Solution 1

(a)

I. Market Risk – Risk to value of derivative due to change in market. Components include:
   a) Delta – Absolute Price
   b) Gamma – Convexity
   c) rho – Discount Rate
   d) Vega – Volatility
   e) Theta – Time Decay
   f) Basis Risk – Tracking Error
   g) Investing and Funding Risk

II. Liquidity Risk – Derivatives may not be able to be sold, unwound at need time or at needed price

III. Credit Risk – The risk of counter party default.

IV. Operational Risk – Risk of human error, system error, or lack of documentation

V. Legal Risk – Risk that a contract is unenforceable. Possible ways:
   a) contract is illegal
   b) lack of documentation
   c) Risk that one party could not contract
   d) unenforceability of contract in insolvency

(b)

I. LifeCo has exposure limit to counter party to mitigate credit risk, and has credting rating minimum.

II. LifeCo has good documentation procedures to mitigate legal and operational risk, but they should also have internal audit to verify adherence to policies.

III. For market risk, LifeCo should calculate and measure things such as delta, sigma, vega, rho, etc.

Solution 1 (continued)

IV. LifeCo has an outline of responsibilities and committees.

V. LifeCo has set guidelines for approved derivatives and permitted uses.

VI. Board of Directors are notified of transactions

VII. Contract and counterparty requires chief counsel approval

VIII. LifeCo needs to consider credit enhancement clauses

IX. LifeCo has ALM committee

X. LifeCo requires master contract

Solution 2

(a) Offered GMDB, which is greater of acct value and accumulative value at 5%

(i) actual approach=guarantee benefit=exposure × election rate × Pr of claim
Election rate = 1
Pr of claim based on mortality
Exposure = max (0, DB-acct value)
use historical data to estimate probability

(ii) capital market approach
use option approach, risk neutral
pay off like a put option
LifeCo's GMDB is type II guarantee

(b) VA funds invested 70% in equity, 15% bond, and 5% money market
lower equity return make guarantee more likely in the money
lower equity return reduces M&E fee since it often based on asset under management
probably means more surrender/lapse
high credit loss makes guarantee more likely in the money
could be more default
lower interest reduces money market return
might be positive for equity and bonds
but could mean low return in long run

(c) Static hedge
buy put options for equity
buy CDs for bonds
buy floors to hedging lower rate
idea is to buy options or exchange risk of guarantee with counterparty risk

Dynamic hedging
synthetic manufacture option payoff
can be flexible process
uses short-dated option

Can use integrated risk management approach
need risk attitude, risk expertise, risk size and correlation
market price dynamics

May just running the risk
or reinsure
or make market by develop products that offset GMDB risk

Solution 2 (continued)

(d) Payout annuity includes pension buyout annuities,
structured settlements
cashflow usually is very predictable
cashflow is usually interest rate insensitive other than some COLA benefit
usually long term

(e) PA portfolio has more than 6% equity, 10% in below investment, 5% in MBS/CMO
low equity return may not support crediting rate
not enough current income
but long term may be good
6% may be too much
Could be large defaut from below investment grade
10% allocation is very large
down grade from investment grade make things worse
may need more capital
low interest rate means low reinvestment income
pre-payment from MBS/CMO
A&L duration match will get worse
right now already asset duration is shorter than liab duration

(f)

(i) reduce equity allocation
reduce below investment allocation
reduce MBS/CMO
could increase investment grade corporate
callable bonds not good, so reduce
diversify
consider real estate, CML, CMBS, ABS etc

(ii) use put option to hedge equity return
use CDS to hedge credit risk
use floor to hedge low interest rate

Solution 3

(a)

Dollar duration gives an idea of how the assets MV will change relative to the liabilities PV for a parallel shift in the interest rate curve, assuming the cashflows will be the same as before the shift.

Effective duration will give an idea of how the assets MV and liabilities PV will change (in %) for a parallel shift, taking into account the presence of interest rate sensitive cashflows on both sides of the balance sheet.

The liabilities in the Non-traditional Life segment have embedded options:

- Guaranteed interest rate of 3-6%
- Max loan rate of 7% on older policies

A non-negligible % of the assets are callable

Effective duration will be a better metric due to the presence of interest rate sensitive cashflows. Dollar duration will not give a true picture.

(b)

Investment Strategy

- Allocation of assets to eliminate excessive and unacceptable risk
- Bulk of portfolio should be in high quality fixed income instruments. This would allow the company to meet the minimum credited rate guarantees. Should also increase the term of the bonds so the durations would increase and this would help mitigate the mismatch problem.
- Should also include equities and derivatives. These would help back the portfolio rates that are being credited. The derivatives if selected appropriately would help with the embedded options of the non-traditional life segment.

Product Development

- Should correspond to investment strategy
- Must balance competitiveness and risk assumption
- LifeCo should manage the risks better in this product, and make sure it is not guaranteeing minimums that are too high

Solution 3 (continued)

Reinsurance

- Help control liability cash flows
- Reinsurance usually takes advantage of diversification by pooling risks, but with respect to asset/investment risk, it might actually increase the risk.
- Securitization is the selling of liabilities at a discount – may help reduce risk, but not really appropriate
- This is a good opportunity for LifeCo. They could use coinsurance to reduce risk… or if they wanted to be risk of risk entirely, could fully reinsure

Holism

- Need to take advantage of the synergies of the different lines
- Seeks to leverage diversification of business. Sell products with offsetting risk.
- Enterprise wide rather than at product level.
- Difficult to know that products in fact have offsetting risks.
- LifeCo doesn't currently use this
- This would work if there are other product lines with different duration requirements. You can then combine them with the UL line and then address the overall mismatch problem.

