Solution 1

(a)

Aggregate Normal Cost = Present Value of Future Normal Cost ÷ Present Value of Future Service
Present Value of Future Normal Cost = Present Value of Benefits - Assets

Participant A values at January 1, 2004

PVB = Annual Projected Benefit x $\ddot{a}_{65}^{(12)}$ x $D_{65} \div D_{50}$

Annual Projected Benefit(AB) = 12 x 50 x min(65-30,20) = 12,000
PVB=12,000 x 10.1 ÷ $(1.07)^{15}$
PVB = 43,928
PV Future Service= $\ddot{a}_{\overline{15}|}$ = 9.75

Participant B values at January 1, 2004

PVB = Annual Benefit x $\ddot{a}_{70:70}^{(12)}$

Monthly Life Annuity at Retirement = 50 x min(65-50,20) = 750

Monthly 50% Joint & Survivor Benefit = 750 x $\ddot{a}_{65}^{(12)} \div (\ddot{a}_{65}^{(12)} + .5 \times (\ddot{a}_{65}^{(12)} - \ddot{a}_{65:65}^{(12)}))$

Monthly Benefit = 750 x 10.1 ÷ (10.1 - .5(10.1 - 8.5))=695
PVB = 12 x 695 x (8.9 + .5(8.9 - 7.2))
PVB = 81,315

Normal Cost at January 1, 2004
Total PVB = 43,928 + 81,315 = 125,243
PVFNC = 125,243 - 80,000 = 45,243
NC = 45,243 ÷ 9.75 = 4,642

(b)

Participant A values at January 1, 2005

PVB = Annual Projected Benefit x $\ddot{a}_{65}^{(12)}$ x $D_{65} \div D_{51}$

Annual Projected Benefit(AB) = 12 x 50 x min(65-30,20) = 12,000
PVB=12,000 x 10.1 ÷ $(1.07)^{14}$
PVB = 47,003
PV Future Service = $\ddot{a}_{\overline{14}|}$ = 9.36

Solution 1 (continued)

Participant B values at January 1, 2005

Participant died & Spouse is receiving half of monthly benefit

PVB = $12 \times 695 \div 2 \times \ddot{a}_{71}^{(12)}$

PVB = $12 \times 695 \div 2 \times 8.7$

PVB = 36,331

Asset Value at January 1, 2005

Asset Value = Assets BOY x $(1 + i)$ + Contribution x $(1+i)^{½}$ - Benefits Paid with interest

Asset Value = $80,000 + 4,642 - 695 \times 6 - 695 \div 2 \times 6$

Asset Value = 78,387

Normal Cost at January 1, 2005

Total PVB = 47,003 + 36,331 = 83,334

PVFNC = 83,334 - 78,387 = 4,947

NC = $4,947 \div 9.36$ = 529

Solution 2

(a)

Determine accrued liability and normal cost.

EAN normal cost = PVFB at Entry Age × (1+salary scale) ^ (val age - hire age) divide by present value of future salary at entry age factor
PVFNC = NC × present value of future salaries at valuation age
AL = PVFB at val age - PVFNC

Participant X
PVFB at Entry Age = 3% × 150,000 × (1.05)^15 × 25 × 9.6 × (1.07)^-25
PVFB at Entry Age = 413,684

PV of Future Salary at Entry Age Factor = 20.12
EAN normal cost = 413,684 × (1.05)^(10) / 20.12
EAN normal cost = 33,491

PVFB at val age = 3% × 150,000 × (1.05)^15 × 25 × 9.6 × (1.07)^-15
PVFB at val age = 813,779

PV of Future Salary at Val Age Factor = 13.19
PVFNC = 33,491 × 13.19
PVFNC = 441,746

AL = 813,779 - 441,746
AL = 372,033

(b)

Determine 1/1/2005 Contribution and 1/1/2006 Contribution
Contribution = Normal Cost + Ten Year Amortization of Initial AL + 5 Year Amortization of Gains and Losses

10 Year Amortization Factor = 7.52
Amortization of AL = 372,033 / 7.52 = 49,472

Contribution = 33,491 + 49,472 = 82,963

Fund on 1/1/2006 = 82,963 × 1.15
Fund on 1/1/2006 = 95,407

Expected Assets = 82,963 × 1.07
Expected Assets = 88,770

Solution 2 (continued)

