Solution 1

(a)

Accounting
- Effects on different accounting basis

Tax
- Tax gain/loss carry forward position
- Netted within calendar year
- Netted between affiliates

EV/EVA
- Need to reproduce cash flows

ALM

Credited Rate
- Effect on credited rate

Policyholder Equity
- Effect on segment's credit quality, maturity structure, concentration

(b)

Description of UL product characteristics:
- Product has embedded options
- Liability cash flows are interest sensitive

Asset Mix

- Duration of 12 is CRAZY long. The company is exposed to HUGE disintermediation risk if interest rates rise.
- A/L dollar-duration mismatch is way outside Investment Policy Constraints
- Sell much of the government and public corporate portfolios, reinvest much shorter, to bring duration down toward 4.0

- Need to consider product liquidity needs
- Given privates' low liquidity a higher quality would be appropriate
- As the pvts would be very difficult to sell, reinvest any excess cash flow on maturity in short-dur, non-callable publics

Derivatives

Policy is in place to permit use

Solution 1 (continued)

Use to quickly correct duration mismatch

Protect with a floor or pay-float swap

Solution 2

(a) Option pricing method:

- Also known as direct method or multi-scenario method
- Generate stochastic economic scenarios
- Project liability CF along each scenario path
- Calculate pathwise liability PV by discounting liability CF along each path
- May reflect risk by adjusting either the discount rate or the projected liability CF
- If adjusting discount rate, may add a spread that reflects
  - ⇒ Issuer's credit
  - ⇒ Quality of issuer's bond portfolio
  - ⇒ Risk contingency margin of the liability
  - ⇒ Liquidity of the liability
- Assign probability to each scenario path
- Calculate fair value of liability as weighted sum of pathwise liability present values
- Not often used for transfer pricing for a block of insurance liabilities

Actuarial Appraisal Methods:

- Also known as indirect method
- Generate stochastic economic scenarios
- Project free CF under each scenario
- Free CF may reflect:
  - ⇒ Investment earnings
  - ⇒ income tax
  - ⇒ RBC
- Calculate pathwise PV by discounting free CF at risk adjusted firm's cost of capital
- Calculate DDE as average of pathwise PVs
- FV of liability = MV of assets – DDE – DTL
- Method of choice for transfer pricing for a block of insurance liabilities
- Reconcilable with Option Pricing Method under certain assumptions

Solution 2 (continued)

(b)

- Volume of computation could be prohibitive
- System may not be available to do this routinely
- Expertise may be lacking
- Market data for calibration may not exist for certain insurance options

(c)

Scenario 1:

LV_0 (liability value at year 0) = 1,500

LV_1 before withdrawal (WD) = $1{,}500 \times (1+6.6\%) = 1{,}599$

$CF_1 = 1{,}599 \times 4\% = 64$

LV_1 after WD = $1{,}599 - 64 = 1{,}535$

LV_2 before WD = $1{,}535 \times (1+6.6\%) = 1{,}636$

$CF_2 = 1{,}636$

$PVCF_{scen1} = 64/1.01 + 1{,}636/\left[(1.01)(1.03)\right] = 1{,}636$

Scenario 2:

$LV_0 = 1{,}500$

LV_1 before WD = 1,599 - 32 = 1,599

$CF_1 = 1{,}599 \times 2\% = 32$

LV_1 after WD = $1{,}599 - 32 = 1{,}567$

LV_2 before WD = $1{,}567 \times (1+6.6\%) = 1{,}670$

$CF_2 = 1{,}670$

$PVCF_{scen2} = 32/1.01 + 1{,}670/\left[(1.01)(1.02)\right] = 1{,}653$

$FV = 0.3 \times 1{,}636 + 0.7 \times 1{,}653 = 1{,}648$

Solution 3

(a)

LifeCo's reported effective durations based on:

- parallel yield curve shift
- small (1bp) yield curve shift

They do reflect interest-sensitive cash flows, but:

- cash flow models may not be perfect
- may mean significantly higher convexity and also optionality, so impact under large shift may be very different than predicted by effective duration

(b)

Impact of 100bp -drop- based on reported durations

Using effective duration * PV cash flows:

Assets:	d=9.26	so impact = 416,600 * 9.3 * 1% = + 38,744 (or 38.7 million)
Liabs:	d=4.00	so impact = 406,000 * 4.00 * 1% = + 16,240 (or 16.2 million)

Total: 38.7 - 16.2 = 22.5 i.e. economic gain of 22.5 million

(c)

Reported "margin squeeze" impact based on 100bp drop was –10.3 million loss
Estimate using effective duration in (b) was 22.5 million gain
Reasons for difference:

- margin squeeze modeled under 100bp shift, not estimated from 1bp shift
- other reasons from part (a) like convexity, optionality
- convexity wouldn't account for opposite direction of impact
- but optionality could: may hit minimum guarantees under 100bp shift but not for 1bp
- if min guarantees "in the money" under 100bp, would explain large negative "margin squeeze" impact

(d)

What "bets" are there...