Solution 4

(a) differences:

	Rationale	Assumptions	Credit Standing
Fair Value	Exit value; Immediate Settlement	Market-based	Reflected
Entity-Specific Value	Orderly settlement over the life of the liability	Specific to Entity	Not reflected

Similarities:

- both use JWG hierarchy
- both use PV techniques
- both update assumptions on each valuation date

(b)

$$r_L = r_A - e\left(\frac{r_e}{1\text{-}t} - r_A\right)$$

assumptions from market participants holding similar assets and liabilities

$r_A = .08$

e=.05

$r_e = .12$

t=.35

$$r_l = .08 - (.05)\left(\frac{.12}{1-.35} - .08\right)$$

$r_l = 7.48\%$

Liability at time 0= $\dfrac{(1{,}000)(1.23)}{(1.0748)^3} = \$990.65$

Solution 4 (continued)

(c) Under the Total Return Approach, the liability value at issue is the single premium = $1,000

The requires spread "s" is such that:

$$\text{initial liability value} = \frac{\text{Maturity Value}}{(1+d_3+S)^3}$$

d_3 = 3 year risk-free spot rate=6.21%

where $$1{,}000 = \frac{1{,}230}{(1+.0621+S)^3} = 93\text{bps}$$

(d) Arguments for:

- Liability is someone else's asset
- Not reflecting it means a company can manipulate its earnings by trading in its own dept
- No compelling reason why insurance liabilities should be treated any differently than publicly issued debt
- FV of liabilities from owner's perspective can never be greater than assets-requiring FV to reflect credit standing

Arguments against:

- if credit reflected, earnings can go up when credit rating goes down-counterintuitive
- Insurers can't exit liabilities except through settlement with policyholders. Trying to do reflecting credit standing violates "unfair trade practices" laws
- The effect of credit standing is small because of the presence of guarantee funds, and the priority status of policyholders in insolvency cases
- Not reflecting it provides more useful information to financial statement users, including creditors and potential buyers.
- Insurance liabilities are not traded-no market value. Asset valuation models don't necessarily apply to insurance liabilities

Solution 4 (continued)

(e) The cost of capital approach incorporates credit standing by discounting at a rate other than the risk-free rate, which takes into account the riskiness of the cash flows.

The impact of this on LifeCo would be to reduce the valuation of their liabilities because of their weakened capital position and potential for downgrade.

Solution 5

(a)

$D_s * S = (D_A * A\text{-}D_L * L)$, where A, L and S are market values

$D_s = (D_A * A\text{-}D_L * L)/S$

$D_s = (7.7 * 5457.1 - 9.8 * 5220)/237.1$

D_s = -38.53 years

(b) Surplus would be reduced to zero if:

Delta S $= {}^{-}S_0 * D_s * \leq (\text{Delta i})$

(0-237.1) $= -237.1 * (-38.53) * (\text{Delta i})$

1 $= (-38.53) * (\text{Delta i})$

Delta i = 1/(-38.53)

Delta i = -2.595%, i.e. if rates decrease by 2.6%

(c) Surplus would be immunized if $D_s = 0$

$D_s * S = (D_A * A\text{-}D_L * L)$, where A, L and S are market values

$0 = (D_A * A\text{-}D_L * L)$

$D_L = (D_A * A)/L$

$D_L = (7.7 * 5457.1)/5220$

D_L = 8.05 years

(d) Use of modified duration assumes fixed cash flows. Assets and liabilities have interest-sensitive cash flows (or embedded options). Should use effective duration.

Solution 6

(a)

D(a) = V(r) D(r) (1+k) / V(a) - V(FR) D(FR) / V(a)

Where r=reserves, FR=future retentions = future prems – future benefits, a=assets, k=required capital, V( ) = value of

If prems interest sensitive (v=1)

Future premium will decrease as int rates rise => D(FR) > 0

This implies that the target duration must be reduced

But, if competitor is not interest sensitive

Negative impact on D(FR), causes an increase in D(a)

If competitor is interest sensitive, no further impact

If prems not interest sensitive (v=0)

D(FL) > D(FP) => D(FR) < 0

So D(a) must be increased

But, if competitor is interest sensitive

Positive impact on D(FR), causes a decrease in D(a)

If competitor not interest sensitive, no further impact

(b)

1. Accounting Constraint
   - Impact varies by accounting system
   - RBC impact
2. Tax Constraint
   - Sale will realize a capital gain or loss
   - Manage at company level
3. Value/EVA
   - Sale may decr. distributable earnings. Post-tax cash flows must be replaced, otherwise decr. EVA
4. ALM
   - Effect on selling segment's duration matching
   - Yield curve shape
5. Credited Rate
   - If sell asset with high yield, it may decr. credited rate

Solution 6 (continued)

6. Policyholder equity issues
   - Sale may create additional risks or mismatches (counterparty, issuer exposure)
   - May be contrary to company philosophy

Solution 7

(a) utility functions, u(x)
are increasing functions

$$u'(x) > 0$$

(utility increases as wealth, x increases)
and are concave

$$u''(x) < 0$$

(utility increases at a decreasing rate as wealth increases)

(b) two investments or decisions are the same to an investor if they have the same utility
Example: insurance coverage that costs a premium an investor will be indifferent to purchasing this insurance if the utility is the same with or without it.

if $u(x) = E[u(W+P-S)]$

where $u(x)$ is utility

and $E[u(W+P-S)]$ is utility with the insurance that give coverage of claims S for a premium P

Solution 8

(a) Let ω_{1i} and ω_{2i} be the weight in % of total portfolio of the i^{th} bond in the first and second portfolio respectively.

The effective duration of portfolio 1 is then calculated by

$D_1 = W_{11}1 + W_{12}2 + W_{13}3$

Similarly, the duration of the portfolio #2 is given by $D_2 = \omega_{21}1 + \omega_{22}2 + \omega_{23}3$

in order to match the durations, we must have

1. $\omega_{11}1 + \omega_{12}2 + \omega_{13}3 = \omega_{21}1 + \omega_{22}2 + \omega_{23}3$

in addition, in order to have the same market valued, we must have

2. $\omega_{11}(1.04)^{-1} + \omega_{12}(1.05)^{-2} + \omega_{13}(1.06)^{-3} = \omega_{21}(1.04)^{-1} + \omega_{22}(1.05)^{-2} + \omega_{23}(1.06)^{-3}$

Many portfolio could be found that meet those equations, for example, let's choose portfolio 1 such that weight are equals:

that is $\omega_{11} = \omega_{12} = \omega_{13} = \frac{1}{3}$

in addition let's assume the market value of the portfolios is \$300, thus:

