

Course 8V - Illustrative Solutions

Solution 1

- (a) A derivative is a financial contract whose payoff depends on the underlying asset or underlying reference rate or index. The derivative in US GAAP should have a notional amount, underlying asset or provision payment. The initial payment for a derivative is zero or very small. The derivative may permit net settlement. Some derivatives may be embedded in other contracts.

The derivative defined in the Case Study is not consistent with that defined in US GAAP. It does not mention that the initial premium is zero or very small. It does not define the settlement method, either.

- (b) Operational risks arise from system inefficiency, human errors, management failure, and model errors. To manage operational risk, Life Co does the following:
1. Set up an ALM committee with clear definition of responsibility
 2. Set up guideline on which derivatives can be used
 3. Set limits on risk exposure
 4. Has written request in place for derivative acquisition
 5. Good transaction documentation and control procedures
 6. Follow required accounting practices
 7. Disclose balance sheet effect
 8. Has Investment administration system to maintain derivatives
 9. If system cannot support, it is documented in a report

All these measures control the operational risks

Legal risks include

1. Unenforceability – derivative contract cannot be enforced
2. Illegality – terms of the contract violate regulation
3. Close-out netting and insolvency.

To address this risk, Life Co

1. Set requirement for approved counterparties
2. Has transaction documentation and control procedure to verify terms of transaction and review them
3. Need approval from Investment Department employee. Make employee accountable for transactions
4. Set limits on risk exposure

Solution 1 (continued)

- (c) Life Co can add these operational risk controls, independent risk management, independent audit and checks.

For legal risk controls, Life Co can address the uncertain legality and authority of counterparties.

Solution 2

(a)

1. OAS is a dynamic value, sensitive to the input/assumptions made.
2. haven't quite established that high OAS securities must be better than lower OAS's.
3. The result is only as good as the model exposed to model risk and no model is a fully realistic representation of reality, assumption/simplification must be made.
4. OAS is a single number. We should consider it within a big background, how the distribution of "PASS" looks like
"PASS" = Price and spread specific Price
(Ave of "PASS" = MV of Asset) gives you OAS.

Jump Z-bond: Have event risk under certain conditions (high prepayment period). Z-bond could jump from interest-accrual to principal payment status. Could be sticky or non-sticky, non cumulative or cumulative. Jump-Z bonds are the support bonds for PACS so Jump Z-bond is very interest-sensitive, even modeling under stochastic model still not 100% sure its behavior. So "profit" = OAS- Required of Jump-Z bond is not a 100% reliable profit that should be gathered at the inception. Better see the market conception of specific securities.

(b)

Similar to 2 (a) all models have limitations.

1. Simplification
2. Assumption
3. Time horizon
4. Sampling space
5. Human error
6. Bad solution scheme for good model
7. Software/hardware bugs
8. Observed or deduced value of market variable

Also, nature of CF of securities:

1. callable
2. putable
3. extendable
4. jump or tricky
5. seniority

All these have impact on the calculation and analysis

Solution 2 (continued)

OAS will vary over time due to
level of yield curve
prepayments
credit risk

OAS provides only a summary of what may happen, actual results may be an outlier

- (c) Regular Z-bond does have longer duration. Is a good matching to long liability. High OAS is a credit, but based on 2 (a) (b), we know it is 100% reliable measure Regular Z-bond less call/extension risk, more stable CFs, high yield, increase liquidity.

So the student is sort of right this time.

But have to understand the payment structure better to make the final decision:

1. collateral
2. Prepayment history
3. Coupon Rate vs. Market Rate
4. Seasoned or new issue
5. Agency
6. Definition of other classes in the same pool.

High OAS does not necessarily mean better investment

It is difficult to compare OAS of different securities

Solution 3

GMBD: the policyholder is guaranteed a minimum benefit upon death based on the accumulation of premiums at 5%.