- durations themselves indicate bet on parallel interest rate decrease, hopefully partials are consistent with this!
- Partial sensitivities show $ change under a 1bp increase
- Short (1-5 year) partials show a gain if short rates rise
- Long (7-20 year) partials show a loss if long rates rise (gain if long rates fall)
- So overall, bet is "flattening" of curve

Solution 3 (continued)

(e)

Only 15-year partial affected, since going to cash (d=0) won't recalculate 0.25-yr partial

Will show change to partial sensitivity since that's what's used in the case study

15-yr sensitivity is reported as –698,000 for 1bp increase

The 50 million of 15-yr zeros alone contribute approx 1bp * -15 * 50 million = -75,000

We should do exact calc using 15-yr spot rate of 5.42%......

PV under 1bp increase is 50 million * (1.0542 ^ 15 / 1.0543 ^ 15) = 49,929,910

So zeros contribute –70,090

Selling them will *increase* 15-yr partial by about 70,000 to –628,000 (not much change)

(f)

Revised margin squeeze..... doing exact calc, the zeros contributed a *gain* under the 100bp drop:

PV under 100 bp drop is 50 million * (1.0542 ^ 15 / 1.0442 ^ 15) = 57.7 million

gain of 7.7 million for zeros

So selling them...... margin squeeze impact will be 7.7 million worse or about 18 million loss!

Proposed sale doesn't help margin squeeze!

(g)

Margin squeeze showed a "bet" that rates wouldn't drop 100bp (or at least showed a loss if they did!)

Looking only at effective duration and partials said gain if longer rates drop 1bp.... these didn't tell the whole story

Solution 4

(a)

ELGIC: Call on 75% of increase of S&P500 over 5 year period

Variable annuity: put, contingent on death, on the invested funds with strike price $S_0 = (1.05)^T$

Additional option provided by dollar for dollar partial surrender

(b)

Potential changes to reduce the impact of a large equity market decline just before maturity:

- Annual resets to lock-in gain at each policy anniversary
- Monthly averaging: use average increase over period
- High water mark to lock-in maximal gain

Reduce cost by using:

- Simple annual ratchet instead of compound one
- Averaging when calculating the actual return

(c)

Evaluate cost and efficiency of dynamic hedging by comparing the alternatives (no hedging or static hedging) over many economic scenarios. Use Monte Carlo simulation techniques to model hedging including:

- The impact of rebalancing and the hedging error introduced by the drift
- Transaction costs (proportional to change in stock position)

Stress testing is also important to highlight potential risks and exposure.

(d)

Dynamic hedging is a viable alternative for large blocks of business with embedded options that are difficult to replicate with standard options. This might be the case for Life Co ELGIC. With small notional amounts, transaction costs may deteriorate any benefit. The simplicity of the ELGIC option is such that it may be available in the market. One benefit of hedging is to combine offsetting exposure. Compare costs and availability of both alternatives.

Solution 4 (continued)

(e)

Can hedge ELGIC by buying a bond and a 5 year European call option on 75% of the notional amount. No need to rebalance unless early withdrawals are very different from initial estimates.

Variable Universal Life investment options depend on timing and amount of premiums and withdrawals. A static hedge will not work. Dynamic hedging on the portfolio of investment options would be preferable.

Solution 5

(a)

1) Lognormal model

A standard model for evaluating equity returns, assumes percentage returns are normally distributed, which is a reasonable assumption in many situations

Allows analytical solution for European call and put options in a form that provides the way to construct a replicating portfolio consisting of underlying stocks and risk-free bonds. However, the model does not account for:

- big jumps in stock prices
- auto regression effects in returns
  volatility clustering effect

The limitations result from the assumption of constant volatility of return. experience data shows that in practice volatility of returns is stochastic with some autoregressive features.

2) Regime Switching (RS)

One of the proposed models is a regime switching-between two lognormal distribution, with μ_1, σ_1 and μ_2, σ_2. The model is shown to fit well to actual results for returns on broad-based indices (like S&P500). The data shows that in fact there are periods in time, when volatility of index returns are relatively low which are then switch to periods with much higher volatility. The model give fatter tails for the distribution of returns and thus provides a better fit to the actual data than the simple lognormal distribution. The RS – distribution can be easily simulated, like the lognormal one, but – it does allow for analytical solutions Also, the model does not incorporate auto regression. More parameters than ARCH or GARCH.

