Course 6
May 2003

Multiple-Choice Answer Key

1	C	26	E
2	B	27	D
3	B	28	A
4	C	29	A
5	E	30	C
6	D		
7	D		
8	E		
9	B		
10	D & E		
11	B		
12	A		
13	C		
14	D		
15	D		
16	D		
17	C		
18	B		
19	B & D		
20	D		
21	E		
22	A		
23	E		
24	C		
25	C		

May 2003 Course 6

1. *(6 points)* You are given the following with respect to corporate bonds:

Rating	Spread Over Treasuries (basis points)
AAA	20
AA	30
A	40

The one-year rating transition matrix is as follows:

	Rating at End of Year		
Rating at Beginning of Year	AAA	AA	A
AAA	0.8	0.1	0.1
AA	0.1	0.7	0.2
A	0.0	0.1	0.9

(a) Describe the top down value-added strategies for active bond management.

(b) Describe the corporate bond sector selection strategies.

(c) Calculate the expected two-year horizon spread over Treasuries for a AAA-rated bond.

Show all work.

Question #1 Solution

(a) Top-down value added bond strategies:

Interest Rate Anticipation
Anticipate changes in the future interest rates and adjust the duration of your portfolio accordingly. If rates are expected to drop, increase duration (by purchasing zeros) and vice versa if rates rise.

Yield Curve Strategies
Stagger the maturities of your bonds based on the shape of the yield curve. Use a barbell, bullet, or ladder- type portfolio depending on which provides the optimal return for the current shape of the curve.

Inter and Intra- Sector Selection Strategies
Allocate your bonds in different sectors based on the expected changes in the spreads between sectors (inter- sector strategies).

Intra- sector bond strategies include allocating your bonds differently within a sector according to changes in credit quality (how you think the spreads will change).

(b) Some corporate bond sector selection strategies (3 of them)

1. Spread Analysis- Analyze how the current spreads are moving and anticipate spread changes. Allocate between sectors based on how you expect the spreads to move.

2. Credit Analysis- Analyze the probability of a downgrade in certain sectors.

The return on a bond over an investment horizon is affected by:

- Current spread over treasuries
- Expected change in credit quality
- Expected change in spreads

Method to compute expected yield on a bond

- Compute current price
- Determine changes in quality
- Determine changes in spreads
- Compute the return in each class
- Weight the returns

3. Valuation Analysis-

- Compute the default free value of a bond
- Use regression to find the sensitivity of the spread to different factors
- Find a value for the bonds based on sensitivities and current market levels
- Compare to current market value

(c) Use probability transition matrix to find probabilities of having each rating at the end of 2 years.

$$\begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}0.8 & 0.1 & 0.1\\ 0.1 & 0.7 & 0.2\\ 0.0 & 0.1 & 0.9\end{bmatrix}=\begin{bmatrix}0.8\\ 0.1\\ 0.1\end{bmatrix}$$

$$\begin{bmatrix}0.8 & 0.1 & 0.1\end{bmatrix}\begin{bmatrix}0.8 & 0.1 & 0.1\\ 0.1 & 0.7 & 0.2\\ 0.0 & 0.1 & 0.9\end{bmatrix}=\begin{bmatrix}0.8^2+0.1^2\\ 0.8(0.1)+0.1(0.7)+0.1^2\\ 0.8(0.1)+0.1(0.2)+0.1(0.9)\end{bmatrix}=\begin{bmatrix}0.65\\ 0.16\\ 0.19\end{bmatrix}$$

$\therefore$ Expected spread at the end of 2 years is $0.65(20)+0.16(30)+0.19(40)=25.4$ basis points

2. *(5 points)* You are given the following:

	Probability	One Year Return
Stock X	0.60 0.20 0.20	10% 5% -10%
Stock Y	0.75 0.25	20% -20%

- risk-free rate is 4%
- the investor has a one-year horizon
- the investor is indifferent between investing in Stock Y and earning the risk-free rate

Determine whether or not the investor would purchase Stock X.

Show all work.

Question #2 Solution

Solve for utility function parameter A

$U = E[r] - 0.005As^2$

For a risk- free investment, $U = E[r]$ as no volatility $= 4$

First need to determine $E[r]$ and s^2 for Y

$E[r]$ for Y is $.75 \times 20 + .25 \times (-20) = 10$

s^2 for Y is $.75 \times (20 - 10)^2 + .25 \times (-20 - 10)^2 = 75 + 225 = 300$

Now sub into utility equation with $U = 4$ to solve for A

$4 = 10 - 0.005 \times A \times 300$

$A = 4$

Can now calculate utility from asset X using $A = 4$

First, solve for $E[r]$ and s^2 for asset X

$E[r]$ for X is $.6 \times 10 + .2 \times (-10) = 5\%$

s^2 for X is $.6 \times (10 - 5)^2 + .2 \times (5 - 5)^2 + .2 \times (-10 - 5)^2 = 15 + 45 = 60$

Now sub into utility equation with $A = 4$

$U = 5 - .005 \times 4 \times 60$

$U = 3.8$

$\therefore$ since $U = 3.8$ for asset X, it is below $U = 4$ for the risk free investment, therefore the investor will not purchase it.