$100 \cdot 1.04 = 104$ of face value in 1-yr bond

$100 \cdot 1.05^2 = 110.25$ of face value in 2-yr bond

$100 \cdot 1.06^3 = 119.10$ of face value in 3-yr bond

To get portfolio 2, first note that market value of each bond must add to 300 and duration must be 2 thus,

$\omega_{21} \cdot 1 + \omega_{22} \cdot 2 + \omega_{23} \cdot 3 = 2$ (duration)

$300(\omega_{21} + \omega_{22} + \omega_{23}) = 300$ (market value)

Choosing an arbitrary value for ω_{22} of ${}^{150}\!/_{300}$, we get

$\omega_{21} = {}^{75}\!/_{300}$ $\quad$ $75 \cdot 1.04 = 78$ of face in 1-yr

$\omega_{22} = {}^{150}\!/_{300}$ $\quad \Rightarrow \quad$ $150 \cdot 1.05^2 = 165.38$ of face in 2-yr

$\omega_{23} = {}^{75}\!/_{300}$ $\quad$ $75 \cdot 1.06^3 = 89.33$ of face in 3-yr

Solution 8 (continued)

(b) Since duration =2

a 1% increase will lead to a charge of $-1\% \cdot 2 \cdot 300 = {}^{-}\$6$
or a decrease of \$6 in the portfolio value for both portfolios

(c) for portfolio 1 we have $\omega_{11} = \frac{1}{3}$

so

Δ in portfolio $= -\omega_{11} \cdot D \cdot 300 \cdot 0.01 = {}^{-}\$2$
or a decrease in value of \$2

for portfolio 2: Δ in portfolio $= -\omega_{21} \cdot D \cdot 300 \cdot 0.01 = \frac{-75}{300} \cdot 2 \cdot 300 \cdot 0.01 = {}^{-}\$1.50$

or a decrease in value of \$1.50

(d) need key rate duration!

portfolio #1

a shift of all curve by 1% gives a value of

$$P^* = \frac{104}{1.05} + \frac{110.25}{(1.06)^2} + \frac{119.10}{(1.07)^3} = 294.39 \text{ vs } {}^{-}\$6 \text{ in answer b}$$

so effective duration of

$$D = \frac{P^* - P}{-P \cdot 0.01} = 1.86972$$

for 1-yr rate only

$$P^* = \frac{104}{1.05} + 100 + 100 = 299.04 \text{ vs } {}^{-}\$2 \text{ in answer c}$$

so D(1) = 0.31746

for 2 yr rate:

$D(2) = 0.62596$

for 3-yr rate:

$D(3) = 0.9263$

Solution 8 (continued)

Portfolio #2

Similarly we get

all rate shift $\Rightarrow P^* = 294.3856$ vs $^-$\$6 in answer b D=1.8714

1 yr shift $\Rightarrow P^* = 299.2857$ D(1)=0.23809

2 yr shift $\Rightarrow P^* = 297.18316$ D(2)=0.938946

3 yr shift $\Rightarrow P^* = 297.9168$ D(3)=0.6944

Solution 9

(a) In this context, value at risk can be defined as follows: "We are 95% certain that over the maximum increase value of our GMAB liability is $X where X is the Value at Risk". The key limitations are as follows:

Results vary widely with methodology
May give false sense of security to senior management
Does not reflect liquidity risk, operational risk, etc
Assumes a normal distribution

(b) Stress Testing – use predefined deterministic scenarios that reflect some of the most extreme event observed historically.
This addresses the fat tails issue
Avoids the statistical distribution issues altogether.
Back-testing – test the model against the past and addresses all of the model integrity issues – policyholder behaviour, and statistical distributions

Solution 10

(a)

(i) FX Risk

- Transactional Exposures
  - Transactions in foreign jurisdictions have to be translated over to base currency. Income risk
  - Commodities priced in other currencies could become expensive if rates move against you.
  - Revenues of course have a major impact if they have to be consolidated.
- Translational Exposures
  - Asset values have to be converted over
  - Affects equity account
- Competitive exposure-change in relative competitive position
- Contingent Exposures
  - For transactions that may take place in the future

(ii) Problem is in Cash Flow Statement where TargetCo now has CAD as an expense for corporate. Other than that there isn't a big problem other than:

Asset values would have to be converted to CAD and this could affect the balance sheet for CanCo.

(b)

(i) 10% depreciation

$\Rightarrow$ 1 USD = 1.35 CAD

	CanCo		TargetCo		before devel TargetCo		after dev TargetCo	
Assets	CAD	11,000	USD	2,500	CAD	3,750	CAD	3,375
Lials	CAD	9,000	USD	2,000	CAD	3,000	CAD	2,700
Surplus	CAD	2,000	USD	500	CAD	750	CAD	675

Consolidated

	Before Combined	After Combined
Assets	14,750	14,375
Lials	12,000	11,700
Surplus	2,750	2,675

So surplus goes down CAD 75

Solution 10 (continued)

(ii) 1 USD = 1.35

	NetCash Flow of TargetCo Before	After
Premiums	USD150	USD 150
Sales&Mktg & Policy Ben	(USD 70)	(USD 70)
Corporate	(CAD75 = USD 50)	(CAD75 = USD 55.56)
	USD 30	24.44

So a reduction of USD 5.56 – makes sense because expenses got a little bigger.

(c) External Concerns:

Analysts (Share Price)

FX is only a 2nd order effect on operations acc to analysts so they may ignore

Investors

Will be concerned with vol of earnings. Still since investors can create own hedges by taking derivative/margin positions this may not be relevant.

Income Investors (Maintain Dividend Policy)

If CanCo pays a dividend is the volatility in earnings going to limit dividend flexibility by putting a strain on cash flow.

What about dividend growth? Problem if too much fluctuation in income/cash flow.

(d) Options

- Gives an option on the move in rates
- Exposed to positive benefits if guess right
- "Insurance" protection at a cost

Solution 10 (continued)

Futures

- Less flexible
- Partial or full hedge
- Could cause cash flow issues if obligation has to be fulfilled but there are cash flow problems operationally.

