

Solution 1

(a)

- Fixed TAP/Subsidy
 - Payments are made in addition to earned Commissions
 - Payments do not vary by production level
 - Advantages
 - Income stable as production increases
 - Strong incentive to produce because commissions are paid
 - Disadvantages
 - High producers not rewarded proportionately
 - Production does not have to be smooth
 - Income can fall below the level of income needs

- Variable TAP
 - Payments are made in addition to earned Commissions
 - Payments vary by production level
 - Advantages
 - Production driven – high producers awarded proportionately
 - Agent experiences the effect of production on income
 - Less costly
 - Disadvantages
 - Income can fluctuate more than for established agents

- Advances
 - Loans made in anticipation of future commission earnings
 - Advantages
 - Low cost to the company
 - Disadvantages
 - Varying degree of security against advances (commissions, all agent assets)
 - Indebtedness must be repaid.

- Line-of-credit plan
 - A drawing account used to smooth out fluctuations in income
 - Advantages
 - Income is relatively stable as long as account credits and debits are constant
 - Large fluctuations in production may still yield a relatively stable income
 - Fairly flexible by incorporating advantages from other types of plans

Solution 1 (continued)

- Disadvantages
 - Agent could experience a decline in income after financing because commissions are withheld
 - Production does not have to be smooth
 - More costly due to development and administration
- Salary plan
 - Payments made in lieu of part or all earned commissions
 - Advantages
 - Level income regardless of production with additional payments made if production exceeds certain limits
 - Attractive to prospective agents, making it easier for management to recruit
 - Disadvantages
 - High producers not always rewarded proportionately – may lead to retention problems
 - Costly if agents do not produce at expected levels but continue on plan
 - Income may change considerably when agent goes off financing straight to commissions
 - Requires close supervision and strict adherence to validation schedules

(b)

- Fraction of Agents = $1 * (1 - \text{agent termination rate})$
- Fraction of agents = 1, .5, .375
- Commission Payment = Validation Schedule * Fraction of Agents
- Commission Payment = 20,000, 12,500, 11,250
- Subsidy Payments = Commission Payment * Subsidy %
- Subsidy Payments = 24,000, 10,000, 4,500
- Unvested Recoveries = 0
- Cost of Financing = Subsidy Payments - Unvested Recoveries
- Cost of Financing = 24,000, 10,000, 4,500
- Financing as percentage of First Year Premium = Cost / First year premium
- Financing as percentage of First Year Premium = 40%, 12.5%, 4.5%

Solution 1 (continued)

(c)

- Calculate cost of financing less price margin 6% of first year premium less cost of financing adjusted for termination
- Financing costs for 100 agents = $(100)(24000+10000+4500) = 3,850,000$
- Year 1 Pricing Support = $(6\%)(60,000)(100) = 360,000$
- Year 2 Pricing Support = $(6\%)(80,000)(100)(1-.5) = 240,000$
- Year 3 Pricing Support = $(6\%)(100,000)(100)(1-.5)(1-.25) = 225,000$
- Year 4+ Pricing Support = $(6\%)(120,000)(100)(1-.5)(1-.25)(1-.1) = 243,000$
- Breakeven = $3 + (3,850,000 - 360,000 - 240,000 - 225,000)/243,000$
- Breakeven = 15.44 years, or if rounded to nearest full year, 16 years

(to review the calculation of the correct answer it is helpful to build a spreadsheet, a systematic method similar to the development of a spreadsheet can be an effective approach to answering the question on the exam)

Solution 2

(a)

- not an immediate annuity as less than 5 annual payments (although payments are level and start within 13 months)
- therefore not the SPIA rates
- defined under AG IX-A and IX-B (IX-B specifically relates to the annuity interest rates)
- criteria for selecting:
 - no cash settlement options
 - issue year basis (as no cash settlement options)
 - no consideration to be received in future, so no future interest guar
 - guarantee duration is # of years from issue to first pmt =1
 - plan type A as there is no withdrawal permitted
 - (plan types are functions of rate adjustments, payout periods, ...)
- rates are functions of 12 & 36 month trailing avgs of Moodys Corp bond yield
- CARVM requires PV of future annuity benefits, so discount rate is needed

(b)

- EV = PV distrib earnings
- start with solvency earnings = prems - comms - benefits + inv inc - res
incr: -264, 345, 175, 59
- subtract change in RC (b)
- RC = (stat res) x (.05): 493, 251, 85
- change in RC: 493, -243, -166, -85
- add inv inc on RC (c)
- IIRC = .04 x RC: 20, 10, 3
- to give distrib earnings :
- t=0: -264 -493 + 0 = -757
- t=1: 345 +243 + 20 = 608
- t=2: 175 + 166 + 10 = 351
- t=3: 59 + 85 + 3 = 148
- discount by hurdle rate (15%)
- EV: -757 + 528 +265 + 97 = 134

Solution 2 (continued)

(c)

- calculate the change in solvency earnings for each marginal death
- statutory reserve released: $5,011 / 75 = 66.8$
- benefits not needed to be paid: 71.2
- each marginal death gives 138,000 more solvency earnings
- 2004 solv earnings = $10000 - 400 - 5346 + 838 - 5011 = 82$
- $250,000 - 82,000 = 168,000$, or more than 26 deaths (at least 2 more than 25 expected)
- use formula $f(y) = f(y-1) \times \text{fratio}(y)$, where $\text{fratio}(y) = (q) \times (n-y+1) / (1-q) \times (y)$
- $f(26) = .0918 \times .25 \times (100-26+1) / (1 - .25) \times 26 = .08827$
- $F(26) = F(25) + f(26)$
- $.5535 + .08827 = .6418$
- probability of more than 26 deaths is $1 - F(26) = .358$
- $f(y)$ long way: $(n!/(n-y)!y!) \times (q^y) \times (1-q)^{n-y}$
- binomial best used when contracts are very similar
- lives should be independent
- given only 100 lives, seriatim projections may be more efficient

(d)

- max assets = $RC(t=\text{jan 1, 2005}) + \text{Stat res}(t) - \text{embedded value}(t)$
- embedded value = $351/(1.15)^1 + 148/(1.15)^2 = 417$
- $RC(t) = 250.6$
- $\text{stat res}(t) = 5,011$
- $MA = 250.6 + 5,011 - 417 = 4.84$ million

Solution 3

(a)