3. *(5 points)* You are given the following:

- margin requirement on short sales: 50%
- maintenance margin: 30%
- an investor's account with a broker currently holds:
  - value of T-bills: 10,000
  - number of shares of XYZ stock: 500
  - stock prices:

Date	ABC Stock Price	XYZ Stock Price
June 2, 2003	103	75
June 3, 2003	102	76
June 4, 2003	99	77
June 5, 2003	100	75
June 6, 2003	101	80
June 9, 2003	105	72
June 10, 2003	115	65

The investor tells the broker to short 1,000 shares of the ABC stock on June 3, 2003. The broker executes the order on the first day allowed. Shares are traded once per day.

(a) Calculate the additional cash (if any) necessary to satisfy the margin requirement.

(b) Calculate the amount of the margin calls (if any) between June 3, 2003 and June 10, 2003.

Show all work.

Question #3 Solution

(a) Order is executed on June 5th since short sales are only allowed after an uptick. Investor borrows 1000 shares from broker and sells at 100 each (total of 100,000). Needs 50% margin for the purchase. T-bills on account with broker count towards margin as well as long position in other stock.

$$\text{Needs } 50\% = \frac{10{,}000 + 37{,}500 + x}{100{,}000}$$

$$x = 2{,}500$$

Needs 2,500 additional cash

(b) $\text{Margin} = \dfrac{\text{Equity}}{\text{Market value of shared stock}}$ must be greater than 30% maintenance margin

$$\text{June 6: } \frac{100{,}000\text{-}101{,}000 + 10{,}000 + 2{,}500 + 40{,}000}{101{,}000} = 51\% \text{, so no margin call}$$

$$\text{June 9: } \frac{100{,}000 - 105{,}000 + 10{,}000 + 2{,}500 + 36{,}000}{105{,}000} = 41\% \text{, so no margin call}$$

$$\text{June10: } \frac{100{,}000 - 115{,}000 + 10{,}000 + 2{,}500 + 32{,}500}{115{,}000} = 26\% \text{, margin call made}$$

$$\frac{30{,}000 + x}{115{,}000} = 30\%$$

$$x = 4{,}500$$

Investor must pay broker 4,500 to meet margin call on June 10.

4. *(9 points)* You are given the following with respect to an Extended Vasicek Trinomial Lattice Model:

- $s = 0.02$
- $\Delta t = 1$ year
- $R(1) = 0.08$
- $R(2) = 0.09$
- $R(3) = 0.10$
- $a = 0.4$

(a) Describe the key characteristics of this model.

(b) Calculate the value of $q(0)$ using the Hull and White approximation.

(c) Calculate the value of $p_2(0,0)$.

(d) Calculate the value of a one-year cap with a notional amount of 100 and a strike interest rate of 9.5%.

Show all work.

Question #4 Solution

(a) Extended Vasicek is a constant volatility model

It is an arbitrage free model

Provides exact fit to current term structure of spot rates of interest

The single factor is the short rate rt which is defined to be the continuously compounded yield on a zero coupon bond maturing in Δtime periods.

The model is additive and the set of achievable interest rates are evenly spaced.

Any rate can be expressed as r0 + jΔr where r0 is the initial value of the short rate at time 0 and j is an integer that can be positive, zero, or negative.

Time is also equally spaced with nodes at zero and iΔt where i is a non-negative integer.

-Δr and Δt must be set so that Δr is between σ(3Δt)0.5/2 and 2σ(3Δt)0.5 and they set in their model equal to σ(3Δt)0.5 where σ is the volatility parameter.

(b)

By Hull and White Approximation,

$e^{-2 * R(2)} \sim e^{-R(1)} * e^{-r0} (1- \mu(0,0) * ?t^2)$

$e^{-2(0.09)} \sim e^{-0.08} * e^{-0.08} (1- \mu(0,0))$ since ?t = 1

$\mu(0,0) = 0.0198014$

$?(0) = \mu(0,0) + a*r_0$

$?(0) = 0.0198014 + 0.4 (0.08)$

$?(0) = 0.0518014$

(c)

$P_2 (0,0) = 1 - \sigma^2\Delta t/\Delta r^2 - \eta^2/\Delta r^2$

$\Delta r = \sigma (3\Delta t)^{0.5}$

$\Delta r = 0.02*1.7320508$

$\Delta r = 0.034641$

$\eta = \mu(0,0) \Delta t + (j-k) \Delta r$

k is chosen so that the short rate reached by the middle branch, r_k, is as close as possible to $r_0 + \mu(0,0)*\Delta t$