Forwards or swaps

- Similar to futures but credit risk may be an issue
- Futures/forwards/swaps effectively lock in the current exchange rate of 1.50,hedging against movements in f/x rate either way

(e) Need to hedge the 75 CAD in 1 year (protect against rising Canadian dollar) Assume we want a put option on US$ (or call on CAN$) – solution is done with put on US$

At the strike price of 1.50

Solution 10 (continued)

$$S_o = 1.5$$
$$K = 1.5$$
$$\sigma = 0.04$$
$$r = 0.04$$
$$r_f = 0.03$$
$$T = 1.0$$
$$p = Ke^{-rT} N(-d_2) - S_o e^{-r_f T} \; N(-d_1)$$
$$d_1 = \frac{\ln(S_o / K) + (r - r_f + \sigma^2/2)T}{\sigma\sqrt{T}}$$
$$= \frac{0 + \left(0.04 - 0.03 + (0.04)^2/2\right)(1)}{(0.04)(\sqrt{1})} = +0.27$$
$$d_2 = d_1 - \sigma\sqrt{T} = +0.27 - (0.04)\sqrt{1} == 0.23$$
$$N(-d_1) = N(-0.27) = 0.3936$$
$$N(-d_2) = N(-0.23) = 0.4090$$
$$\Rightarrow p = (1.50)e^{(-0.04)}(0.4090) - 1.50e^{-(0.03)}(0.3936)$$
$$\Rightarrow p = 1.50[.3929631 - .3819691]$$
$$p = 0.016491$$

Since we have to protect 75M CAD or $50M USD we will need 50 contracts for a cost of:

$50x0.016491 =$ \$825,000 CAD or \$550,000 US

Solution 11

(a) Value of one-month call option implies following value for rf:

Using a one-period binomial tree, movement is either up or down

Call is in the money only for an upward movement, and payoff is the amount of this movement

C = S * p * (u-1) * exp(-rf/12)

25.025=800*p*(u-1)*exp(-rf/12)

d=exp(-sigma * sqrt (1/12))= exp(-.20 *sqrt (1/12)) = .9439

u=1/.9439=1.059434

So p= 25.025/800/(1.059434-1)*exp(rf/12)= .526319*exp(rf/12)

Also, p = (a-d)/(u-d) =(exp(rf/12)-d)/(u-d)=(exp(rf/12)-.9439)/(1.059434-.9439)=8.65544 *exp(rf/12)-8.16987

So exp(rf/12)*8.129121=8.16987

Thus rf= 6%

(b) Upper and lower bounds

Upper bound:
Current value of index (800)

Lower bound:

Look at lower bound as for 2-month European call option since strike will be no more than strike on European call, of 800.

That is, S0-K * exp(-rf*T)= 800*(1-exp(-.06*2/12))= 7.96

And of course, value is greater than one period call option in (a) worth 25.025

The upper bound is the value of the stock index, $800.

Solution 11 (continued)

(c) Price of lookback:

Construct binomial tree with 2 time-steps

Have all assumptions from (a) above.
u=1.05943, d=0.9439, p=0.528956

Index value (scaled to start at 1)

		1.12239
	1.05943	
1.0000		1.0000
	0.9439	
		0.8909

Path-dependent minimum index values (last col. shows lower path's value first)

		1.0000
	1.0000	
1.0000		0.9439,1.0000
	0.9439	
		0.8909

Path-dependent payoff if exercised at given node

		0.12239
	0.05943	
0		0.0561, 0
	0	
		0

Option values at each node, assuming no early exercise

		0.12239
	exp(-.005)*0.528956*0.12239+0=0.064416	
exp(-.005)*(0.528956*0.064416+0.471044*0.029526)		0.0561,0
=0.047742	exp(-.005)*0.528956*0.0561+0=0.029526	
		0

Solution 11 (continued)

Option values at each node, reflecting early exercise right – early exercise not optimal at any node, so same values. So option value = 800*0.047742 = 38.19

Solution 12

- Duration is usually measured to account for interest rate sensitivity of a fixed income instrument.

- The assets are equities. The factors affecting the value of equities are much more than interest rates. Interest rate changes are a main factor affecting the performance of fixed-income instruments, but it is not so dominating in explaining the equity value change.

- According to the DDM motel, duration of equities can be much longer than what is observed in the market. Also, the Franchise Factor model solved the equity duration paradox, it is still an area requiring more research and empirical study.

- Using equity to back long-term liabilities without participation into investment returns is a bad idea, using a metric not even theoretically sound to measure the interest sensitivity of the assets will make things even worse by giving the unreliable indication of performance results.

Solution 13

- Define risk limits
  - to avoid catastrophic losses

- Monitor risk limits seriously
  - penalize when the risk limits are not satisfied otherwise, feeling that the risk limits are not to be taken seriously

- Do not underestimate the benefit of diversification
  - diversification can significantly reduce risk

- Do not think you can outguess the market
  - gain is only obtained by luck

- Carry out scenario analysis and stress testing
  - help to assign the risk present in derivatives

<u>For financial entities</u>

- Monitor traders carefully
  - position limits, concentration
- Separate the back, middle, and front office
  - to ensure that all functions are adequately performed
- Do not blindly trust model
- be conservative in the recognition on inception profits
- do not sell clients inappropriate products
  - dangerous for the LT life of the business
- do not ignore liquidity risk
  - important in period of stress liquidity (crisis)
- beware when everyone following the same trading strategy
  -could result in the future at market instability

<u>For non-financial institutions</u>

- Understand the trades that you are executing
  - large losses occur when a trade was made and the senior management does not understand it
- Be cautious by making the Treasury department, a profit center ⇒ manage liquidity (goal) not produce profit.
- Be cautious that a hedger does not become a speculator, hedge is dull ⇒ speculation is fun however, large risk involved

Solution 14

(a) Critical Factors

- size ÷ composition of international allocation in portfolio
- cost of hedging
- risk tolerance of management
- potential purchase of foreign goods

(b) As size of international allocation ↑'s, hedge ratios ↑'s
As size of hedging ↑'s, optimal hedge ratio ↓'s
As risk tolerance ↑'s, optimal hedge ratio ↑'s

(c) Purchasing Power Parity

Economic theory that predicts in long run equilibrium, the cost of goods in local and foreign markets will be equivalent. Exchange rates tend to offset inflation rates.

