	1131.8	1300.5
III	211.1	237.0

(ii) The critical value at the 0.10 significance level for the *F* distribution calculated by Dickey and Fuller is 5.47.

For which series do you reject at the 0.10 significance level the hypothesis of a random walk?

(A) None

(B) Series I and II only

(C) Series I and III only

(D) Series II and III only

(E) Series I, II and III

10-11. *Use the following information for questions* 10 *and* 11.

(i) The claim count and claim size distributions for risks of type A are:

Number of Claims	Probabilities
0	4/9
1	4/9
2	1/9

Claim Size	Probabilities
500	1/3
1235	2/3

(ii) The claim count and claim size distributions for risks of type B are:

Number of Claims	Probabilities
0	1/9
1	4/9
2	4/9

Claim Size	Probabilities
250	2/3
328	1/3

(iii) Risks are equally likely to be type A or type B.

(iv) Claim counts and claim sizes are independent within each risk type.

(v) The variance of the total losses is 296,962.

A randomly selected risk is observed to have total annual losses of 500.

10. Determine the Bayesian premium for the next year for this same risk.

(A) 493

(B) 500

(C) 510

(D) 513

(E) 514

10-11. *(Repeated for convenience) Use the following information for questions* 10 *and* 11.

(i) The claim count and claim size distributions for risks of type A are:

Number of Claims	Probabilities
0	4/9
1	4/9
2	1/9

Claim Size	Probabilities
500	1/3
1235	2/3

(ii) The claim count and claim size distributions for risks of type B are:

Number of Claims	Probabilities
0	1/9
1	4/9
2	4/9

Claim Size	Probabilities
250	2/3
328	1/3

(iii) Risks are equally likely to be type A or type B.

(iv) Claim counts and claim sizes are independent within each risk type.

(v) The variance of the total losses is 296,962.

A randomly selected risk is observed to have total annual losses of 500.

11. Determine the Bühlmann credibility premium for the next year for this same risk.

(A) 493

(B) 500

(C) 510

(D) 513

(E) 514

12. You are given the following random observations:

0.1 0.2 0.5 1.0 1.3

You test whether the sample comes from a distribution with probability density function:

$$f(x) = \frac{2}{(1+x)^3}, \quad x > 0$$

Calculate the Kolmogorov-Smirnov statistic.

(A) 0.01

(B) 0.06

(C) 0.12

(D) 0.17

(E) 0.19

13. Applied to the model $Y_i = b_1 + b_2 X_{2i} + b_3 X_{3i} + e_i$, the method of least squares implies:

$$\sum (Y_i - \overline{Y})(X_{2i} - \overline{X}_2) = \hat{b}_2 \sum (X_{2i} - \overline{X}_2)^2 + \hat{b}_3 \sum (X_{2i} - \overline{X}_2)(X_{3i} - \overline{X}_3)$$

$$\sum (Y_i - \overline{Y})(X_{3i} - \overline{X}_3) = \hat{b}_2 \sum (X_{2i} - \overline{X}_2)(X_{3i} - \overline{X}_3) + \hat{b}_3 \sum (X_{3i} - \overline{X}_3)^2$$

You are given:

(i) $$r_{YX_2} = \frac{\sum (Y_i - \overline{Y})(X_{2i} - \overline{X}_2)}{\sqrt{\sum (Y_i - \overline{Y})^2 \sum (X_{2i} - \overline{X}_2)^2}} = 0.4$$

(ii) $$r_{YX_3} = \frac{\sum (Y_i - \overline{Y})(X_{3i} - \overline{X}_3)}{\sqrt{\sum (Y_i - \overline{Y})^2 \sum (X_{3i} - \overline{X}_3)^2}} = 0.9$$

(iii) $$r_{X_2 X_3} = \frac{\sum (X_{2i} - \overline{X}_2)(X_{3i} - \overline{X}_3)}{\sqrt{\sum (X_{2i} - \overline{X}_2)^2 \sum (X_{3i} - \overline{X}_3)^2}} = 0.6$$

Determine the value of $\hat{b}_2^*$, the standardized coefficient associated with X_2.