This is a payoff of the fund value with an embedded put option

(b)

i) $\sigma_n^2 = \frac{1}{m} \sum_{i=1}^m u_{n-i}^2$ in its simplified form where $u_t = \log(S_t / S_{t-1})$

Equal weight to all past data, dismissing the fact that the more recent data may be more relevant. The estimate does not incorporate a long-term variance.

ii) EWMA: $\sigma_n^2 = \lambda \sigma_{n-1}^2 + (1-\lambda) u_{n-1}^2$
Reflects autocorrelation or variance
Does not incorporate a long-term variance
Special case of GARCH

iii) GARCH: $\sigma_n^2 = \gamma V_t + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2$
mean-reversion to a long-term average V_t
model may have parameter estimation problem but in general better than EWMA

- (c) Current program only hedges delta, ignoring other risks like
- gamma (rate of change of delta vs. underlying stock price)
 - vega (measure impact of change in volatility)
 - rho (measure impact of change in interest rate)
 - Theta (time decay impact) is usually ignored

These greeks should be monitored and eventually hedged. We would need to use options to hedge gamma and vega.

Consider estimating volatility using a GARCH model. The lognormal assumption should be reviewed as well and compared to other models like RSLN

Solution 3 (continued)

- (d) Develop model using an inter-disciplinary approach since many areas of expertise are needed (finance, pricing, investments, etc.)
Test boundary cases and simple cases first.
Do not ignore small discrepancies as they may signal bigger problems.
Pride of ownership is important for modelers.
Distribute model outward slowly to increase chance of catching errors.
- (e) - Running the risk naked: assume that guarantees will be covered by fees
Risky approach as potential loss can be high.
- Static hedging: use put options to create an initial hedge. Still exposed to over and under hedging and counterparty risk. May be expensive.
- Reinsurance: transfer risk to reinsurer. Subject to availability and counterparty risk.
- Securitization
- Make market by offering offsetting risks
- (f) Buying exchange traded options is a better alternative as it will be cheaper than reinsurance. This will provide a reasonable hedge given a good estimation of policyholder deaths and withdrawals. Credit enhancement can be used to reduce the credit risk.

Solution 4

- (a) The CEO, an accountant by training, emphasizes the importance of statutory and GAAP measures to determine the economic value of the insurer.
Critique this standpoint
- Statutory and GAAP measures:
 - o are open to manipulation by management to smooth earnings
 - o ignore realized and unrealized gains and losses
 - o ignore new business
 - The real economic value of the firm equals:
 - o $PV(\text{Asset Cash Flows}) - PV(\text{Liabilities Cash Flows})$
 - The market sees through earnings and reporting management and focuses on free cash flows
 - However, we cannot ignore the impact of statutory and GAAP measures because
 - o these measures are reviewed by regulators and rating agencies (GP=1)
 - o these measures impact taxes (GP=1)
- (b) Explain how to coordinate LifeCo's investment and product management strategies for future business for this new product to protect LifeCo's shareholder value from interest rate risk.
- To immunize shareholder value against interest rate changes, we should set the duration of assets such that
$$V(A) D(A) + V(FR) D(FR) = (1+k) V(R) D(R)$$
Where $V(A)$ = market value of assets
 $D(A)$ = duration of assets
 $V(FR)$ = market value of future retentions
 $D(FR)$ = duration of future retentions
 $V(R)$ = present value of reserves
 $D(R)$ = duration of reserves
 $kV(R)$ = committed value of surplus
 - If the premiums of the new product are interest sensitive, then the duration of future retentions is positive (i.e. $D(FR) > 0$); if our premiums are not interest sensitive, then $D(FR) < 0$

Solution 4 (continued)

- The duration of assets, $D(A)$ should be increased or decreased to compensate for changes in $D(FR)$ as per the above formula
 - The product management and investment areas should ensure that the liability crediting rates are supportable
- (c) The pricing actuary expects to increase credited rates as interest rates increase. Devise an appropriate investment strategy to protect shareholder value from interest rate risk.
- Since future credited rates vary directly with changes in interest rates, then future retentions will have a positive duration, $D(FR) > 0$
 - The duration of assets, $D(A)$, should be reduced to compensate
- (d) One of LifeCo's key competitors, ACME Life is more prone to maintain premiums regardless of changes in interest rates. Assess how their approach would impact your strategy in part c).
- If competitors maintain premiums regardless of interest rate changes, Life Co's $D(FR)$ will be reduced
 - To compensate, LifeCo's $D(A)$ should be increased
- (e) Propose a method to implement the changes in parts c) & d) that would minimize transaction
- Interest rate swaps may be used to adjust the durations of bonds
 - Transaction costs to enter into a swap are minimal
 - Swaps have counterparty risk
 - Since the changes to asset duration to parts c) & d) are offsetting, we can adjust duration on a net basis rather than completing two separate transactions

Solution 5

The α -quantile risk measure for a loss random variable L is V_α such that, for $0 \leq \alpha \leq 1$

$$V_\alpha = \min \{V : P_r[L \leq V] \geq \alpha\}$$

(b)

α	Real World α -Quantile	Risk Neutral α -Quantile
91%	-0.35%	-0.30%
95%	-0.10%	-0.21%

(c) The α -CTE Risk measure for a loss random variable L , for $0 \leq \alpha \leq 1$, is $CTE_\alpha = E[L|L > V_\alpha]$ provided V_α does not lie in a probability mass for the loss L , where V_α is the α -quantile.