Solution 5 (continued)

3) <u>Time Series with GARCH volatility</u>

In GARCH (1, 1) model the volatility is calculated from:

$\sigma_t^2 = \gamma V + \propto \sigma_{t-1}^2 + \beta \left(\Upsilon_{t-1} - \mu\right)^2$, where V – is the long-term average variance; σ_{t-1} and Υ_{t-1} - values of the volatility and return at the previous time moment

Constants $\propto, \beta, \gamma$ are found by the regression to the actual data. This model is very general in nature and allows for:

- mean reversion; through long-tem average term V
- auto regression, through dependence on the prior value of volatility; σ_{t-1}
- effect on high volatility when returns are far from their long-term mean, μ, through term $\beta\left(\Upsilon_{t-1} - \mu\right)^2$,

The model can be easily simulated but does not allow for analytical solutions

The volatility in the model is (unconditionally) stochastic due to Υ_{t-1}

4) <u>Empirical</u>

Actual data for returns are recorded for some period of time and then used as a sample space, from which the values for future returns are taken. The model is:

- fit well to the past experience by definition; each observation is equally likely
- is limited, since the data is limited
- does not allow new developments in the future; only the returns that were recorded in the past are allowed for the future
- If the return values are sampled with replacement randomly the model does not produce any auto correlations.

This can be improved by sampling number "in bunches".

5) <u>Wilkie</u>

This is an econometric model, which combines processes for different economic factors, such as inflation level; short and long-term interest rates; divided yields and stock returns. The model has a cascade structure. A process for each new factor includes a term, connecting it to the parameters from the prior (upper) levels; and also a stochastic term.

The model is very complex.

<u>Main advantage</u> – combines different economertic parameters in one inter dependant model; for example, short interest rates and stock returns, which can be very useful for many actuarial applications (ALM).

<u>Drawback</u> – very difficult to estimate model parameters. Contains a lot of them and requires a lot of experience data for estimation.

Solution 5 (continued)

6) Stable distributions
This is a particular class of functions, which satisfy specific conditions on their linear transformations One example – normal distribution.
Advantages – wide class of functions with a convenient "convolution" feature; allows to model very "fat" tails
Drawbacks – difficult to simulate
- does not allow autocorrelation

b) GMDB in the simplest form is the return of premiums (less withdrawals) as the minimum death benefit It is a put option on the fund value at the time of death. The payoff equals max (P-F, 0), where P=premiums net of withdrawals, F-fund value at the time of death The value of the option should be adjusted for survivorship.
The total value of the GMDB at $t = 0$:
$\int_0^n p(t) \cdot {}_t p_x^{\tau} \cdot \mu_{x\,t}^{(d)} \, dt$; where the value of the European put option with term t; $p(t)$, is multiplied by probability of surviving to time t and dying in the interval dt; and summarize for the term of the annuity, n years.
We assume that mortality and lapsing functions are known.
To hedge GMDB dynamically we need:
to simulate the fund returns
to calculate the value of the put option for each t
to have a possibility to present the option value in terms of the replicating portfolio.
Of all the models, only the lognormal model gives the analytical value of the put option in terms of the hedge portfolio, split between the stock fund and risk-free bonds.
So - to calculate the parameters of the replicating portfolio, the lognormal model (Black-Scholes) should be used.
- to simulate the fund returns – lognormal, RS, GARCH or empirical can be used.
To better represent actual fund behaviour- use RS or GARCH
Need - to rebalance frequently and calculate hedging errors and transaction costs.

Solution 6

(a)

$$r_A = r_f + OAS - D_{OAS} \Delta OAS - \Sigma D(i) \Delta r(i) + r/c + pa - e_a$$

$$r_\ell = r_f + ROAS - \Sigma D_\ell(i) \Delta r(i) + e_\ell$$

$$r_f = 3\%; OAS = SObps; D_{OAS} \cdot \Delta OAS = 0; r/c, pa, e_a = 0; D(2) = 2$$

Asset return: expected forward rate = 6.01%

actual rate = 4.00%

$$r_A = 3.0\% + 0.5\% - 0 - (4\% - 6\%) \cdot 2 = 7.5\%$$

Liability Return

Liability Yield = $\left(110/100\right)^{\wedge} \cdot 5 - 1 = 4.88\%$

$ROAS$ = .88%

expected forward rate=5.01%

actual rate = 4.00%

$$r_\ell = 3.0\% + .88\% - (4\% - 5\%) \cdot 1 = 4.88\%$$

(b)

	Assets	Liabilities	Net
risk free	3%	3%	0%
C1	OAS = .5%	0	.5%
C2	0	$ROAS$ = .88%	-.88%
C3	$\Sigma D_a \Delta r(i)$	$\Sigma D_\ell(i) \Delta r(i)$	
	4%	1%	3%
Total	7.5%	4.88%	2.62%

Solution 6 (continued)

(c) Total Return Approach:
- splits return into components (e.g., c risks)
  - performance measurement
  - setting consistent goals in managing both assets & liabilities
  - prospective & retrospective analysis
  - measure results relative to
    - bond selection
    - interest rate anticipation
    - sector rotation
    - r/c expenses
  - market value measure

Solution 7

(a) Purpose is to singe number V, says that over the next N days, we will not lose more then V dollars of value with $x\%$ confidence.

