# November 2000
# Course 3

Society of Actuaries/Casualty Actuarial Society

*Questions 1 through 36 are each worth 2 points;*
*questions 37 through 44 are each worth 1 point.*

**1.** For independent lives $(x)$ and $(y)$:

(i) $q_x = 0.05$

(ii) $q_y = 0.10$

(iii) Deaths are uniformly distributed over each year of age.

Calculate $_{0.75}q_{xy}$.

(A) 0.1088

(B) 0.1097

(C) 0.1106

(D) 0.1116

(E) 0.1125

**2.** In a clinic, physicians volunteer their time on a daily basis to provide care to those who are not eligible to obtain care otherwise. The number of physicians who volunteer in any day is uniformly distributed on the integers 1 through 5. The number of patients that can be served by a given physician has a Poisson distribution with mean 30.

Determine the probability that 120 or more patients can be served in a day at the clinic, using the normal approximation with continuity correction.

(A)    $1 - \Phi(0.68)$

(B)    $1 - \Phi(0.72)$

(C)    $1 - \Phi(0.93)$

(D)    $1 - \Phi(3.13)$

(E)    $1 - \Phi(3.16)$

**3.** A person age 40 wins 10,000 in the actuarial lottery. Rather than receiving the money at once, the winner is offered the actuarially equivalent option of receiving an annual payment of $K$ (at the beginning of each year) guaranteed for 10 years and continuing thereafter for life.

You are given:

(i) $i = 0.04$

(ii) $A_{40} = 0.30$

(iii) $A_{50} = 0.35$

(iv) $A^{1}_{40:\overline{10|}} = 0.09$

Calculate $K$.

(A) 538

(B) 541

(C) 545

(D) 548

(E) 551

**4.** Mortality for Audra, age 25, follows De Moivre's law with $w = 100$. If she takes up hot air ballooning for the coming year, her assumed mortality will be adjusted so that for the coming year only, she will have a constant force of mortality of 0.1.

Calculate the decrease in the 11-year temporary complete life expectancy for Audra if she takes up hot air ballooning.
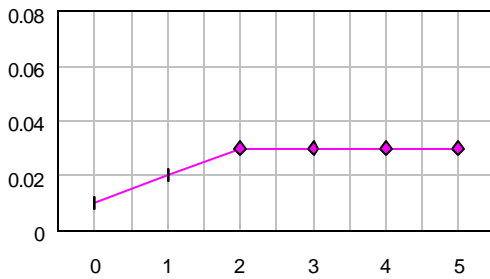
(A)    0.10

(B)    0.35

(C)    0.60

(D)    0.80

(E)    1.00

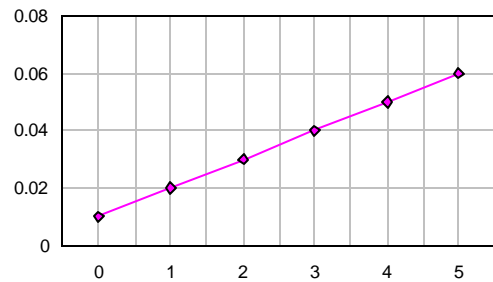**5.** For a 5-year fully continuous term insurance on (x):

(i)     $d = 0.10$

(ii)    All the graphs below are to the same scale.

(iii)   All the graphs show $m_x(t)$ on the vertical axis and t on the horizontal axis.

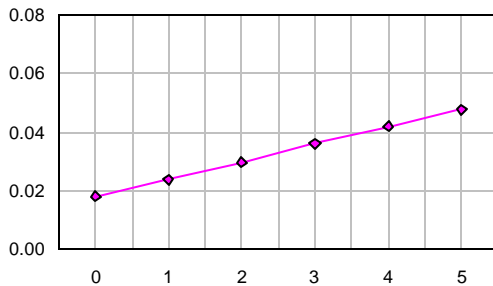Which of the following mortality assumptions would produce the highest benefit reserve at the end of year 2?

(A)

(B)

(C)

(D)
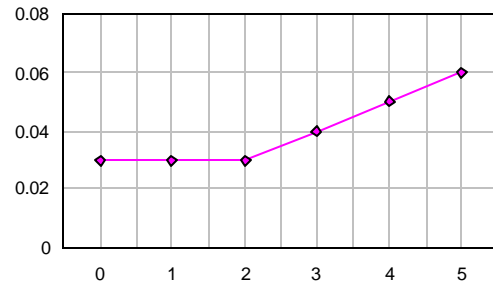
(E)
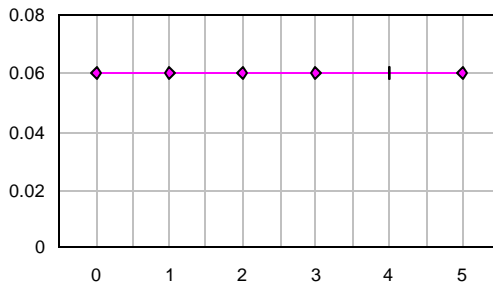
**6.** An insurance company has two insurance portfolios. Claims in Portfolio $P$ occur in accordance with a Poisson process with mean 3 per year. Claims in portfolio $Q$ occur in accordance with a Poisson process with mean 5 per year. The two processes are independent.

Calculate the probability that 3 claims occur in Portfolio $P$ before 3 claims occur in Portfolio $Q$.

(A)     0.28

(B)     0.33

(C)     0.38

(D)     0.43

(E)     0.48

**7.** For a multiple decrement table, you are given:

(i)     Decrement $(1)$ is death, decrement $(2)$ is disability, and decrement $(3)$ is withdrawal.

(ii)    $q_{60}'^{(1)} = 0.010$

(iii)   $q_{60}'^{(2)} = 0.050$

(iv)    $q_{60}'^{(3)} = 0.100$

(v)     Withdrawals occur only at the end of the year.

(vi)    Mortality and disability are uniformly distributed over each year of age in the associated single decrement tables.

Calculate $q_{60}^{(3)}$.

(A)     0.088

(B)     0.091

(C)     0.094

(D)     0.097

(E)     0.100

**8.** The number of claims, $N$, made on an insurance portfolio follows the following distribution:

| $n$ | Pr($N=n$) |
|---|---|
| 0 | 0.7 |
| 2 | 0.2 |
| 3 | 0.1 |

If a claim occurs, the benefit is 0 or 10 with probability 0.8 and 0.2, respectively.

The number of claims and the benefit for each claim are independent.

Calculate the probability that aggregate benefits will exceed expected benefits by more than 2 standard deviations.

(A)    0.02

(B)    0.05

(C)    0.07

(D)    0.09

(E)    0.12

**9.** For a special whole life insurance of 100,000 on $(x)$, you are given:

(i)   $d = 0.06$

(ii)   The death benefit is payable at the moment of death.

(iii)   If death occurs by accident during the first 30 years, the death benefit is doubled.

(iv)   $m_x^{(\tau)}(t) = 0.008, \ t \geq 0$

(v)   $m_x^{(1)}(t) = 0.001, \quad t \geq 0$, where $m_x^{(1)}$ is the force of decrement due to death by accident.

Calculate the single benefit premium for this insurance.

(A)   11,765

(B)   12,195

(C)   12,622

(D)   13,044

(E)   13,235

**10.** You are given the following extract from a select-and-ultimate mortality table with a 2- year select period:

| $x$ | $l_{[x]}$ | $l_{[x]+1}$ | $l_{x+2}$ | $x+2$ |
|-----|-----------|-------------|-----------|-------|
| 60 | 80,625 | 79,954 | 78,839 | 62 |
| 61 | 79,137 | 78,402 | 77,252 | 63 |
| 62 | 77,575 | 76,770 | 75,578 | 64 |

Assume that deaths are uniformly distributed between integral ages.

Calculate $_{0.9}q_{[60]+0.6}$.

(A)     0.0102

(B)     0.0103

(C)     0.0104

(D)     0.0105

(E)     0.0106

**11.** Customers arrive at a service center in accordance with a Poisson process with mean 10 per hour. There are two servers, and the service time for each server is exponentially distributed with a mean of 10 minutes. If both servers are busy, customers wait and are served by the first available server. Customers always wait for service, regardless of how many other customers are in line.

Calculate the percent of the time, on average, when there are no customers being served.

(A)    0.8%

(B)    3.9%

(C)    7.2%

(D)    9.1%

(E)    12.7%

**12.** For a special single premium 20-year term insurance on (70):

(i) The death benefit, payable at the end of the year of death, is equal to 1000 plus the benefit reserve.

(ii) $q_{70+t} = 0.03, \quad t \geq 0$

(iii) $i = 0.07$

Calculate the single benefit premium for this insurance.

(A) 216

(B) 267

(C) 318

(D) 369

(E) 420

**13.** A claim count distribution can be expressed as a mixed Poisson distribution. The mean of the Poisson distribution is uniformly distributed over the interval [0,5].

