COURSE 4
MAY 2001

MULTIPLE-CHOICE ANSWER KEY

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Course 4 Exam Solutions
May 2001

Item Number: 1
Key: D

Solution

Because the autocorrelation function is zero starting with lag 2, this must be an MA(1) model. Then,

$$-.4 = r_1 = \frac{-q_1}{1+q_1^2}, -.4 - .4q_1^2 = -q_1, .4q_1^2 - q_1 + .4 = 0.$$

This quadratic equation has two roots, 0.5 and 2. Because the coefficient's absolute value must be less than 1, only 0.5 is acceptable.

Item Number: 2

Key: B

Solution

The posterior distribution is $p(l\,|2) \propto \dfrac{e^{-l} l^{2}}{2!}\dfrac{1}{3}e^{-l/3} \propto l^{2}e^{-4l/3}$ which is a gamma distribution with parameters 3 and 3/4. The variance is $3(3/4)^2$ = 27/16.

Item Number: 3

Key: C

Solution

$$m_2 = s^2 = \frac{(1-3)^2 + (2-3)^2 + (3-3)^2 + (4-3)^2 + (5-3)^2}{5} = 2$$

$$m_4 = \frac{(1-3)^4 + (2-3)^4 + (3-3)^4 + (4-3)^4 + (5-3)^4}{5} = 6.8$$

$$g_2 = \frac{m_4}{s^4} = \frac{6.8}{4} = 1.7$$

Item Number: 4
Key: E

Solution

The calculations for each possibility are:

	Censored at 1			Censored at 2			Censored at 3			Censored at 4			Censored at 5		
Time	d	Y	$\hat{S}$	d	Y	$\hat{S}$	d	Y	$\hat{S}$	d	Y	$\hat{S}$	d	Y	$\hat{S}$
1	0	5	1	1	5	.80	1	5	.80	1	5	.80	1	5	.80
2	1	4	.75	0	4	.80	1	4	.60	1	4	.60	1	4	.60
3	1	3	.50	1	3	.53	0	3	.60	1	3	.40	1	3	.40
4	1	2	.25	1	2	.27	1	2	.30	0	2	.40	1	2	.20
5	1	1	0	1	1	0	1	1	0	1	1	0	0	1	.20

This result may also be obtained by general reasoning. If the lapse occurs late, then the “death” times come earlier, leading to lower survival probabilities. If the lapse occurs early, then the “death” times come later, leading to higher survival probabilities.

Item Number: 5

Key: C

Solution

This is testing $H_0 : b_4 = b_5 = 0$, which is a joint test on several regression coefficients.

The key formula is 5.21 (p.130 of Pindyck and Rubinfeld).

We are given: $R^2_{UR} = 0.94$

Also, we know: $N = 42$, $k = 5$ (total number of coefficients to be estimated) and $q = 2$

Using 5.21, we obtain $F = \dfrac{(0.94 - 0.915)/2}{(1 - 0.94)/(42 - 5)} = 7.7$.

Item Number: 6

Key: C

Solution

The number of expected claims (e) is proportional to the number of exposure units (n). Let $e = cn$. Using Bühlmann credibility and partial credibility gives:

$$\sqrt{\frac{25}{100}} = \frac{1}{2} = \frac{25/c}{25/c+k} = \frac{25}{25+ck}$$

Therefore $ck = 25$.

When we have 100 expected claims, $Z = \frac{100/c}{100/c+k} = \frac{100}{100+ck} = \frac{100}{100+25} = 0.80$.

Item Number: 7

Key: D

Solution

Following Example 2.71 in *Loss Models* (with no denominator because this problem has no deductible), the contribution to the likelihood function for a loss (x) below 1000 is $f(x) = \theta^{-1}e^{-x/\theta}$ while for observations censored at 1000 it is $1 - F(1000) = e^{-1000/\theta}$. The likelihood function is:

$$L(\theta) = \left(\prod_{j=1}^{62} \theta^{-1} e^{-x_j/\theta} \right) \left(\prod_{j=63}^{100} e^{-1000/\theta} \right) = \theta^{-62} e^{-(28,140+38,000)/\theta}$$

The logarithm and its derivative are:

$$l(\theta) = -62\ln(\theta) - 66,140\theta^{-1}$$
$$l'(\theta) = -62\theta^{-1} + 66,140\theta^{-2}$$

Setting the derivative equal to zero yields $\hat{\theta} = 66,140/62 = 1067$.