(b)

(i) using histogram (1-day)
$254 * 5\% \approx 13$ days
the 15^{th} worst return is -1.5%
so $VaR = -1.5\% - 0.04\% = -1.54\%$
or $\$10m * 1.54\% \approx 154K$

(ii) using normal distribution (10-day)
use $N(x) = 0.95 \Rightarrow x = 1.645$
So 10-day $VaR = 10m \times 1.645\sigma \overline{)10}$
$= 10m \times 1.645 \times 0.0107 * \overline{)10}$
$= 556K$

(c) Histogram
adv: avoid use cash-flow mapping
use historical data

disadv: Computing slow
does not allow volatility updating
sensitive to historical data

Normal:
adv: quick to calculate
can use volatility update scheme

disadvan: normal distribution assumption
give poor result for low-delta portfolios

Monte-Carlo:
adv: any model can be used

disadvan: Computing intensive

Solution 7 (continued)

(d) - assess use the estimation errors
- for quantile-based

$$se(q) = \sqrt{\frac{c(1-c)}{Tf(q)^2}}$$

- for sigma-based

$se(\alpha s) = \alpha \times se(s)$

For normal $se(s(\Phi)) = \alpha\sqrt{\frac{1}{2T}}$

Also, can use back testing and stress testing.

(e) Limitation:
- only 1 point, no tail distribution
- results depend on methodologies
- results depend on assumptions
- results depend on time horizon
- many factors not captured such as legal, operations
- may give management false sense of security

CTE:
- more robust
- consider the shape of tail distribution
- it is the expected loss give loss happens
- meet criteria

Stress testing:
- can test extreme cases not captured in *VaR*

(f) $CTE_{\alpha}(L) = E(L_0 / L_0 > V_{\alpha})$

13 worst losses

$CTE = 10m * (1*4.75\% + 1*3.75\% + 2*2.75\% + 4*2.25\% + 5*1.75\%)/13 = 244K$

Solution 8

(a)

- A stable value fund if an options offered by 401(k) and other DC plans
- It is typically the most conservative investment options
- It is good with ERISA (fiduciary duty)
- Invested mostly in GIC and other medium-term fixed-income contracts issued by a high-quality financial institution
- Provide participants ability to withdraw and transfer funds (subject to plan rules) without penalty or market value risk. Principal is guaranteed by issuer (so there is credit risk)
- Good for participant seeking a safety investment option (near retirement) / reduce volatility
- Good for participant seeking to diversify their investment (low correlation with equity)

(b)

- Credit risk of the GIC/BIC issuer
- For Synthetic GIC / SA GIC we also have
  - Gains and losses amortized into the credited rate
  - Underlying asset default
  - Reinvestment, interest or market risk (performance)
  - Call/extension risk / withdrawal risk / competing funds

(c)

- Asset risk:
  (i)Default / credit risk
  (ii)Call or extension risk
  (iii)Performance, interest, market or Reinvestment risk
- Liability risk:
  (i)Contribution risk
  (ii)Withdrawal / liquidity risk

(d)

- Good Underwriting / reinsurance
- Cash flow matching / Duration Matching / Convexity Matching
- Risk management by ALM techniques / use derivatives
- Computer monitoring / Stochastic projection (Monte Carlo)
- Stress testing / scenarios testing
- Contract design

Solution 9

(a) 1 year forward values are:

$$V_{aa} = \frac{100}{(1.05)^4} = 82.27$$

$$V_a = \frac{100}{(1.053)^4} = 81.34$$

$$V_{bb} = \frac{100}{(1.057)^4} = 80.11$$

$$V_b = \frac{100}{(1.074)^4} = 75.16$$

$$V_c = \frac{100}{(1.09)^4} = 70.84$$

(b) First percentile is at rating C
(since prob(c) = 0.5% and prob(B or C) = 1.25%)
$\Rightarrow 99\% \;\; VaR = 80.11 - 70.84 = -9.27$

(c) Capital charge = Expected forward value – First Percentile value
$EV = .01(82.27) + .02(81.34) + .9575(80.11) + .0075(75.16) + .005(70.84) = 80.07$
$\Rightarrow$ charge = $80.07 - 70.84 = 9.23$

Solution 10

(a)

1) Hedge investment has an underlying notional and is a derivative
2) Must be carried at Market Value
3) Hedge is matched to when underlying item affects income
4) Is only cash flow or Fair Value hedge
5) Risk is Market Price, Interest Rate, or foreign currency
6) Must have well documented use and purpose
7) Must be judged to be effective

(b)

1) DM Life pays fixed payments as premium and will receive at time of defined credit event a payoff of par or other agreed delivery of protection

   Risks being hedged move with change in credit quality of assets and is a fair value hedge

   CDS is a derivative

Solution 11

(a) Black-Scholes model's assumptions, which differ from real world:

- Geometric Brownian motion for stock price changes
- Smooth price changes
- Constant interest rate
- Constant volatility
- no penalty for short selling
- no penalty for borrowing at risk-free rate
- fractional securities are allowed
- European option (Exchange-traded options are mostly American)
- no dividends
- no takeover
- no taxes
- no transaction costs

(b) No, ∴ Black-Scholes model does not use Expected Return $E(r)$. $E(r)$ is used in hedging, but not option valuation.