If V_α lies in a probability mass

$$CTE_\alpha = \frac{(1 - \beta^1)E[L|L > V_\alpha] + (\beta^1 - \alpha)V_\alpha}{1 - \alpha}$$

$$\beta^1 = \max \{ \beta : V_\alpha = V_\beta \}$$

(d)

	Real World α -CTE	Risk Neutral α CTE
91%	0.36%	-0.07%
95%	0.80%	0.07%

Solution 5 (continued)

- (e) Using the Normal approximation to the binomial distribution, the 64% Confidence Interval is $(L_{(95-A)}, L_{(95+A)})$ where

$$\Phi\left(\frac{A}{\sqrt{100(0.95)0.05}}\right) - \Phi\left(\frac{-A}{\sqrt{100(0.95)0.05}}\right) = 0.64$$

$$\Rightarrow \frac{A}{\sqrt{100(0.95)(0.05)}} = \Phi^{-1}\left(\frac{1.64}{2}\right)$$

$$\Rightarrow A = 1.99 \text{ use } A = 2 \text{ for } (L_{(93)}, L_{(97)})$$

that is (-0.25%, 0.10%) is a non-parametric 64% CI for the 95% quantile

- (f) Risk-neutral scenarios should not be used for calculating risk measures. The risk-neutral distribution is for pricing and valuation under hedging.

The CTE is a more robust measure than the quantile measure. It would be better to start and stick with the CTE.

The cost of changing measure standards after a market downturn would be substantial – the numbers above would no longer apply.

The idea of this risk measure is to choose a standard (e.g. 95%) and an approach (e.g. CTE) and stay with it.

Solution 6

(a)

Group into exposure bands:

Band j	# Obligors	Expected Loss	Default Frequency
2	2	.0066 + .0094 = .0160	.0080
7	1	.0105	.0015
9	1	.0180	.0020

CreditRisk+ approach uses the Poisson distribution,

For loss to be ≤ 300000 can have at most one default, which must be in band 2.

$$P(\text{no defaults in bands 2,7,9}) = (e^{-.008})(e^{-.0015})(e^{-.002})$$

$$= .9920 * .9985 * .9980 = .9885$$

$$P(\text{1 default in band 2, none in 7,9})$$

$$= (.008 * e^{-.008})(e^{-.0015})(e^{-.002})$$

$$= .0078$$

$$P(\text{loss} \leq 300,000) = P(\text{no defaults in bands 2,7,9}) + P(\text{1 default in band 2, none in 7,9})$$

$$= .9885 + .0078$$

$$= .9963$$

(b)

CreditRisk+ has advantages

- easy to implement
- closed form expression possible
- few inputs needed
- CreditMetrics can't reflect firm-specific factors
- KMV needs assumptions for capital structure plus measurement of equity value

Disadvantages:

- Relies on historical information
- Ignores downgrade/migration risk

Also shares some limitations of KMV, CreditMetrics

- no relationship between credit risk and market risk
- can't handle nonlinear product like options

Solution 7

Factors to consider:

- understand the market
- understand the index
- understand the trading cost
- compare nationally
- rule, regulations, procedures may not be published
- know how closing level is calculated
- know how index level is calculated
- small index may be dominated by a few issues
- fee and commission
- bid-ask spread
- price differential below quote driven and market-driven
- how well they are managed
- compare historical data

Factor with the most effect

- Legal protection to investor
- corporation management objective goal
- reliability of information/communication with investor

To improve the above factors

- understand local market
- understand legal structure
- understand goal of management
- effective communication, obtain reliable information
- experience staffs
- research
- location (centrally)
- use of analytical tool

Financial info should be questioned/cannot rely totally on if not understand local accounting procedures

Historical return data cannot be meaningfully compared if you do not understand how closing level determined (index dividend may other cost may impact closing level).