Item Number: 8

Key: E

Solution

$$\hat{B}\left(14\right) = \sum_{t_i \le 14} \frac{d_i}{Q(t_i)} = \frac{1}{0.25 + 0.15 + 0.5} + \frac{1}{0.15 + 0.5} + \frac{1}{0.5} = 4.65$$

Item Number: 9

Key: D

Solution

$$F \text{ statistic} = (N-k)\frac{(ESS_R - ESS_{UR})}{q(ESS_{UR})}$$

$N = 100$

$k = 3$

$q = 2$

Series I: $F = 97\frac{(3552.2 - 3233.8)}{2(3233.8)} = 4.78$, Fail to reject

Series II: $F = 97\frac{(1300.5 - 1131.8)}{2(1131.8)} = 7.23$, Reject

Series III: $F = 97\frac{(237.0 - 211.1)}{2(211.1)} = 5.95$, Reject

Item Number: 10

Key: A

Solution

$$E[L|A] = E[N|A]E[X|A] = (2/3)(990) = 660$$
$$E[L|B] = E[N|B]E[X|B] = (4/3)(276) = 368$$
$$P(L = 500|A) = P(N = 1|A)P(X = 500|A) = (4/9)(1/3) = 4/27 = 12/81$$
$$P(L = 500|B) = P(N = 2|B)P(X_1 = X_2 = 250|B) = (4/9)(2/3)(2/3) = 16/81$$
$\Rightarrow P(A|500) = 12/28 = 3/7$ and $P(B|500) = 4/7$
$$E[L_2|L_1 = 500] = (3/7)\,660 + (4/7)\,368 = 493$$

Item Number: 11
Key: D

Solution

$$VHM = \left[(1/2)660^2 + (1/2)368^2\right] - [514]^2 = 285{,}512 - 264{,}196 = 21{,}316$$

Total variance = 296,962

EPV = Total variance – VHM = 275,646

$$k = \frac{EPV}{VHM} = 12.9314 \Rightarrow Z = \frac{1}{1+k} = 0.07178$$

Bühlmann credibility premium $= 500Z + 514(1 - Z) = 513$

Item Number: 12
Key: E

Solution

The distribution function is $F(x)=\int_0^x 2(1+t)^{-3}\,dt = 1-\frac{1}{(1+x)^2}$. From it, the following table yields the K-S statistic:

x	.1	.2	.5	1.0	1.3
$F(x)$	.174	.306	.556	.75	.811
$F_n(x-)$	0	.2	.4	.6	.8
$F_n(x+)$	.2	.4	.6	.8	1.0
Max diff.	.174	.106	.156	.15	.189

The K-S statistic is 0.189.

Item Number: 13
Key: B

Solution

Divide the first equation by $\sqrt{\sum\left(Y_i-\bar{Y}\right)^2\sum\left(X_{2i}-\bar{X}_2\right)^2}$ and the second equation by $\sqrt{\sum\left(Y_i-\bar{Y}\right)^2\sum\left(X_{3i}-\bar{X}_3\right)^2}$ to get the equations:

$$r_{YX_2}=\hat{\beta}_2\frac{s_{X_2}}{s_Y}+\hat{\beta}_3\frac{s_{X_3}}{s_Y}r_{X_2X_3}=\hat{\beta}_2^*+\hat{\beta}_3^*r_{X_2X_3}$$

$$r_{YX_3}=\hat{\beta}_2\frac{s_{X_2}}{s_Y}r_{X_2X_3}+\hat{\beta}_3\frac{s_{X_3}}{s_Y}=\hat{\beta}_2^*r_{X_2X_3}+\hat{\beta}_3^*$$

Solve these equations for $\hat{\beta}_2^*=\dfrac{r_{YX_2}-r_{YX_3}r_{X_2X_3}}{1-r_{X_2X_3}^2}=\dfrac{0.4-(0.9)(0.6)}{1-(0.6)^2}=-0.22$.