(A) –0.7

(B) –0.2

(C) 0.3

(D) 0.8

(E) 1.0

14-15. *Use the following information for questions* 14 *and* 15.

For a mortality study with right-censored data, you are given:

t_i	d_i	Y_i	$\dfrac{d_i}{Y_i(Y_i - d_i)}$	$\hat{S}(t_i)$	$\int_{t_i}^{\infty} \hat{S}(t)dt$
1	15	100	0.0018	0.8500	14.424
8	20	65	0.0068	0.5885	8.474
17	13	40	0.0120	0.3972	3.178
25	31	31	---	0.0000	0.000

14. Calculate $s_H(20)$, the Aalen estimate of the standard deviation of the Nelson-Aalen estimator of the cumulative hazard function at time 20.

(A) Less than 0.05

(B) At least 0.05, but less than 0.10

(C) At least 0.10, but less than 0.15

(D) At least 0.15, but less than 0.20

(E) At least 0.20

14-15. *(Repeated for convenience) Use the following information for questions* 14 *and* 15.

For a mortality study with right-censored data, you are given:

t_i	d_i	Y_i	$\frac{d_i}{Y_i(Y_i - d_i)}$	$\hat{S}(t_i)$	$\int_{t_i}^{\infty} \hat{S}(t)dt$
1	15	100	0.0018	0.8500	14.424
8	20	65	0.0068	0.5885	8.474
17	13	40	0.0120	0.3972	3.178
25	31	31	---	0.0000	0.000

15. Determine the symmetric 95% confidence interval for the mean survival time.

(A) (13.5, 17.4)

(B) (13.7, 17.2)

(C) (13.9, 17.0)

(D) (14.1, 16.8)

(E) (14.3, 16.6)

16. A sample of ten losses has the following statistics:

$$\sum_{i=1}^{10} X^{-2} = 0.00033674 \qquad \sum_{i=1}^{10} X^{0.5} = 488.97$$

$$\sum_{i=1}^{10} X^{-1} = 0.023999 \qquad \sum_{i=1}^{10} X = 31{,}939$$

$$\sum_{i=1}^{10} X^{-0.5} = 0.34445 \qquad \sum_{i=1}^{10} X^{2} = 211{,}498{,}983$$

You assume that the losses come from a Weibull distribution with $t = 0.5$.

Determine the maximum likelihood estimate of the Weibull parameter q.

(A) Less than 500

(B) At least 500, but less than 1500

(C) At least 1500, but less than 2500

(D) At least 2500, but less than 3500

(E) At least 3500

17. You are using an ARMA(1,1) model to represent a time series of 100 observations.

You have determined:

$$\hat{y}_{100}(1) = 197.0$$

$$\hat{s}_e^2 = 1.0$$

Later, you observe that y_{101} is 188.0.

Determine the updated estimate of s_e^2.

(A) 1.0

(B) 1.2

(C) 1.4

(D) 1.6

(E) 1.8

18. You are given:

(i) An individual automobile insured has annual claim frequencies that follow a Poisson distribution with mean l .

(ii) An actuary's prior distribution for the parameter l has probability density function:

$$p(l) = (0.5)5e^{-5l} + (0.5)\frac{1}{5}e^{-l/5}.$$

(iii) In the first policy year, no claims were observed for the insured.

Determine the expected number of claims in the second policy year.

(A) 0.3

(B) 0.4

(C) 0.5

(D) 0.6

(E) 0.7

19-20. *Use the following information for questions* 19 *and* 20.

During a one-year period, the number of accidents per day was distributed as follows:

Number of Accidents	Days
0	209
1	111
2	33
3	7
4	3
5	2

19. You use a chi-square test to measure the fit of a Poisson distribution with mean 0.60. The minimum expected number of observations in any group should be 5. The maximum possible number of groups should be used.

Determine the chi-square statistic.

(A) 1

(B) 3

(C) 10

(D) 13

(E) 32

19-20. *(Repeated for convenience) Use the following information for questions* 19 *and* 20.

