Solution 1

(a) Retirement pension before amendment Dec 31 $\rightarrow$ 30 yrs svc age 60
at 65 = 0.015(100,000)(30)=45000
at 60 $B_{60}\ddot{a}_{60}^{(12)} = B_{65\ 5}/\ddot{a}_{65}^{(12)}$ b/c actuarially equivalent

$$B_{60}(11.2) = 45000\ \ {}_5^{v^5}p_{60}\ddot{a}_{65}^{(12)}$$

$$B_{60}(11.2) = 45000(1.06)^{-5}(0.945)(9.9)$$

$$B_{60} = 28,089$$

retirement pension (annual) before amendment is 28089
after = unreduced at 60

$$B^*_{60} = 45000$$

increase in pension due to amendment = 45000-28089=16911

(b) need NC and AL before amendment $e = 30$

$$NC_{30}\ddot{a}_{\overline{65-30|}j} = pvFB$$

$$\ddot{a}_{\overline{35|}j} = 25.789$$

$$\text{J}=\frac{1.06}{1.04}-1 = 1.92\%$$

$$pvFB = 0.015(100000)(1.04)^{64-59}(35)v^{35}\ddot{a}_{65}^{(12)}$$

$$=0.015(121665)(35)(1.06)^{-35}(9.9)$$

$$=82273$$

$$NC_{30}(25.789) = 82273$$

$$NC_{30} = 3190$$

$$NC_{59} = 3190(1.04)^{29} = 9949$$

$$AL_{59} = NC_{59}\ddot{s}_{\overline{29|}j} = 9949(39.08) = 388836$$

need AL and NC after amendment

$NC_{30}\ddot{a}_{\overline{30|}} = pvFB$ retire at 60

$$\ddot{a}_{\overline{30|}j} = 23.07$$

$pvFB = 0.015(100000)(30)v^{30}\ddot{a}_{60}^{(12)}$ unreduced at 60

$$= 45000(1.06)^{-30}(11.2)$$

$$=87752$$

Solution 1 (continued)

$$NC_{30} = 3804$$

$$NC_{59} = 3804(1.04)^{29} = 11863$$

$$AL_{59} = NC_{59}\ddot{S}_{\overline{29}|i} = 11863(39.08) = 463620$$

change in NC = 11863-9949=1914, normal cost ↑ by 1914 after amendment
change in AL = 463620-388836=74784
AL ↑ by 74784 after amendment

Solution 2

(a) AL@1/1/2004

x=61 earn 2003 = 400,000

svc=25 pv account =1450000

$AL = Ben\, v^{r-x}{}_{r-x}P\chi^{(\lambda)} \times \ddot{a}_r^{(12)}$

$$\Rightarrow AL = \left[0.02 \times 400000(1.0) \times 25 - \frac{1450000}{9.55}\right] v_{6.5\%}^4 * 9.55$$

$=357568.62$ annuity provided by notional account

(b) $${}^{exp}AL_{1/1/2005} = \left[0.02 \times 400000 \times 26 - \frac{1450000}{9.55}\right] \times v_{6.5\%}^3 * 9.55$$

$=444058.25$

$${}^{act}AL_{1/1/2005} = \left[0.02x400000 \times 26 \times 0.91 - \frac{1150000}{10.32}\right] \times 10.69$$

$=832172.58$

Gain (Loss) $= {}^{exp}AL_{1/1/2005} - {}^{act}AL_{1/1/2005}$

$=(388114)$

Solution 3

data @ 1/1/04
$(\chi) = 60$ svc=10

ret ben $M/month/yrs svc
bridge ben $100/month until 65

UC: $AL_\chi = B_\chi \ \ddot{a}_r^{(12)} \frac{D_r}{D_\chi}$ $NC_\chi = \Delta B_x \ddot{a}_r^{(12)} \frac{D_r}{D_\chi}$

(a) $AL_{1/1/04}$ before plan Δ

1400M = PVB_{60}

PVB = ^{PV} RetBen + ^{PV} BridgeBen

$1400M = M \times 12 \times 10 \times \ddot{a}_{62}^{(12)} \times v^2 + 100 \times 12 \times v^2 \times \ddot{a}_{\overline{62:3}|}^{(12)}$

$1400M = M \times 12 \times 10 \times 11.6067 \times 1.06^{-2} + 100 \times 12 \times 1.06^{-2} \times \left[\ddot{a}_{62}^{(12)} - v^3 {}_3p_{62} \ \ddot{a}_{65}^{(12)}\right]$

1400 = 1240.M+2913 2.7272

M = 18.21

(b) $AL_{1/1/04}$ before Δ = $AL_{1/1/04}$ after Δ

18.21×1400 = PVB w/ Δ

$25{,}494 = 18.21 \times 12 \times 10 \times v^2 \times$ JSJ% factor

J% J$\dot{\varepsilon}$S $= \ddot{a}_{62:62} + J\left[\ddot{a}_{62} - \ddot{a}_{62:62}\right] + \left[\ddot{a}_{62} - \ddot{a}_{62:62}\right]$