Item Number: 14
Key: C

Solution

$$s_H^2(20) = \sum_{t_i \le 20} \frac{d_i}{Y_i} = \frac{15}{100^2} + \frac{20}{65^2} + \frac{13}{40^2} = 0.0144$$

$$s_H(20) = \sqrt{0.0144} = 0.12$$

Item Number: 15

Key: A

Solution

$$\hat{m}_t = \int_0^t \hat{S}(t)dt$$

$$= (1.0 \times 1) + (0.85 \times 7) + (0.5885 \times 9) + (0.3972 \times 8) = 15.42$$

$$\hat{V}[\hat{m}_t] = \sum_{i=1}^{D} \left[\int_{t_i}^{t} \hat{S}(t)dt \right]^2 \frac{d_i}{Y_i(Y_i - d_i)}$$

$$= 14.424^2 \times 0.0018 + 8.474^2 \times 0.0068 + 3.178^2 \times 0.0120$$

$$= 0.9840$$

The 95% confidence interval is:

$$15.42 \pm 1.96 \times \sqrt{0.9840}$$

$$(13.5, 17.4)$$

Item Number: 16

Key: C

Solution

The Weibull density function is $f(x) = .5(xq)^{-.5} e^{-(x/q)^{.5}}$. Therefore the likelihood function is

$$\begin{aligned} L(q) &= \prod_{j=1}^{10} .5(x_j q)^{-.5} e^{-(x_j/q)^{.5}} \\ &= (.5)^{10} \left(\prod_{j=1}^{10} x_j \right)^{-.5} q^{-5} e^{-q^{-.5} \sum_{j=1}^{10} x_j^{.5}} \\ &\propto q^{-5} e^{-488.97 q^{-.5}}. \end{aligned}$$

The logarithm and its derivative are:

$$\begin{aligned} l(q) &= -5 \ln q - 488.97 q^{-.5} \\ l'(q) &= -5q^{-1} + 244.485 q^{-1.5}. \end{aligned}$$

Setting the derivative equal to zero yields

$$\hat{q} = (244.485/5)^2 = 2391.$$

Item Number: 17
Key: E

Solution

The estimated variance of the forecast errors is the sum of the squares of the error terms divided by $T - p - q$. In this case, after 100 observations, the sum of the squares of the error terms must equal 98, because the sum divided by $(100 - 1 - 1)$, or 98, is 1.0.

The 101st observation introduces a new error term equal to $188 - 197$, or -9. The square of that term is 81. Adding 81 to the previous sum of 98 gives a new total of 179. Dividing 179 by $(101 - 1 - 1)$, or 99, gives a new estimated variance of 1.8.

Item Number: 18
Key: A

Solution

The posterior distribution is $p(l\,|\,0) \propto e^{-l}\left[(.5)5e^{-5l} + (.5).2e^{-.2l}\right] = 2.5e^{-6l} + .1e^{-1.2l}$. The normalizing constant can be obtained from $\int_0^\infty \left[2.5e^{-6l} + .1e^{-1.2l}\right]dl = .5$ and therefore the exact posterior density is $p(l\,|\,0) = 5e^{-6l} + .2e^{-1.2l}$. The expected number of claims in the next year is the posterior mean, $E(\Lambda\,|\,0) = \int_0^\infty l\,[5e^{-6l} + .2e^{-1.2l}]\,dl = \frac{5}{36} + \frac{5}{36} = \frac{5}{18} = .278$.

Item Number: 19
Key: B

Solution

There are 365 observations, so the expected count for k accidents is $365p_k = 365\frac{e^{-.6}(.6)^k}{k!}$ which produces the following table:

No. of accidents	Observed	Expected	Chi-square
0	209	200.32	0.38
1	111	120.19	0.70
2	33	36.06	0.26
3	7	7.21	1.51**
4	3	1.08	
5	2	0.14*	

*This number is 365 minus the sum of the other expected counts.
**The last three cells must be grouped to get the expected count above 5. The calculation for the test statistic is $(12 - 8.43)^2/8.43 = 1.51$. The total of the chi-square numbers is the test statistic of 2.85.

Item Number: 20
Key: E

Solution

The likelihood ratio test statistic is $\left(-382.4+385.9\right)\cdot 2=7$.