Interest Rate Parity

Economic theory stating that adjusted for foreign exchange rates, interest rates in foreign and local markets are equivalent.

(d) If true-preserving future purchasing power, not a consideration in deciding whether to hedge because cost of goods in local and foreign markets will be equivalent.

If false – In light of preserving purchasing power of foreign products, consider hedging less to protect against local cost of imports due to exchange rate fluctuations. Also if false implies possible arbitrage opportunities.

Solution 15

(a) prepayments=relocations and re-financings-assumptions and curtailments

relocations-when an individual moves out of one house and into another. Relocations are high during strong housing markets. They tend to be higher in the spring and summer. Low current interest rates increase relocations since existing payment can buy more home. Higher relocations increase prepays. Demographic reasons increase relocations. Higher tax deductibility with relocations.

Refinancings-when an individual prepays his mortgage by refinancing into a lower interest or shorter term mortgage. High after a fall in interest rates. However, burnout occurs after the same low is hit a few times. Also, when the yield curve steepens, there is incentive to refinance into a shorter term mortgage. The impact of refinancings are high, higher refinancings increase prepays. Surge is refinancing when rates increase after steady decrease and homeowner thinks they missed the bottom.

assumptions-when the buyer of a house "assumes" the mortgage from the seller of the house. This occurs mostly when interest rates have gone up, so the current mortgage is a discount mortgage. Higher assumptions decreases prepays. GNMA mortgage may be assumed rather than prepaid. Minimal transaction costs, little judgment or interest rate timing, and easier to qualify.

curtailments-partial extra principal payments above what is scheduled. While they do not have a large impact on new mortgage pools, their cumulative impact can be very high. They are higher when homeowners feel there are no better investment opportunities for their extra money. Increasing curtailments increase prepays. WAM calculations ignore future curtailments and may be overstated/

(b) Pool A: relocations – Likely fairly low since 80% of pool has only 3 years on their current mortgage. However there is a strong housing market. Slower prepays since not in summer anymore.

refinancings- High as interest rates fall below the weighted average coupon. Pool A on cusp of being economic to refinance Due to steep curve would be good time to refinance to shorter term mortgage.

Solution 15 (continued)

	Pool A younger with higher LTV and higher incentive to refinance which would increase prepays over b. Present increase in rates could cause surge in prepays.
assumptions:	Very slow since interest rates are falling. However, they are allowed since it's a GNMA pool.
curtailments -	Small effect since pool is not very seasoned.
Pool B: relocations	Very high since high demographic seasoning. Strong housing market, relatively old mortgages, 50% balloon mortgages, which appeal to short-tenure homeowners, falling interest rates all contribute. Slower prepays since not in summer anymore.
refinancings	Somewhat high. Although 80% of loans have already refinanced, the weighted average coupon is very high relative to end-of-2003 rates. Due to the steep yield curve, would be a good time to refinance into a shorter term mortgage. Recent increase in rates could cause surge in prepays.
assumptions	Not allowed since it's not a GNMA pool
curtailments	Have likely had a large impact over time, especially on those mortgages that are very seasoned. Curtailments will likely continue to be strong as homeowners look to gain equity.

Solution 16

(a) $\text{Cap Rates} = \dfrac{\text{Net operating Income}}{\text{MV property}}$

Cap rates:

1) Provide yield similar to debt instruments adjusted for risk and uncertainty
   Can use fixed rate mortgages which are positively correlated with cap rates
2) Have similar characteristics to equities
   - proxy with S&P index or E/P ratio
     - E/P ratio is positively correlated with cap rates
3) Influenced by inflation
   - proxy with T-note/T-Bill differential
   - inflation negatively correlated with cap rates
4) Include expectation for tax sheltered returns
   - tax effect not very significant
5) Include risk premium that varies with supply/demand cycle of Real estate market.
   - proxy with national vacancy rate or Δ in GNP

Also consider

Trends in operating income
Expectation for market rents

(b)

1) Need appropriate return & volatility measures
   Consider:
   a) Appraisal Based Index
      Drawbacks
      Volatility of appraisal data less than half
      true volatility due to
      Appraisal smoothing-use stale info
      Temporal Aggregation properties appraise annually but updated quarterly

Solution 16 (continued)

Seasonal Patterns properties reappraise in 4^{th} Q (about 55%)

Can correct using

$$\text{Adjusted Index} = \frac{\text{Previous Annual Index Return} - \lambda(\text{Current Annual Index Return})}{(1-\lambda)}$$

higher λ means more smoothing

λ should be between 0.5 and 0.7

b) Index of equity Real Estate stocks including:
REITS, Real estate operating companies not in trust form, land subdividers and commercial developers and general contractors

2) Consider risk aversion of portfolio owner, diversification requirements, and any other constraints.

3) Maximize return for each level of risk and construct efficient frontier. Use analysis to determine optimal level.

Solution 17

(a)

Set up default payoff and probability - QRFexp(-Y)
Set up no-default payoff and probability - (1-Q)Fexp(-Y)
equate sum to riskless bond value - Fexp(-r)
solve for formula - Q=(1-exp-(Y-r))/(1-R)
Use Merton model to value company equity
as an option on the company assets
E0=V0N(d1)-Dexp(-rT)N(d2)
this imposes one condition on company value and asset volatility

$$\sigma_E E_0 = N(d_1)\sigma_V V_0$$

Ito's lemma gives the other condition

Now solve 2 equations in 2 unknowns

(b)

use Q=(1-exp-(Y-r))/(1-R)
substitute the right numbers for each bond (r from C, then into B)
Produces illogical numbers (risky yields lower than risk free) - model is wrong

(c)

Higher default probabilities from bonds
1) Liquidity premium on corporates
2) Extreme scenario anticipation
3) Bond driven calculation is a risk neutral world estimate
4) Historical default probabilities are real world

(d)

First to default pays off only on the first default
- cheap if high correlations
- but bad if high correlation and low number of bonds
- bad if significant low quality concentration
Add-up CDS is equivalent to the
sum of simple CDS' on each bond
- expensive (risky bonds)
- no correlation reduction

Solution 17 (continued)

(e)