- Current Target Markets
 - All relevant Aspects
- Competition
 - They are offering all types of designs from accumulated % to ratchets, step ups and resets
- Economy
 - Investors remember the stk mkt down drafts of 2000-2002
 - Stk mkts have shown significant recoveries in 2003
 - Interest rates are at exceptionally low levels
 - US and other economies are showing GDP growth
- Society
 - North American populations continue to increase in average age (demographics)
 - Many investors are reevaluating their risk tolerances
 - Net worths are recovering
- Technology
 - Advances in health care are increasing life expectancies
 - Greater computing capabilities allow for increasingly complex product designs
- Regulation
 - US statutory and RBC methodologies continue to evolve
- Labor
 - Falling unemployment
 - Rising disposable income
- Distributors
 - Great knowledge and acceptance of various GMDB designs
- International Conditions
 - N/A or global economies continue to rebound

(b)

(i)

- Return of prem - less risk because it is a subset of the step up
- Ratchet X - if less frequent than ann then it is a subset than step up and less risky otherwise if more frequent it will have more points to compare and be riskier
- Ratchet Xth - it has one evaluation point and is therefore a subset of step up and less risky

Solution 3 (continued)

- X% Roll up - Can't tell if riskier. It depends on the % and the actual returns
- X% Simple - Can't tell if riskier. It depends on the % and the actual returns
- Reset X - Less risky since the benefit can go down
- Reset Xth - Less risky since one data point and it can go down
- Combinations of the benefits listed above

(ii)

- Guaranteed Benefit Risk controls
- Stochastic modeling can help us understand the risk
- Cost increases by Issue Age
- Cost increases with volatility
- Roll-ups are generally most expensive form
- Frequency of Ratchet increases cost
- Limits on guarantee amount
- Cap on absolute amount
- Attained Age limit on adjustment
- Attained age limit on benefit
- Issue Age availability limit
- Proportionate vs. \$ for \$ w/d adjustment
- Cap based on factor times premiums less withdrawals
- Limitations on Investment options
- Require specific fund allocation
- Exclude certain funds
- Use options/puts or hedges to control the risk
- Use reinsurance to control the risk
- Maximum annuitization age
- Monitor expected policyholder behavior

(c)

Separate Account Reserve Calculation

- $AV(t) = AV(t-1) * (1 + \text{Assumed Return}(t) - \text{Net Asset Charges})$ or
Separate Account Reserve is the calculated CARVM reserve ignoring the presence of the GMDB
- $YR1 = 1038.8 = 980(1 + .07 - .01)$
- $YR1 = 1101.13 = 1038.8(1 + .07 - .01)$
- $CSV(t) = AV(t) - \text{Surrender Charge}(t) * \text{Premium Paid}$

Solution 3 (continued)

- CSV(0)=980, CSV(1)=1038.8, CSV(2)=1101.13 No Surrender Charge
- Average AV(t) = (AV(t-1) + AV(t))/2
- Average AV(1)=1009.4=(980+1038.8)/2
- Average AV(2)=1069.96=(1038.8+1101.13)/2
- Separate Account Reserve(t) = CSV(t) / {(1 + Valuation Rate(t))}
- SAR(0)=980
- SAR(1)=970.84=1038.8/(1.07)
- Rest<980 therefore max is 980

Integrated Reserve Calculation

- AV(t) = AV(t-1) * (1 + Assumed Drop) * (1 + Assumed Recovery(t) – Net Asset Charges)
- AV(0)=842.8=980*(1-.14)
- AV(1)=952.36=842.8*(1+.14-.01)
- AV(2)=1076.17=952.36*(1+.14-.01)
- Average AV(t) = (AV(t-1) + AV(t))/2
- Average AV(1)=897.58=(842.8+952.36)/2
- Average AV(2)=1014.27=(952.36+1076.17)/2
- DB @ EOY = (Max(GMDB=1000 and AV)
- DB(0)=1000, DB(1)=1000, DB(2)=1076.17
- Average DB(t) = (DB(t-1) + DB(t))/2
- Average DB(1)=1000
- Average DB(2)=1038.09=(1000+1076.17)/2
- Average NAR= (Max (Average DB(t) - Average AV(t)) and 0)
- Average NAR(1)=102.42=1000-897.58
- Average NAR(2)=23.82=1038.09-1014.27
- PV NAR(t)=NAR(t) * Survival Function(t-1) * Mortality Rate(t) / {(1 + Valuation Rate(t-.5))}
- PV NAR(1)=1.68=102.42*1*.017/(1+.07)^1-.5
- PV NAR(2)=.40=23.82*.983*.019/(1+.07)^2-.5
- Cumulative NAR(1)=1.68, Cumulative NAR(2)=2.09
- PV Base Average AV(t) = Average AV(t) * Survival Function(t-1) * Mortality Rate(t) / {(1 + Valuation Rate(t-.5))}
- PV Base Average AV(1)=16.59=1009.4*1*.017/(1+.07)^1-.5 or 1.68/102.42 x 1009.4
- PV Base Average AV(2)=18.06=1069.96*.983*.019/(1+.07)^2-.5 or .40/23.82 x 1069.96
- Cumulative PV Base Average AV(1)=16.59, Cumulative PV Base Average AV(2)=34.64
- PV Base CSV paid to Surv(t) = CSV(t)*Survival Function(t)/(1+Val Int)^t
- PV Base CSV paid to Surv(0)=980

Solution 3 (continued)

- PV Base CSV paid to Surv(1)=954.34=1038.8*.983/(1+.07)
- PV Base CSV paid to Surv(2)=927.14=1101.13*.964/(1.07)^2
- "Integrated Reserve(t) = PV Base CSV paid to Surv(t) + Cumulative PV Base Average AV(t) + Cumulative PV NAR(t) or
 - The calculation of the Integrated Reserve is based on the combination of three benefit streams:
 - Projected net amounts at risk paid to those projected to die during the calculation period based on the valuation mortality table.
 - Projected unreduced account values paid to those projected to die during the calculation period based on the valuation mortality table.
 - Base benefit streams projected during the calculation period and discounted for survivorship based on the valuation mortality table. The base benefit streams do not reflect the assumed drop and recovery."
- Integrated Reserve(0)= 980
- Integrated Reserve(1)=954.34+16.59+1.68 = 972.61
- Integrated Reserve(2)=927.14+34.64+2.09 = 963.87
- Integrated Reserve = Max(Integrated Reserve(t))=980
- General Account GMDB Reserve=Separate Account Reserve-Integrated Reserve
- General Account GMDB Reserve=0=980-980

Solution 4

(a)

- YRI advantages
 - simplest to administer as reinsurer doesn't need records for premiums, allowances, reserves
 - usually obtainable at lower effective cost due to reinsurer's lower profit objectives
- YRI disadvantages
 - only mortality risk is reinsured
 - does not provide relief for deficiency reserves
 - provides little surplus relief as reserves quite small
- Coinsurance advantages
 - reinsurer shares proportionally in all risks: mortality, lapse, surrenders, investment
 - reinsurer shares proportionally in surplus strain of new issues
 - reinsurer shares proportionally in deficiency reserves
- Coinsurance disadvantages
 - administration is relatively complex, as it involves determination of allowances, reserves, surrenders in addition to premiums and death benefits