$= 9.8213 + (1+J)(11.6067 - 9.8213)$

$= 9.8213 + (1+J)(1.7854)$

$= 11.6067 + 1.7854J$

$25494 = 1944.82\left[11.6067 + 1.7854J\right]$

$J = 84.13\%$

Solution 3 (continued)

$$= \ddot{a}^{(12)}_{62:62} + 1.5\left(\ddot{a}^{(12)}_{62} - \ddot{a}^{(12)}_{62:62}\right)$$

$$= 12.5$$

(c) $J = 50\%$ $\uparrow$

$$AL_{1/1/04} = 18.21 \times 12 \times 10 \times v^2 \times JS50\% \text{ factor}$$

$$= 24{,}310$$

$$\Delta AL = 24310 - 1400(18.21) = -1184$$

$$\text{amort} = -\frac{1184}{\ddot{a}_{\overline{2}|}}$$

$$= -609$$

$$\text{NC}_{1/1/04} = 18.21 \times 12 \times v^2 \times 12.5$$

$$= 2{,}431$$

Cont = 2,431-609=1,822

Solution 4

(a) At 1/1/04 By Fil

$$NC_x = \frac{\sum PVFB_x - \sum AL_x}{\sum PVFS_x} \times Sal_x$$

AL_x calculated by EAN method

For A,

$$PVFB_x = 1\% \times 40000 \times 1.03^{17} \times \ddot{a}\frac{(12)}{60:5|} \times 0.75 \times 30 \times v^{18}$$

$$\ddot{a}^{(12)}_{60:\overline{5}|} = \ddot{a}^{(12)}_{\overline{5}|\ 6\%} + \ddot{a}^{(12)}_{65} \times v^5\ {}_5P_{60}$$

$$= 4.3480 + 10 \times 1.06^5 \times 0.95$$

$$= 11.4470$$

$$PVFB_x = 59657$$

$$PVFB_w = 59657 \times v^{12} = 29648$$

$$NC_x = \frac{PVFB_w}{\ddot{a}_{\overline{30}|j}} \times (1.03)^{12} \qquad j = \frac{1.06}{1.03} - 1$$

$$= \frac{29648}{20.401} \times 1.03^{12}$$

$$= 2072$$

$$PVFNC_x = 2072 \times \ddot{a}_{\overline{18}|j}$$

$$= 29545$$

$$AL_x = PVFB_x - PVFNC_x$$

$$= 59657 - 29545$$

$$= 30112$$

$$PVFS_x = 40000 \times \ddot{a}_{\overline{18}|j} = 570369$$

For B,

$$PVFB_x = 1\% \times 25000 \times 1.03^{29} \times \ddot{a}^{(12)}_{\overline{60\ 5}|} \times 0.75 \times 32 \times v^{30}$$

$$= 28180$$

Solution 4 (continued)

$$PVFB_w = 28180 \times v^2 = 25080$$

$$NC_x = \frac{PVFB_w}{\ddot{a}32\rceil j} \times 1.03^2$$

$$=1253$$

$$PVFNC_x = 1253 \times \ddot{a}_{\overline{30}\rceil j}$$

$$=25563$$

$$AL_x = PVFB_x - PVFNC_X$$

$$=2617$$

$$PVFS_x = 25000 \times \ddot{a}_{\overline{30}\rceil j} = 510027$$

$$NC_x = \frac{\sum PVFB_x - \sum AL_x}{\sum PVFS_x} \times \sum Sal_x$$

$$= \frac{87837 - 32729}{1080396} \times [40000 + 25000]$$

$$=5.1007\% \qquad \text{x65000}$$

$$=3315$$

$\therefore NC$ for 2004 = 3315

Amortization for $AL_x = 32729 / \ddot{a}_{\overline{5}\rceil 6\%}$

$$=7330$$

(b) At 1/1/2005

For A,

$$PVFB_x = 59657 \times 1.06 = 63236$$

$$PVFS_x = 40000 \times 1.03 \times \ddot{a}_{\overline{17}\rceil j}$$

$$=562191$$

For B,

$$PVFB_x = 28180 \times 1.06 = 29871$$

$$PVFS_x = 25000 \times 1.03 \times \ddot{a}_{\overline{29}\rceil j}$$

$$=514129$$

$$UAL_0 = AL_0 : F_0 = 0$$

$$=32729$$

Solution 4 (continued)