$r_0 + \mu(0,0) * \Delta t = 0.08 +$
0.0198014

$= 0.0998014.$

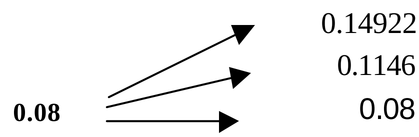
$r_0 + i * \Delta r$

$0.08 + (0)\ 0.03461 = 0.08$

$0.08 + (1)\ 0.03461 = 0.1146$

$0.08 + (2)\ 0.03461 = 0.14922$

0.0998014 is closest to 0.1146, so set k=1



$\mu(0,0) = 0.0198014$ (from part b)

$\eta = 0.0198014\ (1) + (0-1) * 0.034641$

$\eta = -0.0148396$

$P_2\,(0,0) = 1 - (0.02)^2\ (1)/(0.034641)^2 - (-0.0148396)^2/(0.034641)^2$

$= 0.4831664$

(d)

Value of Cap

$= ((r_0 + 2\Delta r) - 0.095) * 100 * P_1\,(0,0) * e^{-rt} + ((r_0 + \Delta r) - 0.095) * 100 * P_2\,(0,0) * e^{-rt} + (0)$

$P_1\,(0,0) = \sigma^2 \Delta t/2\Delta r^{2} + \eta^2/2\Delta r^{2} + \eta/2\Delta r$

$= (0.02)^2/2*(0.034641)^2 + (-0.0148396)^2 / 2*(0.034641)^2 + (-0.0148396)/2*(0.034641)$

$= 0.04423103$

Value of Cap

$= 0.14922 - 0.095) * 100 * 0.04423 * e^{-.08} + ((0.1146 - 0.095) * 100 * 0.4831664\ e^{-.08} + (0)$

$= 1.095574$

5. *(7 points)* You are given the following with respect to non-callable default-free zero-coupon bonds:

Time to Maturity (t)	Current Price
1 year	980.392
2 years	942.596
3 years	888.996
4 years	838.561
5 years	765.134

You are given the following with respect to a five-year option-free bond:

- annual coupons of $(2+t)\%$ are paid at the end of each year
- par value of 1,000
- yield-to-maturity (y) of 5.50%

For the five-year annual coupon bond:

(a) Calculate the Macaulay duration.

(b) Calculate the Macaulay convexity.

(c) Calculate the Fisher-Weil duration.

(d) Calculate the Fisher-Weil convexity.

(e) Calculate the elasticity of the price using a discount factor of $1/(1+y)$.

(f) Calculate the value of M-squared.

Show all work.

Question #5 Solution

(a) Macaulay Duration

$$= \frac{\sum_{t=1}^{5} t \times PVCF_t}{PVTCF}$$

$$\text{Price of Bond} = PVTCF = \frac{(2+1)\% \times (1000)}{1.055} + \frac{(2+2)\% \times (1000)}{1.055^2} + \ldots + \frac{(2+5)\% \times (1000)}{1.055^5}$$

$$= \frac{30}{1.055} + \frac{40}{1.055^2} + \frac{50}{1.055^3} + \frac{60}{1.055^4} + \frac{1070}{1.055^5}$$

=974.08

Macaulay Duration

$$= \frac{1\left(\frac{30}{1.055}\right) + 2\left(\frac{40}{1.055^2}\right) + 3\left(\frac{50}{1.055^3}\right) + 4\left(\frac{60}{1.055^4}\right) + \left(\frac{1070}{1.055^5}\right)}{974.08}$$

= 4.635

(b) Macaulay Convexity

$$= \frac{\sum_{t=1}^{5} t^2 \times PVCF_t}{PVTCF}$$

$$= \frac{1\left(\frac{30}{1.055}\right) + 2^2\left(\frac{40}{1.055^2}\right) + 3^2\left(\frac{50}{1.055^3}\right) + 4^2\left(\frac{60}{1.055^4}\right) + 5^2\left(\frac{1070}{1.055^5}\right)}{974.08}$$