(b)

- COMMON DATA
 - ceded amount = FA * portion reinsured
 - $100,000,000 * .9 = 90,000,000$
 - premium tax = % x estimated premium = $.02 \times 1000\ 000 = 20\ 000$
 - $.02 \times 1000\ 000 = 20\ 000$
 - commission = % x estimated sales
 - $.5 \times 1000\ 000 = 500\ 000$
- YRI REINSURANCE
 - ceded premium = ceded amount x yrt rate per 1000 / 1000
 - $= 90\ 000\ 000 / 1000 \times 0.2 = 18\ 000$
 - premium tax reimbursement = ceded premium * % tax
 - $= 18\ 000 * .02 = 360$
 - reserve credit = $1/2 * \text{one year term insurance benefit (YRI premium)}$
 - $= 18\ 000 * 1/2 = 9\ 000$
 - FY STRAIN = premium - ceded premium + reins allow - (acquisition expense + commission + premium tax - premium tax reimbursement + reserve increase - reserve credit)

Solution 4 (continued)

- $= 1000\ 000 - 18\ 000 + 0 - (750\ 000 + 500\ 000 + 20\ 000 - 360 + 50\ 000 - 9\ 000) = (328\ 640)$

- COINSURANCE
 - ceded premium = premium * ceded percentage
 - $= 1000\ 000 * .9 = 900\ 000$
 - premium tax reimbursement = ceded premium * % tax
 - $= 900\ 000 * .02 = 18\ 000$
 - reserve credit = gross reserve * ceded percentage
 - $= 50\ 000 * .9 = 45\ 000$
 - reinsurance allowance = ceded premium * allowance percentage
 - $= 900\ 000 * .9 = 810\ 000$
 - FY STRAIN = premium - ceded premium + reinsurance allowance - (acquisition exp + commission + premium tax - premium tax reimbursement + reserve increase - reserve credit)
 - $= 1000\ 000 - 900\ 000 + 810\ 000 - (750\ 000 + 500\ 000 + 20\ 000 - 18\ 000 + 50\ 000 - 45\ 000) = (347\ 000)$

- YRI reinsurance results in lower strain

Solution 5

- Types of Reports: New business, acquisition costs and commissions, policyowner option utilization, persistency, mortality, maintenance expense, reinsurance, inforce statistics, profitability reporting, investments, value-based reporting
- New business:
 - Includes new sales premium and counts, by channel and plan, comparison to forecasts
 - Uses include feedback on competitive position, monitor channel mix.
 - Give feedback on Saturn's direct channel,
 - identify financial / cash flow / capital use issues
- Acquisition costs and commissions:
 - Express as per face, per policy, per premium, and by channel and plan
 - Uses include validation of pricing assumptions, DAC recoverability (US), different cost structure of various distribution channels
- Option utilization:
 - Gives feedback on success of design
- Persistency:
 - Report by plan, age, duration, and channel
 - Feedback to product development
 - Implications for DAC recoverability or PPM valuation
 - Impact on earnings forecasts
 - Important for conservation efforts
 - Saturn's experience is deteriorating
- Mortality:
 - Report by plan, age, duration, underwriting class
 - Needed for reinsurance management
 - Needed to evaluate preferred underwriting
 - Saturn's underwriting is aggressive and needs monitoring
 - Feedback to valuation assumption setting
- Maintenance costs:
 - By source, cost center, activity
 - Used for feedback to managers responsible for activities

Solution 5 (continued)

- Reinsurance:
 - Cost of reinsurance – tracking ceded premiums and benefits and allowances

- Inforce Statistics:
 - By plan and channel
 - Helps evaluate areas for future market concentration

- Profitability reporting:
 - At plan level
 - Uses ROI, ROE, other performance measures
 - Shows where attention should be focused

- Investments:
 - Not too important for term

- Value-base reporting:
 - Measures change in economic value
 - Overcomes shortcomings of GAAP/Stat reporting

Solution 6

(a)

- Exact Age Approach
 - rates determined from first principle
 - based on exact age and risk classification of each pair of lives
 - often developed by formula
 - by formula is economical for co
 - if co calculates all possible combo of rates and stores them can be disk storage problems
 - if done as needed by formula co cannot preview all rates for consistency checks
- Joint Equal Age Approach
 - each combo of lives assigned rates based on equivalent combo of 2 lives at equal ages
 - simple to administer
 - jurisdiction may require certification that values exceed those rates from actual age combo
- Equivalent Single Age Approach
 - equate rates for joint pair to one single life
 - can result in mortality overcharge early years
 - Anticipated level of single life mortality for target market
- Underlying mortality of individual lives in survivorship market may be a lot different than single lives
 - degree of underwriting concessions provided
 - contagion - provision for joint accident risk
 - contagion - broken heart syndrome
 - significant mortality increase upon 1st death due to heartbreak
 - impact of financial hardship due to death of 1 individual
 - impact of medically underwritten
 - socio-economic class of lives insured
- impact of low lapses on long-term mortality
- implication of married - if widowed mortality is higher than aggregate mortality then married mortality should be lower than aggregate
- may not have credible basis for female mortality at advanced ages
- impairments that result in substandard rating more common with survivorship
- increase in cost from a rating is much smaller
- may issue even if one life is uninsurable

Solution 6 (continued)

(b)

- initial persistency results for the last survivor market are good
- lapse supported pricing can be very dangerous
- difficult to provide projected benefits if too many lives persist
- experience lapse rates on survivor business less than 3%
- companies have reported better persistency for survivors than for a corresponding block of single life policies
- propose to change lapse assumption to 3%
- modify base lapse rate to reflect difference between market (competitor) rate and rate credited to policy
 - subject lapse rate formula to some overall maximum
- modify base lapse rate to reflect any surrender charges
- since flexible premium may want uniform distribution of lapses by month vs. annual
- sensitivity by scenario may vary by distribution channel

(c)

- i. GMP calculated from the issue date (valuation date is incorrect)
- i. GMP uses policy guaranteed assumptions (not valuation)
 - ii. GMFs are calculated from the issue date (not valuation)
 - ii. GMFs are calculated using policy guaranteed assumptions (not valuation)
 - ii. GMFs assume the GMP is paid (not the gross premium)
- iii. PVFB should project forward the larger of the GMF and the actual fund
- iii. PVFB should be projected using guaranteed assumptions
- iii. PVFB should project forward assuming the GMP is paid
- iv. Net level premiums should be calculated using guaranteed assumptions
- iv. Net level Premiums should be calculated assuming the GMP is paid
- v. Subtract off p.v. of net level premiums at valuation rate ($PNL \times \ddot{a}_{x+t}$)
- v. whole quantity multiplied by ratio(r) of actual fund at t / GMF at t
- v. r may not exceed 1
- vi. Use guaranteed assumptions to calculate EA
- vi. Assume GMPs are paid to calculate EA