Calculate the probability that there are 2 or more claims.

(A)    0.61

(B)    0.66

(C)    0.71

(D)    0.76

(E)    0.81

**14.** For a double decrement table with $l_{40}^{(t)} = 2000$:

| $x$ | $q_x^{(1)}$ | $q_x^{(2)}$ | $q_x'^{(1)}$ | $q_x'^{(2)}$ |
|---|---|---|---|---|
| 40 | 0.24 | 0.10 | 0.25 | $y$ |
| 41 | -- | -- | 0.20 | $2y$ |

Calculate $l_{42}^{(t)}$.

(A)    800

(B)    820

(C)    840

(D)    860

(E)    880

**15.** You own one share of a stock. The price is 18. The stock price changes according to a standard Brownian motion process, with time measured in months.

Calculate the probability that the stock reaches a price of 21 at some time within the next 4 months.

(A)     0.0668

(B)     0.0735

(C)     0.0885

(D)     0.1096

(E)     0.1336

**16.** For a fully discrete whole life insurance of 10,000 on (30):

(i) $p$ denotes the annual premium and $L(p)$ denotes the loss-at-issue random variable for this insurance.

(ii) Mortality follows the Illustrative Life Table.

(iii) $i=0.06$

Calculate the lowest premium, $p'$, such that the probability is less than 0.5 that the loss $L(p')$ is positive.

(A) 34.6

(B) 36.6

(C) 36.8

(D) 39.0

(E) 39.1

**17.** You are such a fan of a weekly television drama about a big city hospital that you begin tracking the arrival and departure of patients on the show. Over the course of a season, you determine the following:

(i)      Patients arrive according to a Poisson process with mean 5 per hour.

(ii)     The time from arrival to departure for each patient is exponentially distributed with mean 30 minutes.

(iii)    There is no limit to the number of doctors available to treat patients.

(iv)    Patients who do not depart the show by the end of an episode appear on the following week's show without a loss of time.

Calculate the limiting probability that there are fewer than three patients on the show at any given time.

(A)    0.544

(B)    0.597

(C)    0.651

(D)    0.704

(E)    0.758

**18.** For a fully discrete whole life insurance of 1000 on (40), you are given:

(i) The contract premium is equal to the benefit premium.

(ii) $L$ is the loss variable for the one-year term insurance in the eleventh contract year with a benefit equal to the amount at risk on the whole life insurance.

(ii) Mortality follows the Illustrative Life Table.

(iii) $i = 0.06$

(iv) $K(40)$ is the curtate-future-lifetime random variable.

Calculate the conditional variance of $L$, given that $K(40) \geq 10$.

(A) 3900

(B) 4000

(C) 4100

(D) 4200

(E) 4300

**19.** A community is able to obtain plasma at the continuous rate of 22 units per day. The daily demand for plasma is modeled by a compound Poisson process where the number of people needing plasma has mean 20 and the number of units needed by each person is approximated by an exponential distribution with mean 1. Assume all plasma can be used without spoiling.

At the beginning of the period there are 20 units available.

Calculate the probability that there will not be enough plasma at some time.

(A)   0.11

(B)   0.12

(C)   0.13

(D)   0.14

(E)   0.15

**20.** $Y$ is the present-value random variable for a special 3-year temporary life annuity-due on $(x)$. You are given:

(i) $\quad {}_t p_x = 0.9^t, \quad t \geq 0$

(ii) $\quad K$ is the curtate-future-lifetime random variable for $(x)$.

(iii) $\quad Y = \begin{cases} 1.00, & K = 0 \\ 1.87, & K = 1 \\ 2.72, & K = 2,3,.... \end{cases}$

Calculate Var$(Y)$.

(A) 0.19

(B) 0.30

(C) 0.37

(D) 0.46

(E) 0.55

**21.** A claim severity distribution is exponential with mean 1000. An insurance company will pay the amount of each claim in excess of a deductible of 100.

Calculate the variance of the amount paid by the insurance company for one claim, including the possibility that the amount paid is 0.

(A)    810,000

(B)    860,000

(C)    900,000

(D)    990,000

(E)    1,000,000

**22.** For a fully continuous whole life insurance of 1 on $(x)$:

(i)  $p$ is the benefit premium.

(ii)  $L$ is the loss-at-issue random variable with the premium equal to $p$.

(iii)  $L^*$ is the loss-at-issue random variable with the premium equal to 1.25 $p$.

(iv)  $\bar{a}_x = 5.0$

(v)  $d = 0.08$

(vi)  $\text{Var}(L) = 0.5625$

Calculate the sum of the expected value and the standard deviation of $L^*$.

(A)  0.59

(B)  0.71

(C)  0.86

(D)  0.89

(E)  1.01

**23.** Workers' compensation claims are reported according to a Poisson process with mean 100 per month. The number of claims reported and the claim amounts are independently distributed. 2% of the claims exceed 30,000.

Calculate the number of complete months of data that must be gathered to have at least a 90% chance of observing at least 3 claims each exceeding 30,000.

(A)    1

(B)    2

(C)    3

(D)    4

(E)    5

**24.** For students entering a three-year law school, you are given:

(i)     The following double decrement table:

| Academic Year | For a student at the beginning of that academic year, probability of | | |
| --- | --- | --- | --- |
| | Academic Failure | Withdrawal for All Other Reasons | Survival Through Academic Year |
| 1 | 0.40 | 0.20 | -- |
| 2 | -- | 0.30 | -- |
| 3 | -- | -- | 0.60 |

(ii)    Ten times as many students survive year 2 as fail during year 3.

(iii)    The number of students who fail during year 2 is 40% of the number of students who survive year 2.

Calculate the probability that a student entering the school will withdraw for reasons other than academic failure before graduation.

(A)    Less than 0.35

(B)    At least 0.35, but less than 0.40

(C)    At least 0.40, but less than 0.45

(D)    At least 0.45, but less than 0.50

(E)    At least 0.50

**25.** Given:

(i) Superscripts $M$ and $N$ identify two forces of mortality and the curtate expectations of life calculated from them.

(ii) $$m_{25}^{N}(t) = \begin{cases} m_{25}^{M}(t) + 0.1 * (1-t) & 0 \le t \le 1 \\ m_{25}^{M}(t) & t > 1 \end{cases}$$

(iii) $e_{25}^{M} = 10.0$

Calculate $e_{25}^{N}$.

(A) 9.2

(B) 9.3

(C) 9.4

(D) 9.5

(E) 9.6

**26.** A fund is established to pay annuities to 100 independent lives age $x$. Each annuitant will receive 10,000 per year continuously until death. You are given:

(i) $d = 0.06$

(ii) $\overline{A}_x = 0.40$

(iii) $^2\overline{A}_x = 0.25$

Calculate the amount (in millions) needed in the fund so that the probability, using the normal approximation, is 0.90 that the fund will be sufficient to provide the payments.

(A) 9.74

(B) 9.96

(C) 10.30

(D) 10.64

(E) 11.10

**27.** Total hospital claims for a health plan were previously modeled by a two-parameter Pareto distribution with $a = 2$ and $q = 500$.

The health plan begins to provide financial incentives to physicians by paying a bonus of 50% of the amount by which total hospital claims are less than 500. No bonus is paid if total claims exceed 500.

Total hospital claims for the health plan are now modeled by a new Pareto distribution with $a = 2$ and $q = K$. The expected claims plus the expected bonus under the revised model equals expected claims under the previous model.

Calculate $K$.

(A)     250

(B)     300

(C)     350

(D)     400

(E)     450

**28.** A decreasing term life insurance on (80) pays (20-$k$) at the end of the year of death if (80) dies in year $k+1$, for $k=0,1,2,\ldots,19$.

You are given:

(i)   $i=0.06$

(ii)   For a certain mortality table with $q_{80} = 0.2$, the single benefit premium for this insurance is 13.

(iii)   For this same mortality table, except that $q_{80} = 0.1$, the single benefit premium for this insurance is $P$.

Calculate $P$.

(A)   11.1

(B)   11.4

(C)   11.7

(D)   12.0

(E)   12.3

**29.** Job offers for a college graduate arrive according to a Poisson process with mean 2 per month. A job offer is acceptable if the wages are at least 28,000. Wages offered are mutually independent and follow a lognormal distribution with $m = 10.12$ and $s = 0.12$.

Calculate the probability that it will take a college graduate more than 3 months to receive an acceptable job offer.

(A)  0.27

(B)  0.39

(C)  0.45

(D)  0.58

(E)  0.61

**30.** For independent lives (50) and (60):

$$\mu(x) = \frac{1}{100 - x}, \qquad 0 \le x < 100$$

Calculate $\overset{\circ}{e}_{\overline{50:60}}$.