= 22.378

(c) Fisher Weil Duration

$$= \frac{\Sigma t A_t P(0,t)}{\Sigma A_t P(0,t)}$$

$$= \frac{30(980.392) + 2(40)(942.596) + 3(50)(888.996) + 4(60)838.561 + 5(1070)(765.134)}{30(980.392) + 40(942.596) + 50(888.996) + 60(838.561) + 1070(765.134)}$$

$$= \frac{4532890.38}{980572.44}$$

$$= 4.623$$

(d) Fisher Weil Convexity

$$= \frac{\Sigma t^2 A_t P(0,t)}{\Sigma A_t P(0,t)}$$

$$= \frac{1^2(30)(980.392) + 2^2(40)(942.596) + 3^2(50)(888.996) + 4^2(60)(838.561) + 5^2(1070)(765.134)}{980572.44 \text{ (from 5c)}}$$

$$= \frac{21852628.38}{980572.44}$$

$$= 22.286$$

(e) Elasticity

Elasticity = Macaulay Duration = 4.635

(f)

$$M^2 = C - D^2$$

$$= \text{Macaulay Convexity} - (\text{Macaulay Duration})^2$$

$$= 22.378 - 4.635^2$$

$$= .895$$

6. *(8 points)* You are given the following with respect to a defined benefit pension plan:

- final pension is based on career average salaries without an explicit provision for post-retirement indexation
- assets of the pension plan are actively managed by two external managers
- no contributions were made to the fund in 2002

The plan balance sheet for the previous two years is as follows:

	December 31, 2001	December 31, 2002
Assets		
Fixed Income	100	104
Equities	100	80
Liabilities		
Accumulated Benefit Obligation	180	198
Projected Benefit Obligation	195	213

The plan sponsor is concerned with the volatility of the pension plan surplus and is reviewing the investment portfolio structure and asset allocation.

(a) Describe the fundamental decisions involved in the construction of a pension plan investment portfolio.

(b) Compare active to passive investment management strategies.

(c) Recommend a strategy for the pension plan.

(d) Calculate the surplus rate of investment return for 2002.

(e) Formulate the surplus optimization analysis for the plan and recommend an asset allocation.

(f) Propose stress tests to assess the sensitivity of the surplus to sharp changes in asset prices.

Show all work.

Question #6 Solution

(a) Objectives concerning funding level and investment return
Risk Definition
Selection of asset classes
Legal restrictions on permitted investments
Number of managers
Internal/External
Active/passive

(b) Active: frequent trading, passive: buy-and-hold or reduced trading
Active: higher risk, passive: market risk only
Passive: lower management fees
Passive: return equal to market, active: potential for added value.

(c) Recommendation: to limit volatility of surplus consider cash flow matching or immunization for bonds.

(d) Surplus return = (change in surplus value)/(initial asset value)
Initial surplus = 20 (ABO) or 5 (PBO)
Ending surplus = -14 (ABO) or –29(PBO)
Change in surplus = -34
Surplus return = -34/200
Surplus return = -17%

(e) ABO would be appropriate liability measure (career avg., no indexation)
ABO should be based on current interest rates
Objective function (utility) is : $E(SR_M) - \sigma(SR_M)^2 /t$
$E(SR_M)$ = expected surplus return for mix M
$\sigma(SR_M)$ = st. deviation of surplus return for mix M
t = investor's risk tolerance
Objective is maximize the utility
When utility deals with ABO surplus, optimization tends to favor long bonds
If risk tolerance low, mixes with low equity content would be preferred.

(f) Stock market crash
Parallel shift in yield curve
Long term interest rates declines
Widening of credit spreads
Emerging markets declines
Could be hypothetical or based on historical events

7. *(5 points)* With respect to auto loan asset-backed securities (ABS) and auto lease ABS, describe the risks associated with each of these securities and explain how these risks can be mitigated.

Question #7 Solution

<u>Auto Loan ABS</u>

- External risks:
  Bankruptcy of issuer
  - Investor may not be able to get back his money
  - Should use a Special Purpose Entity (SPE) to mitigate

- Internal risks:
  Default and delinquency
  - Mitigate by
    - Using rating agencies to conduct review of servicer
      - Quality of underwriting
      - Ability to service portfolio
    - Using rating agencies collateral analysis
    - Use of credit enhancement

<u>Auto Lease ABS</u>

- External risks
  - Bankruptcy of the lessor
    - Lessor owns the vehicle
    - Should transfer title
    - Can be difficult and time consuming
  - Vicarious tort liability
    - Lessor can be sued for lessee’s actions
    - Mitigate by forcing lessee to get insurance
  - Securitization doesn’t directly own the vehicle
    - Could have claims against trust
    - e.g. from pension plans (PBGC)

- Internal risks
  - Default and delinquency
  - Lease residuals
    - Amount received at lease end
    - Factors affecting rate vehicle turn in:
      - Depreciation of vehicle
      - Marketing strategies of manufacturers
      - Change in model
    - Mitigated by:
      - Underwrite leases conservatively
      - Residual value insurance

8. *(4 points)* You are given the following with respect to shares of Bre-XYZ:

State of Economy	Probability	Share Price on May 1, 2004
Boom	0.20	100
Normal Growth	0.65	50
Recession	0.15	20

- share price on May 1, 2003: 45
- semi-annual cash dividend: 2
- rate of inflation: 2.5%

(a) Calculate the expected holding-period return.