Solution 6 (continued)

- vii Multiply EACRVM by same ratio(t) actual fund t / GMF t
- vii EA should be unamortized EA ($EA \times \ddot{a}_x / \ddot{a}_{x+t}$)
- vii Where \ddot{a}_x and \ddot{a}_{x+t} are calculated using valuation assumptions

Alternatively

- Step 1 – replace “valuation date” with “issue date”
- replace “valuation assumptions” with “policy guaranteed assumptions”

- Step 2 – should be from “issue date” and using “policy guaranteed assumptions”
- should be “GMP” not “gross premiums”

- Step 3 – should project MAX(GMF, actual fund) with guaranteed assumptions
- assumes “GMP” are paid

- Step 4 – “GMP” not “gross premiums”

- Step 5 – calculate $r = \text{MIN}(\text{actual fund}/\text{GMF}, 1)$
- formula should be ${}_tV^{\text{NL}} = {}_t(\text{PVFB}_t - P^{\text{NL}} \ddot{a}_{x+t})$

- Step 7 - ${}_tV^{\text{CVRM}} = {}_tV^{\text{NL}} - r \cdot EA \cdot \ddot{a}_{x+t} / \ddot{a}_x$

Solution 7

(a)

- Investment income assumptions need to be based on current rates available. Spot rates are normally used.
- The annuity payments are not contingent on the life of the annuitant
- Agent commissions may not be required
- Average size needs to be considered
- Use of a policy fee in pricing can help reflect economies of scale
- Depending on state law, premium tax may or may not apply
- Need to consider statutory surplus requirements (surplus strain)
 - i. Reserve at issue will typically exceed premium charged, leading to surplus strain.

(b)

(i) Surrender Provision

- Advantages:
 - Winners may be more willing to choose an annuity over single payment if can change minds later on.
 - Feature may make proposal more competitive relative to lump sum option
- Disadvantages:
 - C-3 risk will increase cost of annuity
 - Increased costs make company's proposal less competitive
 - Surrender provision may lead to disintermediation
 - In high interest rate environment, may surrender in favor of higher yield alternative
 - Investments backing annuity have declined in value
 - May be necessary to liquidate investments at capital loss
 - Early surrenders may make it difficult to cover first year costs.

(ii) Medical Bailout Provision

- Advantages:
 - Winners more likely to choose annuity if know can get single payment if needed medical reasons
 - Cost of well designed benefit is low
 - Under 10 basis points

Solution 7 (continued)

- Disadvantages:
 - Will tend to increase costs
 - Especially if trigger is subjective or easily abused
 - Can lead to costly investigations
 - High costs may make company's proposal less competitive
 - True cost of provision may include surplus strain caused by statutory reserve requirements.

(iii) Variable Payout

- Advantages:
 - Attractive to some annuitants who may be willing to take greater risk for a potential greater return
 - Shifts investment risk to annuitant, reduces cost to company, makes annuity more competitive.
- Disadvantages:
 - Will be increase in costs due to:
 - Need for management attention (SEC/brokers training)
 - Increases complexity to consumer, may need additional agent explanation
 - Increased administrative costs and/or systems costs
 - Increased risk may be unattractive to some annuitants

Solution 8

(a)

Application info is divided into 3 categories

PERSONAL HISTORY

- one of most frequently used criteria
- most commonly used criteria are diabetes and heart disease, then
- high cholesterol, non-skin cancers, stroke, hypertension, and melanoma
- each of these may preclude an applicant from preferred
- prescription drugs is the least used criterion
- more useful than family history in distinguishing preferred risks from other risks
- for applicants below age 50, family history more commonly used for evaluating heart disease

FAMILY HISTORY

- less reliance on family history of a parent or sibling due to difficulties in eliciting, verifying or clarifying family history
- only unfavorable family history of heart disease may preclude applicant from preferred class by most responders
- most use family history of occurrence of death rather than the diagnosis prior to death as the tool
- Can use natural parents or both natural parents and siblings.
- Number of incidences of death or diagnosis allowed
- Age limit for incidence of death or diagnosis
- Offset family history with good applicant health or negative stress test in the past year.
- Using gender specific cancers only

LIFE STYLE

- some life style criteria to eliminate an applicant from preferred are alcohol abuse, use of illegal drugs,
- adverse driving record, DUI, participation in private aviation, and in hazardous sports or avocations
- may be evaluated from motor vehicle report or lab testing of body fluids
- may reconsider applicants some time after they discontinue hazardous life style
- may allow preferred but include extra premium for hazardous life style
- drug and alcohol abuse will usually preclude issue of preferred
- except for those 2, none of others will preclude preferred issue
- Can use regular exercise as a criteria

Solution 8 (continued)

- most common driving record criterion was no more than 2 moving violations in a 3 year period
- most will not allow more than 1 DUI violation
- most allow no DUI convictions within past 5 years
- difficult to verify since motor vehicle reports do not go back more than 3 years
- more than half allow no cigarette use in past 12 months
- may be that cigarette use is a factor in base smoker/nonsmoker distinction rather than in preferred class
- few do vary their preferred criteria by smoking status or gender (e.g., build criteria, blood pressure, cholesterol)
- may preclude an applicant based on Tot-C/HDL-C ratio (cholesterol)
- GGT(test on liver enzyme) can be used
- SGOT can be used
- SGPT can be used
- PSA can be used
- most allow for treated hypertension
- Height and weight
- usually there is a maximum number of debits allowed for any risk to be considered further as preferred

(b)

- Preferred qx = (1-Discount) * Aggregate qx
 - = (1-.15) * .006
 - = .0051 or 5.1 per 1000
- Standard qx = [Aggregate qx - (Preferred qx * % Qualifying)]/[1- % Qualifying]
 - = [.006 - .0051 * .30]/[1- .30]
 - = .0064 or 6.4 per 1000