Company in need of capital inflow may be willing to disclose all the information in order to attract capital should watch out for the reliability of that info.

Can also hedge foreign currency exposure in foreign market
Use option – for company with fair amount of capital (w/upside potential)
Use future – lock in fix spread, (w/o upside potential)
Consider how much CF generated from overseas

Solution 7 (continued)

Can also look at political stability in foreign market by looking at:

- Human capital (higher more stable)
- Agriculture (higher as a % of GDP less stable)
- Competitiveness (Export/import)/GDP (higher, more stable)
- Per capita DGP (higher more stable)
- Trauma (country with historical trauma usually stable (more successful)
- Democracy (lack of, less stable)
- Quality of life (life expectance) (higher, more stable)
- Distribution of income (infant mortality) (less equally distributed, less stable)
- Rental income (higher, less stable)
- Predictability (ability to predict wholesale price Δ for stability)

Solution 8

First find the state prices for the known securities

$$1 = 1.5\tau(1) + 0.7\tau(2)$$

$$\frac{15}{11} = 2\tau(1) + \tau(2)$$

$$\text{solving system we get } \tau(1) = \tau(2) = \frac{5}{11}$$

Now solve for the risk free rate, r

$$1 = (1+r)\left(\frac{5}{11}\right) + (1+r)\left(\frac{5}{11}\right) \Rightarrow r = 10\%$$

Using security 'A' calculate the probability of 'Up' and 'Down'

$$(1+r) = 1.5p + 0.7(1-p) \Rightarrow p = 0.5$$

The Deflator for state 's' is $U(s) = \frac{\tau(s)}{p(s)}$

$$\text{So } D(\text{'Up'}) = D(\text{'Down'}) = \frac{\frac{5}{11}}{0.5} = \frac{10}{11}$$

$$\begin{aligned} \text{Now, } C &= \sum_s p(s)D(s)C(s) \\ &= .5\left(\frac{10}{11}\right)(1.8) + .5\left(\frac{10}{11}\right)(.85) \\ &= \frac{53}{44} \end{aligned}$$

So, the price of security 'C' is $\frac{53}{44}$

Solution 9

(a)

1. Cash flows from new Premium
2. Fund allocation levels between accumulation fund and payment annuity
3. Overall level of future expected cash flows in and out
4. Lapse, surrender and mortality expected
5. Guaranteed rates
6. Assumed earned rate in pricing
7. Product features that may effect required return and future cash flows

(b)

1. Cash flow levels
2. Possible alternative investment types
3. Any special risk or regulatory limitations
4. Effect of investments on product competitiveness
5. Any features of existing portfolio fit this product

(c)

1. This benchmark is not appropriate because it includes non-investment grade assets
2. Also does not include corporate bonds which is 65%
3. Because of this will not track with returns or credit risk
4. Will not be able to measure effectiveness of manager

(d)

- Consideration when stop selling
1. no future premium coming in
 2. Must forecast match of asset and liability cash flows
 3. can change investment strategy to shorten duration and more definite match
 4. Keep more liquidity in assets
 5. If have to sell assets, consider
accounting concerns
tax concerns
ALM concerns
Effect of existing policy holders

Solution 10

- (a) i.) **Asset Swaps**
Hedges out interest rate risk by swapping fixed to floating
- Advantages
Reduces interest rate exposure
Takes advantage of mispricings in FRN market
Over the counter (very liquid)
Can purchase credit protection in future cheaply through a forward asset swap
Gains access to foreign currency market with little interest or credit risk
- Disadvantages
Retains credit exposure
- ii) **Single Name default swaps**
Bilateral contract to buy credit protection against default of specific asset
After credit event buyer receives payment of par – recovery value
Protection buyer pays fee or premium up front over life of contract
Negotiated contract – must define reference asset credit event and payoff
- Advantages
Customized
Access to credit market with no up front cost (unfunded)
Hedges large concentrations of risk
Asset protection without shorting asset
Arbitrage possibilities
digital or binary can reduce recovery risk
- Disadvantages
Par product only
Replicating strategy does not always exist

Solution 10 (continued)

iii) **Basket Default Swap**

Similar to single Name Default Swap
Except credit event is a combination of assets in a basket
Can be first-to-default or second to default
Pricing dependent on correlations of assets