Use monte carlo simulation
At each trial calculate for each of A,B,C
(a) PV (Payoff)
(b) PV of payments until min(default,1 year)
Swap spread=(average of all (a))/(average of all (b))
To incorporate correlation
assume their joint is multivariate normal
then joint distribution of times to default
can be described in terms of
cumulative probability distributions of default and
pairwise correlations of inverse normal transforms
this is the gaussian copula assumption
it allows estimation of correlation separately from marginals

(f)

[1] Excess of loss contract is a bull spread on total losses
long call with strike at lower limit of the layer and
short call with strike at higher limit of the layer
[2] Issue bonds that curtail principal or interest payments
based on the value of earthquake losses
(I) issue bond with principal of (350-100=250) million
and high bond holder interest
the principal is decreased by losses in the layer
(II) issue a much larger amount with interest rates
reduced by excess layer losses
[3] CAT bonds are attractive because of lack of
any significant correlation to market returns

Solution 18

(a) To split the performances between the four sources we need:

Return of liabilities: 6.2% (AAA,duration=7)
Return of assets: 5.7% (A, duration =10)

Benchmark 1, must replicate credit of liabilities and duration of liabilities
Benchmark 2, must replicate credit of liabilities but duration of asset
Benchmark 3, must replicate the duration and credit quality of asset

Benchmark 1 = 7.3%
Benchmark 2 = 7%
Benchmark 3 = 5.5%

(b) Company performance $= r_A - r_I = {}^{-}0.5\%$
Liability performance = benchmark 1-r_L = 1.1%
Asset performance-interest rate risk=benchmark 2-benchmark 1 = -0.3%
Asset performance-credit risk=benchmark 3 – benchmark 2 = -1.5%
Asset performance-selection=r_A -benchmark 3

(c)
- Organizational issues
- Short term vs long term measures
- Handling interest rate sensitive cashflows
- Take into account duration of asset and liabilities and convexity
- Selecting the appropriate market curve for insurance co. Cash flows are long term
- Take into account embedded options in the products

Solution 19

(a) Multivariate normal simulation is

- Inherently single period
- Does not allow dynamic asset allocation
- Asset return series are not assumed to be autocorelated
    - Not good for fixed-interest assets (FI)
        - Common to assume mean reversion or some autocorrelation for FI
- Modeling interest rates as return on an asset class is problematic
    - Model term structure of interest rates more appropriate
- Variance is assumed to be constant
    - Empirical evidence of volatility clustering

(b)

(i) Use $S_0 = A_0 - L_0$ to buy a 5 year put on the risky asset portfolio with strike value L_5

L_5 = liability value in 5 years

Invest L_0 in a risky portfolio

i_e = uncertain return of risky portfolio in 5 years

$L_0(1+i_e) + \max\left[L_5 - L_0(1+i_e), 0\right] = L_0(1+i_e)$ if $L_0(1+i_e) > L_5$

$L_0(1+i_e) + L_5 - L_0(1+i_e) = 5$ if $L_0(1+i_e) < L_5$

(ii) Buy a 5 year call on the risky asset portfolio with strike value L_5

Invest $L_5/(1+i_f)$ in duration-matched Treasury fund

i_f = risk-free return on Treasury in 5 years

- Long dated option not viable
    - Market is illiquid
    - High cost (spread)

Solution 19 (continued)

(c) by put-call parity $P+L_0 = C + L_5/(1+i_f)$

(d) replicate put option using B-S formula

$P=L_5/(1+i_f)N(\text{-d2}) - L_0N(\text{-d1})$

$L_5/(1+i_f)N(\text{d-2})$ represent amount invested in risk-free Treasury fund

$-L_0N(\text{-d1})$ represent short holding of risky portfolio

The total portfolio is

Risky asset = $L_0 - L_0N(\text{-d1}) = L_0(1-N(\text{-d1}))$

Treasury fund = $L_5/(1+i_f)N(\text{-d2})$

- Require sale and purchase of equity portfolio as the hedge ratio changes
  - Can be expensive because of transaction cost and illiquidity
- Not possible to trade when prices jump as market "gaps"
- For long run investor fixed time horizon is unlikely to be appropriate
- Gamma of put is high and hedge ration, $N(\text{-d1})$, changes significantly as time horizon moves close to expiry date and value of asset close to strike price
  - Increased trading activities and transaction costs

Solution 20

(a) Common hedging strategies for derivatives

- Naked hedging: do nothing; exposure when the option in-the-money
- Covered position: short call while long the underlying asset; exposure when stock price decreases more than option price earned
- Stop-loss strategies: When stock price higher than strike price, then buying stock; when stock price lower than strike price, then selling the stock. may incur substantial transaction cost and suffered due to high bid-ask spread.

Above three methods are all inappropriate. We always using delta neutral may be with gamma-neutral, vega-neutral strategies to hedging the call. May hold underlying assets in an amount governed by calls delta, gamma and vega.

(b) Due to no dividends:

For European call = $C=S_0N(d_1)-ke^{-rT}N^{(d_2)}$

$$S_0\ \Delta = N(d_1) \text{ where } d_1 = \frac{\ln(S_0/K)+\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = \frac{\ln\left(\frac{100}{110}\right)+\left(0.05+\frac{0.2^2}{2}\right)}{0.2\sqrt{2}} \doteq 0.158$$

$$\therefore N(d_1) = N(0.158) = N(0.15)+0.8\left[N(0.16)-N(0.15)\right] = 0.5628$$

Because portfolio consist of three short call options, and each option's contract size is 1,000 shares

$\therefore$ delta of portfolio=$-3\times 1000\times 0.5628 \doteq -1688.4$

$$\Theta = -\frac{S_0N^1(d_1)\sigma}{2\sqrt{T}} - rke^{-rT}N(d_2)$$

$$d_1 = 0.158 \qquad d_2 = d_1 - \sigma\sqrt{T} \doteq -0.1248$$

$$N^1(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}} \qquad \therefore N^1(d_1) = 0.394 \qquad N(d_2) = 0.4503$$

Solution 20 (continued)

$$\therefore \Theta = -\frac{100\times 0.394\times 0.2}{2\sqrt{2}} - 0.05\times 110\times e^{-0.05\times 2}\times 0.4503 \doteq -5.027$$