$$F_1 = 15000 \times 1.1 = 16500$$

$$UAL_1 = (UAL_0 + NC_0)(1+i) - \text{Cont with i}$$

$$= (32729 + 3315)(1.06) - 16500$$

$$= 21707$$

At 1/1/2005

$$NC_x \frac{\sum PVFB_x - UAL_1 - F_1}{\sum PVFS_x} \times \sum Sal_x$$

$$= \frac{93107 - 21707 - 16500}{1076320} \times 65000 \times 1.03$$

$$= 3415$$

NC@1.1.2005 is 3415

Solution 5

1/1/04 eff date

ILP

t=2004 x=44=a w=30 y=65

x

$PVFNCa = PVFBa$

$$NC_{44}\ddot{a}_{\overline{21}|7\%} = B(65)\ddot{a}^{|12}_{65} \times \frac{D_{65}}{D_{44}}$$

$$NC_{44}(11.59) = (30)(12)(35)(10.5)(1.07)^{-21}$$

NC_{44} =2,756 88

$NC^{x}_{04} = NC_{44}$

t = 2005

x x=45 w=30 y=65

$$AL_{05} = NC_{04}(1.07) = 2756\ 88 \times 1.07$$

$$=2949.86$$

$$NC_{05} = \frac{PVFB_{05} - AL_{05}}{PVFY_{05}}$$

$$= \frac{(32)(12)(35)(10.5)(1.07)^{-21} - 2949.86}{\ddot{a}_{\overline{20}|2\%}\ (=11.34)}$$

$$=2,955.75$$

Solution 5 (continued)

$t = 2005$

$\underline{Y} \qquad PVFB_a = PVFNC_a$

$x = 30 = a \qquad y = 65$

$$NC_{30}\ddot{a}_{\overline{35}|7\%} = (32)(12)(35)(10.5)(1.07)^{-35}$$
$$NC_{30}(13.85) = 13,217.71$$

$$NC_{30} = 954.35$$
$$\therefore NC_{05}^{Y} = 954.35$$

$\therefore$ Employer Normal Cost for 2005 = $NC_{05}^{x} + NC_{05}^{Y}$
=3,910 10

Solution 6

(a) July 1 cont = $NC * .15\left(UAL_{1/1/04}\right)$

$UAL_0 = AL_0 - A_0$

$A_0 = 1,000,000$ given

$AL_0 = \sum AL_x$

A

$AL = (10,000)(12)\ddot{a}_{\overline{67:3|}} = 1,165,753$

$\ddot{a}_{\overline{67:3|}} = \ddot{a}^{(12)}_{\overline{3|}} + {}_3 1\ddot{a}^{(12)}_{67}$

$= 2.71+7.0=9.71$

B

$AL = (.02)(10)(100,000)(1.0)\ v^{13}\ 10.1$

$=83,822$

C

$AL = .02(4)(40,000)(1)v^{24}\ 10\ 1$

$=6,371$

$AL = 1,255,947 = \left(\sum AL\right)$

$UAL = AL - A = 255,947$

amort$= UAL * .15 = 38,392$

$NC = \sum NC$

$A \Rightarrow ret\ \ NC = 0$

$B \Rightarrow NC = {}^{AL}/_{10} = 8,382$

$C \Rightarrow NC \Rightarrow {}^{AL}/_{4} = 1,593$

$NC = 9,975$

July contrib =9,975+38,392=48,366

Solution 6 (continued)

(b) at 05

A

$$AL = 120,000 * \ddot{a}_{\overline{2}|}^{(12)} = 224,654$$

B

$$Al = (105,000)(.02)(11)v^{12} \cdot 10 \cdot 1$$
$$=103,592$$

C

$$AL = 0 \text{ nonvested}$$

$$AL = \sum AL = 328,246$$
$$A_1 = A_0(1.10)$$
$$+48,366(1.10)^{1/2}$$
$$-120,000(1.10)^{1/2}$$
$$=1,024,869$$
$$UAL_{05} = 328,246 - 1,024,869$$
$$=(696,623) \quad \text{surplus}$$

Solution 6 (continued)

(c) Investment Gain

$$A^{exp} = 1,000,000(1.07)$$

$$-(120,000 - 48,366)(1.07)^{1/2} = 995,901$$

$$\text{gain} = A^{act} - A^{exp} = 28,968$$

Withdrawal gain

$$AL^{exp} = (6,371 + 1593)(1.07) = 8521$$

$$AL^{act} = 0$$

gain from withdrawal of C = 8521

Death gain

$$AL^{exp} = AL_0(1+i) - BP(1+i)^{1/2}$$

$$= 1,165,753\ (1.07) - 120,000\ (1.07)^{1/2}$$

$$= 1,123,227$$

$$AL^{act} = 224,654$$

death gain = 898,572

Salary loss

$$AL^{exp} = (83,822 + 8,382)(1.07) = 98,656$$

$$AL^{act} = 103,592 \qquad \text{loss} = 4,936$$