Advantages

Diversification
Cheaper to use second to default

Disadvantages

Historical correlations difficult to measure
Basket spread is 2-3 times avg spread of assets (very expensive)
No static hedge strategy, need dynamic

iv) **Portfolio Default swap**

Similar to basket but takes a portfolio of credit names
Redistributes to a tranche structure
Riskier tranche is that which reduces principal on first default, similar to equity
Works best for 40-100 names in portfolio
Redistribution of risk is tied to amount of loss (% of portfolio) rather than 3 of assets

Advantages

Easier to understand than Bucket default swap
Investors can choose their own risk/reward profile

Disadvantage

will need to pay higher spread on equity tranche

(b) **Accounting Considerations**

Must follow FAS 133 or IAS39 if adopted
Must mark-to-market (fair value) derivative contracts
Hedge accounting to apply if can demonstrate effectiveness
If fair value hedge, both MTM of derivative and offsetting item booked through income
If cashflow hedge, MTM booked through OCI in equity
Must document all hedging transactions and compliance with FAS 133

Solution 10 (continued)

Default swap is treated as a fair value hedge

Must separate interest component from credit risk component

Credit risk defined as full price risk less benchmark interest rate risk

Any hedging gains/losses recorded in income as ineffectiveness

Solution 11

The cost of equity

- based on CAPM, $r_E = r_f + \beta(r_m - r_f)$
- where r_f = risk free rate
- β = company beta
- $r_m - r_f$ = market risk premium
- Therefore, $r_e = 0.06 + 1.2(0.05) = 12\%$

$$\begin{aligned}\text{The cost of capital} &= (1 - \text{Tax Rate}) (\text{Cost of debt}) (\text{proportion of debt}) \\ &\quad + (\text{cost of equity}) (\text{proportion of equity}) \\ &= (1 - 0.35)(10\%)(20\%) + (12\%)(80\%) \\ &= 10.9\%\end{aligned}$$

The economic model approach will consider the cash flows of the company instead of just the income level.

The total cash flows to the company

$$\begin{aligned}&= (\text{Net income}) + (\text{goodwill amortization}) + (\text{interest after tax expense}) - \\ &\quad (\text{capital expenditures}) \\ &= 250,000 + 40,000 + 45,000 \times (1 - 35\%) - 30,000 \\ &= 289,250\end{aligned}$$

Under a perpetual growth scenario, the market value of the firm =

$$\begin{aligned}&\frac{\text{cash flow} (1 + \text{growth rate})}{\text{cost of capital} - \text{growth rate}} \\ &= \frac{289,250(1 + 0.04)}{0.109 - 0.04} \\ &= 4,359,710\end{aligned}$$

$$\begin{aligned}\text{The share price} &= \frac{\text{firm market value} - \text{market value of debt}}{\text{number of shares outstanding}} \\ &= \frac{4,359,710 - 2,000,000}{100,000} \\ &= 23.60\end{aligned}$$

Given that the stock has been trading around \$25.5, the share price is currently over valued.

Solution 12

(a) For

- a liability is someone's asset, thus should be under the same valuation platform
- Fail to include credit rate in liability means company can manipulate earnings by trading on its own debt
- No compelling reason to suggest that financial liabilities should be treated differently from publicly issued debt.
- Value of liability cannot be greater than value of asset from the owner's point of view.

Against

- Involve credit rate means higher earnings when credit falls.
- Liabilities are not traded, thus asset valuation does not apply
- Actual exit price does not make sense to insurance liability since insurance liability must be settled.
- Financial Statement with no credit adjustment to liability is more useful to user.

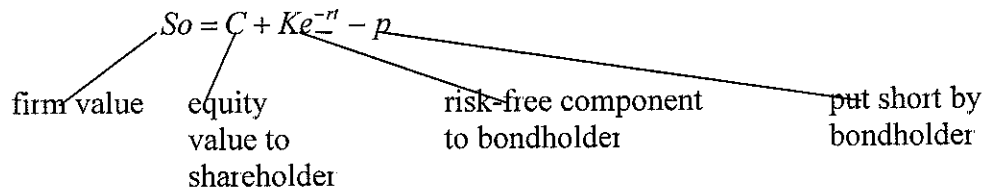
(b) Indirect method: $FML_t = MVA_t - DDE_t - DTL_t$

Put call parity: $S_0 + p = Ke^{-rt} + C$

According to Modigliani and Miller (MM), value of a firm does not change by amount or structure of debt. It assumes a world where market is perfect, no tax.