Investor decisionmaking is based on $E(r)$ & risk (proxied by variance) as per Markowitz.

∴ use a Generalized Actuarial model

Assumption: Normal Distribution for Stock Return
Can handle combination of securities
Calculates E(r) for the investment strategy,
we then compare alternative strategies

Solution 12

(a) His statement is false in the sense that nobody could possible "know" exactly which properties are great performers. To measure his performance, you need to compare his returns to the index return. Compare in two ways: his individual property selection and property-type (market timing).

1. Property type weighting
   SUM[portf wght × indx rets] − SUM[indx wght × indx rets]
   He got 6.05% vs. 5.70% = 35bps better than index
   He outperformed the index by increasing his weigh in the high-returning retail segment and decreasing his weight in the lower-returning office segment

2. Individual property selection-compare his property type returns to the index returns using the index weights
   SUM[indx wght × portf rets] − SUM[indx wght × indx rets]
   He got 5.70% vs. 5.70% for the index, so he exactly matched the index return. His better performance in picking retail and office properties was offset by his poorer performance in picking warehouse and apartment properties.

   So his skill in weight in the portfolio to higher – returning segments and not in individual property selections.

(b) **On the recommendation about selling the apartment holdings:**
Reducing the apartment holdings might be a good idea since apartments have been low yielding. But if demand is high (since occupancy rates are high) you may be able to increase rent (depending on lease agreements) and improve yields. Plus, the property management must be fairly good if occupancy rates are high. I would stay in this holding and increase rent to see what would happen.

On the recommendation about investing in the new retail complex:
Retail is more risky and already over-weighted
Would change risk/return profile.
New complex means higher risk and return versus the apartment, especially since it is only a proposal.
Should perform scenario analysis

Solution 13

(a) Statement is incorrect. Current rates are not the sole determinate of prepayments The pace of prepayments is also driven by general housing turnover and refinancing.

General housing turnover is driven by relocation, seasonal variations, the aging process and curtailments.

Refinancing. Rate of refinancing is influenced by the shape of the yield curve, credit quality of borrower, mortgage characteristics (i.e. LTV, equity build up) and is path dependent (burnout)

(b)

i.) This bond has a 6 year period before first principal payment is made. Principal payments will follow a schedule as long as prepayment stay within 100 PSA to 250 PSA. This bond has more certain cashflows then pass-thru or standard CMOs, it provides call protection and has better convexity than most CMO structures.

ii.) this bond will not start paying principal until the earlier tranches have been paid down to zero. This bond will have less prepayment variability than support tranches but more volatility than PAC's. It typically offers higher yield than a comparable PAC.

iii.) The bond is not paid until all senior bonds are paid. It has a period of principal and interest lockout or an accrual phase and payment phase. It has long duration and is good for long liabilities.

iv.) Similar to Z-bond but based on some event it will stop accruing and begin paying P&I. This jump can be sticky or non-sticky.

Buy the newly issued PAC bond with lock out period for this interest forecast. This PAC has the least negative convexity. The rising interest rates will cause all the other bonds to extend more than the PAC. The PAC's support or companion bonds will help re-direct prepayments and keep the PAC on schedule.

Solution 14

(a) The student's response is incorrect. This product has the effective duration less than 5 years as embedded options are included

- right to surrender policy at book value
- rate reset feature after the initial 5 year guarantee period
- minimum crediting rate
- interest-sensitive cash flows
  - higher new money rate leads to higher lapse
  - lower new money rate leads to lower lapse

(b) The MVA mitigates the disintemediation risk to the policy holders in the event of rising interest rates by discouraging anti-selective surrender which requires capital loss on asset sales
The MVA removes the embedded put option from SPDA which reduces the convexity.
The company can match the liability better with option-free bonds.
The MVA allows the company to invest longer which enables the company to credit higher rates.

(c) Without the MVA, the put option embedded in the SPDA reduces the effective duration. Without the MVA, when the interest rates rise, higher lapse/surrender shortens the duration.
In other word, the value of the liability doesn't decrease much comparing to the SPDA with MVA.

