Solution 1

Determine NC as at 1.1.04

Normal cost for Giles = 0 (Retired)

Normal cost for Faith

$$^{nc}ILP(04) = \frac{\Delta PVFBx}{a_{(65-x)}} + {}^{nc}ILP(03)$$

$^{nc}ILP(03) = 2780$

$\Delta PVFB = \Delta B_{65} x\ a_{65}\ x\ v^{65-44} = 4800 \times 9 \times (1.07)^{-(65-44)} = 10{,}433$

$\Delta B_{65} = 50*12*35 - 45x30x12 = 4{,}800$

$$^{nc}ILP(04) = \frac{10{,}433}{\ddot{a}_{\overline{21}|}} + 2{,}780 = 3{,}680$$

$[PVFS = \ddot{a}_{\overline{21}|} = 11.5940]$

Total NC = 3,680 + 0 = 3,680

Determine liability @ 1.1.04

Faith

$AL^{04} = NC^{03} \times 1.07 = 2{,}780 * 1.07$
$= \$2{,}975$

Giles

$AL = 45 \times 12 *\text{service at } 63 * [1-.04 \times (65-x)] \times \ddot{a}_{63}$
$= 45 \times 12 \times 23 \times [1-(.04x2)]x10$
$= 114{,}264$

Total $AL^{04} = 117{,}239$

Solution 1 (continued)

Assets as at 1.1.04

$Assets^{04}$= contribution @ 1.1.03 x fund earnings
= (35,320 + 2,780) x 1.06
= 40,386

Amortization

$UAL^{04} = Assets^{04} - AL^{04}$
= 40,386 – 117,239
= 76,853

$$\text{Amortization Payment} = \frac{UAL^{04}}{\ddot{a}_{\overline{5}|}} = \frac{76,853}{4.3872} = 17,518$$

Total ER 1.1.04 Contribution = 17,518 + 3,680 = 21,198

Solution 2

(a) Accrued Liability = $\Sigma B(x) \ddot{a}_y^{(12)} D_y / D_x$

at 1/1/2003 as active = 50 x 12 x Min[30, service at 1/1/2003] x $\ddot{a}_{65}^{(12)}$ x v^{65-62}

= 50 x 12 x 30 x 9.9166 x $(1 / 1.06)^3$

= $149,871.03

$ERF_{62} = N_{65}^{(12)} / N_{62}^{(12)} = ({}_3p_{62} \times v^{65-62} \times \ddot{a}_{65}^{(12)}) / \ddot{a}_{62}^{(12)}$

= .9631 x $(1 / 1.06)^3$ x 9.9166 / 10.7330

= .747129

AL at 1/1/2003 as retired = 50 x 12 x Min[30, service at 1/1/2003] x ERF_{62} x $\ddot{a}_{62}^{(12)}$

= 50 x 12 x 30 x .747129 x 10.7330

= $144,340.84

Gain/(Loss) = AL as active – AL as retired

= 149,871.03 – 144,340,84

= 5,530.19 gain

(b) There is a gain because there is no probability of death before 65 assumed for actives, but there is probability of death between age 62 and age 65 for this early retiree.

Since there is a chance that the retiree will die prior to 65, there is a chance that less payments will be made and thus the liability will be less.

Gain = (AL as active) * ${}_3q_{62}$ = 149,871 * (1 - .9631)

= 5,530

Solution 2 (continued)

(c) B is the annual life only benefit payable to the member

Y is the annual optional amount payable while both member and spouse are alive

(½ xY) is the amount payable while only the spouse is alive, if the member predeceases the spouse