Item Number: 21
Key: C

Solution

We need for $S(a)=\sum_{t=1}^{8}\left(\frac{Y_t-a}{\sqrt{0.4}}\right)^2+\sum_{t=9}^{20}\left(\frac{Y_t-a}{\sqrt{0.6}}\right)^2$ to be a minimum. Setting the derivative equal to zero produces the equation $S'(a)=\frac{1}{.4}\sum_{t=1}^{8}2(Y_t-a)+\frac{1}{.6}\sum_{t=9}^{20}2(Y_t-a)=0.$ Multiplying by 0.6 produces the equation:

$$0=3(8\overline{Y}_1-8a)+2(12\overline{Y}_2-12a)$$
$$0=24\overline{Y}_1+24\overline{Y}_2-48a$$
$$\hat{a}=.5\overline{Y}_1+.5\overline{Y}_2$$

Item Number: 22
Key: E

Solution

$\text{O} = 4$

$E = (.29 - .27) + (.35 - .27) + (.41 - .27) + 7\left(.465 - .27\right) = 1.605$

Chi-square statistic $= (4 - 1.605)^2 / 1.605 = 3.57$

The 0.05 level of significance is 3.84, so the answer is (E).

Item Number: 23
Key: B

Solution

The Bühlmann-Straub credibility factor is $\dfrac{n}{n+\dfrac{w+\frac{v}{m}}{a}}$ which goes to $\dfrac{n}{n+\frac{w}{a}}$ as m goes to infinity.

Item Number: 24

Key: B

Solution

With a lognormal error component, the linear model should be for the logarithm of the observation. A model that conforms to the description is

$$\ln Y = a_1^* + a_2^* D + b_1^* X + e^* .$$

Exponentiating both sides yields

$$Y = e^{a_1^*} e^{a_2^* D} e^{b_1^* X} e^{e^*}$$

and then defining an unstarred quantity as its starred version exponentiated, we have

$$Y = a_1 a_2^D b_1^X e.$$

Note that when D is 1, the value of Y is multiplied by a_2 and so the hypothesis to test is if this value is equal to 0.8.

Item Number: 25
Key: C

Solution

The function of interest is $f(a, b) = a + 10b$. The partial derivatives are 1 and 10 and so the variance can be estimated as

$$\begin{aligned}
\hat{s}_f^2 &= \begin{bmatrix} 1 & 10 \end{bmatrix} \widehat{\mathrm{Var}} \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} \begin{bmatrix} 1 \\ 10 \end{bmatrix} \\
&= \begin{bmatrix} 1 & 10 \end{bmatrix} \begin{bmatrix} 0.00055 & -0.00010 \\ -0.00010 & 0.00002 \end{bmatrix} \begin{bmatrix} 1 \\ 10 \end{bmatrix} \\
&= 0.00055 - 2 \times 0.00010 \times 10 + 0.00002 \times 100 \\
&= 0.00055
\end{aligned}$$

and so the standard deviation is the square root, or 0.02345.

Item Number: 26
Key: A

Solution

Following Klein and Moeschberger with the time scale shifted by 1997:

T_i	L_i	R_i	d_i	Y_i	$P\left[L < l_i \mid L < 3\right]$
0	0	2	6		
1	0	2	5		
2	0	2	4	15	$(99/160) \cdot 0 = 0$
0	1	1	3		
1	1	1	2	16	$(9/10) \cdot (11/16) = 99/160$
0	2	0	1	10	$9/10$

$$P\left[L = 1 \mid L < 3\right] = P\left[L < 2 \mid L < 3\right] - P\left[L < 1 \mid L < 3\right] = \frac{9}{10} - \frac{99}{160} = 0.28125$$

Item Number: 27

Key: E

Solution

From formula 15.41:

$$z_t = \frac{y_t}{\tilde{y}_t},\ z_6 = \frac{825}{843},\quad z_{18} = \frac{784}{804},\ z_{30} = \frac{710}{740},\ z_{42} = \frac{918}{905}$$

From formula 15.42:

$$\tilde{z}_6 = \tfrac{1}{4}\left(z_6 + z_{18} + z_{30} + z_{42}\right)$$

$$\tilde{z}_6 = \tfrac{1}{4}\left(0.9786 + 0.9751 + 0.9595 + 1.0144\right) = 0.9819$$

Adjustment factor = 12/11.9607 =1.0033

$$\bar{z}_6 = .9819 * 1.0033 = 0.9851$$

$$y_{42}^a = \frac{918}{0.9851} = 932$$

Item Number: 28
Key: E

Solution

Die/Spinner	Prior probability	Probability of getting a 12	Posterior Probability
AX	1/4	$3/4\times 1/2=3/8$	1/4
AY	1/4	$3/4\times 1=3/4$	1/2
BX	1/4	$1/4\times 1/2=1/8$	1/12
BY	1/4	$1/4\times 1=1/4$	1/6
Total	1	3/2	1

Die/Spinner	Expected Value	Posterior Probability	Expected Value $\times$ posterior probability
AX	(3/4)(1/2)(12+c)	1/4	1.125+3/32c
AY	(3/4)(12)	1/2	4.5
BX	(1/4)(1/2)(12+c)	1/12	0.125+1/96c
BY	(1/4)(12)	1/6	0.5
Total		1	6.25+10/96c

Because the expected value is 10, 6.25+10/96c = 10, so c = 36.