During a one-year period, the number of accidents per day was distributed as follows:

Number of Accidents	Days
0	209
1	111
2	33
3	7
4	3
5	2

20. For these data, the maximum likelihood estimate for the Poisson distribution is $\hat{l} = 0.60$, and for the negative binomial distribution, it is $\hat{r} = 2.9$ and $\hat{b} = 0.21$.

The Poisson has a negative loglikelihood value of 385.9, and the negative binomial has a negative loglikelihood value of 382.4.

Determine the likelihood ratio test statistic, treating the Poisson distribution as the null hypothesis.

(A) –1

(B) 1

(C) 3

(D) 5

(E) 7

21. Twenty independent loss ratios $Y_1, Y_2, \ldots, Y_{20}$ are described by the model

$$Y_t = a + e_t$$

where:

$$\text{Var}(e_t) = 0.4,\ t = 1,2,\ldots,8$$
$$\text{Var}(e_t) = 0.6,\ t = 9,10,\ldots,20$$

You are given:

$$\overline{Y}_1 = \frac{1}{8}(Y_1 + Y_2 + \ldots + Y_8)$$
$$\overline{Y}_2 = \frac{1}{12}(Y_9 + Y_{10} + \ldots + Y_{20})$$

Determine the weighted least squares estimator of a in terms of $\overline{Y}_1$ and $\overline{Y}_2$.

(A) $0.3\overline{Y}_1 + 0.7\overline{Y}_2$

(B) $0.4\overline{Y}_1 + 0.6\overline{Y}_2$

(C) $0.5\overline{Y}_1 + 0.5\overline{Y}_2$

(D) $0.6\overline{Y}_1 + 0.4\overline{Y}_2$

(E) $0.7\overline{Y}_1 + 0.3\overline{Y}_2$

22. For a mortality study, you are given:

(i) Ten adults were observed beginning at age 50.

(ii) Four deaths were recorded during the study at ages 52, 55, 58 and 60. The six survivors exited the study at age 60.

(iii) H_0 is a hypothesized cumulative hazard function with values as follows:

t	$H_0(t)$
50	0.270
51	0.280
52	0.290
53	0.310
54	0.330
55	0.350
56	0.370
57	0.390
58	0.410
59	0.435
60	0.465

Determine the result of the one-sample log-rank test used to test whether the true cumulative hazard function differs from H_0.

(A) Reject at the 0.005 significance level

(B) Reject at the 0.01 significance level, but not at the 0.005 level

(C) Reject at the 0.025 significance level, but not at the 0.01 level

(D) Reject at the 0.05 significance level, but not at the 0.025 level

(E) Do not reject at the 0.05 significance level

23. You are given the following information about a single risk:

(i) The risk has m exposures in each year.

(ii) The risk is observed for n years.

(iii) The variance of the hypothetical means is a.

(iv) The expected value of the annual process variance is $w+\frac{v}{m}$.

Determine the limit of the Bühlmann-Straub credibility factor as m approaches infinity.

(A) $\dfrac{n}{n+\frac{n^2 w}{a}}$

(B) $\dfrac{n}{n+\frac{w}{a}}$

(C) $\dfrac{n}{n+\frac{v}{a}}$

(D) $\dfrac{n}{n+\frac{w+v}{a}}$

(E) 1

24. Your claims manager has asserted that a procedural change in the claims department implemented on January 1, 1997 immediately reduced claim severity by 20 percent. You use a multiple regression model to test this assertion.

For the dependent variable, *Y*, you calculate the average claim costs on closed claims by year during 1990-99.

You define the variable *X* as the year.

You also define a variable *D* as:

$$D = \begin{cases} 0 & \text{for years 1996 and prior} \\ 1 & \text{for years 1997 and later} \end{cases}$$

Assuming a lognormal error component and constant inflation over the entire period, which of the following models would be used to test the assertion?