Solution 9

- (a)
- Assumption for Disability Waiver Experience Premiums:
 - premiums waived
 - should be net cost to insurer of premiums actually waived
 - adjust for commissions and premium taxes not paid
 - waiver cost will cover average premium of converted policies
 - may be appropriate for cost to cover the waiver of the higher premium
 - if insured can choose from several permanent policies, may be appropriate to assume average premium
 - weighted average of all underwritten premium classes
 - appropriate to use weighted average of all premium classes or distinct premiums
 - if distinct, use distinct morbidity and mortality assumptions
 - interest
 - should use conservative long-term rate
 - can have significant impact on calculation
 - expenses
 - include cost of claim investigation
 - accounting and reserving for coverage
 - appropriate to cover premium taxes and commissions paid on the waiver premium
 - active life mortality
 - use to determine survivorship of non-disabled lives for
 - PV of benefits at issue
 - annuities used in net annual premium calculation
 - should use same mortality assumption as for basic insurance contract
 - select and ultimate
 - male/female
 - smoker/non-smoker
 - lapse rates
 - no nonforfeiture value for coverage
 - conservative not to reflect lapses
 - if used, use same rates as for basic coverage
 - morbidity
 - rates of disablement
 - chance of becoming disabled at particular age and remaining alive and disabled until end of waiting period

Solution 9 (continued)

- should be sex distinct and
 - smoker distinct
- based on company data or
 - inter-company data,
- adjusted to company's experience
- rates of termination of disability
 - probability of leaving group of disabled lives due to
 - recovery or death
 - should be select by duration since disablement
 - should use monthly rates for first 2 years
 - appropriate for type of coverage
 - waiting period
 - economic conditions expected
 - should be sex distinct
 - smoker/non-smoker distinct

(b)

$$B = P \left\{ \frac{D_{\overline{55}|1/2} + 1/2}{D_{\overline{55}|1/2} + 1/2} + \frac{D_{\overline{55}|1/2} + 1 1/2}{D_{\overline{55}|1/2} + 1/2} + \frac{D_{\overline{55}|1/2} + 2 1/2}{D_{\overline{55}|1/2} + 1/2} + \frac{D_{\overline{55}|1/2} + 3 1/2}{D_{\overline{55}|1/2} + 1/2} \right\} + .5 * P$$

$$= 100 \left\{ \frac{97.80}{100.16} + \frac{93.19}{100.16} + \frac{88.72}{100.16} + \frac{84.38}{100.16} \right\} + .5 * 100$$

413.5083866

OR

$$B = P \left\{ \frac{D_{\overline{55}|1/2} + 1/2 + D_{\overline{55}|1/2} + 1 1/2 + D_{\overline{55}|1/2} + 2 1/2 + D_{\overline{55}|1/2} + 3 1/2}{D_{\overline{55}|1/2} + 1/2} \right\} + 5P$$

$$B = 100 \left\{ \frac{97.8 + 93.19 + 88.70 + 84.38}{100.16} \right\} + .5 * 100$$

$$B = 100 * 364.09 / 100.16 + 50 = 413.5083866$$

Solution 10

(a)

- Calculate IAV at the end of the Index Period (time 3)
- The best format is a table with the following columns: time (t), GMAV, index / SP500, index growth %, participation rate, apply margin, apply cap, indexed-based interest %, IAV

At time 0:

- $IAV = 100\%$

At time 1:

- Index growth % = $1050 / 1000 - 1 = 5\%$
- 90% partic'n rate = $90\% * 5\% = 4.5\%$
- apply margin = $4.5\% - .5\% = 4\%$
- apply cap = $\min(15\%, 4\%) = 4\%$
- index-based interest = $\max(0, 4\%) = 4\%$ (due to annual ratchet)
- $IAV = 100\% * (1.04) = 104\%$

At time 2:

- Index growth % = $1250 / 1050 - 1 = 19.05\%$
- 90% partic'n rate = $90\% * 19.05\% = 17.14\%$
- apply margin = $17.14\% - .5\% = 16.64\%$
- apply cap = $\min(15\%, 16.64\%) = 15\%$
- index-based interest = $\max(0, 15\%) = 15\%$ (due to annual ratchet)
- $IAV = 104\% * (1.15) = 119.6\%$

At time 3:

- $GMAV = 90\% * (1.03)^3 = 98.35\%$
- Index growth % = $1175 / 1250 - 1 = -6\%$
- 90% partic'n rate = $90\% * -6\% = -5.4\%$
- apply margin = $-5.4\% - .5\% = -5.9\%$
- apply cap = $\min(15\%, -5.9\%) = -5.9\%$
- index-based interest = $\max(0, -5.9\%) = 0\%$ (due to annual ratchet)
- $IAV = 119.6\% * (1.00) = 119.6\%$

At the end of the index period, we take the greater of the GMAV and IAV based index-based interest = 119.6%

Solution 10 (continued)

(b)

- i) The price elasticity of demand is affected by the following factors:
- Number of substitute products available.
 - demand for products with close substitutes is usually more elastic and vice versa
 - Need associated with the product.
 - demand for products that are necessities (e.g. home & auto insurance) is typically inelastic
 - life & health insurance is typically elastic since people are not required to carry them
 - as a result, many people have no insurance or are insured for less than they would like
 - Level of expenditure required to purchase the product
 - demand for products that constitute a large % of person's budget (e.g. housing, cars) is normally elastic
 - demand for inexpensive items (e.g. milk) tends to be inelastic
 - demand for relatively inexpensive forms of insurance (e.g. accidental death) tends to be inelastic
- ii) Evaluate proposed "customer appreciation campaigns" assuming company uses profit-oriented pricing objectives
- For EIAs, credited rate is made up of 2 components - GMAV and the index-based interest component
 - Easy way to visualize is to fund the GMAV, expenses and profit and use the remainder to determine an option budget to fund the index-based interest
 - we are given the PV of the index-based interest costs, so must solve for PV of profit
 - Also need to determine PV of expenses (fixed & variable)
 - a variable cost varies directly with the amount of volume sold
 - a fixed cost remains constant regardless of the volume sold (at least over some determined period or range of volume)
 - Without "campaigns":
 - cost of GMAV = $90\% * (1.03)^3 / (1.055)^3 = 83.75\%$
 - PV of expenses = $4\% + 1,000,000 * [1 + 1/(1.055) + 1/(1.055)^2] / 100,000,000 = 6.85\%$
 - PV of index-based interest costs = 6% (given)
 - margin for profit = $100\% - 83.75\% - 6.85\% - 6\% = 3.40\%$
 - the present value of dollar profits = $3.4\% * 100,000,000 = 3.4$ million

Solution 10 (continued)

- With "campaigns":
 - we are given that a 1% increase in the PV of index-based interest costs generates 1.5% increase in additional sales
 - therefore, expected sales = $100,000,000 * (1 + (1.5 / 1) * (9\% / 6\% - 1)) = 175,000,000$
 - cost of GMAV is same = 83.75%
 - PV of expenses = $4\% + 1,000,000 * [1 + 1/(1.055) + 1/(1.055)^2] / 175,000,000 = 5.6265\%$
 - PV of index-based interest costs = 9% (given)
 - margin for profit = $100\% - 83.75\% - 5.63\% - 9\% = 1.62\%$
 - the present value of dollar profits = $1.62\% * 175,000,000 = 2.835$ million

Therefore, the change in PV dollar profits = $2.835 - 3.4 = -.575$ million.