(A)     30

(B)     31

(C)     32

(D)     33

(E)     34

**31.** For an industry-wide study of patients admitted to hospitals for treatment of cardiovascular illness in 1998, you are given:

(i)

| Duration In Days | Number of Patients Remaining Hospitalized |
|---|---|
| 0 | 4,386,000 |
| 5 | 1,461,554 |
| 10 | 486,739 |
| 15 | 161,801 |
| 20 | 53,488 |
| 25 | 17,384 |
| 30 | 5,349 |
| 35 | 1,337 |
| 40 | 0 |

(ii) Discharges from the hospital are uniformly distributed between the durations shown in the table.

Calculate the mean residual time remaining hospitalized, in days, for a patient who has been hospitalized for 21 days.

(A) 4.4

(B) 4.9

(C) 5.3

(D) 5.8

(E) 6.3

**32.** For an individual over 65:

(i)     The number of pharmacy claims is a Poisson random variable with mean 25.

(ii)    The amount of each pharmacy claim is uniformly distributed between 5 and 95.

(iii)   The amounts of the claims and the number of claims are mutually independent.


Determine the probability that aggregate claims for this individual will exceed 2000 using the
normal approximation.

(A)     $1 - \Phi(1.33)$

(B)     $1 - \Phi(1.66)$

(C)     $1 - \Phi(2.33)$

(D)     $1 - \Phi(2.66)$

(E)     $1 - \Phi(3.33)$

**33.** For a fully discrete three-year endowment insurance of 10,000 on (50), you are given:

(i)      $i = 0.03$

(ii)      $1000 q_{50} = 8.32$

(iii)      $1000 q_{51} = 9.11$

(iv)      $10{,}000 \, {}_1V_{50:\overline{3}|} = 3209$

(v)      $10{,}000 \, {}_2V_{50:\overline{3}|} = 6539$

(vi)      ${}_0L$ is the prospective loss random variable at issue, based on the benefit premium.

Calculate the variance of ${}_0L$.

(A)      277,000

(B)      303,000

(C)      357,000

(D)      403,000

(E)      454,000

**34.** The Town Council purchases weather insurance for their annual July 14 picnic.

(i) For each of the next three years, the insurance pays 1000 on July 14 if it rains on that day.

(ii) Weather is modeled by a Markov chain, with two states, as follows:
- The probability that it rains on a day is 0.50 if it rained on the prior day.
- The probability that it rains on a day is 0.20 if it did not rain on the prior day.

(iii) $i = 0.10$

Calculate the single benefit premium for this insurance purchased one year before the first scheduled picnic.

(A) 340

(B) 420

(C) 540

(D) 710

(E) 760

**35.** For a special 30-year deferred annual whole life annuity-due of 1 on (35):

(i) If death occurs during the deferral period, the single benefit premium is refunded without interest at the end of the year of death.

(ii) $\ddot{a}_{65} = 9.90$

(iii) $A_{35:\overline{30|}} = 0.21$

(iv) $A^{1}_{35:\overline{30|}} = 0.07$

Calculate the single benefit premium for this special deferred annuity.

(A) 1.3

(B) 1.4

(C) 1.5

(D) 1.6

(E) 1.7

**36.** Given:

(i)    $m(x) = F + e^{2x}$,    $x \geq 0$

(ii)    $_{0.4}p_0 = 0.50$

Calculate $F$.

(A)    -0.20

(B)    -0.09

(C)    0.00

(D)    0.09

(E)    0.20

*Questions 1 through 36 are each worth 2 points;*
*questions 37 through 44 are each worth 1 point.*

**37-38** *Use the following information for questions 37 and 38.*

A complaint department has one worker. Customers arrive according to a non-homogeneous Poisson process with intensity function $l(t) = 5 - t$, for $t < 5$. Customers always wait to be served.

(i)     Arrival times are simulated using the thinning algorithm with $l = 5$.

(ii)    Random numbers are from the uniform distribution on [0,1].

(iii)   Use the following random numbers to simulate the potential arrival times, where low random numbers correspond to longer times:
              0.10,   0.05,   0.24,   0.01

(iv)    Use the following random numbers to determine acceptance or rejection where low numbers indicate acceptance:
              0.50,   0.94,   0.62,   0.47

(v)     Simulated service times are:
              1.1,    0.8,    0.9

**37.** Calculate the arrival time of the third person.

(A)     1.6

(B)     1.7

(C)     1.9

(D)     2.0

(E)     2.3

**37-38** *(Repeated for convenience.) Use the following information for questions 37 and 38.*

A complaint department has one worker. Customers arrive according to a non-homogeneous Poisson process with intensity function $\mathbf{1}(t) = 5 - t$, for $t < 5$. Customers always wait to be served.

(vi)     Arrival times are simulated using the thinning algorithm with $\mathbf{1} = 5$.

(vii)     Random numbers are from the uniform distribution on [0,1].

(viii)     Use the following random numbers to simulate the potential arrival times, where low random numbers correspond to longer times:
          0.10,   0.05,   0.24,   0.01

(ix)     Use the following random numbers to determine acceptance or rejection where low numbers indicate acceptance:
          0.50,   0.94,   0.62,   0.47

(x)     Simulated service times are:
          1.1,   0.8,   0.9

**38.**   Determine the status of the system at time 2.

(A)     0 have completed service, 1 is being served, 0 are waiting for service

(B)     0 have completed service, 1 is being served, 1 is waiting for service

(C)     1 has completed service, 1 is being served, 0 are waiting for service

(D)     1 has completed service, 1 is being served, 1 is waiting for service

(E)     2 have completed service, 1 is being served, 0 are waiting for service

**39-40** *Use the following information for questions 39 and 40.*

For a new car dealer commencing business:

(i)     The dealer has neither assets nor liabilities.

(ii)    In exchange for continuous payments at the rate of expected sales, the car dealer receives cars on demand from the manufacturer.

(iii)   Car sales occur in accordance with a compound Poisson process at the rate of 12 cars per month.

(iv)    The price of each car sold is either 20,000 or 30,000, with equal probability.

(v)     The dealer is in a positive position when cumulative sales exceed cumulative payments to the car manufacturer.

**39.**  Calculate the probability that the dealer will ever be in a positive position.

(A)     0.00

(B)     0.25

(C)     0.50

(D)     0.75

(E)     1.00

**39-40** *(Repeated for convenience.)  Use the following information for questions 39 and 40.*

For a new car dealer commencing business:

(vi)    The dealer has neither assets nor liabilities.

(vii)   In exchange for continuous payments at the rate of expected sales, the car dealer receives cars on demand from the manufacturer.

(viii)  Car sales occur in accordance with a compound Poisson process at the rate of 12 cars per month.

(ix)    The price of each car sold is either 20,000 or 30,000, with equal probability.

(x)     The dealer is in a positive position when cumulative sales exceed cumulative payments to the car manufacturer.

**40.**  Given that the dealer will attain a positive position, calculate the conditional expected value of the amount of the first positive position.

(A)    10,000

(B)    13,000

(C)    15,000

(D)    17,000

(E)    20,000

**41-42** *Use the following information for questions 41 and 42.*

An insurer has excess-of-loss reinsurance on auto insurance. You are given:

(i)     Total expected losses in the year 2001 are 10,000,000.

(ii)    In the year 2001 individual losses have a Pareto distribution with

$$F(x) = 1 - \left(\frac{2000}{x+2000}\right)^2, \; x > 0.$$

(iii)   Reinsurance will pay the excess of each loss over 3000.

(iv)    Each year, the reinsurer is paid a ceded premium, $C_{year}$, equal to 110% of the expected losses covered by the reinsurance.

(v)     Individual losses increase 5% each year due to inflation.

(vi)    The frequency distribution does not change.


**41.**  Calculate $C_{2001}$.


(A)    2,200,000

(B)    3,300,000

(C)    4,400,000

(D)    5,500,000

(E)    6,600,000

**41-42** *(Repeated for convenience.)  Use the following information for questions 41 and 42.*

An insurer has excess-of-loss reinsurance on auto insurance.  You are given:

(vii)     Total expected losses in the year 2001 are 10,000,000.

(viii)    In the year 2001, individual losses have a Pareto distribution with

$$F(x) = 1 - \left( \frac{2000}{x + 2000} \right)^{2}, \ x > 0.$$

(ix)      Reinsurance will pay the excess of each loss over 3000.

(x)       Each year, the reinsurer is paid a ceded premium, $C_{year}$, equal to 110% of the expected losses covered by the reinsurance.

(xi)      Individual losses increase 5% each year due to inflation.

(xii)     The frequency distribution does not change.


**42.**   Calculate $C_{2002} / C_{2001}$ .


(A)     1.04

(B)     1.05

(C)     1.06

(D)     1.07

(E)     1.08

**43-44** *Use the following information for questions 43 and 44.*

Lucky Tom finds coins on his 60 minute walk to work at a Poisson rate of 2 per minute. 60% of the coins are worth 1 each; 20% are worth 5 each; 20% are worth 10 each. The denominations of the coins found are independent.