(d) The return of premium feature would be in the money if the value of contract after applying the MVA factor is less than the initial deposit.
Let initial deposit to be P.
Account value at the end of the year 1=P$\times(1.04)$

The MVA factor $= \left(\dfrac{1+j}{1+i}\right)^{T-t}$

where j = the current fixed crediting rate = 4%
i = the current market rate
$T-t$ = the fixed rate period remaining = 4 yr

Solve i for Premium (Deposit)$\geq$Account Value $*$(MVA factor)

$$P \geqq P(1.04)\left(\frac{1.04}{t+i}\right)^4$$

Solution 14 (continued)

$$(1+i)^4 \geq 1.04^5$$
$$1+i \geq 1.04^{(5/4)}$$
$$i \geq 1.04^{(5/4)} - 1 \approx 0.0502$$

If the current market rate is higher than 5.02% at the end of year 1, the option is in-the-money

(e) Since the policyholders could have a higher first year cash surrender value, the policyholders are more likely to surrender in the year 1 and Return of Premium decreases the effective duration. ROP works like a put option.

(f) Since there is no MVA when the current market rate (j) is lower than fixed rate (i), the minimum guarantee has no effect during the initial guarantee period. After the 5 year initial guarantee period, the minimum guarantee should extend the effective duration when the current rate is below the minimum crediting rate. The policyholders would keep their contract. It acts similar to the interest floor.

Solution 15

(a) risk free rate = 0.05
V0=market value of company's assets today = 21
D=Company's debt interest and principal due to be
σ v=volatility of assets = 0.20
T=1 year

$$d2=\left[\ln\left(V0/D\right)+\left(r-\sigma^2 v/2\right)T\right]/\sigma v^*\left(T^\wedge \tfrac{1}{2}\right)$$

d2=[ln (21/17) + (.05 – (0.2^2)/2)*1] / 0.2*1
d2=1.2065

Calculate N(-d2) to obtain risk-neutral probability of default
N(-d2) = 1-N(d2)
$N(d2)=N(1.20) + .65^* \left[N(1.21) - N(1.20)\right]$
=0.8849+0.65 * (0.8869-0.8849)
=0.1138

(b) A=De ^ (-rt)
A=17e ^ (-0.05 * 1)
A=16.1709

B=V0-E0
B=21-5=16

Expected loss on the debt = (16.1709-16)/16.1709=1.06%

0.0106=1.1138 * (1-R)
(1-r)=0.0106/0.1138
R=1-0.0106/0.1138=(0.1138-0.0106)/0.1138

R=90.7%

Solution 15 (continued)

(c) $Q(T) = (1 - \exp[-(y(T) - y^*(t))T]) / (1- R)$
$Q(5) = (1 - \exp[-0.035*5] / (1 - 0.4)$
$Q(5) = 0.26757$
$Q(1) = 0.26757/5 = 5.35\%$

Lower than Merton's default probability

Reasons for discrepancy:

- Provision for liquidity premium
- Provision for possibility of recession or depression scenario
- Merton's model impacted by volatility

Solution 16

(a)

p = K * exp[rt] * N(-d2) – S_0*N(d1)

d1= [ln(S_0/K) + (r+ σ^2/2)T] / $\sigma\sqrt{T}$

$=1100e^{-.025}(.37336)-1200(.28331)$

=400.56-339.972

=60.58

60.58×5000=302.919

profit=302919-242900

=60019

$d_2 = d_1 - \sigma\sqrt{T}$

d1 = [ln(1200/1100) + (0.25 + .25^2/2)]/0.25

=0.5730

$d_2 = 0.3230$

$N(d_1) = 0.71669$ $N(d_2) = 0.62664$

$N(-d_1) = 0.28331$ $N(-d_2) = 0.37336$

(b) Delta is $N(d_1)-1$ for a put option

d1 = [ln(1200/1100) + 0.02S + .22^2/2]/0.22= 0.6191

$N(d_1) = 0.7321$

delta=0.7321-1=-0.2679

$\times 5000 = -1339$

need to buy 1339 notional amt of index = 1339×1200=1,607,382

Solution 16 (continued)

(c) At each quarter $\Rightarrow$ they need to recalculate Δ of option, then sell or purchase shares of index to make Δ of portfolio = 0.