(B-Y) is the amount payable to the member, in addition to Y, if the spouse predeceases the member

$$B \times \ddot{a}_{member}^{(12)} = Y \times (\ddot{a}_{member}^{(12)} + \tfrac{1}{2} \times (\ddot{a}_{spouse}^{(12)} - \ddot{a}_{member:spouse}^{(12)})) + (B-Y) \times (\ddot{a}_{member}^{(12)} - \ddot{a}_{member:spouse}^{(12)})$$

$$B = 50 \times 12 \times 30 \times .747129$$

$$= \$13,448.32$$

$$13,448.32 \times \ddot{a}_{62}^{(12)} = Y \times (\ddot{a}_{62}^{(12)} + \tfrac{1}{2} \times (\ddot{a}_{57}^{(12)} - \ddot{a}_{62:57}^{(12)})) + (13,448.32 - Y) \times (\ddot{a}_{62}^{(12)} - \ddot{a}_{62:57}^{(12)})$$

$$13,448.32 \times 10.7330 = Y \times (10.7330 + \tfrac{1}{2} \times (13.2893 - 10.0844)) + (13,448.32 - Y) \times (10.7330 - 10.0844)$$

$$144,340.82 = (Y \times 12.3355) + 8,722.58 - (Y \times .6486)$$

$$Y = 135,618.24 / 11.6869 = 11,604.29 \text{ / year}$$

Solution 3

(a) Derive the earnings used for the January 1, 2002 valuation.

Employee X: Age 45, Service 10, 2002 Earnings not provided; outside of assumed termination zone

$$1/1/2002 \text{ PUC AL} = 2.0\% \times \text{Earnings} \times (1.04)^{59-45} \times (1 - 3\% \times 5 \text{ yrs}) \times 10 \text{ yrs of service} \times N^{(12)}_{60}/D_{60} \times (1.07)^{45-60}$$

$$= 2.0\% \times \text{Earnings} \times 1.73168 \times 0.85 \times 10 \times 10.11 \times 0.36245 = \$50,000$$

Therefore,

$$2002 \text{ Earnings (X)} = \$50,000 / [2.0\% \times 1.73168 \times 0.85 \times 10 \times 10.11 \times 0.36245]$$

$$= \underline{\$46,350}$$

Employee Y: Age 35, Service 4, 2002 Earnings not provided; Within assumed termination zone

$$1/1/2002 \text{ PUC AL} = \text{PUC AL-term} + \text{PUC AL-ret} =$$

$$2.0\% \times \text{Earnings} \times 4 \text{ yrs of service} \times 5\% \text{ probability of terminating} \times N^{(12)}_{65}/D_{65} \times (1.07)^{35-65}$$

$$+$$

$$2.0\% \times \text{Earnings} \times (1.04)^{59-35} \times 95\% \text{ probability of retiring} \times (1 - 3\% \times 5 \text{ yrs}) \times 4 \text{ yrs of service} \times N^{(12)}_{60}/D_{60} \times (1.07)^{35-60} = \$5,000$$

$$= [2.0\% \times \text{Earnings} \times 4 \times 0.05 \times 9.22 \times 0.13137] + [2.0\% \times \text{Earnings} \times 2.5633 \times 0.95 \times .85 \times 4 \times 10.11 \times 0.18425] = \$5,000$$

Therefore,

$$0.00484 \times \text{Earnings} + 0.30845 \times \text{Earnings} = \$5,000$$

And,

$$\underline{2002 \text{ Earnings (Y)} = \$15,960}$$

Employee Z: Age 57, Service 12, 2002 Earnings not provided; Outside of assumed termination zone

Solution 3 (continued)

1/1/2002 PUC AL = 2.0% x Earnings x $(1.04)^{59-57}$ x (1 – 3% x 5 yrs) x 12 yrs of service x $N^{(12)}_{60}/D_{60}$ x $(1.07)^{57-60}$

= 2.0% x Earnings x 1.08160 x 0.85 x 12 x 10.11 x 0.81630 = $100,000

Therefore,

2002 Earnings (Z) = $100,000 / [2.0% x 1.08160 x 0.85 x 12 x 10.11 x 0.81630]

= $54,916

(b) Calculate the funding policy contribution for 2003, as of January 1, 2003.