$\therefore$ theta of portfolio = $-3\times 1000\times(-5.027) = 15{,}081$

$$\Gamma = \frac{N^{1(d_1)}}{S_0\sigma\sqrt{T}} = \frac{0.394}{100\times 0.2\times\sqrt{2}} \doteq 0.01393$$

$\therefore$ gamma of portfolio = $-3\times 1000\times 0.01393 = -41.79$

(c) BSM differential equation: $\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial s} + \frac{1}{2}\sigma^2 s^2\frac{\partial^2 f}{\partial s^2} = rf$

then $\Theta + rs\Delta + \frac{1}{2}\sigma^2 s^2\Gamma = r\Pi$

from b)

$\Theta + rs\Delta + \frac{1}{2}\sigma^2 s^2\Gamma = {}^{-}5.027 + 0.05\times 100\times 0.5628 + \frac{1}{2}0.2^2\cdot 100^2\cdot 0.01393 = 0.573$

and c= $\pi = S_0N(d_1) - ke^{-r\tau}N(d_2) = 100\times 0.5628 - 110e^{-0.05\times 2}\times 0.4503 = 11.46$

$\therefore r\pi = 0.05\times 11.46 = 0.573 = \Theta + rs\Delta + \frac{1}{2}\sigma^2 s^2 p$

This verify the BSM differential equation

Solution 21

(a) Garch (1,1)

$$\sigma_n^2 = \gamma V_L + \alpha\mu^2(n-1) + B\sigma^2(n-1)$$

$\sigma_0^2 = 9\%$ $\quad$ $V_L = 4\%$ $\quad$ $\mu = 20\%$

$B = 80\%$ $\quad$ $\gamma = 10\%$

$\gamma + \partial + B = 1$ $\quad$ $\alpha = 10\%$

$$\sigma_n^{\ 2} = .004 + .1(.2)^2 + .8\sigma^2(n-1) = .008 + .8\sigma^2(n-1)$$

$$\sigma_0^{\ 2} = .09$$

$$\sigma_1^{\ 2} = .008 + .8 \times .09 = .08$$

$$\sigma_2^{\ 2} = .008 + .8 \times .08 = .072$$

$$\sigma_0^{\ 2} = .08$$

$$\sigma_1^{\ 2} = .072$$

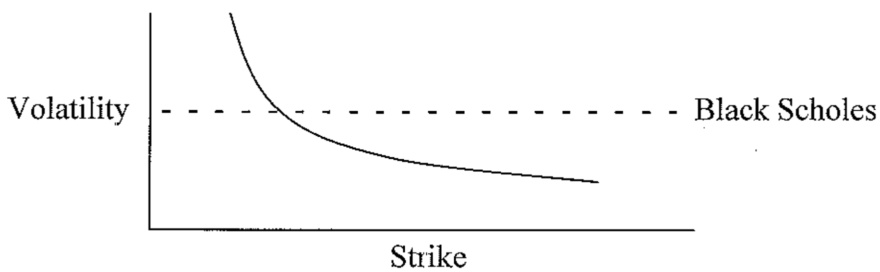
$$\sigma_2^{\ 2} = .008 + .8 \times .072 = .0656$$

so

$$\Delta\sigma_1 = \sqrt{.072} - \sqrt{.08} = -1.45\%$$

$$\Delta\sigma_2 = \sqrt{.0656} - \sqrt{.072} = -1.22\%$$

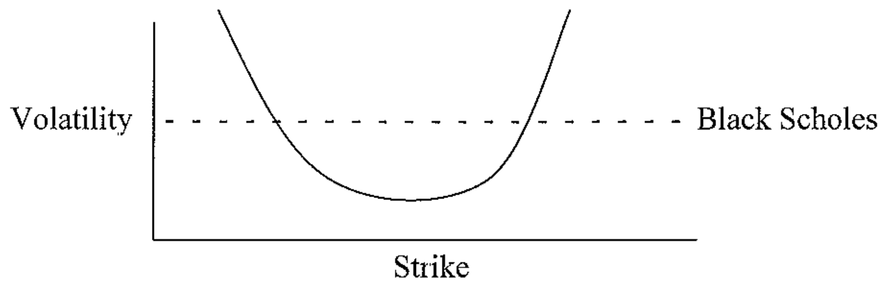
(b) Equity has the following volatility smile



For low prices the volatility is above constant volatility log normal, Black Scholes. For higher strike prices the volatility is below lognormal, lighter tail. If the option were valued by Black Scholes it would overestimate the price of an equity option at a high strike price and underestimate it at a low strike price.

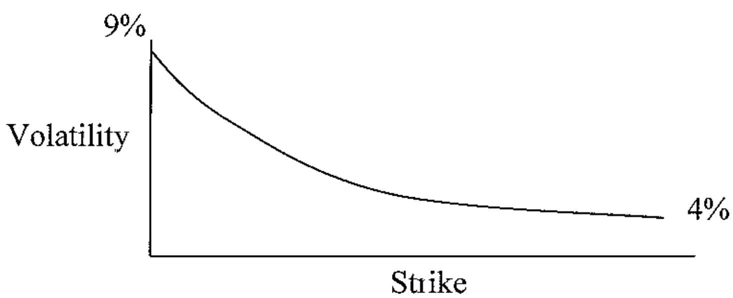
Solution 21 (continued)

Currency Options have the following volatility smile:



When the option is deep in or deep out of the money its volatility is higher than lognormal, heavier tails. While if it is around the strike price its volatility is below lognormal. If valued by Black Scholes with constant volatility, deep in or out of the money options would be underpriced while at the money options would be overpriced.

The given model has the following volatility smile:



This is more appropriate for an equity option.

(c) Find α, B and γ to maximize the following function

$$\sum_{i=1}^{m} -\ln\sigma(i) - \frac{\left(u(i)-u\right)^2}{\partial\sigma^2(i)}$$

Where $\sigma(i)$ is the volatility estimate for period i with observation $\mu(i)$.

This can be simiplified by assuming $\gamma V_L = .008$, then you only need to find α and B that maximizes the sum.