Time	Index	$N(d)_1$	Δ(option) = 1-N(d1)	Shares purchased	Cost (# shares × index level)	
0	1200	0.7321	0.2679	-1339.5	-1,607,400	negative
1	1250	0.8051	0.1949	365	456,250	implies
2	1150	0.6700	0.3300	-675.5	-776,800	profit
3	1050	0.3783	0.6217	-1458.5	-1,531,400	
4	1000		1.000	-1891.5	-1,891,500	
			in the money	total= $^-$5000		

at time = 1, Δ=0.1949$\Rightarrow$so you want $(5000)\left({}^-0.1949\right) = {}^-974.5$ shares,

so you need to buy $\left({}^-974.5\right)-\left({}^-1339.5\right)=365$ shares

time = 2$\Rightarrow$sell(5000)(0.33)-974.5=675.5 shares

time = 3$\Rightarrow$sell (5000)(0.6217)-(5000)(0.33)=1458.5 shares

time = 4$\Rightarrow$sell (5000)(1.000)-(5000)(06217)=1891.5 shares

(negative = profit)

Time	Stock Purchase Cost	Trading Cost	Cum Cost	Int Cost = $\left(\frac{0.025}{4}\right)$(Cum Cost)
0	-1,607,400	1607	-1,605,800	-10036
1	456,250	456	-1,159,130	-7245
2	-776,800	777	-1,942,398	-12140
3	-1,531,400	1531	-3,484,407	-21777
4	-1,891,500	1892	-5,395,942	

so firm has cumulative cost of –5,395,792$\Rightarrow$profit =$5,395,792
Now, to settle put, you're obligated to purchase 5000 shares at price of 1100$\Rightarrow$so, you need 5000 (1100)=5,500,000 (You've sold and then purchased 5000 shares so net gain in shares)
Therefore, net loss=5395,792-5,500,000=104,200

Solution 16 (continued)

(d) Securitization package CF's expected and sell at market to offset risky CF's

Market maker - sell products that counterbalance risks of this product

Reinsurance - hard to find good price and willing counterparty

Naked position - do nothing. Okay if option is out-of-the money, but in trouble if it is in-the-money

Covered position - A hedge initially, but do nothing afterwards-risky if Δ changes dramatically

Stoploss - only change holdings if option is in or out of the money

gamma hedging - make portfolio gamma neutral $\Rightarrow$ requires position in another instrument

rho,vega hedging - similar to gamma hedging

Solution 17

(a) Risks in Global investing:

- legal protection for investor
- corporate objectives of management
- communication with shareholders
- political risk
- currency risk
- credit risk

Solutions:

- understand local market conditions
- understand local legal framework
- understand reliability of communications with corporations
- understand corporate objectives
- use experienced staff
- do research
- currency hedging strategies
- credit hedging strategies

(b) Political Risk model: 10 variables, correlations show:

1. Democracy: if lack democracy & legitimacy, then less stable
2. quality of life: higher means more stable
3. GDP: higher means more stable
4. Rental Income: higher means less stable
5. Distribution of Income: if inequality, then less stable
6. Predictability of wholesale prices: higher means more stable
7. Agriculture (as a % of GDP): higher means less stable
8. Trauma: countries had trauma can be successful
9. Competition (measure= $\left(\frac{\text{Import} + \text{Export}}{\text{GDP}}\right)$: higher means more stable
10. Human Capital: higher means more stable

Solution 17 (continued)

(b)

		A	B	C
	Country:	A	B	C
Agriculture (as % GDP)		$\frac{1+3}{26}=15.4\%$	$\frac{150+450}{1500}=40\%$	$\frac{15+40}{600}=9\%$
	Rank:	middle	worst	∴ Best
Competition $\left(=\frac{\text{Import+Export}}{\text{GDF}}\right)$		$\frac{1+1+8}{26}=38.4\%$	$\frac{150+150+150}{1500}=30\%$	$\frac{5+15+300}{600}=53\%$
	Rank:	middle	worst	∴ Best
Democracy	Rank:	Stable	Stable	Worst (∵ Emerging)
Infant deaths	Rank:	Best	middle	worst
Rental:Oil/GDG	Rank	3.8% middle	10% worst	0.8% Best
Life Expectancy	Rank:	Best	middle	Worst
Inflation range	Rank:	middle	worst	Best
D=GDP/Capita	Rank:	$26/3=8.7$ middle	$14/4=3.8$ worst	$600/50=12$ Best

∴ Country C is the most stable
A is the next most stable
B is the least stable

Solution 18

(a) Reduces volatility of CFs

Reduces cost of financial distress

- Legal & Accounting costs
- Higher costs with customers, employers and suppliers

Reduces Taxes

- Taxes reduced if tax schedule convex

Improve Investment Decision

- Improve incentives to undertake only profitable projects

Improve Debt Capacity

- Reduces conflicts with stockholders & bond holders

Dividend Policy

- Important means to express confidence in company's growth

Managerial Self Interest

- Management has incentive to mange strategic exposure otherwise will not have job

Imperfect Market Conditions

- External capital more expensive than internal capital

(b) Purchase 1000 call options with strike=400 at $t = \frac{1}{2}$

Purchase 1000 call options with strike=400 at t=1

Enter into swap to receive LIBOR and pay 3.25%

- Assume company is payor of LIBOR

Enter a forward contract to hedge forward currency rate

$$= .75\left(\frac{1.05}{1.03}\right) = .764 \quad \text{This guarantees US payoff of } \$7M/.764 = 9.156M$$

(c) Value of Risk = Cost to hedge

Call options = 1000(4)+1000(6) = 10,000

LIBOR swap = no cost to hedge however.