May not proceed due to negative result. There could be marketing reasons that will still result in proceeding even with negative result

(c)

- Considerations in selecting lapse rate assumption for cash flow testing
- Actuary must select a best estimate lapse assumption and make an assumption for how lapses will depart under changing economic conditions
 - little experience available for life and annuity products
 - usually base on common sense principle that lapses will increase as ph's other options become more attractive

Key items to consider:

- presence and level of SCs
 - grading off pattern might be recognized by ph and cause him to hold onto policy longer than otherwise
 - no SC's in this product; expect higher excess lapses
- marketing techniques and loyalty of field force
 - the proposed campaigns may reduce excess lapses, provided they are well communicated and applied consistently
 - expect higher excess lapses as this product will be marketed through independent stockbrokers

Solution 10 (continued)

- prominence of the interest rate in marketing and maintenance of the policy
 - the index-based return is front and center in the marketing of an EIA
- duration from issue
 - some believe that there is a core group of policyholders that are not sensitive to interest rates
- type of product
 - pure investment (e.g. single-premium EIA) more sensitive to excess lapses than protection-oriented products
- Usually, the actuary makes an estimate of excess lapses at different differentials between the credited rate and the market rate and then adjusts for impact of surrender charges
 - actuary will need to understand better the details behind the campaigns to properly estimate future credited rate
 - should assess the impact on lapses from poor management of campaigns (e.g. poor communication)

Solution 11

MORTALITY

- Direct Response business typically underwritten on non-medical or guaranteed issue basis
 - Results in higher mortality
- Brokers business typically fully underwritten
 - Results in lower mortality
- Direct Response reaches middle to lower income markets (lower face amounts)
 - Results in higher mortality
- Brokers reach affluent markets (higher face amounts)
 - Results in lower mortality
- Brokers tend to shop for rates
 - Results in anti-selection and higher mortality

LAPSE

- Direct Response has no face-to-face contact
- Results in higher lapses
 - Brokers have face-to-face contact
- Results in lower lapses
 - Brokers work with multiple companies and tend to shop around
- Results in replacement sales
- Results in higher lapses
 - Direct Response often targets senior citizens (higher issue age)
- Results in lower lapses
 - Direct Response reaches middle to lower income markets (lower face amounts)
- Results in higher lapses
 - Brokers reach affluent markets (higher face amounts)
- Results in lower lapses

INTEREST

- Same investment strategy for both distribution channels; no difference in interest assumptions

Solution 11 (continued)

EXPENSE

- Direct Response is primarily direct/fixed costs
 - Salaried company employees
 - No commissions paid
 - Expense of media used to reach consumers
- Brokers have few fixed costs; costs vary with production
 - Paid first-year and renewal commissions
 - May be paid bonuses based on sales/profitability

(b)

First Year Commissions

- Compensation paid during first policy year
- Expressed as a percentage of 1st year premium. Is most common
- Amount per new policy
- Amount per thousand dollars of death benefit or “sum insured”
- Percentage of policy’s cash value
- Usually substantially higher than commission paid in renewal years
- Often annualized and fully vested
- Commissions on unpaid premium are charged to the agent if policy lapses in first year

Renewal Commissions

- Compensation paid during renewal policy years
- Expressed as: a percent of premium
- Pattern of Compensation:
 - Level
 - Variable by policy year
 - Graded by policy size
 - Graded by volume of new business
- Conditions for payment
- Fully vested
 - Payable regardless of whether or not the agent still represents the company when the commission is due
- Partially vested
- Non-vested
 - Payable only if the agent still represents the company when commissions are due
- Generally fully vested for Non-agency building brokers

Solution 11 (continued)

Bonuses

- Focus on product profitability / sharing profitability with the agent
- Expressed as:
 - Percentage of first-year premium
 - Percentage of first-year commissions
 - Percentage of renewal premium
 - Percentage of renewal commissions
- Based on various measures of performance
 - First-year premiums or commissions
 - Net first-year commissions
 - Total (first-year plus renewal) premiums or commissions
 - Persistency – short and/or long-term
 - Number of new clients or policies sold
 - A combination of the above

Expense Allowances

- To offset costs of the agents office and other business expenses
- Based on:
 - New Sales
 - Renewals/Persistency

(c)

Buyer-Related

- Higher persistency is found among buyers who are:
 - Older
 - Earn a higher income
 - Professionals and executives
 - Already owners of life insurance in the same company
 - Insuring the lives of juveniles or students
 - Initiating the sale themselves

Product-Related

- Higher persistency is found among policies that are:
 - Permanent rather than term
 - Higher in premium and/or account value (for cash value plans)
 - Without policy loans
 - Underwritten on a medical basis
 - Sold with an insurability option rider
 - Larger policies issued on a preferred risk basis
 - Business policies rather than personal policies
 - Issued as applied for

Solution 11 (continued)

- Sold with a waiver of premium provision in the home service marketplace
- Annual premium mode
- Monthly Bank Draft Premium

Producer-Related

- Persistency increases as the agent's length of service increases
- New agents who are subsequent terminators sell business with poor persistency
- Persistency is positively related to the level of the agent's product knowledge
- Orphan business has poor persistency
- Agent contact with policyholder increases persistency
- Persistency bonuses have their intended effect

Sales Process-Related

- Higher lapsation occurs when:
 - Only a partial premium or no cash is paid with the application
 - Needs selling is not employed
 - The agent stress savings and thrift
 - Policies are not delivered personally to the insured
 - Post-sale service is not employed

Related to Outside Environment

- Persistency tends to be poorer during periods of unemployment and high interest rates
- Persistency tends to improve when personal savings and effective buying incomes are high
- Competition increases replacements leading to high lapses
- Regional differences: persistency is better in mid-Atlantic and northern part of the United States
- Build persistency bonus into agent compensation
- Use level commission scale
- Offer bonuses to agents based on length of service with Saturn Life
- Educate consumers on the value of their policy
- Educate agents so they are knowledgeable about product
- Set mode factors to discourage non-annual premiums
- Do not offer monthly direct bill / only offer monthly bank draft
- Manage replacements
- Provide quality customer service

Solution 11 (continued)

- Recruit high quality brokers
- Encourage higher face amount policies by setting minimum issue limit
- Encourage higher face amount policies by using banded premium rate

Solution 12

Policyholder Benefits (alternative answer..... give 3 points for Assumptions in Reserves)

- Both FAS60 & FAS120 use a net level premium reserve for benefits
- FAS60 uses assumptions based on best-estimate experience at policy issue
- FAS120 uses only guaranteed mortality and dividend fund assumptions
- FAS120 mortality and dividend fund assumptions are usually more conservative than assumptions used in FAS60
- Bonus point if candidate mentions PADs required in FAS60 reserving