(b) Calculate the standard deviation of the holding-period return.

(c) Calculate the purchasing power of 1,000 to be received in 10 years.

Show all work.

Question #8 Solutions

(a) Holding period return =

$$\frac{\text{Share price on May 1, 2004} + 2\times \text{ Semi-Annual Cash Dividend} - \text{Share price on May 1, 2003}}{\text{Share price on May 1, 2003}}$$

$$\text{HPR}_{\text{Boom}} = \frac{(100+2\times 2-45)}{45} = 131.111\%$$

$$\text{HPR}_{\text{Normal Growth}} = \frac{(50+2\times 2-45)}{45} = 20\%$$

$$\text{HPR}_{\text{Recession}} = \frac{(20+2\times 2-45)}{45} = -46.667\%$$

$$\therefore \text{ Expected Holding Period Return} = .20(131.111\%)+.65(20\%)+0.15(-46.667\%)$$

$$=32.222\%$$

(b) Standard Deviation of Holding Period Return =

$$\sqrt{0.20(131.111\%-32.222\%)^2+0.65(20\%-32.222\%)^2+0.15(-46.667\%-32.222\%)^2}$$

$$=\sqrt{0.298641975}$$

$$=54.648145\%$$

(c) Purchasing Power of 1000 to be received in 10 years =

$$\frac{1000}{(1.025)^{10}} = \$781.20$$

9. *(8 points)* You are given the following with respect to European style options on a common stock:

- strike price: 100
- current market price of the underlying stock: 95
- standard deviation of the underlying stock returns: 0.14
- dividend rate of the underlying stock: 3% payable continuously
- time to maturity for the options: 3 months

You are also given the following selected values from the Standard Normal Cumulative Distribution Function:

Z	N(Z)
0.1	0.5398
0.2	0.5793
0.3	0.6179
0.4	0.6554
0.5	0.6915
0.6	0.7257
0.7	0.7580
0.8	0.7881
0.9	0.8159
1.00	0.8413

The risk-free rate is 3%.

(a) List the assumptions underlying the Black-Scholes option model.

(b) Calculate the value of the call option ignoring the dividend.

(c) Calculate the value of the put option ignoring the dividend.

(d) Calculate the value of the call option including the dividend.

(e) Calculate the value of the put option including the dividend.

Show all work.

Question #9 Solution

(a) Black-Scholes Assumptions:

1) Investors can borrow and lend at the risk-free rate
2) Short selling, borrowing, and fractional shares are allowed
3) No taxes or transaction costs
4) No dividends or underlying asset
5) No arbitrage
6) Returns follow a random walk
7) Continuous returns are normally distributed with mean μ(T-t) and variance σ^2(T-t)

(b) Black-Scholes value of put and call- IGNORING DIVIDEND

$$\text{Call} = SN(d_1) - Xe^{-r(T-t)}N(d_2)$$

$$\text{Put} = Xe^{-r(T-t)}N(-d_2) - SN(-d_1)$$

$$\text{where} \quad d_1 = \frac{1n\left(-\frac{S}{k}\right) + \left(r + \frac{s^2}{2}\right)(T-t)}{s\sqrt{T-t}}$$

$$d_2 = d_1 - s\sqrt{T-t}$$

Given that $S = 95$

$k = 100$

$s = .14$

$T\text{-}t = 3 \text{ months } \left(\frac{1}{4} \text{ of a year}\right)$

Call option value is as follows:

$$d_1 = \frac{1n\left(\frac{95}{100}\right) + \left(.03 + \frac{(.14)^2}{2}\right)\left(\frac{1}{4}\right)}{(.14)\sqrt{\frac{1}{4}}} = -.59$$

$$d_2 = -.59 - (.14)\sqrt{\frac{1}{4}} = -.66$$

$$N(d_1) = N(-.59) \approx 1 - \left[(.9)(.7257) + (.1)(.6915)\right] = .27772$$
$$N(d_2) = N(-.66) \approx 1 - \left[(.6)(.7580) + (.4)(.7257)\right] = .25492$$

$$SN(d_1) - Xe^{-r(T-t)}N(d_2) = 95(.2772) - 100e^{-.03\left(\frac{1}{4}\right)} * .25492$$
$$= 1.08$$

(c) Put option value is as follows:

$$N(-d_1) = N(.59) = .72228$$
$$N(-d_2) = N(.66) = .74508$$

$$Xe^{-r(T-t)}N(-d_2) - SN(-d_1) = 100e^{-.03\left(\frac{1}{4}\right)}.74508 - 95(.72228)$$
$$= 5.33$$