Funding policy = NC + 5-yr amortization of UAL at 1/1/2003
Therefore, will need NC, AL under the PUC method and MV as of 1/1/2003.

Step 1: Based on the above 2002 Earnings and 2002 experience, determine the 2003 liabilities:

Employee X, still active:

Actual AL = 2.0% x [$46,350 x 1.06] x $(1.04)^{59-46}$ x (1 – 3% x 5 yrs) x 11 yrs of service x $N^{(12)}_{60}/D_{60}$ x $(1.07)^{46-60}$

= $59,980

NC = $59,980/11

= $5,453

Employee Y, terminated vested:

Actual AL = 2.0% x $15,960 x 5 yrs of service x $N^{(12)}_{65}/D_{65}$ x $(1.07)^{36-65}$ =

2.0% x $15,960 x 5 yrs of service x 9.22 x 0.14056

= $2,068

NC = $0

Solution 3 (continued)

Employee Z, retired:

Actual AL = 2.0% x \$54,916 x 13 yrs of service x 0.79 x $N^{(12)}_{58}/D_{58}$

= \$116,858

NC = \$0

Total AL: \$59,980 + \$2,068 + \$116,858 = \$178,906

Total NC: \$5,453

Step 2: Based on 2002 experience, determine MV of assets as of January 1, 2003:

MV at 1/1/2003 = MV at 1/1/2002 + Contribution during 2002

= \$150,000 + [NC + five-year amortization of \$5,000 (=UAL at 1/1/2002)]x (1.035)

= \$150,000 + [\$14,583+\$1,140]x(1.035) = \$166,273

Step 3: Determine 2003 Funding Policy Contribution Based on Above:

Funding Policy Contribution for 2003 = \$5,453 + five-year amortization of (\$178,906 - \$166,273) = \$5,453 + \$12,633/4.3875

= \$8,332

(c) Calculate the gains/losses, by source, for 2002.

Sources of gain/loss:

1. Salary experience
2. Mortality experience
3. Termination experience
4. Retirement experience
5. Investment experience

Salary experience:

For Employee X, determine the expected AL:

Solution 3 (continued)

Expected AL = ($50,000 +$5,000) x 1.07

= $58,850 vs. $59,980 = $1,130 loss

Mortality experience:

None expected, none occurred. No gain/loss.

Termination experience:

For Employee Y: determine the expected AL

= ($5,000 + $1,250) x 1.07 = $6,688

vs. $2,068 actual

Therefore, gain = $6,688 - $2,068 = $4,620

Retirement experience:

For Employee Z: determine the expected AL

= ($100,000 + $8,333) x 1.07 = $115,916

vs. $116,858 actual

Therefore, loss = $115,916 - $116,858 = $942

Investment experience:

Expected MV = $150,000 x 1.07 + $16,273 x 1.035 = $177,343

Vs. $166,273 actual = $11,070 loss.

Check in total:

Expected UAL = ($5,000 + $14,583) x 1.07 - $16,273 x 1.035 = $4,111
Actual UAL = $12,633
Total Loss = $12,633 - $4,111 = $8,522

Total loss from above = $1,130 - $4,620 + $942 + $11,070 = $8,522 Ok.

Solution 4

a) Employer Normal Cost at January 1, 2003:

Aggr NC = $(\Sigma \text{ PVFB} - \text{F}) / \Sigma \text{ PVFS} \times \text{s}$

F = 0

PVFB = $2\% \times \$100{,}000 \times (1.04)^{(59-40)} \times 20 \times \ddot{a}_{60}^{(12)} \times v^{(60-40)}$
= 333,718

PVFS = $\ddot{a}_{20\overline{0}|j} \times \$100{,}000$ where $j = (1.06/1.04) - 1$
= $16.7903 \times \$100{,}000$ = $1,679,030

Aggr NC = 333,718/16.7903
= $19,876 = Employer Normal Cost

b) Employer Normal Cost at January 1, 2004:

F = $20,000

PVFB = $2\% \times \$110{,}000 \times (1.04)^{(59-41)} \times 20 \times \ddot{a}_{60}^{(12)} \times v^{(60-41)}$
= 374,149

PVFS = $\ddot{a}_{19\overline{0}|j} \times \$110{,}000$ where $j = (1.06/1.04) - 1$
= $16.0939 \times \$110{,}000$ = $1,770,329

Aggr NC = $(374{,}149 - 20{,}000)/16.0939$
= $22,005 = Employer Normal Cost

Solution 5

(a) Determine the funding policy contribution for 2003, as of January 1, 2003.