Annual Policyholder Dividends

- FAS120, dividends recognized as amounts earned by policyholder
- FAS60, dividends can be part of benefits or a deferred dividend liability can be set up

Acquisition Costs

- definitions of deferred acquisition expenses consistent between FAS60 & FAS120
- FAS120 - amortized in relation to estimated gross margins
- FAS60 - amortized in relation to gross premiums recognized
- FAS120 - DAC trued-up to reflect prior and future expected experience
- FAS60 - DAC assumptions locked-in unless a loss recognition situation occurs

Terminal Dividends

- FAS120 - TD recognized as level amount related to gross margins
- FAS120 - recognition of TD based on prior and expected future experience
- FAS60 - recognition of TD in part of benefits or can set up a terminal dividend liability can be set up

Solution 12 (continued)

(b)

Formula for Expected Gross Margins

- $EGM(t) = (GP(t) - P(t)) + (I(t) - IR(t)) + (BR(t) - B(t)) - E(t) - Div(t)$
- $Alt...EGMs = Loading + Interest Margin + Benefits Margins - Expenses - Dividends$
- Where:
 - GP is expected gross premium
 - P is change in NLP reserve due to premiums
 - I is investment income on NLP reserve (may accept II as well)
 - IR is increase in NLP reserve due to interest
 - BR is decrease in NLP reserve due to benefits
 - B is expected benefits
 - E is expected non-deferrable expenses
 - Div is expected annual p/h dividends

Calculate EGMs

	Loading	Interest Margin	Benefit Margin	Expenses	Div	EGM
1	400	9	113	-350	-70	102
2	360	13	119	-225	-98	169
3	324	11	131	-203	-124	139
4	308	9	138	-193	-149	113
5	292	8	144	-183	-175	86

Calculate PV EGMs

Discount Factor	EGMs	PV EGMs
0.943	102	96
0.890	169	151
0.840	139	117
0.792	113	89
0.747	86	64
Sum		517

$$PV\ EGMs(t) = \text{Discount Factor} * EGMs$$

Calculate K Factor

- $K = PVDE / PVEGMs$
- $K = 550 / 517 = 106\%$

Solution 12 (continued)

Interpretation of Results and Recommendation

- $K > 100\%$ indicates that not all deferrable expenses are recoverable
- Need to classify deferrable expenses as nondeferrable until $K = 100\%$
- Recommended amount of deferrable expenses to classify as nondeferrable
 $= 550 - 517 = 33$
- 33 charged to income in year 1

Solution 13

(a)

Conversion

- less healthy lives will convert
- remaining lives for ART renewal will be healthier, leading to lower mortality

Termination

- healthier lives will terminate coverage
- remaining lives will be less healthy, leading to higher mortality

Re-underwriting

- healthier lives will be re-underwritten for new coverage
- remaining lives will be less healthy, leading to higher mortality

(b)

- $q_{AS}(x,t) = [q(x,t) - A * q(x+r,t-r)] / (1-A)$

- where :

$q(x,t)$ = duration t mortality for issue age x , in absence of selective lapses

A = percentage of policies that lapse at duration r to buy newly underwritten policy

$QAS(x,t)$ = mortality rate at duration t reflecting effect of anti-selection

$$A = 0.75 * 0.20 = 0.15$$

$$q(40,5) = 0.90 * 2.14 = 1.926$$

$$q(45,0) = 0.80 * 1.926 = 1.541$$

$$\begin{aligned} q_{AS}(40,5) &= [q(40,5) - A * q(45,0)] / (1-A) \\ &= [1.926 - 0.15 * 1.541] / 0.85 \\ &= 1.994 \end{aligned}$$

(c)

- Premium deficiency = PV future benefits & expenses – PV future gross premiums – (Benefit Reserve – DAC)

$$= 15,000,000 - 50,000,000 - (7,388,657 - 52,559,076)$$

$$= 10,170,419$$

Solution 13 (continued)

- If there is a deficiency, first reduce DAC
- If DAC reduced to zero and there is still a deficiency, benefit reserve is increased
- Reduce DAC to 42,388,657
- No change to benefit reserves required

Solution 14

(a)

i) Market Size and Growth Potential

- Because market size and growth potential are linked to profitability, some companies target those segments that have high current sales, high growth rates and high profit margins.
- This can lead to the “majority fallacy” – where you assume the largest is the best
- The largest segment may be unprofitable compared to a smaller segment due to intense competition
- The company should choose markets that meet the company’s needs

ii) Market Attractiveness

- Depends on the level of competition and the customer buying power
- Fierce competition may limit the company’s ability to price its product effectively
- The company should only enter markets where they can gain a competitive advantage or distinguish themselves from the competition

iii) Compatibility with Company Goals and Resources

- Need to choose a target market that is a good match with the company’s long term goals and can be reached with the company’s resources (human, financial, technological, etc)

(b)

Mercury Life’s primary markets are the Affluent and Very Wealthy and the Pre-Retired and Retired

i) Market Size and Growth Potential

- The market size and the growth potential is high for all of these markets:
 - i. The baby boomers are a large segment of the population and they are starting to reach retirement age; they also are fairly affluent.
 - ii. The affluent market is growing at 5 times the general population rate
 - iii. They tend to have liquid assets
 - iv. There are a large number of people aged 65 and over, and as the boomers hit retirement, this number will continue to increase

Solution 14 (continued)

- ii) Market Attractiveness
 - Affluent people are very sophisticated and make use of financial planners and brokers. Mercury should expand to alternative distribution systems; including financial planners. They are moving into the bank distribution
 - Affluent require a full array of traditional and non-traditional products. Mercury needs to start offering variable products in addition to their traditional products.
 - The company has starting selling mutual funds and this is a non-traditional product, so that is good. They also sell LTCI and other investment products
 - Competition will be aggressive which will make it difficult to price their products effectively.
 - The affluent may have bargaining power over the sellers given the size of their liquid assets.
 - Baby boomers need financial security for their family and they want to build and protect their retirement savings.
 - The affluent boomers may have more bargaining power over the sellers.
 - There will be intense competition which will make it difficult to price products
 - Mercury Life does not have good name recognition in the younger market
 - Seniors tend to require additional life and health insurance (eg long term care); they may purchase insurance for their grandchildren.
 - The affluent seniors like second-to-die policies for estate planning purposes

- iii) Company goals and Resources
 - For Mercury's markets, they have a goal of increasing the number of products owned from 1.5 to 2.0 per customer
 - Mercury needs to offer more variable products for the affluent market or they will not be successful in the affluent market
 - However, their sale of LTCI and investments products will help Mercury be successful in this market
 - This is also true for the pre-retired and retired market; similarly the sale of mutual funds and other investment products
 - Pre-retired market wants life insurance; annuities; LTCI and retirement plans
 - For the Retired market, Mercury needs to offer estate planning products and it also needs to beef up it's portfolio of health products