Two actuaries are simulating Tom's 60 minute walk to work.

(i) The first actuary begins by simulating the number of coins found, using the procedure of repeatedly simulating the time until the next coin is found, until the length of the walk has been exceeded. For each coin found, he simulates its denomination, using the inverse transform algorithm. The expected number of random numbers he needs for one simulation of the walk is $F$.

(ii) The second actuary uses the same algorithm for simulating the times between events. However, she first simulates finding coins worth 1, then simulates finding coins worth 5, then simulates finding coins worth 10, in each case simulating until the 60 minutes are exceeded. The expected number of random numbers she needs for one complete simulation of Tom's walk is $G$.

**43.** Which of the following statements is true?

(A) Neither is a valid method for simulating the process.

(B) The first actuary's method is valid; the second actuary's is not.

(C) The second actuary's method is valid; the first actuary's is not.

(D) Both methods are valid, but they may produce different results from the same sequence of random numbers.

(E) Both methods are valid, and will produce identical results from the same sequence of random numbers.

**43-44** *(Repeated for convenience). Use the following information for questions 43 and 44.*

Lucky Tom finds coins on his 60 minute walk to work at a Poisson rate of 2 per minute. 60% of the coins are worth 1 each; 20% are worth 5 each; 20% are worth 10 each. The denominations of the coins found are independent.

Two actuaries are simulating Tom's 60 minute walk to work.

(i)     The first actuary begins by simulating the number of coins found, using the procedure of repeatedly simulating the time until the next coin is found, until the length of the walk has been exceeded. For each coin found, he simulates its denomination, using the inverse transform algorithm. The expected number of random numbers he needs for one simulation of the walk is $F$.

(ii)     The second actuary uses the same algorithm for simulating the times between events. However, she first simulates finding coins worth 1, then simulates finding coins worth 5, then simulates finding coins worth 10, in each case simulating until the 60 minutes are exceeded. The expected number of random numbers she needs for one complete simulation of Tom's walk is $G$.

**44.** Determine which of the following ranges contains the ratio $F/G$.

(A)     $0.0 < F/G \leq 0.4$

(B)     $0.4 < F/G \leq 0.8$

(C)     $0.8 < F/G \leq 1.2$

(D)     $1.2 < F/G \leq 1.6$

(E)     $1.6 < F/G$

**\*\* END OF EXAMINATION \*\***

# Solutions for November 2000 Course 3 Exam

**Test Question:   1**          **Key:  B**

$$_{0.75}p_x = 1 - (0.75)(0.05)$$
$$= 0.9625$$
$$_{0.75}p_y = 1 - (0.75)(.10)$$
$$= 0.925$$
$$_{0.75}q_{xy} = 1 - _{0.75}p_{xy}$$
$$= 1 - \left(_{0.75}p_x\right)\left(_{0.75}p_y\right) \text{ since independent}$$
$$= 1 - (0.9625)(0.925)$$
$$= 0.1097$$

**Test Question:   2**          **Key:  A**

| | | |
|---|---|---|
| N = number of physicians | E(N) = 3 | Var (N) = 2 |
| X = visits per physician | E(X) = 30 | Var (X) = 30 |
| S = total visits | | |

$$E(S) = E(N)\,E(X) = 90$$
$$Var(S) = E(N)\,Var(X) + E^2(X)\,Var(N) =$$
$$= 3 \cdot 30 + 900 \cdot 2 = 1890$$
Standard deviation (S) = 43.5

$$Pr(S > 119.5) = Pr\left(\frac{S-90}{43.5} > \frac{119.5-90}{43.5}\right) = 1 - \Phi(0.68) \text{ Course 3: November 2000}$$

**Test Question: 3**        **Key: A**

The person receives $K$ per year guaranteed for 10 years $\Rightarrow K\ddot{a}_{\overline{10|}} = 8.4353K$

The person receives $K$ per years alive starting 10 years from now $\Rightarrow {}_{10|}\ddot{a}_{40}K$

*Hence we have $10000 = \left(8.4353 + {}_{10}E_{40}\ddot{a}_{50}\right)K$

Derive ${}_{10}E_{40}$:

$$A_{40} = A^1_{40:\overline{10|}} + \left({}_{10}E_{40}\right)A_{50}$$

$${}_{10}E_{40} = \frac{A_{40} - A^1_{40:\overline{10|}}}{A_{50}} = \frac{0.30 - 0.09}{035} = 0.60$$

Derive $\ddot{a}_{50} = \dfrac{1 - A_{50}}{d} = \dfrac{1 - 0.35}{\dfrac{.04}{1.04}} = 16.90$

Plug in values:

$$10{,}000 = \left(8.4353 + (0.60)(16.90)\right)K$$
$$= 18.5753K$$
$$K = 538.35$$


**Test Question: 4**        **Key: D**

STANDARD: $\mathring{e}_{25:\overline{11|}} = \int_0^{11}\left(1 - \dfrac{t}{75}\right)dt = t - \dfrac{t^2}{2\times75}\Big|_0^{11} = 10.1933$

MODIFIED: $p_{25} = e^{-\int_0^1 0.1ds} = e^{-1} = 0.90484$

$$\mathring{e}_{25:\overline{11|}} = \int_0^1 {}_tp_{25}dt + p_{25}\int_0^{10}\left(1 - \dfrac{t}{74}\right)dt$$

$$= \int_0^1 e^{-0.1t}dt + e^{-0.1}\int_0^{10}\left(1 - \dfrac{t}{74}\right)dt$$

$$= \dfrac{1 - e^{-0.1}}{0.1} + e^{-0.1}\left(t - \dfrac{t^2}{2\times74}\right)\Big|_0^{10}$$

$$= 0.95163 + 0.90484(9.32432) = 9.3886$$

Difference        $= 0.8047$

**Test Question: 5**     **Key: B**

Comparing B & D: Prospectively at time 2, they have the same future benefits. At issue, B has the lower benefit premium. Thus, by formula 7.2.2, B has the higher reserve.

Comparing A to B: use formula 7.3.5. At issue, B has the higher benefit premium. Until time 2, they have had the same benefits, so B has the higher reserve.

Comparing B to C: Visualize a graph C* that matches graph B on one side of t=2 and matches graph C on the other side. By using the logic of the two preceding paragraphs, C's reserve is lower than C*'s which is lower than B's.

Comparing B to E: Reserves on E are constant at 0.

**Test Question: 6**     **Key: A**

For each claim, probability it is from portfolio $P = \dfrac{3}{8}$.

Prob (3 from $P$ before 3 from $Q$) = Prob (3 or more of first 5 are from $P$)

$$
\begin{aligned}
&= \binom{5}{3}\left(\tfrac{3}{8}\right)^3\left(\tfrac{5}{8}\right)^2 + \binom{5}{4}\left(\tfrac{3}{8}\right)^4\left(\tfrac{5}{8}\right)^1 + \binom{5}{5}\left(\tfrac{3}{8}\right)^5\left(\tfrac{5}{8}\right)^0 \\
&= \frac{5 \times 4 \times 3^3 \times 5^2}{2 \times 8^5} + \frac{5 \times 3^4 \times 5}{8^5} + \frac{3^5}{8^5} \\
&= \frac{6750 + 2025 + 243}{32,768} \\
&= 0.2752
\end{aligned}
$$

**Test Question: 7**     **Key: C**

Since only decrements (1) and (2) occur during the year, probability of reaching the end of the year is
$$
p_{60}'^{(1)} \times p_{60}'^{(2)} = (1 - 0.01)(1 - 0.05) = 0.9405
$$

Probability of remaining through the year is
$$
p_{60}'^{(1)} \times p_{60}'^{(2)} \times p_{60}'^{(3)} = (1 - 0.01)(1 - 0.05)(1 - 0.10) = 0.84645
$$

Probability of exiting at the end of the year is
$$
q_{60}^{(3)} = 0.9405 - 0.84645 = 0.09405
$$

**Test Question:   8**          **Key:  E**

$E(N) = .7$

$Var(N) = 4 \times .2 + 9 \times .1 - .49 = 1.21$

$E(X) = 2$

$Var(X) = 100 \times .2 - 4 = 16$

$E(S) = 2 \times .7 = 1.4$

$Var(S) = E(N)\,Var(X) + E^2(X)\,Var(N) =$

$\qquad = .7 \times 16 + 4 \times 1.21 = 16.04$

Standard $Dev(S) = 4$

$E(S) + 2 \times$ Standard $Dev(S) = 1.4 + 2 \times 4 = 9.4$

Since there are no possible values of $S$ between 0 and 10,

$Pr(S > 9.4) = 1 - Pr(S = 0)$

$\qquad = 1 - .7 - .2 \times .8^2 - .1 \times .8^3 = .12$

**Test Question:   9**          **Key:  D**

APV of regular death benefit $= \int_0^\infty (100000)\left(e^{-\boldsymbol{d}\,t}\right)(0.008)\left(e^{-\boldsymbol{m}t}\right)dt$