- In one year earn $10M×3% risk free
- pay $10M×3.25% through swap
- cost = (3.25%-3.0%) ∗ $10M/1.03 = 24,272

Foreign currency exchange forward

- Receive 9.16M in 1 yr (hedged at no cost)
- Expect $7M/.75 = 9.33M in 1 yr
- Risk = $177,777 / 1.03 = 172,600

Total Risk = 10,000+24,272+172,600 = 206,872

Solution 19

(a) From Ito's lemma, if G is a function of S and t, the process is

$$dG = \left((\partial G/\partial S)\mu S + \partial G/\partial t + 0.5\left(\partial^2 G/\partial S^2\right)dt + (\partial G/\partial S)\sigma S dz\right)$$

Since G = S exp (r(T-t)),

$$\partial G/\partial S = \exp(r(T\text{-}t))$$

$$\partial G/\partial t = -rS\ \exp(r(T\text{-}t))$$

$$\partial^2 G/\partial S^2 = 0$$

This gives

$$dG = \left(\exp(r(T-t))\mu S - rS\ \exp(r(T-t)) + 0\right)dt + \exp(r(T\text{-}t))\sigma S dz$$

or

$$dG = (\mu G - rG)\,dt + \sigma G dz = (\mu - r)Gdt + \sigma G dz$$

(b) If G is a stock with an instantaneous dividend payout, μ would be the risk free rate, r is the instantaneous dividend yield, and σ the volatility of the stock. The stock is growing at the rate of $\mu - r$ at the risk-neutral world.

(c) $dG = (r - r_f)Gdt + \sigma G dz$

where r is the domestic risk-free rate and r_f is the foreign risk-free rate.

(d) Each unit of call option on 1 Japanese Yen is worth

$$C = G_0\ \text{Exp}(-r_f(T-t))N(d_1) - K\ \text{Exp}(-r(T-t))N(d_2)$$

$$d_1 = \left[\ln(G_0/K) + (r - r_f + \sigma^2/2)*(T-t)\right]/\left(\sigma*(T-t)^{0.5}\right)$$

$$d_2 == d_1 - \sigma*(T-t)^{0.5}$$

Where r is the domestic risk-free interest rate,
r_f is the foreign (Japanese) risk-free rate,
σ is the exchange rate's annualized volatility,
T-t is the time to maturity of the option
S_0 is the current price of Japanese Yen,
and K is the strike price on the Japanese Yen of the option
$G_0 = K = 1/110$ US$
r=0.01
r_f=0.0005
T-t=0.25

Solution 19 (continued)

Annualized volatility σ = (# of trading days in a year)$^{0.5}\times$ daily volatility

$$=(4\times 65)^{0.5}\times 0.62\% = 10\%$$

$$d_1 = \left[\ln(1)+\left(0.01-0.0005+0.1^2/2\right)*0.25\right]/\left(0.1*0.25^{0.5}\right)=0.0725$$

$$d_2 = 0.0725 - 0.1*0.25^{0.5} = 0.0225$$

$$C=(1/110)*Exp(-0.0005*0.25)N(0.0725)-(1/110)*Exp(-0.01*0.25)N(0.0225)$$

$$=(1/110)*0.999875*0.528898-(1/110)*0.997503*0.5090$$

=0.004808-0.004616=0.000192 US$

Value of the contract on 220 billion Yen
=0.000192 × 220 billion US$
=42,240,000 US$ or about 42 million US$

Solution 20

The guarantee at maturity: $L_T^* = L_0 \cdot e^{rT}$
$=66,673,640 \times e^{0.03\times 10}$
$=90,000,000$

$$\alpha = \frac{Lo}{Ao} = \frac{66,673,640}{74,081,822} = 0.9$$

$$\therefore (1-\alpha)A_o = C_e(A_o, L_T^*) - \delta \cdot \alpha C_E\left(A_0, \frac{L_T*}{\alpha}\right)$$

δ is the equilibrium participation level

$$\frac{L_T^*}{\alpha} = \frac{90,000,000}{0.9} = 100,000,000$$

From the given table we have,

$$\therefore C_E(A_o, L_T^*) = C_E(A_o, 90,000,000) = 7,625,000$$

$$C_E\left(A_o, \frac{L_T^*}{\alpha}\right) = C_E(A_0, 100,000,000) = 1,204,330$$

$$\therefore \delta = \frac{C_E(A_0, L_T^*) - (1-\alpha)A_0}{\alpha C_E\left(A_0, \frac{L_T^*}{\alpha}\right)} = \frac{7,625,000 - (1-0.9)\times 74,081,822}{0.9 \times 1,204,330}$$

$$=0.2$$

hence the equilibrium participation level is 0.2