Solution 15

(a)

MARKET ANALYSIS

- study of all environmental factors that might affect the sales of a product
- factors include potential value of product to customers, nature and size of target market,
- factors include:
 - potential value to customers
 - nature and size of target market
 - potential value to company
 - nature of competition
 - customer appeal
 - appeal to distributors
 - relationship to co's other products
 - legal or regulatory problems
 - economic considerations
 - company fit
 - tax considerations

PRODUCT DESIGN OBJECTIVES

- specify characteristics, features, benefits, issue and age limits, underwriting classes, and manner benefits provided
- death benefit, renewability, convertibility, cash value, minimum face amount,
- premiums: range, guaranteed or not, min/max, modes and methods, age bands, policy fees or volume discounts
- marketing costs: commissions, overhead, advertising expenses, field expenses, chargeback schedules
- risks: mortality, investment, legal and regulatory, financial, inflation, interest rate, persistency
- underwriting standards: nonmedical limits, guaranteed or simplified issue limits, smoker/nonsmoker, min/max range
- riders, other benefit features, interest, persistency, and expense assumptions, reinsurance, premium taxes

FEASIBILITY STUDY

- research designed to determine the operational and technical viability of producing and offering the product
 - is the product compatible with co goals
 - is there a real need for product?

Solution 15 (continued)

- can existing product be modified?
- will product generate new sales or displace existing product sales?
- market potential large enough?
- can product be marketed thru existing distribution systems?'
- will product support adequate commission scale to appeal to distributors
- personnel and systems
- desirable and easy to understand for target market
- offer through affiliate or subsidiary?

A MARKETING PLAN

- specific, detailed, action - oriented activities involving the pricing, promotion, and distribution of product
- a plan for each product
- info from each plan is incorporated into co overall marketing plan

PRELIMINARY SALES AND FINANCIAL FORECASTS

- estimate potential unit sales, revenues, costs, and profits
- these are modified as additional info becomes available

(b)

- Used to stabilize earnings
- Transfer / offset risk
- Virtually all kinds of risks
- Most commonly is to stabilize mortality risk
- Reinsurer sometimes takes more mortality risk than direct company
- Good way to reduce unwanted pricing risks
- Reinsurers may be experts with pricing advice
- Affects incidence of earnings
- Reins premiums may not match pattern of mortality rate
- Can minimize cash flows between reinsurer and direct co.
- Required capital is transferred along with risk
- Extent depends on reinsurance regulations
- Tax planning Can affect taxes especially for unused tax losses
- Can affect solvency and taxable earnings
- Different designs may not affect pattern of shareholder earnings
- Obtain financing more quickly than debt or equity capital
- Finances new business strain/expense allowances
- Enables company to write higher amounts of new business
- Other types of financing have to be obtained well in advance

Solution 15 (continued)

- Targeted change to assets or liabilities
- Can leverage the company's returns
- Can help with Strategic business planning. Eg supporting new lines of business

Solution 16

(a)

- Modified duration
- equals $\text{Summation}(tv^{(t+1)} \text{CashFlow}(t) / \text{Summation}(v^t * \text{CashFlow}(t))$
- equals Macauley duration / (1+i)
- matching of duration of assets to duration of liabilities
- Can combine with 2nd/convexityorder (or higher) measures for more accuracy
- small changes in interest rates will have an equal affect on assets and liabilities
- Problems
- Portfolio will need to be rebalanced from time to time due to:
- defaults, sales of assets, changes in interest rates, emerging diff of actual and expected CF
- There is not exact matching of assets and liabilities
- Implies A/L mismatches in each period can be offset by invest/borrow at interest rate used to calc the duration
- It is possible to match duration, but have a terrible mismatch of cashflows

(b)

t	$(1.05)^t$	CashFlow(t)	PV CashFlow (t)	tPV	$tV^{(t+1)}$ CashFlow (t)
.5	1.0198	50,000	49,029	24,515	23,572
1.0	1.0400	60,000	57,692	57,692	55,473
1.5	1.0606	55,000	51,858	77,786	74,795
2.0	1.08716	70,000	67,719	129,438	1124,459
2.5	1.10302	60,000	54,396	135,990	130,760
3.0	1.12486	80,000	71,120	213,359	205,153
		Totals	348,814	638,781	614,212

- Maccauley duration = $638,781 / 348,814 = 1.83129$
- Modified duration = $\text{maccauley} / 1.05 = 1.76086$
- or modified duration = $614,212 / 348,814 = 1.76086$

(c)

Exact Matching

- exactly matching asset and liability cashflows
- start with longest duration (Final Liability Cashflow) and work backwards
- Often no assets long enough to match longest liabilities
- once longest liability is matched with noncallable assets, proceed to next longest, etc.

Solution 16 (continued)

- Problems
- Premium paying products generate positive cashflows for a number of years leading to future interest rate risk
- if no disintermediation risk, strategy is to match longest liabilities first, otherwise match shortest liabilities first
- if an asset defaults or repays prematurely, matching is thrown out of balance
- if liability cash flows deviate significantly from expected, the portfolio will need to be rebalanced

(d)

exact matching example

- $\text{BondUnits}(b) = \{ \text{LiabCashFlow}(b) - \text{summation} [\text{BondUnits}(x) \text{ Coupon}(x) \text{ (x is greater than b)}] \} / (\text{Coupon}(b) + \text{Par Value}(b))$
- $\text{BondUnits}(b)$ = number of units of the bond that will be purchased to match liability cash flows
- start at longest duration first, $t=3$
- $\text{BondUnits}(3) = \text{LiabCashFlow}(3) / [\text{Coupon}(3) + \text{Par Value}(3)] = 80,000 / (.055 * 100 * .5 + 100) = 778.5888$ units
- using formula above $\text{Bond Units}(2.5) = (60,000 - 778.588 (.055 * 100 * .5)) / (.05 * 100 * .5 + 100) = 564.4769$ units
- using formula above $\text{Bond Units}(2.0) = (70,000 - 778.588 (.055 * 100 * .5) - 564.4769 (.05 * 100 * .5)) / (.045 * 100 * .5 + 100) = 649.8551$ units

(e)

horizon matching

- hybrid between exact matching and duration matching
- assets are purchased to closely match liability cashflows for first 5 or 10 years
- remaining liability cash flows are then matched using duration matching
- as later CF become nearer term CF, matching is adjusted to cover
- would have limited use for this situation as the liabilities are only for 3 years and are predictable
- exact matching can be used more effectively in this situation