$= \int_0^\infty (100000)\left(e^{-0.06t}\right)(0.008)\left(e^{-0.008t}\right)dt$

$= 100000\left[0.008 / (0.06 + 0.008)\right] = 11{,}764.71$

APV of accidental death benefit $= \int_0^{30} (100000)\left(e^{-\boldsymbol{d}\,t}\right)(0.001)\left(e^{-\boldsymbol{m}t}\right)dt$

$= \int_0^{30} (100000)\left(e^{-0.06t}\right)(0.001)\left(e^{-0.008t}\right)dt$

$= 100\left[1 - e^{-2.04}\right] / 0.068 = 1{,}279.37$

Total APV $= 11765 + 1279 = 13044$

**Test Question: 10**      **Key: B**

$$l_{[60]+.6} = (.6)(79,954) + (.4)(80,625)$$

$$= 80,222.4$$

$$l_{[60]+1.5} = (.5)(79,954) + (.5)(78,839)$$

$$= 79,396.5$$

$$_{0.9}q_{[60]+.6} = \frac{80222.4 - 79,396.5}{80,222.4}$$

$$= 0.0103$$

**Test Question: 11**      **Key: D**

$$\mathbf{l}_i = 10 \qquad i \geq 0 \quad - \quad \text{customers arrive at rate of 10/hr.}$$

$$\mathbf{m}_0 = 0$$

$$\mathbf{m}_1 = 6 \qquad\qquad \text{if 1 customer is being served, customer leaves at 6/hr.}$$

$$\mathbf{m}_i = 12 \qquad i \geq 2$$

$$\mathbf{l}_i P_i = \mathbf{m}_{i+1}P_{i+1}$$

$$6P_1 = 10P_0$$

$$12P_{i+1} = 10P_i \qquad i \geq 1$$

$$\sum_{i=0}^{\infty} P_i = 1$$

$$P_1 = \tfrac{5}{3}P_0$$

$$P_2 = \tfrac{5}{6}P_1 = \tfrac{5}{6} \cdot \tfrac{5}{3}P_0$$

$$P_i = \left(\tfrac{5}{6}\right)^{i-1}\tfrac{5}{3}P_0$$

$$\sum P_i = P_0 + \tfrac{5}{3}\sum_{i=1}^{\infty}\left(\tfrac{5}{6}\right)^{i-1}P_0$$

$$= P_0 + \tfrac{5}{3}\left(\frac{1}{1-\tfrac{5}{6}}\right)P_0$$

$$11P_0 = 1$$

$$P_0 = \tfrac{1}{11} = 9.09\overline{09}\%$$

**Test Question: 12**    **Key: C**

(Per 1 instead of per 1000)

$$v_{h+1}V = {}_hV + p_h - vq_{x+h} \qquad \text{for all } h; \; p_h = 0 \text{ for } h > 0$$

$$v^{h+1}{}_{h+1}V = v^h{}_hV + v^h p_h - v^{h+1}q_{x+h}$$

$$v^{h+1}{}_{h+1}V - v^h{}_hV - v^h p_h = -v^{h+1}q_{x+h}$$

$$\sum_{h=0}^{19}(v^{h+1}{}_{h+1}V - v^h{}_hV - v^h p_h) = -\sum_{h=0}^{19} v^{h+1}q_{x+h}$$

$$0 - p_0 = -\sum_{h=0}^{19} v^{h+1}q_{x+h}$$

$$p_0 = .03\sum_{h=0}^{19} v^{h+1}$$

$$= .03 a_{\overline{20}|}$$

$$= 0.31782$$

$$1000 p_0 = 317.82$$


**Test Question: 13**    **Key: A**

$$P(0) = \tfrac{1}{5}\int_0^5 e^{-l}\,dl = \tfrac{1}{5}\left(-e^{-l}\right)\Big|_0^5 = \tfrac{1}{5}\left(1 - e^{-5}\right) = 0.1987$$

$$P(1) = \tfrac{1}{5}\int_0^5 l e^{-l}\,dl = \tfrac{1}{5}\left(-l e^{-l} - e^{-l}\right)\Big|_0^5 = \tfrac{1}{5}\left(1 - 6e^{-5}\right) = 0.1919$$

$$P(N \geq 2) = 1 - .1987 - .1919 = .6094$$

**Test Question: 14**    **Key: A**

$$q_{40}^{(t)} = q_{40}^{(1)} + q_{40}^{(2)} = 0.34$$
$$= 1 - p_{40}^{\prime(1)} p_{40}^{\prime(2)}$$
$$0.34 = 1 - 0.75 p_{40}^{\prime(2)}$$

$$p_{40}^{\prime(2)} = 0.88$$
$$q_{40}^{\prime(2)} = 0.12 = y$$

$$q_{41}^{\prime(2)} = 2y = 0.24$$
$$q_{41}^{(t)} = 1 - (0.8)(1 - 0.24) = 0.392$$
$$l_{42}^{(t)} = 2000(1 - 0.34)(1 - 0.392) = 803$$

**Test Question: 15**    **Key: E**

For standard Brownian motion:
$$\boldsymbol{m} = 0$$
$$\text{Var} = 1 \text{ per month}$$
$$= 4 \text{ per 4 months}$$

Thus distribution of stock price at time 4 is normal with mean 18 and variance 4.

$$\text{Prob}\left[\text{Price at time } 4 > 21\right] =$$
$$= \text{Prob}\left[\frac{\text{Price} - 18}{2} > \frac{21 - 18}{2}\right]$$
$$= 1 - \Phi(1.5)$$
$$= 1 - 0.9332 = 0.0668$$

$$\text{Prob}\left[\text{Maximum price during}[0,4] > 21\right]$$
$$= 2 \times \text{Prob}\left[\text{Price at } 4 > 21\right]$$
$$= (2)(0.0668)$$
$$= 0.1336$$

**Test Question: 16**      **Key: C**

$$\Pr\left[L(\boldsymbol{p}') > 0\right] < 0.5$$

$$\Pr\left[10{,}000 v^{K+1} - \boldsymbol{p}' \ddot{a}_{\overline{K+1}|} > 0\right] < 0.5$$

From Illustrative Life Table, $_{47}p_{30} = 0.50816$ and $_{48}p_{30} = .47681$

Since $L$ is a decreasing function of $K$, to have
$\Pr\left[L(\boldsymbol{p}') > 0\right] < 0.5$ means we must have $L(\boldsymbol{p}') \le 0$ for $K \ge 47$.
Highest value of $L(\boldsymbol{p}')$ for $K \ge 47$ is at $K = 47$.

$$L(\boldsymbol{p}')[\text{at } K = 47] = 10{,}000\, v^{47+1} - \boldsymbol{p}' \ddot{a}_{\overline{47+1}|}$$

$$= 609.98 - 16.589 \boldsymbol{p}'$$

$$L(\boldsymbol{p}') \le 0 \Rightarrow (609.98 - 16.589\boldsymbol{p}') \le 0$$

$$\Rightarrow \boldsymbol{p}' > \frac{609.98}{16.589} = 36.77$$

**Test Question: 17**      **Key: A**

This is an $M/M/\infty$ queuing model.

$\boldsymbol{l}_n = \boldsymbol{l}$, $n \ge 0$,   $\boldsymbol{m}_n = n\boldsymbol{m}$, $n \ge 1$, since patients recover individually and number of doctors is unlimited.

$$\boldsymbol{l} P_0 = \boldsymbol{m} P_1$$
$$\boldsymbol{l} P_1 = 2\boldsymbol{m} P_2$$
$$\boldsymbol{l} P_2 = 3\boldsymbol{m} P_3$$
$$\vdots$$
$$\boldsymbol{l} P_{n-1} = n\boldsymbol{m} P_n$$

$$P_n = \left(\tfrac{\boldsymbol{l}}{\boldsymbol{m}}\right)^n \tfrac{1}{n!} P_0 = \tfrac{x^n}{n!} P \quad \text{where } x = \tfrac{\boldsymbol{l}}{\boldsymbol{m}} \ \left(\text{so for this problem } x = \tfrac{5}{2} = 2.5\right)$$

Also, $\displaystyle\sum_0^\infty P_n = 1 \Rightarrow P_0 + P_0 \sum_1^\infty \tfrac{x^n}{n!} = 1$

$$P_0 = \frac{1}{1 + \sum\limits_1^\infty \frac{x^n}{n!}} = \frac{1}{e^x} = e^{-x}$$

$$\Pr\{\# \text{ patients} < 3\} = P_0 + P_1 + P_2$$

$$= P_0\left[1 + x + \tfrac{x^2}{2}\right]$$

$$= e^{-x}\left(1 + x + \tfrac{x^2}{2}\right)$$

$$= 0.543813 \text{ for } x = 2.5$$

**Test Question: 18**          **Key: C**

You may find equation 8.5.3 and exercise 8.31 helpful in conjunction with this solution. The random variable $\Lambda$ in question 18 is $\Lambda_{10}$ in those references.