(d) Call option including dividend

$$d_1 = \frac{1n\left(\frac{5}{k}\right) + \left(r\text{-}0 + \frac{s^2}{2}\right)(T\text{-}t)}{s\sqrt{T\text{-}t}}$$

$$d_2 = d_1 \text{-} s\sqrt{T\text{-}t}$$

$$d_1 = \frac{1n\left(\frac{95}{100}\right) + \left(.03\text{-}.03 + \frac{(.14)^2}{2}\right)\left(\frac{1}{4}\right)}{(.14)\left(\sqrt{1/4}\right)} = -.70$$

$$d_2 = \text{-}.70\text{-}(.14)\sqrt{1/4} = \text{-}.77$$

$$N(d_1) = N(\text{-}.70) = 1\text{-}.7580 = .2420$$

$$N(d_2) = N(\text{-}.77) = 1\text{-}\left[(.7)(.7881) + (.3)(.7580)\right] = .22093$$

$$Se^{-f(T-t)}N(d_1)\text{-}Xe^{-r(T-t)}N(d_2) = (242)95e^{-.03\left(\frac{1}{4}\right)} - 100e^{-.03\left(\frac{1}{4}\right)}.22093$$

$$= .089$$

(e) Put option value including dividend

$$N(\text{-}d_1) = N(.70) = .758$$

$$N(\text{-}d_2) = N(.77) = .77907$$

$$Xe^{-r(T-t)}N(\text{-}d_2)\text{-}Se^{-f(T-t)}N(\text{-}d_1) = 100e^{-.03\left(\frac{1}{4}\right)}.77907\text{-}95e^{-.03\left(\frac{1}{4}\right)}.758$$

$$= 5.85$$

10. *(6 points)* You are given the following with respect to an option-free bond portfolio:

- the value of the bond portfolio using the current yield curve is 800
- the value of the bond portfolio using the current yield curve with a parallel shift upwards of 20 basis points is 788
- the value of the bond portfolio using the current yield curve with a parallel shift downwards of 20 basis points is 813

(a) Using the methodology outlined in the Fabozzi textbook, estimate the value of the bond portfolio for a parallel shift upwards of 200 basis points in the yield curve.

(b) Explain how the inclusion of convexity impacts your estimate.

Show all work.

Question #10 Solution

(a) $D=\dfrac{V_- - V_+}{2V_0 ?\, y}$ $\qquad C=\dfrac{V_- + V_+ - 2V_0}{2V_0 ?\, y^2}$

New value change = $\left(\text{-D}\right)\times ?y + C\times ?\,y^2$

$$D=\frac{813-788}{2\times 800\times\left(0.002\right)}=7.8125$$
$$C=\frac{813+788-2\times 800}{2\times 800\times\left(0.002\right)^2}=156.25$$

Now $\Delta y = 200$ bps = 0.02 for up shift

$$\left(\text{-D}\right)\times\Delta y + C\times\Delta y^2 = -7.8125\times 0.02 + 156.25\times 0.02^2$$
$$= -0.09375$$

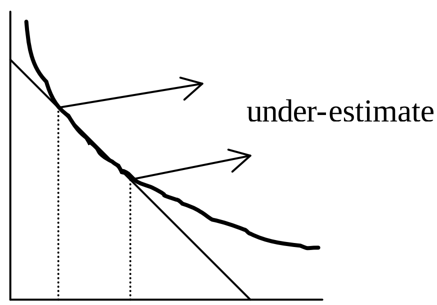
So value of bond $= \left(1-0.09375\right)\times 800 = 725$

(b) Without convexity, value will be

$$-7.8125\times 0.02 = -0.15625$$

$\left(1-0.15625\right)\times 800 = 675$ which under-estimates the bond value by $725\text{-}675 = 50$

This is because for option free bonds, the convexity is always positive. So always under-estimate value if no convexity adjustment. See:



Also, duration approximation is good only for small yield change. For large shock (this is 2%) we need convexity adjustment.

11.

(a) *(4 points)* Describe the issues and practical considerations in immunizing a portfolio of insurance liabilities.

(b) *(1 point)* Describe cash flow matching.

(c) *(1 point)* Describe contingent immunization.

Question #11 Solution

(a) Idea behind immunization is to get interest sensitivity of assets and liabilities in sync.

For a single liability, best way to immunize would be to support the liability with a zero coupon instrument of same maturity.

For a portfolio of coupon bonds to be immunized against a single liability, 3 conditions needed:

1. PV Assets $\geq$ PV liabilities
2. Dur Assets = Dur liabilities
3. Dispersion of assets slightly > Disp liab where dispersion (aka M^2)

$$= \frac{\Sigma \, \text{PVCF}_i \bullet (t_i - D)^2}{\Sigma \, \text{PVCFi}}$$

Note: depending on which text you read, the discount rate used for liab could be:

-The yield on assets (handbook)
-Treasury yield curve plus spread (bond portfolio mgt)
-Treasury spot rate curve plus spread (bond portfolio mgt)
-Yield curve of assets

For multiple liabilities, or "multi-period immunization" you can think of it as n single period immunizations.