Funding policy = NC using EAN + 10-yr amortization of (AAL – MV) at 1/1/2003

Therefore, will need NC & AAL under EAN and MV as of 1/1/2003.

Step 1: Determine NC and AAL under EAN cost method as of 1/1/2003:

$NC = PVFB_{EA} / PVFS_{EA}$

EA = 30
Retirement Age = 60

$PVFB_{EA} = 30 \times (\$50) \times 12 \times (1.07)^{60-30} \times [\ddot{a}^{(12)}_{\overline{10}|} + {}_{10}p_{60} \times \ddot{a}^{(12)}_{70}] / (1.07)^{10}$

$= \$18,000 \times 0.13137 \times [7.28714 + (0.91 \times 9/1.96715)]$

$= 2,364.66 \times [11.4505]$

$= \$27,077$

$PVFS = 13.2777$

$NC = \$27,077/13.2777 = \$2,039$

AAL = PVFB at 1/1/2003 – PVFNC at 1/1/2003

$PVFB \text{ at } 1/1/2003 = \$27,077 \times (1.07)^{45-30} = \$74,706$

PVFNC at 1/1/2003 = NC at 1/1/2003 x PVFS at 1/1/2003 = \$2,039 x 9.7455 = \$19,871

AAL = \$54,835

Step 2: Determine Funding Policy Contribution for 2003

Funding policy contribution = NC + 10-year amortization of UAL at 1/1/2003

$= \$2,039 + (\$54,835 - \$7,500)/7.5152 = \$8,338$

Solution 5 (continued)

(b) Determine the Normal Retirement Benefit dollar multiplier such that the 2003 funding policy contribution would remain the same if given the following:

- the actuarial cost method is changed to Attained Age Normal retroactive to the date the plan was established, and
- the 2003 funding policy is changed to be the Normal Cost plus a 15 year amortization of any unfunded accrued liability

Step 1: Determine AAN NC as of January 1, 2002

NC under AAN = [PVFB – Assets – UAL] / PVFS, all values at 1/1/2002

$$PVFB_B = PVFB_{EA} \times (1.07)^{44-30}$$

$$= \$27,077 \times (1.07)^{44-30}$$

$$= \$69,820$$

$$\text{Assets} = \$0$$

$$\text{UAL} = \text{AAL (unit credit)} - \text{Assets} = \text{AAL (unit credit)}$$

$$\text{AAL (unit credit)} = \text{PVAB}$$

$$\text{PVAB} = 14 \times (\$50) \times 12 \times (1.07)^{44-60} \times [11.4505]$$

$$= \$32,581$$

$$NC_{2002} = [\$69,820 - 0 - \$32,581] / [10.1079] = \$3,684$$

Step 2: Solve for Increase in Multiplier Based 2003 Results (after plan change)

Funding policy (after) – Funding Policy (before) = \$0

Funding policy (after) = Funding Policy (before) = \$8,338

Solution 5 (continued)

Funding policy (after) = AAN Normal Cost at 1/1/2003 (after plan change) + 15-year amortization of UAL = $8,338

Solve for increase in multiplier ("Z"):