(A) $Y = a_1^D b_1^X e$

(B) $Y = a_1 a_2^D b_1^X e$

(C) $Y = a_1 b_1^X b_2^{XD} e$

(D) $Y = a_1 a_2^D b_1^X b_2^{XD} e$

(E) $Y = a_1 a_2^D X^{b_1} e$

25. You have modeled eight loss ratios as $Y_t = a + bt + e_t$, $t = 1,2,\ldots,8$, where Y_t is the loss ratio for year t and e_t is an error term.

You have determined:

$$\begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = \begin{bmatrix} 0.50 \\ 0.02 \end{bmatrix}$$

$$Var\left(\begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix}\right) = \begin{bmatrix} 0.00055 & -0.00010 \\ -0.00010 & 0.00002 \end{bmatrix}$$

Estimate the standard deviation of the forecast for year 10, $\hat{Y}_{10} = \hat{a} + \hat{b} \cdot 10$, using the delta method.

(A) Less than 0.01

(B) At least 0.01, but less than 0.02

(C) At least 0.02, but less than 0.03

(D) At least 0.03, but less than 0.04

(E) At least 0.04

26. You are given the following claims settlement activity from the past three years for a book of auto claims:

Number Of Claims Settled			
Year Reported	Year Settled		
	1997	1998	1999
1997	6	3	1
1998		5	2
1999			4

Let L = (Year Settled !Year Reported) be a random variable describing the time lag in settling a claim.

Calculate $\Pr\left[L=1 \middle| L<3\right]$ by first estimating the survival function for right-truncated data.

(A) 0.28

(B) 0.29

(C) 0.30

(D) 0.31

(E) 0.32

27. Based on 4 years of monthly sales data, you are given:

(i)

	Original Series	12-month Average
t	y_t	$\tilde{y}_t$
June 1993	825	843
June 1994	784	804
June 1995	710	740
June 1996	918	905

(ii) $$\sum_{i=1}^{12} \tilde{z}_i = 11.9607$$

Calculate the seasonally adjusted sales for June 1996.

(A) 892

(B) 904

(C) 915

(D) 919

(E) 932

28. Two eight-sided dice, A and B, are used to determine the number of claims for an insured. The faces of each die are marked with either 0 or 1, representing the number of claims for that insured for the year.

Die	Pr(Claims = 0)	Pr(Claims = 1)
A	$\frac{1}{4}$	$\frac{3}{4}$
B	$\frac{3}{4}$	$\frac{1}{4}$

Two spinners, X and Y, are used to determine claim cost. Spinner X has two areas marked 12 and c. Spinner Y has only one area marked 12.

Spinner	Pr(Cost = 12)	Pr(Cost = c)
X	$\frac{1}{2}$	$\frac{1}{2}$
Y	1	0

To determine the losses for the year, a die is randomly selected from A and B and rolled. If a claim occurs, a spinner is randomly selected from X and Y and spun. For subsequent years, the same die and spinner are used to determine losses.

Losses for the first year are 12.

Based upon the results of the first year, you determine that the expected losses for the second year are 10.

Calculate c.

(A) 4

(B) 8

(C) 12

(D) 24

(E) 36

29. You are given:

Claim History		
Year	Group A Number of Claims	Group B Number of Claims
1989	27	21
1990	14	18
1991	15	20
1992	12	16
1993	13	21
1994	10	12
1995	9	11
1996	12	11
1997	13	19
1998	18	24

Ranked Claim History	
Group	Number of Claims
A	9
A	10
B	11
B	11
A	12
A	12
B	12
A	13
A	13
A	14
A	15
B	16
A	18
B	18
B	19
B	20
B	21
B	21
B	24
A	27

$$\mathrm{Var}_{H_0}(R) = nm\left(\frac{n+m+1}{12}\right)$$

You perform a two-tailed rank-sum test of the hypothesis that the number of claims per year for Group A and Group B come from the same distribution. You use the classical approach for approximating the *p*-value.

Determine *p*.

(A) 0.065

(B) 0.081

(C) 0.131

(D) 0.162

(E) 0.186

30. The following are ten ground-up losses observed in 1999:

18	78	125	168	250	313	410	540	677	1100

You are given:

(i) The sum of the ten losses equals 3679.