Item Number: 29

Key: D

Solution

Summing the ranks of the A's (being careful to average for ties) gives (1+2+6+6+8.5+8.5+10+11+13.5+20) = 86.5 (or 123.5 working with B's).
$E_{H_0}(R) = 105$, $\mathrm{Var}_{H_0}(R) = 175$, so $r^* = \pm 1.398$ giving $p = 0.162$.

Item Number: 30
Key: C

Solution

The maximum likelihood estimate of the mean, q, is the sample mean, 367.9.
Two years of inflation at 5% inflates this scale parameter to $q = 1.05^2(367.9) = 405.6$.
The expected amount paid per loss is

$$
\begin{aligned}
&E(X \wedge 1000) - E(X \wedge 100) \\
&= 405.6(1 - e^{-1000/405.6} - 1 + e^{-100/405.6}) \\
&= 283.
\end{aligned}
$$

Item Number: 31

Key: A

Solution

$$L(\hat{\mathbf{a}}) = \left(\left(\frac{e^{b_1}}{e^{b_1} + e^{b_2} + e^{b_3}}\right)\left(\frac{e^{b_2}}{e^{b_2} + e^{b_3}}\right)\left(\frac{e^{b_3}}{e^{b_3}}\right)\right)$$

Item Number: 32
Key: A

Solution

$$\hat{v} = \frac{\sum_{i=1}^{2}\sum_{j=1}^{3} m_{ij}\left(x_{ij} - \overline{x}_i\right)^2}{\sum_{i=1}^{2}(3-1)} = \frac{2020}{2+2} = 505$$

$$\hat{a} = \frac{\sum_{i=1}^{2} m_i\left(\overline{x}_i - \overline{x}\right)^2 - \hat{v}(2-1)}{m - \frac{1}{m}\sum_{i=1}^{2} m_i^2} = \frac{4800-505}{100-(1/100)\left(25^2+75^2\right)} = \frac{4295}{37.5} = 114.5333$$

$$\hat{k} = \frac{\hat{v}}{\hat{a}} = 4.4092$$

$$\hat{Z}_1 = \frac{25}{25+\hat{k}} = 0.850074 \text{ and } \hat{Z}_2 = \frac{75}{75+\hat{k}} = 0.944475$$

$$\hat{m} = \frac{\hat{Z}_1\overline{X}_1 + \hat{Z}_2\overline{X}_2}{\hat{Z}_1 + \hat{Z}_2} = 105.4208$$

Bühlmann credibility premium $= \hat{Z}_1(97) + \left(1-\hat{Z}_1\right)(\hat{m}) = 98.26$

Item Number: 33
Key: A

Solution

According to a statement on page 159 of Pindyck and Rubinfeld, the estimator is unbiased.

Item Number: 34
Key: D

Solution

The observations are right and left truncated and the truncation depends upon the report year. For report year 1997 only claims settled at durations 1 and 2 can be observed, so the denominator must be the sum of those two probabilities. For 1998, only durations 0 and 1 can be observed and for 1999 only duration 0 can be observed. Calculation of the denominator probabilities is summarized below.

Probabilities			
Year Reported	Year Settled		Sum (Denominator)
	1998	1999	
1997	$(1-p)p$	$(1-p)p^2$	$(1-p)p(1+p)$
1998	$(1-p)$	$(1-p)p$	$(1-p)(1+p)$
1999		$(1-p)$	$(1-p)$

The likelihood function is:

$$L(p)=\left(\frac{(1-p)\,p}{(1-p)\,p\,(1+p)}\right)^3\left(\frac{(1-p)\,p^2}{(1-p)\,p\,(1+p)}\right)^1\left(\frac{(1-p)}{(1-p)\,(1+p)}\right)^5\left(\frac{(1-p)\,p}{(1-p)\,(1+p)}\right)^2\left(\frac{(1-p)}{(1-p)}\right)^4$$

$$=\left(\frac{1}{1+p}\right)^3\left(\frac{p}{1+p}\right)^1\left(\frac{1}{1+p}\right)^5\left(\frac{p}{1+p}\right)^2$$

$$=\frac{p^3}{(1+p)^{11}}$$

The loglikelihood is $l(p)=3\ln(p)-11\ln(1+p)$

Take the derivative with respect to p to obtain the equation to solve:

$$\frac{3}{p}-\frac{11}{(1+p)}=0$$

The solution is $\hat{p}=\frac{3}{8}$.