AAN Normal Cost = [PVFB – AVA – UAL] / PVFS, all values at 1/1/2003

AAN Normal Cost at 1/1/2003 (after plan change) = $[PVFB_{2003} - \$7,500 - {}_eUAL] / PVFS_{2003}$

$$PVFB_{2003} = \$74,706^{[1]} + [\$74,706]/50 \times Z^{[2]}$$

$${}_eUAL^{[3]} = [(\$32,581 + \$3,684) \times 1.07 - (\$7,000 \times 1.035)] + [[(\$32,581 \times 1.07 \times 15/14) / 50] \times Z]^{[4]} = \$31,559 + \$747 \times Z$$

$$PVFS_{2003} = 9.7455$$

Therefore,

$$\$8,338 =$$

$$[\$74,706 + [\$74,706]/50 \times Z - \$7,500 - \$31,559 - \$747 \times Z] / 9.7455$$

$$+ [\$31,559 + 747 \times Z] / \ddot{a}_{\overline{15}|}^{[5]}$$

$$= [\$35,647 + 747 \times Z] / 9.7455 + [31,559 + 747 \times Z] / 9.7455$$

Therefore,

$$\$67,206 + 1,494 \times Z = \$81,258$$

and

$$Z = \$9.41$$

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[1] Roll-forward the PVFB at 1/1/2002 with valuation interest (no experience gain/loss).
[2] New multiplier will increase the PVFB.
[3] Under AAN, roll-forward UAL with actual contributions and expected interest on actual contributions. Investment experience is brought into future normal costs.
[4] Under AAN, plan change will create new UAL amount, determined using UC funding method. UAL impact of plan change is added to UAL in method.
[5] Amortize the UAL over 15 years.

Solution 6

a)

$$\text{Accrued Liability}_{EAN} = \Sigma\, B(y)\, \ddot{a}_y^{(12)}\, D_y / D_x - \Sigma \text{ Normal Cost x } (N_x - N_y)/D_x$$

$$= PVFB - PVFNC$$

which means

$$AL_{aa} = PVFB_{aa} - [(PVFB_{ea} / PVFS_{ea}) \times PVFS_{aa}]$$

Attained Age (aa) = 37 Entry Age (ea) = 27

$$PVFB_{aa} = 1.5\% \times Salary_{final} \times Service \times \ddot{a}_{65}^{(12)} \times v^{65-aa} = \$150{,}000$$

$$PVFB_{ea} = 1.5\% \times Salary_{final} \times Service \times \ddot{a}_{65}^{(12)} \times v^{65-ea}$$

therefore $PVFB_{ea} = PVFB_{aa} \times v^{aa-ea}$

$$= 150{,}000 \times (1/1.06)^{37-27}$$

$$= \$83{,}759.22$$

$$PVFS_{aa} = Salary_{aa} \times \ddot{a}_{\overline{65-aa}|}\, j \text{ where } j = (1.06/1.03)-1$$

$$= Salary_{final} \times (1/1.03)^{64-37} \times \ddot{a}_{\overline{28}|}\, j$$

$$= \mathbf{Salary_{final} \times 8.7870}$$

$$PVFS_{ea} = Salary_{ea} \times \ddot{a}_{\overline{65-ea}|}\, j$$

$$= Salary_{final} \times (1/1.03)^{64-27} \times \ddot{a}_{\overline{38}|}\, j$$

$$= \mathbf{Salary_{final} \times 7.8605}$$

$$AL_{aa} = 150{,}000 - [(83{,}759.22 / (Salary_{final})) \times 7.8605)) \times (Salary_{final} \times 8.7870)]$$

$$AL_{aa} = 150{,}000 - [(83{,}759.22 / 7.8605) \times 8.7870]$$

$$= \$\ 56{,}368.26$$

(b)

$$NC_{1/1/2003} = (PVFB_{ea} / PVFS_{ea}) \times Salary_{1/1/2003}$$

$$= (83{,}759.22 / (Salary_{final} \times 7.8605)) \times (Salary_{final} \times (1/1.03)^{64-aa})$$

$$= (83{,}759.22 / \times 7.8605)) \times (\times .4502)$$

$$= \$\ 4{,}797.20$$