Expected Liability = (372,033 + 33,491)× 1.07
Expected Liability = 433,911

Expected UAL = Expected Liability - Expected Assets
Expected UAL = 433,911 - 88,770 = 345,141

Participant X
PVFB at Entry Age = 3% × 150,000 × 1.10 × (1.05)^14 × 25 × 9.6 × (1.07)^-25
PVFB at Entry Age = 433,383

PV of Future Salary at Entry Age Factor = 20.12
EAN normal cost = 433,383 × 1.05^11 / 20.12
EAN normal cost = 36,841

PVFB at val age = 3% × 150,000 × 1.10 × (1.05)^14 × 25 × 9.6 × (1.07)^-14
PVFB at val age = 912,208

PV of Future Salary at Val Age Factor = 12.42
PVFNC = 36,841 × 12.42
PVFNC = 457,565

AL = 912,208 - 457,565
AL = 454,643

Actual UAL = 454,643 - 95,407 = 359,236
Expected UAL = 345,141

(Gain)/Loss = Actual UAL - Expected UAL
(Gain)/Loss = 359,236 - 345,141 = 14,095

Amortization of Initial Unfunded = 49,472
5 Year Amortization Factor = 4.39
5 Year Amortization Factor of Loss = 14,095 / 4.39 = 3,211

2006 Funding Policy Contribution = 36,841 + 49,472 + 3,211 = 89,524

Solution 3

Alvin age = 30, service = 5

$\ddot{a}^{(12)}_{65:\overline{10}|} = \ddot{a}^{12}_{\overline{10}|} + v^{10} * {}_{10}p_{65} * \ddot{a}^{(12)}_{75} = 7.287 + 0.508 * 0.83 * 7.5 = 10.45$

UCAL = $20 * 12 * 5 * \ddot{a}^{(12)}_{65:\overline{10}|} * v^{35} = 1200 * 10.45 * 0.0937 = 1,175$

UCNC = $20 * 12 * 1 * \ddot{a}_{65:\overline{10}|} * v^{35} = 240 * 10.45 * 0.0937 = 235$

PVFB = $20 * 12 * 30 * \ddot{a}_{65:\overline{10}|} * v^{35} = 7200 * 10.45 * 0.0937 = 7,050$

PVFS = $\ddot{a}_{\overline{35}|} = 13.85$

Simon age = 55, service = 32

UCAL = $20 * 12 * 30 * \ddot{a}^{(12)}_{65:\overline{10}|} * v^{10} = 7200 * 10.45 * 0.5083 = 38,247$

UCNC = 0

PVFB = UCAL

PVFS = $\ddot{a}_{\overline{10}|} = 7.52$

Theodore age = 70, service = 35

$\ddot{a}^{(12)}_{70:\overline{5}|} = \ddot{a}^{(12)}_{\overline{5}|} + v^5 * {}_5p_{70} * \ddot{a}^{(12)}_{75} = 4.254 + 0.713 * 0.89 * 7.5 = 9.01$

AL = $20 * 12 * 30 * \ddot{a}^{(12)}_{70:\overline{5}|} = 7200 * 9.01 = 64,872$

Allocate assets to inactives: 64,872
remaining assets for actives: 100,000 – 64,872 = 35,128

allocate Alvin = 35,128 * (1,175+235)/(1,175+235+38,247) = 1,249
allocate Simon = 35,128 * (38,247)/(1,175+235+38,247) = 33,879

NC (Alvin) = (PVFB – Assets) / PVFS
= (7,050 – 1,249) / 13.85 = 419

NC (Simon) = (38,247 – 33,879) / 7.52 = 581

Total Individual Aggregate NC = 419 + 581 = 1,000

Solution 4

(a)

using PUC AL = $B_{62} * v^2 * \ddot{a}_{62(12)\,5}k$

Bx = 01 * 80,000 * 1 03^2 * (1- 03*3) * 22

= 16,991

$\ddot{a}_{62(12)\,5}k = \ddot{a}_5k(12) + {}_5p_{62} * v^5 * \ddot{a}_{67(12)}$

= 4 348 + 0 9487 * v^5 *10 2489

= 11.6139

AL = 772.34 * 11 6139 * v^2

= 175,625

Annual pension = .01*80000*22 * (1- 15)