For $K(40) \geq 10$, the loss variable $\Lambda =$
$$\begin{cases} 1000(1 - {}_{11}V_{40})v - p\,\ddot{a}_{\overline{1}|}, & K(40) = 10 \\ -p\,\ddot{a}_{\overline{1}|}, & K(40) > 10 \end{cases}$$

where $p$ is the benefit premium for the 1-year term insurance.
$\left[ p = 1000(1 - {}_{11}V_{40})v\,q_{50}, \text{ but you don't need its value.} \right]$

Since $\text{Var}(X + c) = \text{Var}(X)$, where $c$ is any constant, the $-p\,\ddot{a}_{\overline{1}|}$ term can be ignored for computing $\text{Var}(\Lambda)$.

The conditional probabilities, given $K \geq 10$ are
$\Pr(K = 10 | K \geq 10) = q_{50};\quad \Pr(K > 10 | K \geq 10) = p_{50}$
The variance of a binomial distribution with parameters $q,\ 1$ (also called a Bernoulli distribution) is $m\,q(1-q)$, so $\text{Var}(\Lambda | K(40) \geq 10)$

$$= \left[ 1000(1 - {}_{11}V_{40})v \right]^2 p_{50}\,q_{50}$$
$$= \left[ 1000 \left(1 - \left(1 - \tfrac{\ddot{a}_{51}}{\ddot{a}_{40}}\right)\right) \tfrac{1}{1.06} \right]^2 (0.99408)(.00592)$$
$$= \left[ 1000 \left(1 - \left(1 - \tfrac{13.0803}{14.8166}\right)\right) \tfrac{1}{1.06} \right]^2 (.005885)$$
$$= \left[ (1000)(0.88281)\tfrac{1}{1.06} \right]^2 (.005885)$$
$$= 4,083$$

**Test Question: 19**          **Key: E**

$$\Psi(u) = \frac{1}{1+q}\exp\left[\frac{-qu}{(1+q)p}\right]$$

$$1 + q = \frac{22}{20} \Rightarrow q = 0.1$$

$u = 20$
$p = 1$

$$\Psi(u) = \frac{1}{1.1}e^{-\frac{2}{1.1}} = 0.15$$

**Test Question:**      **20**      **Key:**   **B**

$\Pr(K = 0) = 1 - p_x = 0.1$

$\Pr(K = 1) = {}_1p_x - {}_2p_x = 0.9 - 0.81 = 0.09$

$\Pr(K > 1) = {}_2p_x = 0.81$

$E(Y) = .1 \times 1 + .09 \times 1.87 + .81 \times 2.72 = 2.4715$

$E(Y^2) = .1 \times 1^2 + .09 \times 1.87^2 + .81 \times 2.72^2 = 6.407$

$$VAR(Y) = 6.407 - 2.4715^2 = 0.299$$

**Test Question:**    **21**        **Key:**   **D**

Let X be the occurrence amount, Y = max(X-100, 0) be the amount paid.

E[X] = 1,000

$\text{Var}[X] = (1,000)^2$

P(X>100) = exp(-100/1,000) = .904837

The distribution of Y <u>given that X>100</u>, is also exponential with mean 1,000 (memoryless property).

So Y is $\begin{cases} 0 \text{ with prob } .095163 \\ \text{exponential mean 1000 with prob } .904837 \end{cases}$

$E[Y] = .095163 \times 0 + .904837 \times 1,000 = 904.837$

$E[Y^2] = .095163 \times 0 + .904837 \times 2 \times (1,000)^2 = 1,809,675$

$\text{Var}[Y] = 1,809,675 - (904.837)^2 = 990,944$

Alternatively, think of this as a compound distribution whose frequency is Bernoulli with p = .904837, and severity is exponential with mean 1,000.

$\text{Var} = \text{Var}[N] \times E[X]^2 + \text{Var}[X] \times E[N] = p(1-p)(1,000,000) + p(1,000,000)$

**Test Question: 22**      **Key: B**

In general $\text{Var}(L) = \left(1 + \frac{P}{d}\right)^2 \left({}^2\overline{A}_x - \overline{A}_x^2\right)$

Here $\overline{P}(\overline{A}_x) = \dfrac{1}{\overline{a}_x} - d = \dfrac{1}{5} - .08 = .12$

So $\text{Var}(L) = \left(1 + \dfrac{.12}{.08}\right)^2 \left({}^2\overline{A}_x - \overline{A}_x^2\right) = .5625$

and $\text{Var}(L^*) = \left(1 + \dfrac{\frac{5}{4}(.12)}{.08}\right)^2 \left({}^2\overline{A}_x - \overline{A}_x^2\right)$

So $\text{Var}(L^*) = \dfrac{\left(1 + \frac{15}{8}\right)^2}{\left(1 + \frac{12}{8}\right)^2}(0.5625) = .744$

$E[L^*] = \overline{A}_x - .15\overline{a}_x = 1 - \overline{a}_x(d + .15) = 1 - 5(.23) = -.15$

$E[L^*] + \sqrt{\text{Var}(L^*)} = .7125$

**Test Question: 23**      **Key: C**

Serious claims are reported according to a Poisson process at an average rate of 2 per month. The chance of seeing at least 3 claims is (1 – the chance of seeing 0, 1, or 2 claims).

$P(3+) \geq 0.9$ is the same as $P(0,1,2) \leq 0.1$ is the same as $\left[P(0) + P(1) + P(2)\right] \leq 0.1$

$0.1 \geq e^{-\lambda} + \lambda e^{-\lambda} + \left(\lambda^2 / 2\right)e^{-\lambda}$

The expected value is 2 per month, so we would expect it to be at least 2 months $(\lambda = 4)$.
Plug in and try

$\quad\quad e^{-4} + 4e^{-4} + \left(4^2 / 2\right)e^{-4} = .238$, too high, so try 3 months $(\lambda = 6)$

$\quad\quad e^{-6} + 6e^{-6} + \left(6^2 / 2\right)e^{-6} = .062$, okay. The answer is 3 months.

[While 2 is a reasonable first guess, it was not critical to the solution. Wherever you start, you should conclude 2 is too few, and 3 is enough].

**Test Question: 24**     **Key: B**

Let $l_0^{(t)}$ = number of students entering year 1
 superscript (f) denote academic failure
 superscript (w) denote withdrawal
 subscript is "age" at start of year; equals year - 1

$$p_0^{(t)} = 1 - 0.40 - 0.20 = 0.40$$

$$l_2^{(t)} = 10 l_2^{(t)} q_2^{(f)} \Rightarrow q_2^{(f)} = 0.1$$

$$q_2^{(w)} = q_2^{(t)} - q_2^{(f)} = (1.0 - 0.6) - 0.1 = 0.3$$

$$l_1^{(t)} q_1^{(f)} = 0.4 \left[ l_1^{(t)} \left( 1 - q_1^{(f)} - q_1^{(w)} \right) \right]$$

$$q_1^{(f)} = 0.4 \left( 1 - q_1^{(f)} - 0.3 \right)$$

$$q_1^{(f)} = \frac{0.28}{1.4} = 0.2$$

$$p_1^{(t)} = 1 - q_1^{(f)} - q_1^{(w)} = 1 - 0.2 - 0.3 = 0.5$$

$$_3 q_0^{(w)} = q_0^{(w)} + p_0^{(t)} q_1^{(w)} + p_0^{(t)} p_1^{(t)} q_2^{(w)}$$

$$= 0.2 + (0.4)(0.3) + (0.4)(0.5)(0.3)$$

$$= 0.38$$

$$e_{25} = p_{25}(1 + e_{26})$$

$$e_{26}^N = e_{26}^M \quad \text{since same } \boldsymbol{m}$$

$$p_{25}^N = e^{-\int_0^1 \left[ \boldsymbol{m}_{25}^M(t) + 0.1(1-t) \right] dt}$$