(ii) Losses are modeled using an exponential distribution with maximum likelihood estimation.

(iii) 5% inflation is expected in 2000 and 2001.

(iv) All policies written in 2001 have an ordinary deductible of 100 and a policy limit of 1000. (The maximum payment per loss is 900.)

Determine the expected amount paid per loss in 2001.

(A) 256

(B) 271

(C) 283

(D) 306

(E) 371

31. For a study in which you are performing a proportional hazards regression using the Cox model, you are given:

(i) $h(t|\mathbf{Z}) = h_0(t)\exp\left(b^t\mathbf{Z}\right)$

(ii) The covariate vectors for the three individuals studied, in the order in which they die, are as follows:

$$\mathbf{Z}_1 = \begin{pmatrix}1\\0\\0\end{pmatrix} \qquad \mathbf{Z}_2 = \begin{pmatrix}0\\1\\0\end{pmatrix} \qquad \mathbf{Z}_3 = \begin{pmatrix}0\\0\\1\end{pmatrix}$$

Determine the partial likelihood.

(A) $\dfrac{e^{b_1+b_2+b_3}}{\left(e^{b_1}+e^{b_2}+e^{b_3}\right)\left(e^{b_2}+e^{b_3}\right)e^{b_3}}$

(B) $\dfrac{e^{b_1}+e^{b_2}+e^{b_3}}{\left(e^{b_1}+e^{b_2}+e^{b_3}\right)\left(e^{b_2}+e^{b_3}\right)e^{b_3}}$

(C) $\dfrac{e^{b_1+b_2+b_3}}{\left(e^{b_1+b_2+b_3}\right)\left(e^{b_1+b_2}\right)e^{b_1}}$

(D) $\dfrac{e^{b_1+b_2+b_3}}{\left(e^{b_1+b_2+b_3}\right)\left(e^{b_2+b_3}\right)e^{b_3}}$

(E) $\dfrac{e^{b_1}+e^{b_2}+e^{b_3}}{\left(e^{b_1+b_2+b_3}\right)\left(e^{b_2+b_3}\right)e^{b_3}}$

32. You are given the following experience for two insured groups:

Group		Year			
		1	2	3	Total
1	Number of members Average loss per member	8 96	12 91	5 113	25 97
2	Number of members Average loss per member	25 113	30 111	20 116	75 113
Total	Number of members Average loss per member				100 109

$$\sum_{i=1}^{2}\sum_{j=1}^{3} m_{ij}\left(x_{ij}-\bar{x}_i\right)^2 = 2020$$

$$\sum_{i=1}^{2} m_i\left(\bar{x}_i-\bar{x}\right)^2 = 4800$$

Determine the nonparametric Empirical Bayes credibility premium for group 1, using the method that preserves total losses.

(A) 98

(B) 99

(C) 101

(D) 103

(E) 104

33. An actuary uses annual premium income from the previous year as the independent variable and loss ratio in the current year as the dependent variable in a two-variable linear regression model. Using 20 years of data, the actuary estimates the model slope coefficient with the ordinary least-squares estimator $\hat{b}$ and does not take into account that the error terms in the model follow an AR(1) model with first-order autocorrelation coefficient $r > 0$.

Which of the following statements is false?

(A) The estimator $\hat{b}$ is biased.

(B) The estimator $\hat{b}$ is consistent.

(C) The R^2 probably gives an overly optimistic picture of the success of the regression.

(D) The estimator of the standard error of $\hat{b}$ is biased downward.

(E) Use of the Cochrane-Orcutt procedure would have produced a consistent estimator of the model slope with variance probably smaller than the variance of $\hat{b}$.

34. You are given the following claims settlement activity for a book of automobile claims as of the end of 1999:

Number of Claims Settled			
Year Reported	Year Settled		
	1997	1998	1999
1997	Unknown	3	1
1998		5	2
1999			4

$L =$ (Year Settled – Year Reported) is a random variable describing the time lag in settling a claim. The probability function of L is $f_L(l) = (1-p)p^l$, for $l = 0, 1, 2, \ldots$.