Equity can think of as a call on value of a firm
debt can think of as a combination of a risk-free asset plus a put (since firm can default)

In this case we have the following

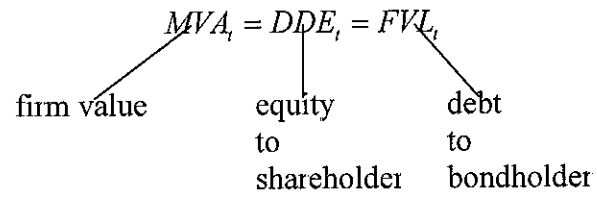


This is the actual put-call parity stated earlier

Solution 12 (continued)

In Indirect method, in a world without tax we have:

$$FVL_t = MVA_t - DDE_t$$



This is in the same structure as shown by the put-call parity above.

Solution 13

$$(a) \quad u = 1 - e^{-(0.055 - 0.05) \times 1} \\ = 0.004988$$

$$(b) \quad v = 130 [FN(d1) - KN(d2)] e^{-rT}$$

$$d1 = \frac{\ln(F/K) + \sigma^2 \cdot T/2}{\sigma \sqrt{T}}$$

$$d2 = d1 - \sigma \sqrt{T}$$

$$F = K = 0.76923$$

$$\sigma = 0.12$$

$$T = 1$$

$$r = 0.05$$

$$d1 = 0.06$$

$$d2 = -0.06$$

$$v = 130 \times 0.035009$$

$$= 4.55111$$

$$(c) \quad \text{cost of default} = uv \\ = 0.004988 \times 4.55111 \\ = 0.02270$$

Solution 14

- (a) NPV: project net cash flows (which derived from project operations net of new capital injection) in the real world, then discounting them at risk-adjusted rate (which may reflect the investor's risk/reward profile, may be derived by CAPM or simply judgment) at initial point. Then subtract the initial investment. If the result is a positive number then this capital investment project is desirable.
Risk-neutral valuation: project net cash flows in risk-neutral world (assume investors are all risk-neutral), then discount them at risk-free rate at inception.
- (b) Advantages of RN valuation: simply, eliminating the subjectivity of selection of appropriate discount rate. Assume every investment project will earn risk-free rate. Avoid difficulties in stochastic simulation of projected cash flows. The spot rate of CFs just equal to short rate more appropriate for real options.
- (c) Moving from real world to risk-neutral world, expected return changes from μ , risk-adjusted return, to r , risk-free rate, while the volatility remains unchanged.
- (d) To value derivatives, risk-neutral approach projects expected net cash flows in risk-neutral world, then discounts at risk-free rate; real world approach (actuarial approach) projects cash flows in real world (using unadjusted probability based on historical data) then also discount at risk-free rate.
- (e) When real world and risk neutral world give the same cash flows, then above two approaches give the same value. This will be the case when derivatives risk is complete unsystematic

Solution 15

(a)

Anticipated Stock Prices

$$\begin{array}{cc} 20 & 30 \\ & 15 \end{array}$$

$$T = 3/12 \quad r = 0.05$$

Option Payoffs

$$\begin{array}{cc} ? & 8 \\ & 0 \end{array}$$

$$f = e^{-rT} [pf_u + (1-p)f_d]$$

$$p = \frac{e^{rT} - d}{u - d}$$

$$c = e^{-0.05 \times 0.25} [0.35 \times 8 + (1 - 0.35) \times 0]$$

$$C = 2.766$$

(b)

Lognormal is not appropriate as it assumes smooth changes in price. Regime switching lognormal might be more appropriate.

Delta-gamma hedge is not appropriate as an announcement is expected that will make the stock price jump.

Dynamic hedging assumes continuous price movement

Solution 16

(a) $r_A = 7\%, r_f = 5\%, r_E = 15\%, e = 20\%, t = 35\%$

$$r_L = r_A - [e \times (r_E / (1 - t) - r_A)]$$

$$r_L = 7\% - [20\% \times (15\% / (1 - 35\%) - 7\%)]$$

$$r_L = 3.7846\%$$

The fair value of the liability at the beginning of the policy year is the present value of cash flows discounted at rate r_L

$$C / (1 + r_L) = 2000 / 1.037846 = 1927.07$$

(b) $(C + MVM) / (1 + r_f) = C / (1 + r_L)$

$$MVM = C \times (r_f - r_L) / (1 + r_L)$$

$$MVM = 2000 \times (5\% - 3.7846\%) / 1.037846$$

$$MVM = 23.42$$

(c) From economic theory, the risk adjustment or insurance should be zero. Yet, the risk adjustment is not zero in the real world. Assumptions that may not hold in the real world include the following:

- i.) The risks may have highly skewed distributions. There may be a very small probability of an extremely large loss. It is difficult to reduce through diversification for such risks.
- ii.) The risks may not be diversifiable. The risk due to the unknown level of true expected claims can be hard to diversify. Financial instruments that allow easy and inexpensive trading of such risks do not exist in most cases.
- iii.) The theory depends on costless bankruptcy, which means that all assets of a failed entity are available to settle its liabilities. In reality, there are substantial legal costs that reduce the amount of assets available to settle liabilities.
- iv.) The theory depends on a well-defined market portfolio. World financial markets have not converged to the point where such a market portfolio can be clearly defined and its covariance matrix determined.

Solution 16 (continued)

- (d) (i) U.S. Treasuries are not really risk-free because they are denominated in dollars. They have inflation risk.
- (ii) U.S. Treasuries are very liquid, thereby implicitly including an option to sell at current market value before maturity. Liquidity is not a requirement of a default-free security with cash flows that are certain.

Solution 17

(a)

- An excess spread is used to measure the profitability in terms of a spread between the earned returns on the assets and the required rate on the assets to satisfy the liabilities.
- RSA is the required spread on assets, which is the spread over Treasuries that must be earned on assets to satisfy the liabilities.
- There are five steps to calculate RSA:
 - Calculate the market value of the asset portfolio as of a certain date
 - Calculate the Treasury forward rates as of the same date
 - for interest-rate sensitive liabilities, develop a set of interest rate paths.
 - Calculate the liability and expense cash flows
 - Determine the spread that, when added to the corresponding Treasury rates, will discount the liability and expense cash flows to the market value of assets. This spread is RSA.
- Excess Spread=Spread on Asset
 - Credit Risk
 - Investment expenses
 - RSA

Where Spread on assets is a spread of the actual assets held over Treasuries.

(b)

- SPDA is an insurance product with the following risks faced by the company:
 - Interest risk
 - Surrender risk
 - New business rate/sales volume
 - Expense risk
 - Mortality risk

(c)

- Interest rate risk can be measured by the effect on excess spread of parallel shocks in interest rates.
- The first step is to select the shock levels
- Then calculate the new market value of the assets at each shock level.
- The second step is to generate a set of interest rate paths for each interest rate shock.
- Then to calculate the RSA as in a)

Solution 17 (continued)

(d)

- ES (static) = $150 - 5 - 15 - 80 = 50\text{BP}$
- ES (dynamic) = $160 - 10 - 20 - 70 = 60\text{BP}$

(e)

- Here is better to choose the dynamic strategy which has a higher ES.
- but the one with the highest ES may not be the best strategy as the goal is to maximize the total profits
- So the effects on sales volume should be considered: the one that maximizes the total profit should be the best strategy.

Solution 18

- (a) The Minimum Surplus Variance Portfolio (MSV) is the portfolio that minimized surplus volatility.
- traditionally ALM has focused on asset side
 - The true health of the plan should look at economic surplus
 - Must decide how much β (hedgeable risk) and α risk to take.
 - Cash flow or duration matched portfolios can be viewed as proxies for the MSV.
 - You use Markowitz mean-variance method to construct surplus efficient frontier
 - The MSV is the point that is at the left most efficient point on the EF
 - In practice asset only EF is very close to surplus EF.
 - Can view liability as short on asset and look at asset only view
 - liabilities have a market β just like all assets

- (b)
- $$r_A = r_f + \beta_A (r_\phi) + \alpha$$
- $$r_L = r_f + \beta_L (r_\phi) + \alpha$$
- r_ϕ is the market risk premium
 r_f is risk free rate
 α is alpha risk (conditional being able to pick skilled managers)

$$r_{s(L)} = \frac{A}{L} r_A - r_L$$

$r_{s(L)}$ is the liability relative surplus return.