$$= e^{-\int_0^1 \boldsymbol{m}_{25}^M(t)dt - \int_0^1 0.1(1-t)dt}$$

$$= e^{-\int_0^1 \boldsymbol{m}_{25}^M(t)\,dt} \; e^{-\int_0^1 0.1(1-t)dt}$$

$$= p_{25}^M \, e^{-\left[ 0.1\left( t - \frac{t^2}{2} \right) \right]_0^1}$$

$$= e^{-0.05} p_{25}^M$$

$$e_{25}^N = p_{25}^N (1 + e_{26})$$

$$= e^{-0.05} p_{25}^M (1 + e_{26})$$

$$= 0.951 \, e_{25}^M = (0.951)(10.0) = 9.5$$

**Test Question: 26**      **Key: D**

$$E\left[Y_{AGG}\right] = 100E\left[Y\right] = 100(10,000)\bar{a}_x$$

$$= 100(10,000)\left(\frac{\left(1-\bar{A}_x\right)}{d}\right) = 10,000,000$$

$$\boldsymbol{s}_Y = \sqrt{\mathrm{Var}\left[Y\right]} = \sqrt{(10,000)^2\frac{1}{d^2}\left(^2\bar{A}_x - \bar{A}_x^2\right)}$$

$$= \frac{(10,000)}{d}\sqrt{(0.25)-(0.16)} = 50,000$$

$$\boldsymbol{s}_{AGG} = \sqrt{100}\boldsymbol{s}_Y = 10(50,000) = 500,000$$

$$0.90 = \Pr\left[\frac{F-E\left[Y_{AGG}\right]}{\boldsymbol{s}_{AGG}} > 0\right]$$

$$\Rightarrow 1.282 = \frac{F - E\left[Y_{AGG}\right]}{\boldsymbol{s}_{AGG}}$$

$$F = 1.282\boldsymbol{s}_{AGG} + E\left[Y_{AGG}\right]$$

$$F = 1.282(500,000) + 10,000,000 = 10,641,000$$

**Test Question: 27**      **Key: C**

Expected claims under current distribution = 500
$q$ = parameter of new distribution
$X$ = claims
$E(X) = q$
bonus $= .5 \times \left[500 - X \wedge 500\right]$

$$E \text{ (claims + bonus)} = q + .5\left(500 - q\left(1 - \frac{q}{500+q}\right)\right) = 500$$

$$q - \frac{q}{2}\left(\frac{500}{500+q}\right) = 250$$

$$2(500+q)q - 500q = 250(500+q)\cdot 2$$

$$1000q + q^2 \cdot 2 - 500q = 2 \times 250 \times 500 + 500q$$

$$q = \sqrt{250 \times 500} = 354$$

**Test Question: 28**    **Key:   E**

$$(DA)^{1}_{80:\overline{20}|} = 20vq_{80} + vp_{80}\left((DA)^{1}_{81:\overline{19}|}\right)$$

$q_{80} = .2$    $$13 = \frac{20(.2)}{1.06} + \frac{.8}{1.06}(DA)^{1}_{81:\overline{19}|}$$

$$\therefore (DA)^{1}_{81:\overline{19}|} = \frac{13(1.06) - 4}{.8} = 12.225$$

$q_{80} = .1$    $$DA^{1}_{80:\overline{20}|} = 20v(.1) + v(.9)(12.225)$$

$$= \frac{2 + 9(12.225)}{1.06} = 12.267$$


**Test Question:   29**    **Key:  B**


Let $T$ denote the random variable of time until the college graduate finds a job
Let $\{N(t), t \geq 0\}$ denote the job offer process

Each offer can be classified as either

$\begin{cases} \text{Type I -  - accept with probability } p \Rightarrow \{N_1(t)\} \\ \text{Type II -  - reject with probability } (1 - p) \Rightarrow \{N_2(t)\} \end{cases}$

By proposition 5.2, $\{N_1(t)\}$ is Poisson process with $\lambda_1 = \lambda \cdot p$
$p = \Pr(w > 28,000) = \Pr(\ln w > \ln 28,000)$

$$= \Pr(\ln w > 10.24) = \Pr\left(\frac{\ln w - 10.12}{0.12} > \frac{10.24 - 10.12}{0.12}\right) = 1 - \Phi(1)$$

$$= 0.1587$$
$\lambda_1 = 0.1587 \times 2 = 0.3174$

$T$ has an exponential distribution with $\theta = \dfrac{1}{.3174} = 3.15$

$\Pr(T > 3) = 1 - F(3)$

$$= e^{\frac{-3}{3.15}} = 0.386$$

**Test Question:   30          Key:  A**

$$_t p_x = \exp\left[-\int_0^t \frac{ds}{100 - x - s}\right] = \exp\left[\ln(100 - x - s)\big|_0^t\right] = \frac{100 - x - t}{100 - x}$$

$$\overset{\circ}{e}_{\overline{50:60}} = \overset{\circ}{e}_{50} + \overset{\circ}{e}_{60} - \overset{\circ}{e}_{50:60}$$

$$\overset{\circ}{e}_{50} = \int_0^{50} \frac{50 - t}{50} dt = \frac{1}{50}\left[50t - \frac{t^2}{2}\right]_0^{50} = 25$$

$$\overset{\circ}{e}_{60} = \int_0^{40} \frac{40 - t}{40} dt = \frac{1}{40}\left[40t - \frac{t^2}{2}\right]_0^{40} = 20$$

$$\overset{\circ}{e}_{50:60} = \int_0^{40}\left(\frac{50 - t}{50}\right)\left(\frac{40 - t}{40}\right)dt = \int_0^{40} \frac{1}{2000}\left(2000 - 90t + t^2\right)dt$$

$$= \frac{1}{2000}\left(2000t - 45t^2 + \frac{t^3}{3}\Big|_0^{40}\right) = 14.67$$
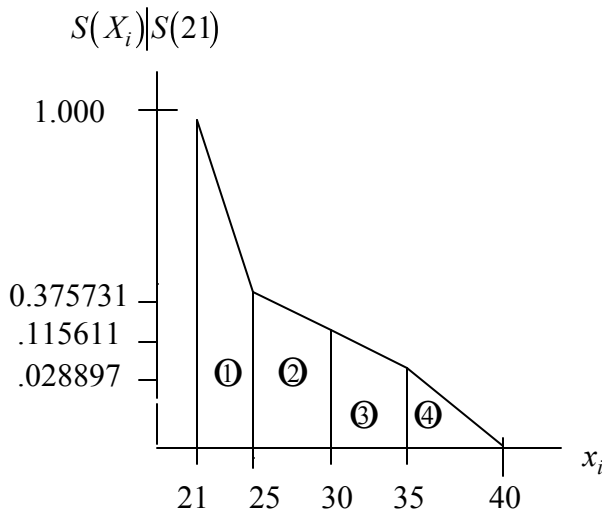
$$\overset{\circ}{e}_{\overline{50:60}} = 25 + 20 - 14.67 = 30.33$$

**Test Question:   31          Key:  A**

$$UDD \Rightarrow l_{21} = (0.8)(53{,}488) + (0.2)(17{,}384) = 46{,}267.2$$

$$Mrl(21) = \overset{\circ}{e}_{21} = \int_0^\infty {}_t p_{21} \int_0^\infty \frac{S(21+t)}{S(21)} dt$$

$$= \sum areas$$
$$= 2.751 + 1.228 + 0.361 + 0.072$$
$$= 4.412$$



**Test:   32       Key: Question          D**

$$m_{n1} = E[N] = 25$$
$$m_{x2} = Var[X] = 675$$
$$m_{N2} = Var[N] = 25$$
$$E[X] = 50$$

$$E[S] = E[X]E[N] = 25 \times 50 = 1250$$

$$Var[S] = E[N]Var[X] + Var[N]E[X]^2$$
$$= 25 \times 675 + 25 \times 2500 = 79{,}375$$

$$Standard\ Deviation[S] = \sqrt{79{,}375} = 281.74$$

$$Pr(S > 2000) = Pr\big[(S - 1250)/281.74 > (2000 - 1250)/281.74\big] = 1 - \Phi(2.66)$$