Determine the maximum likelihood estimate of the parameter *p*.

(A) 3/11

(B) 7/22

(C) 1/3

(D) 3/8

(E) 7/15

35. You are studying the hazard rates associated with the time until settlement of claims. You are given the following nine observations in terms of number of months to settle a claim:

1 3 3 4* 5 7* 8 8 10

* denotes that the claim is right-censored

You are also given:

(i)

t	$\Delta\tilde{H}(t)$	$\Delta\hat{V}\left[\tilde{H}(t)\right]$
1	0.11111	0.01235
3	0.25000	0.03125
5	0.20000	0.04000
8	0.66667	0.22222
10	1.00000	1.00000

(ii) $\hat{h}(t)$, the kernel-smoothed estimate of the hazard rate, is determined using bandwidth 3 and the biweight kernel

$$K(x) = \begin{cases} \frac{15}{16}\left(1-x^2\right)^2, & -1 \le x \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

Determine the 95% log-transformed confidence interval for $h(4)$.

(A) (0.000, 0.240)

(B) (0.023, 0.240)

(C) (0.024, 0.274)

(D) (0.031, 0.320)

(E) (0.035, 0.355)

36. You are using an ARMA(p,q) model to represent a time series. You perform diagnostic checking to test whether the model was specified correctly.

Which of the following statements about diagnostic checking is incorrect?

(A) The autocorrelation function for the simulated series (the time series generated by the model) should be compared to the sample autocorrelation function of the original series.

(B) If the autocorrelation functions are not markedly different, then the next step is to analyze the residuals of the model.

(C) If the model is correctly specified, then the residuals should resemble a white-noise process.

(D) If the model is correctly specified, then the residual autocorrelations are themselves uncorrelated, normally distributed random variables with mean 0 and variance T, where T is the number of observations in the time series.

(E) The Q statistic, where $Q = T\sum_{k=1}^{K}\hat{r}_k^2$, is approximately distributed as chi-square with $(K-p-q)$ degrees of freedom.

37-38. *Use the following information for questions* 37 *and* 38.

You are given the following information about workers' compensation coverage:

(i) The number of claims for an employee during the year follows a Poisson distribution with mean

$$(100 - p)/100,$$

where p is the salary (in thousands) for the employee.

(ii) The distribution of p is uniform on the interval $(0, 100]$.

37. An employee is selected at random. No claims were observed for this employee during the year.

Determine the posterior probability that the selected employee has salary greater than 50 thousand.

(A) 0.5

(B) 0.6

(C) 0.7

(D) 0.8

(E) 0.9

37-38. *(Repeated for convenience) Use the following information for questions* 37 *and* 38.

You are given the following information about workers' compensation coverage:

(i) The number of claims for an employee during the year follows a Poisson distribution with mean

$$(100 - p)/100,$$

where p is the salary (in thousands) for the employee.

(ii) The distribution of p is uniform on the interval $(0, 100]$.

38. An employee is selected at random. During the last 4 years, the employee has had a total of 5 claims.

Determine the Bühlmann credibility estimate for the expected number of claims the employee will have next year.

(A) 0.6

(B) 0.8

(C) 1.0

(D) 1.1

(E) 1.2

39. You are modeling a claim process as a mixture of two independent distributions *A* and *B*.

You are given:

(i) Distribution *A* is exponential with mean 1.

(ii) Distribution *B* is exponential with mean 10.

(iii) Positive weight *p* is assigned to distribution *A*.

(iv) The standard deviation of the mixture is 2.

Determine *p* using the method of moments.

(A) 0.960

(B) 0.968

(C) 0.972

(D) 0.979

(E) 0.983

40. For a two-variable regression based on seven observations, you are given:

(i) $\sum\left(X_i - \overline{X}\right)^2 = 2000$

(ii) $\sum \hat{e}_i^2 = 967$

Calculate $s_{\hat{b}}$, the standard error of $\hat{b}$.

(A) 0.26

(B) 0.28

(C) 0.31

(D) 0.33

(E) 0.35

****END OF EXAMINATION****