Thus is required because if surplus is zero you get division by zero problems

$$\Rightarrow \beta_s = \frac{A}{L} \beta_A - \beta_L$$

given $\beta_L = 8$ $\frac{A}{L} = 70\%$

$$\alpha = 0$$

$$r_\phi = .08$$

$$r_f = .04$$

$$r_A = .04 + \beta_A (.08) + 0 \qquad r_L = .04 + 8(.08) + 0 = .104$$

$$r_{s(L)} > 0 \Rightarrow \frac{A}{L} r_A - r_L > 0$$

$$\Rightarrow .7 r_A - .104 > 0 \Rightarrow r_A > \frac{.104}{.7} = .1485$$

Solution 18 (continued)

$$\begin{aligned}\Rightarrow .14857 &= .04 + .08\beta_A \\ \Rightarrow \beta_A &= 1.35714\end{aligned}$$

(c) $\beta_A = 1.35714$

- You need to take on a large amount of risk. Surplus variance may go up; could do very poorly.
- This may not be appropriate; does depend on size of DB plan relative to corporate assets.
- Investment policy is just one lever. You also have contribution and benefit policy.
- Investment policy is the weakest of the Three levers.
- You may want to consider changing contribution and/or benefit policy.

Solution 19

(a) Tepper arbitrage:

The company will shift assets from equity to debt in the pension plan, and for every dollar the company shifts, the shareholders shift $\$(1 - \text{corporate tax rate})$ from debt to equity in their personal accounts. By doing so, the net gain to

shareholders is $\$(1 - t_c)r_b(t_{pb} - t_{ps})$, or $\frac{(1 - t_c)(t_{pb} - t_{ps})}{(1 - t_{pb})}$ in perpetuity

Black arbitrage:

Instead of shareholders shifting money from debt to equity, company can change its capital structure to achieve the goal. Specifically, the firm shifts from equity, to debt in the pension plan, and for every dollar shifted. The firm issues $\$(1 - t_c)$ debt and buys back $\$(1 - t_c)$ stock. By doing so, the net gain to shareholders (in

perpetuity) is $\frac{(1 - t_{ps})t_c(1 - t_c)}{1 - t_{pb}}$

(b) Tepper:

$$\frac{(1 - t_c)(t_{pb} - t_{ps}) * .60 * \$10,000,000}{1 - t_{pb}}$$

$$t_c = \text{corporate tax rate} = .35$$

$$t_{pb} = \text{personal tax rate on bonds} = .28$$

$$t_{ps} = \text{personal tax rate on stocks} = .18$$

$$\frac{(1 - .35)(.28 - .18)}{1 - .28} * .60 * 10,000,000 = \$541,667$$

Black:

$$\frac{(1 - t_{ps})t_c(1 - t_c)}{1 - t_{pb}} * .60 * \$10,000,000$$

$$\frac{(1 - .18)(.35)(1 - .35)}{1 - .28} * .60 * 10,000,000 = \$1,554,583$$

Solution 19 (continued)

(c)

- Pension liabilities are long term, stocks are expected to earn more than bonds in the long run
- Vested interests by actuaries and fund managers
- current accounting rules allow manipulating earnings with expected returns on the plan portfolio
- compensation ties to plan performance
- Equities considered a good hedge against salary inflation
- ERIS requires prudent person approach, i e., diversifications

Solution 20

The company use VaR to measure risk of portfolio. VaR is attractive as a risk measurement tool in that it provides management a single number to summarize the risk in portfolio. It tried to make the statement like: "we are x% certain that the loss during the next N days would not exceed V dollars."

The company provide 95% CI for VaR over the two-year time horizon. Time horizon is too long, and it's almost meaningless to set such risk measure.

There are many advantage and use of VaR:

1. useful for financial reporting
2. set position limit for traders
3. measure return on risk adjusted basis
4. evaluate model
5. investment recommendation.

There are also some limitations of VaR.

1. VaR measure varies significantly with the assumption and methodology used.
2. Risk management result gives management false sense of security, especially when they see how wildly the risk exposure can change as conditions change
3. There are some risks not captured by quantitative VaR calculation like liquidity risk, operational risk, etc.
4. It's not very meaningful to compare VaR from different analysis
5. VaR should be supplemented with checks, balances, procedures, audits.
6. Quantitative risk management tools is also necessary for correctly manage risk
7. VaR currently lack standardization. Standardization will improve the quality of risk management, but may also inhibit the development of new risk management tools and techniques.

VaR should also be reported with confidence interval.

Stress testing and back testing should also be carried to see how VaR perform under extreme market movement and see how VaR performed in the past.