**Test Question: 33**          **Key: E**

$$\mathrm{Var}(_0L) = \mathrm{Var}(\Lambda_0) + v^2 \mathrm{Var}(\Lambda_1) \quad \text{since } Var(\Lambda_2) = 0$$

$$\mathrm{Var}(\Lambda_0) = \left[v(b_1 - _1V)\right]^2 p_{50}q_{50}$$

$$= \frac{(10{,}000 - 3{,}209)^2(0.00832)(0.99168)}{1.03^2}$$

$$= 358664.09$$

$$\mathrm{Var}(\Lambda_1) = \left[v(b_2 - _2V)\right]^2 p_{50}q_{51}p_{51}$$

$$= \frac{(10{,}000 - 6{,}539)^2(0.99168)(0.00911)(0.99089)}{1.03^2}$$

$$= 101075.09$$

$$\mathrm{Var}(_0L) = 358664.09 + \frac{101075.09}{1.03^2} = 453937.06$$

Alternative solution:

$$p = 10{,}000\,v - _2V_{50:\overline{3|}} = 9708.74 - 6539 = 3169.74$$

$$_0L = \begin{cases} 10{,}000\,v - p\,\ddot{a}_{\overline{1|}} = 6539 & \text{for } K = 0 \\ 10{,}000\,v^2 - p\,\ddot{a}_{\overline{2|}} = 3178.80 & \text{for } K = 1 \\ 10{,}000\,v^3 - p\,\ddot{a}_{\overline{3|}} = -83.52 & \text{for } K > 1 \end{cases}$$

$$\mathrm{Pr}(K = 0) = q_{50} = 0.00832$$
$$\mathrm{Pr}(K = 1) = p_{50}\,q_{51} = (0.99168)(0.00911) = 0.0090342$$
$$\mathrm{Pr}(K > 1) = 1 - \mathrm{Pr}(K = 0) - \mathrm{Pr}(K = 1) = 0.98265$$

$$\mathrm{Var}(_0L) = \mathrm{E}\left[_0L^2\right] - \mathrm{E}\left[_0L\right]^2 = \mathrm{E}\left[_0L^2\right] \quad \text{since } p \text{ is benefit premium}$$

$$= 0.00832 \times 6539^2 + 0.00903 \times 3178.80^2 + 0.98265 \times (-83.52)^2$$

$$= 453{,}895 \text{ [difference from the other solution is due to rounding]}$$

**Test Question: 34** **Key: D**

After 365 days, probabilities have converged to limiting probabilities.
Let p = limiting probability of rain
$$.5p+.2(1-p)=p$$
$$p=\frac{2}{7}$$

$$\text{Benefit premium } =\left[\left(\frac{2}{7}\right)\left(\frac{1}{1.1}\right)+\left(\frac{2}{7}\right)\left(\frac{1}{1.1}\right)^2+\left(\frac{2}{7}\right)\left(\frac{1}{1.1}\right)^3\right]1000$$

$$=710$$

**Test Question: 35** **Key: C**

Let $\boldsymbol{p}$ denote the single benefit premium.
$$\boldsymbol{p}=\,_{30}|\ddot{a}_{35}+\boldsymbol{p}\,A^1_{35:\overline{30}|}$$

$$\boldsymbol{p}=\frac{_{30}|\ddot{a}_{35}}{1-A^1_{35:\overline{30}|}}=\frac{\left(A_{35:\overline{30}|}-A^1_{35:\overline{30}|}\right)\ddot{a}_{65}}{1-A^1_{35:\overline{30}|}}=$$

$$=\frac{(.21-.07)9.9}{(1-.07)}$$

$$=\frac{1.386}{.93}$$

$$=1.49$$

**Test Question: 36** **Key: E**

$$_{0.4}p_0=.5=e^{-\int_0^{0.4}\left(F+e^{2x}\right)dx}$$

$$=e^{-.4F-\left[\frac{e^{2x}}{2}\right]_0^{.4}}$$

$$=e^{-.4F-\left(\frac{e^{0.8}-1}{2}\right)}$$

$$.5=e^{-.4F-.6128}$$

$$\Rightarrow \ln(.5)=-.4F-.6128$$
$$\Rightarrow -.6931=-.4F-.6128$$
$$\Rightarrow F=0.20$$

**Test Question: 37          Key: E**

first $t = \dfrac{-\log(0.1)}{5} = .46 < 5$ (the end of the interval) so continue

       Test:   $.5 < \mathbf{1}(.46) / \mathbf{1} = (5 - .46)/5 = .91$

       True, so first arrival time = .46

Next t = .46 – 1/5 log .05 = 1.06

       Test:   $.94 < \mathbf{1}(1.06) / \mathbf{1} = .79$

       False, so continue

       Next t = 1.06 – 1/5 log .24 = 1.35

       Test:   $.62 < \mathbf{1}(135) / \mathbf{1} = .73$

       True, so second arrival time = 1.35

       Next t = 1.35 – 1/5 log .01 = 2.27

       Test:   $.47 < \mathbf{1}(2.27) / \mathbf{1} = .55$

       True, so third arrival time = 2.27

**Test Question: 38          Key: C**

First person arrives at t = .46. Service time = 1.1, so departs at t = 1.56
Second person arrives at t = 1.35. Service time = 0.8, so departs at t = 1.56 + 0.8 = 2.36
No further arrivals before t = 2, so 1 has been served, 1 is being served.

**Test Question: 39**   **Key: E**

Model is dealer financial position at $t$ = sales through $t$ − payments through $t$

$$= \sum_{j=1}^{N(t)} x_j - (1+q) \times \textbf{\textit{l}} \times p_1 t$$

where

$$(1+q) = \frac{\text{payment rate}}{\text{expected sales rate}} = 1$$

$\textbf{\textit{l}}$ = rate of sales = 12
$p_1$ = average sales price = 25,000

No term needed for dealer's starting position; it is 0.

$$= -\left[(1+q)\textbf{\textit{l}}p_1 t - \sum_{j=1}^{N(t)} x_j\right] = \text{model for surplus process.}$$

So final position > 0 ⇔ surplus process < 0.

By formula 13.5.2, $\Psi(0) = \dfrac{1}{1+q} = 1$ (Since $q = 0$).

**Test Question: 40**     **Key: B**

See solution to #39 for definition and values of $\boldsymbol{q}, \boldsymbol{l}$ and $p_1$.

Dealer's final position $= \displaystyle\sum_{1}^{N(t)} x_j - (1+\boldsymbol{q})\boldsymbol{l}p_1 t$

$\qquad\qquad\qquad = -\left[ (1+\boldsymbol{q})\boldsymbol{l}p_1 t - \displaystyle\sum_{1}^{N(t)} x_j \right] \leftarrow$ surplus process.

Dealer's first final position $> 0 \iff$ first negative for surplus process (denoted $L_1$)

$pdf$ of $L_1 = \dfrac{1}{p_1}\left[ 1 - P(y) \right]$

$P(y) = \begin{cases} 0, & y < 20{,}000 \\ 0.5, & 20{,}000 \le y < 30{,}000 \\ 1.0, & 30{,}000 \le y \end{cases}$

$E[L_1] = \displaystyle\int_0^{20{,}000} \dfrac{1}{25{,}000} y\ dy + \int_{20{,}000}^{30{,}000} \dfrac{0.5}{25{,}000} y\ dy = \dfrac{4\times10^8}{50{,}000} + \dfrac{5\times10^8}{100{,}000} = \dfrac{13\times10^8}{10^5} = 13{,}000$

**Test Question: 41**     **Key: C**

$E[X] = 2000(1!)/(1!) = 2000$

$E[X \wedge 3000] = \left(\dfrac{2000}{1}\right) \times \left[ 1 - \dfrac{2000}{(3000+2000)} \right] = 2000 \times \left(1 - \dfrac{2}{5}\right) = 2000 \times \dfrac{3}{5} = 1200$

So the fraction of the losses expected to be covered by the reinsurance
is $\dfrac{2000 - 1200}{2000} = 0.4$.

The expected ceded losses are $4{,}000{,}000 \Rightarrow$ the ceded premium is $4{,}400{,}000$.

**Test Question: 42**       **Key: E**

$$X_{2002} = 1.05 \times X_{2001}$$

so:
$$F\left(\frac{x_{2002}}{1.05}\right) = 1 - \left[\frac{2000}{(x_{2002}/1.05 + 2000)}\right]^2$$

$$= 1 - \left[\frac{2100}{x_{2002} + 2100}\right]^2$$

This is just another Pareto distribution with $a = 2, q = 2100$.
$E[X_{2002}] = 2100$.

and

$$E[X_{2002} \wedge 3000] = \left(\frac{2100}{1}\right) \times \left[1 - \left(\frac{2100}{(3000 + 2100)}\right)\right]$$

$$= 2100 \times \left[\frac{3000}{5100}\right] = 1235$$

So the fraction of the losses expected to be covered by the reinsurance is
$$\frac{2100 - 1235}{2100} = 0.412.$$

The total expected losses have increased to 10,500,000, so
$$C_{2002} = 1.1 \times 0.412 \times 10,500,000 = 4,758,600$$

And $\dfrac{C_{2002}}{C_{2001}} = \dfrac{4,758,600}{4,400,000} = 1.08$

**Test Question: 43**       **Key: D**

Proposition 5.3 (page 266) of Probability Models makes it clear that the two views of the process are equivalent.

The simulation procedures are consistent with Ross.

It is obvious that the two methods can give different results. (E.g., let the first number be so extreme that actuary 1 finds no coins; actuary 2 is still likely to get some worth 5 or 10)

**Test Question:   44**          **Key:  E**

For F:  Expected coins          $= 60 \times 2 = 120$
Need (expected) 120 + 1 random numbers (because you need the one that exceeds 60 minutes)
Need (expected) 120 to determine the denominations.
Thus   $F = 120 + 1 + 120 = 241$

For G:  $60 \times 1.2 + 1 = 73$          for coins worth 1
$60 \times .4 + 1 = 25$          for coins worth 5
$60 \times .4 + 1 = 25$          for coins worth 10
$\overline{123}$

$F/G = 1.96$