Item Number: 35

Key: E

Solution

t	d	Y	$\Delta \tilde{H}(t)$	$\Delta \hat{V}\left[\tilde{H}(t)\right]$	$t^* = \frac{4-t_i}{b}$	$K(t^*)$	$\frac{\Delta \tilde{H}(t)K(t^*)}{b}$	$\Delta \hat{V}\left[\tilde{H}(t)\right]\left(\frac{K(t^*)}{b}\right)^2$
1	1	9	0.1111	0.01235	1.0000	0.000	0.0000	0.000000
3	2	8	0.2500	0.03125	0.3333	0.741	0.0617	0.001905
5	1	5	0.2000	0.04000	−0.3333	0.741	0.0494	0.002439
8	2	3	0.6667	0.22222	−1.3333	0.000	0.0000	0.000000
10	1	1	1.0000	1.00000	−2.0000	0.000	0.0000	0.000000
							$\hat{h}(4) = 0.1111$	$s^2[\hat{h}(4)] = 0.00434$

Lower confidence limit: $0.1111e^{-1.96\sqrt{0.00434}/0.1111} = 0.035$

Upper confidence limit: $0.1111e^{1.96\sqrt{0.00434}/0.1111} = 0.355$

Item Number: 36

Key: D

Solution

See page 555 of Pindyck and Rubinfeld. The variance is $1/T$, not T, and the statement is then only true for large displacements.

Item Number: 37
Key: B

Solution

The first step is to get the posterior distribution of P given no claims:

$$\begin{aligned} p(p|0) &\propto f(0|p)p(p) \\ &= e^{-1+0.01p}(0.01) \\ &\propto e^{0.01p}, 0 < p < 100 \end{aligned}$$

The normalizing constant can be found from

$$\int_0^{100} e^{0.01p}dp = 100e^{0.01p}\Big|_0^{100} = 100(e-1) = 171.83.$$

The posterior density is $p(p|0) = \frac{1}{171.83}e^{0.01p}, 0 < p < 100.$

Then,

$$\Pr(P > 50|0) = \int_{50}^{100} \frac{1}{171.83}e^{0.01p}dp = \frac{100}{171.83}(e - e^{.5}) = 0.622.$$

Item Number: 38
Key: B

Solution

$$m(p) = 1 - 0.01p$$
$$m = E[m(p)] = E(1 - 0.01p) = 1 - 0.01(50) = 0.5$$
$$a = Var[m(p)] = Var(1 - 0.01p) = 0.0001Var(p) = 0.0001\frac{100^2}{12} = 1/12$$
$$v(p) = 1 - 0.01p$$
$$v = E[v(p)] = 0.5$$
$$k = v / a = (1/2)/(1/12) = 6$$
$$Z = 4/(4+6) = 0.4$$
$$P_c = 0.4(5/4) + 0.6(0.5) = 0.8$$

The number of claims is Poisson, so the mean and variance are the Poisson parameter, 1 – 0.01*p*. The mean and variance of *p* come from the uniform distribution.

Item Number: 39

Key: E

Solution

For a mixture, the mean and second raw moments are a mixture of the component means and second raw moments. Therefore,

$$E(X) = p(1) + (1-p)(10) = 10 - 9p$$
$$E(X^2) = p(2) + (1-p)(200) = 200 - 198p$$
$$Var(X) = 200 - 198p - (10 - 9p)^2 = 100 - 18p - 81p^2 = 4.$$

The only positive root of the quadratic equation is $p = 0.983$.

Item Number: 40
Key: C

Solution

$$s^2 = \frac{\sum \hat{e}_i^2}{N-2} = \frac{967}{5} = 193.4$$

$$s_{\hat{b}}^2 = \frac{s^2}{\sum x_i^2} = \frac{193.4}{2000} = 0.0967$$

$$s_{\hat{b}} = 0.31$$