Solution 22

(a) The best Monte Carlo simulation formula for the stock price is

$$S(T) = S(0)e^{\left[\left(\hat{\mu}-\frac{\sigma^2}{2}\right)T+\sigma\varepsilon\sqrt{T}\right]}$$

Set T = 1/2 , S(0) = 100, σ = 0.25.

Stratified sampling on 3 intervals: the Hull book suggests mid-points of the 3 intervals, i.e. 1/6, 3/6 and 5/6, as random numbers to use.

$$\varepsilon_1 = N^{-1}\left(\frac{1}{6}\right) = -0.97,\ \varepsilon_2 = N^{-1}\left(\frac{3}{6}\right) = 0,\ \varepsilon_3 = N^{-1}\left(\frac{5}{6}\right) = 0.97$$

For risk neutral valuation, set $\hat{\mu} = r = 6\%$

$$S_3\left(\frac{1}{2}\right) = 100e^{\left[\left(6\%-\frac{25\%^2}{2}\right)+25\%\varepsilon_3\sqrt{\frac{1}{2}}\right]} = 100e^{[1.44\%+17.678\%\varepsilon_3]} = 120.43$$, payoff of call = 120.43 - 95 = 25.43

$S_2\left(\frac{1}{2}\right) = 101.45$, payoff of call = 101.45-95 = 6.45

$S_1\left(\frac{1}{2}\right) = 85.46$, payoff of call = 0

Hence, call price at time 0 is

$$\frac{1}{3}e^{-rT}\left[\sum_{i=1}^{3} Payoff\ \ in\ scenario\ i\right] = \frac{1}{3}e^{-6\%\times 0.5}\left[25.43 + 6.45 + 0\right] = 10.31$$

The control variate adjustment is 11.37-10.31 = 1.06

Solution 22 (continued)

(b)

Antithetic variable

- Use -ε with ε

Importance sampling

- For out-of-money options, focus simulation on 'in-the-money' portion of results

Moment matching of random numbers

Low discrepancy sequence

- So random numbers spread out over region more evenly

Representative simulation through a tree

- Ensure number of paths through a node is consistent with probability implied in the tree used

(c)

Method 1: Least Square Approach

Price of American option known at time T.

At time (N-1)Δt, calculate exercise value and holding value V. With values from all simulation runs, relate value V to stock price S using formula $V = aS^2 + bS + c$.

Method 2: Exercise Boundary Parametrization approach

Determine $S^*(t)$ so that exercise immediately for American put if $S_i(t) < S^*(t)$, otherwise hold. Work iteratively to find $S^*(t)$ that maximizes option value.

Solution 23

(a)

- A collar is a combination of a long cap and short floor
- A cap can be recognized as a put option maturing at time τ_k on a zero-coupon bond maturing at time τ_{k+1}. The strike price is the notional amount L. The payoff is

$$Max\left[L - \frac{L(1+R_k\partial_k)}{1+R_k\partial_k}, 0\right]$$

- The same applies to a floor, it can be recognized as a call option on a zero-coupon bond maturing at time τ_{k+1}
- Therefore, the collar will have the same payoffs with a portfolio of bond option contains a put and call option on zero-coupon bond.

(b) $\partial = 0.2 \qquad \tau_k = 1 \qquad \tau_{k+1} = 1.25 \qquad \partial_k = 0.25$

$R_k = 0.045 \qquad F_k = 0.052 \qquad \gamma = 0.0477$

$$\therefore d_1 = \frac{\ln(F_k/R_k) + \partial k^2 \tau_k / 2}{\partial_k \sqrt{\tau_k}} = \frac{\ln\frac{0.052}{0.045} + 0.02}{0.2} = 0.8229$$

$$d_2 = d_1 - \partial_k\sqrt{\tau_k} = 0.6229$$

$$\begin{aligned} N(-d_1) &= N(-0.8229) \\ &= N(-0.82) - 0.29\left[N(-0.82) - N(-0.83)\right] \\ &= 0.2061 - 0.29[0.2061 - 0.2033] \\ &= 0.2053 \end{aligned}$$

$$\begin{aligned} N(-d_2) &= N(-0.6229) \\ &= N(-0.62) - 0.29\left[N(-0.62) - N(-0.63)\right] \\ &= 0.2676 - 0.29(0.2676 - 0.2643) \\ &= 0.2666 \end{aligned}$$

Solution 23 (continued)

$$L\delta_k \rho(o,\tau_{\kappa\tau\iota})\left[R_\kappa N(-d_2) - F_\kappa N(^-d_1)\right]$$
$$=100{,}000{,}000 \times 0.25 \times e^{-0.0477\times1.25} \times (0.045 \times 0.2666 - 0.052 \times 0.2053)$$
$$=31122.86$$

(c)

1) Black's model assumes a log normal distribution of a single variable
   - can elevate the risk in an analytical term. Like option pricing on cap or floor
   - ease to apply

2) Can't model path dependency of cashflows
   - simplified assumption may not measure accurately the prepayment risk
   - Assume interest rate is constant
   - Assume volatility is constant

3) In general Black's model is not suitable to value prepayment option in MBS
   - may also be better to have a model that models the yield curve, not just the short rate.

Solution 24

(a) - Understanding and measuring trading costs is essential to a successful implementation of any international equity strategy.

- Trading cost measurement gives you an indication of how expensive a given stock is to physically trade
- Trading costs can eat into profits and gains if they are large for a given stock

Time	Price	Size	Price Differential	Change x Shares	Impact	Value of Trade
	44 1/8					
1:40	44 1/8	1000	0	$ -	0.00%	$ 44,125.00
1:42	44 1/4	2000	1/8	$ 250.00	0.28%	$ 88,500.00
1:45	44	500	- 1/4	$ 125.00	0.57%	$ 22,000.00
1:46	43 7/8	5000	- 1/8	$ 625.00	0.28%	$ 219,375.00
1:50	43 3/4	2000	- 1/8	$ 250.00	0.29%	$ 87,500.00
1:51	43 7/8	400	1/8	$ 50.00	0.28%	$ 17,550.00
1:54	44 1/4	6000	3/8	$ 2,250.00	0.85%	$ 265,500.00
				$ 3,550.00	0.48%	$ 744,550.00

(b)

Total Dollar Value of the price impact of trading = $3,550 or 0.48% of the Total Value of the Trade