=$14,960

$\ddot{a}_{60(12)\,5}k = \ddot{a}_5k(12) + {}_5p_{60} * v^5 * \ddot{a}_{65(12)}$

= 4 348 + 0.96 * v^5 *10 767

= 12.0719

AL = 14,960 * 12 0719

=$180,596

Total Loss due to early retirement = $180,596 - $175,625

= $4,971

Solution 4 (continued)

(b)

Actuarial equivalent pension forms = value of pension payable under the normal form
= value of pension payable under optional form

Value of pension payable under Normal Form = \$180,596

equivalence formula

$\$180{,}596 = X * \ddot{a}_{60}\,(\text{member}) + .6 * X * (\ddot{a}_{57}\,(\text{spouse}) - \ddot{a}_{60:57})$

$+ (14{,}960 - X) * (\ddot{a}_{60}\,(\text{member}) - \ddot{a}_{60:57})$

$= X * 11.9995 + .6 * X * (13.3128 - 11.0728) +$

$(14{,}960 - X) * (11.9995 - 11.0728)$

$= X\,(12.4168) + 13{,}863.43$

$X = (180{,}596 - 13{,}863.43)/12.4168$

$= \$13{,}428$

Solution 5

(a)

PUC AL $\quad \sum B(y)\left[(x-w)/(y-w)\right]\ddot{a}_y^{(12)}v^{(y-x)}$

Employee A:

B(y) $= \$12{,}000 + \$50{,}000 \times .02 \times [a_{\overline{20}|} \times (1.04)^{20}]$

(i.e. $1+1.04 + 1.04^2 \ldots + 1.04^{19}$)

$= \$12{,}000 + \$1{,}000 \times 29.778 = \$41{,}778$

PUC AL $= \$41{,}778 \times 15/35 \times 11.2 \times v^{20}$

$= \$62{,}528$

Employee B:

B(y) $= \$15{,}000 + \$50{,}000 \times .02 \times [a_{\overline{5}|} \times (1.04)^5]$

$= \$15{,}000 + \$1{,}000 \times 5.416 = \$20{,}416$

PUC AL $= \$20{,}416 \times 30/35 \times 11.2 \times v^5$

$= \$146{,}458$

$Fund_{2005}$ $= PUC\ AL_{2005} - UAL_{2005}$

$= \$62{,}528 + \$146{,}458 - \$10{,}000 = \$198{,}986$

NC AL $= \Delta B(x)\ddot{a}_y^{(12)}v^{(y-x)} = \sum PUC\ AL / (x-w)$

$NC\ AL_{2005}$ $= \$62{,}528 / 15 + \$146{,}458 / 30$

$= \$4{,}168 + \$4{,}882 = \$9{,}050$

$Fund_{2006}$ $= (\$198{,}986 + \$9{,}050) \times 1.05 - \$62{,}000$

$= \$156{,}438$

$PUC\ AL_{2006}$ $= \$20{,}416 \times 31/35 \times 11.2 \times v^4 = \$146{,}458 \times 31/30\ 1.06$

$= \$160{,}420$

UAL_{2006} $= \$160{,}420 - \$156{,}438 = \$3{,}982$

(b)

Asset loss = Exp'd Fund – Actual Fund

Exp'd Fund $= (\$198{,}986 + \$9{,}050) \times 1.06 - \$62{,}000$

$= \$158{,}518$

Asset loss $= \$158{,}518 - \$156{,}438 = \$2{,}080$

Termination loss = Lump-sum payment - Exp'd AL_{2006}

Solution 5 (continued)

Exp'd AL_{2006} $= (AL_{2005} + NC_{2005}) \times 1.06$
$= (\$62,528 + \$4,168) \times 1.06 = \$70,698$

Termination loss = \$62,000 - \$70,698 = (\$8,698)

Termination loss could be split into 2 pieces totaling (\$8,698)

Optional form loss = Lump-sum payment – Deferred Pen Liab
$= \$62,000 - [(\$13,000) \times 11.2 / 1.06^{(65-46)}]$
= \$62,000 - \$48,123 = \$13,877

Termination decrement loss = \$48,123 - \$70,698 = (\$22,575)

Total losses = (\$6,618) = $UAL_{2006} - UAL_{2005} \times 1.06$