If 3 conditions still met, portfolio will be immunized against a one-time-shift in interest rates.

For multi-period iterative process needed to set portfolio and get yield of portfolio to discount liab with.

Rebalancing- as assets' and liab's durations become unsynced (due to time passage and multiple shifts in yield curve), need to rebalance portfolio to meet 3 conditions.

Must weigh need to rebalance vs transaction costs for rebalancing in determining frequency

Embedded options- if bonds have embedded options, cannot use modified duration. Effective duration (aka "option adjusted duration") is better. Modified duration won't work for interest sensitive cash flows.

Other considerations:
Credit quality- defaults can cause need for rebalancing
Liquidity- bonds must be sold near theoretic market value for immunization to work.

In particular, when a liability is due, rebalancing is often necessary

Asset mix and asset selection (diversification)

embedded options in liabilities complicate calculations

(b) Cash flow matching involves setting monthly cash flows from bonds (coupons & maturities) equal to needed cash flows for liabilities.

If successful, eliminates interest rate risk as bonds will never need to be sold- all buy and hold.

Embedded options (calls) are no good as cash flows need to be known With certainty for the technique to work.

Defaults will cause mismatching so credit quality a major concern.

Credit downgrade shouldn't matter; barring default.

Other considerations:
Perfect matching unlikely
Set portfolio constraints as to sector, quality, and size.
Assume a reinvestment rate because cash flows do not exactly match
Select least cost portfolio by using linear programming

(c) Set a safety net return which must be must

Can actively manage as long as safety net still in reach

Once active management puts you in a position of danger with respect to safety net, must switch to immunized portfolio & stay immunized.

Calculate a required terminal value:

$$rtv = I \bullet \left(1+\frac{S}{2}\right)^{2H}$$

Where I = Investment, S = safety, and H = horizon

Then: $ra_t = \frac{rtv}{(1+i_t)^{2(H-t)}}$ = required assets at $_t$ in order to stay active

12. *(6 points)*

(a) With respect to corporate bonds, describe the role of the corporate trustee.

(b) Differentiate the levels of security offered by various corporate bonds.

Question #12 Solutions

(a)

- Contract of the bond is call indenture (very complex)
- Corporate trustee act as a fiduciary, acknowledge its friduary status in writing
- Trustee acts in the interest of bond holders
- Trustee can only act in accordance to the indenture
- Trustee is paid by the bond issuer
- Trustee authenticate the bonds

(b)

1. Mortgage bonds

-The underlying property serves as the collateral
-If the property is sold, then substitute other property or securities.
-Can issue a series of bonds that are secured by the same mortgage with restriction of not over-extending the collateral (blanket mortgage)
-The bond can be secured by the second mortgage
-Low interest for secured bonds

2. Corporate trust bonds

-Use securities (stocks of affiliated company) as collateral
-Have to maintain the value of collateral as certain percentage of par value of the bond.

3. Equipment trust certificates

-Developed by railroad companies
-The trustee owns the equipment and leases it back to the rail-road company.
-The lease payment is used to pay interest and principal on the certificates.
-At the end of lease, the trustee sells the equipment to company for a nominal fee.
-Value of the certificate decreases as the equipment depreciates

4. Unsecured (debentures)

-Most bonds are issued this way
-Some companies are very secure, but others are shaky
-Have to watch out events that increases the leverage (LBO, restructure) and unfriendly takeover.
-Maintaing sufficient capital ratios and limits cash dividend.
-Are rated by S & P, Moody's, etc.

5. Subordinated debentures, e.g. convertibles

-Convertible bonds give the bondholder option to trade in the bond for common stock.
-Last in line (after other creditors) to get money from the issuer if they go bankrupt.
-Offer higher yield or stock options to attract investors.

6. Guaranteed bonds

-A company can guarantee the payment for another company.

13. *(6 points)* You are given the following for an insurance company that currently offers term insurance and fixed deferred annuities:

- corporate pre-tax target return on capital of 18%
- risk-based capital (RBC) formula:
  $1.5 * \sqrt{\left(\text{asset default risk component}^2 + \text{mortality risk component}^2\right)}$

Asset Default Risk Component (C-1)			
Asset Class	Amount (millions)	RBC Factor	Historical Mean Return
Bond	600	1%	7%
Real Estate	300	7%	8%
Common Stock	100	20%	10%

Mortality Risk Component (C-2)		
	Amount (millions)	RBC Factor
Net Amount at Risk	10,000	0.1%

The industry-wide ratio of C-1 to C-2 is 1.5.

The risk-free rate is 6%.

(a) Describe the shortcomings of this RBC formula.

(b) Calculate the RBC-adjusted spread for this company's asset portfolio.

(c) Evaluate the competitive advantage of the company's product lines from a cost of capital perspective.

Show all work.

Question #13 Solution

(a) Shortcomings of this RBC formula $\left(1.5\sqrt{C1^2 + C2^2}\right)$

- Only 2 factors; should include more factors such as interest rate and interest rate spread risks
- No covariance adjustments
- Does not factor in the key risks such as general business risk like accounting, tax management, regulatory risks
- Factors used; not enough refinement to treat sub-categories of the asset class (eg: T-bond vs mortgage bond)
- Just a formula; does not reflect the actual data of company such as underwriting standards

(b) RBC – adjusted spread

= gross spread net of expenses- [Target ROC – Surplus ROR] x RBC factor
= net spread – [.18-.06] x RBC factor for this company

$$\text{RBC} = 1.5\sqrt{600(.01)*(300)\times.07+100(.20)^2+(10000\times.001)^2}$$

$$= 1.5\times\sqrt{(6+21+20)^2+10^2}$$

$$=1.5\times\sqrt{47^2+10^2}$$

$$=1.5\times\sqrt{2309}$$

$$=1.5\times48.05$$

$$=72.08 \text{ million}$$

$$\therefore \text{RBC ratio} = \frac{\text{capital + surplus}}{\text{RBCrequired} - 1}$$

$$= \frac{1000 \text{ mil}}{72} = 13.87$$

For bond, the RBC –adjusted spread

7% - (18%-6%)x1%x1.5 = 6.82%

For real estate

8% - (18%-6%)x(1.5x7%) = 6.74%

For common stock

10% - (18%-6%)x1.5x.20% = 6.4%

For the asset portfolio
Weighted RBC adjusted spread (proportioned to amount)

.60x6.82 +.30x6.74+.10x6.4% = 6.754%

RBC – adjusted spread = 6.754% - 6% = 0.754% or75bps

(c) Competitive advantage if company product lower

Market required Capital = Face Capital + at risk Capital
(earning 6%) (earning 10%)

Cost of Capital

$$\therefore \text{COC} = \text{\% face capital} \times 6\% + (1\text{-\% of face capital}) \times \text{ equity return}$$

Those companies which are more asset unicated, (i.e. with relatively high proportion of C1/RBC) would have advantage to sell more insurance risk type products like term insurance. Those with relatively low ratio have competitive advantage to sell accumulation products.

RBC required can tie down some capital and restrain company's ability to grow and sell certain more capital-intensive products.

This is particularly so if the earning ratio is too near the regulatory or rating agencies threshold level (e.g. trend test zone).

In this case:

$$R = \frac{C_2}{C_1} = \frac{10}{47} = .21 < 0.35$$

So, looks like insurance-oriented$\rightarrow$more competitive to sell insurance-oriented products.

14. *(4 points)*

(a) Describe and compare the various prepayment models used to evaluate a block of mortgage-backed securities (MBS) pass-throughs.

(b) Describe the effects of prepayment rates on the cash flows of MBS pass-throughs.

Question #14 Solution:

(a) 12-year prepaid life

- No prepayments for 1st 12 years, then ALL prepaid at one time
- Very unrealistic, not very useful
- Used because average life of FHA mortgage = 12 years
- Hard to value MBS accurately
- Ignores that prepayment tend to vary with interest rates and mortgage characteristics.

Economic Models

- Projects defaults
- Considers import of economic environment
- Most accurate
- Lots of work and time = costly
- Worth effort if valuing large portfolios

Characteristics: Interest Rate/NOI simulation
Market value/NOI projections
Prepayment assumptions
Loss severity
Deal structure interface
Default

FHA experience

- Based on actual experience from FHA-insured 30 year loans
- Not a simple formula
- Changes when experience is updated
- Contains characteristics specific to FHA loans; doesn't match well for non-FHA
- Difficult to use; no formula
- Prepayment rates are expressed as % of FHA table

PSA model

- Commonly used
- Simple in practice
- Easy to understand
- Matches actual experience while considering its simplicity
- Models increasing prepayment rates to month 30, then steady thereafter
- Easily scalable for faster/slower prepay

prepayment rate = .002 x N for months 1-30
= .06 for months 31+

- Combines the information of FHA and simplicity of CPR.
- Current industry standard, expressed as % of PSA
- Reflects age of prepayments (advantage)

(b) Fast prepay

- More principal repaid quickly
- Good for discount MBS's; increases return
- Generally happens when interest rates fall
- Shortens average life
- Good for PO's, bad for IO's (usually)

Slow prepay

- Less principal repaid quickly
- May happen when rates increase
- Good for premium MBS, usually good for IO's
- Bad for PO's
- Extends average life