MLC Spring 2018

Model Solutions
Written Answer Questions

Question 1 Model Solution

Learning Outcomes: 1(a), 1(b), 1(d), 2(a)

Chapter References: AMLCR Chapter 8, Sections 8.2 -8.6

General comment:
Candidates did very well on this question with many candidates receiving full credit for it.

a)

Pr[Inactive at least once in next 7 months]

= 1 – Pr[remaining active during next 7 months]

$= 1 - {}_{7/12}p_x^{\overline{00}} = 1 - e^{-\frac{7}{12}(.2)} = \mathbf{0.110118}$

Comments:

1. *This part was done correctly by almost all candidates.*
2. *A common error was to use 0.7 instead of 7/12.*

b)

(i) Probability of being inactive at time t after only one transition:

$$\int_0^t {}_sp_x^{\overline{00}} \cdot \mu_{x+s}^{01} \cdot {}_{t-s}p_{x+s}^{\overline{11}}\, ds =$$

$$= \int_0^t e^{-(0.2)s} \cdot (0.2) \cdot e^{-(0.1)(t-s)} ds$$

$$= (.2)e^{-(0.1)t} \int_0^t e^{-(0.1)s}\, ds = (.2)e^{-(0.1)t}\left(\frac{1-e^{-(0.1)t}}{0.1}\right) = 2(e^{-0.1t} - e^{-0.2t})$$

Comments:

1. *This part was done correctly by most candidates.*
2. *Some candidates skipped key steps of the proof or tried to use a memorized formula based on a constant force of transition. Partial or no credit was awarded in those cases.*

(ii) $EPV = 10{,}000 \int_0^1 v^t \cdot 2(e^{-0.1t} - e^{-0.2t}) \cdot \mu_{x+t}^{10}\, dt$

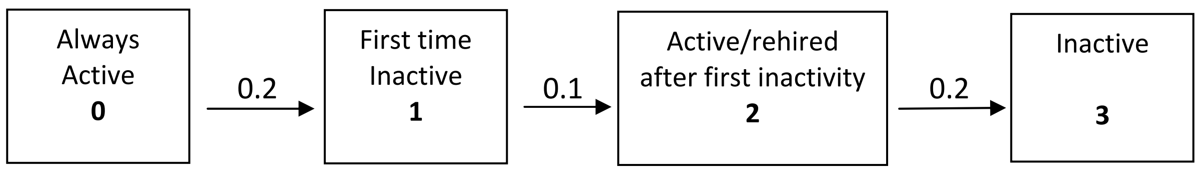
$= 10{,}000 \int_0^1 e^{-0.05t} \cdot 2(e^{-0.1t} - e^{-0.2t}) \cdot (0.1) dt$

$= 10{,}000(0.2) \int_0^1 e^{-0.15t} - e^{-0.25t}\, dt = 2000\left(\frac{1-e^{-0.15}}{0.15} - \frac{1-e^{-0.25}}{0.25}\right) = \mathbf{87.63}$

Comments:

1. *About half the candidates answered this part correctly.*
2. *Many candidates did not recognize that the answer to part (i) was useful in answering this part.*
3. *Candidates who ignored the discounting factor received little or no credit for this part.*

c)



Comments:

1. *Most candidates answered this part correctly.*
2. *The most common error was not differentiating between the two active states.*

d)

The employee is actively employed if in state 0 or state 2 (above diagram).
The probability is ${}_5p_x^{\overline{00}} + {}_5p_x^{02}$.

Here, ${}_5p_x^{\overline{00}} = e^{-0.2(5)} = 0.367879$

$$ {}_5p_x^{02} = \int_0^5 {}_tp_x^{01} \cdot \mu_{x+t}^{12} \cdot {}_{5-t}p_{x+t}^{22}\,dt = \int_0^5 {}_tp_x^{01} \cdot \mu_{x+t}^{12} \cdot {}_{5-t}p_{x+t}^{\overline{22}}\,dt $$

${}_tp_x^{01}$ is the answer to part b)(i)

$$ {}_5p_x^{02} = \int_0^5 2(e^{-0.1t} - e^{-0.2t}) \cdot (0.1) \cdot e^{-(0.2)(5-t)}\,dt = (0.2)e^{-1} \int_0^5 (e^{0.1t} - 1)\,dt $$

$$ = (0.2)e^{-1}\left(\frac{e^{0.5}-1}{0.1} - 5\right) = 0.109423 $$

Finally, ${}_5p_x^{\overline{00}} + {}_5p_x^{02} = 0.367879 + 0.109423 = \mathbf{0.477302}$

Comments:

1. *Most candidates only received partial credit for this part.*
2. *Setting up the integral to calculate the probability of being in State 2 at time 5 proved to be challenging for most candidates.*
3. *Candidates who did well on this part recognized that, while not required, a probability similar to the one used in part b)(ii) was useful in solving this part.*

Question 2 Model Solution

Learning Outcomes: 2(a)

Chapter References: AMLCR Chapter 9, Sections 9.2-9.4, Exercise 9.14

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a)

$\bar{a}_x = (1 - \bar{A}_x)/\delta$ => $\bar{a}_x = 12.8333$ and $\bar{a}_y = 13.5$

$\bar{a}_{xy} = (1 - \bar{A}_{xy})/\delta = 10.8333$

$\bar{a}_{\overline{xy}} = \bar{a}_x + \bar{a}_y - \bar{a}_{xy} = 15.5$

Alternatively,

$\bar{A}_{\overline{xy}} = \bar{A}_x + \bar{A}_y - \bar{A}_{xy} = 0.07$ and $\bar{a}_{\overline{xy}} = (1 - \bar{A}_{\overline{xy}})/\delta = 15.5$

Let *P* be the annual payment rate.

$200{,}000 = P \cdot \bar{a}_{\overline{xy}} \Rightarrow P = \mathbf{12{,}903.226}$

Comment: Candidates did very well on this part, most receiving full credit.

b)

$$Var\left[\bar{a}_{\overline{T_{\overline{xy}}}|}\right] = Var\left[\bar{a}_{\overline{\max(T(x),T(y))}|}\right] = Var\left[\frac{1-v^{\max(T(x),T(y))}}{\delta^2}\right] = \frac{{}^2\bar{A}_{\overline{xy}} - \bar{A}_{\overline{xy}}^{\,2}}{\delta^2}$$

where ${}^2\bar{A}_{\overline{xy}} = {}^2\bar{A}_x + {}^2\bar{A}_y - {}^2\bar{A}_{xy} = .02$ and $\bar{A}_{\overline{xy}} = .23 + .19 - .35 = .07$

SD of annuity:

$SD\left[12{,}903.226\, \bar{a}_{\overline{T_{\overline{xy}}}|}\right] = 12{,}903.226\ [(.02 - (.07)^2)\ /\ (.06^2)]^{0.5} = \mathbf{26{,}426.25}$

Comments:

1. *Many candidates did well on this part.*
2. *The two most common errors were:*
    a. *Calculating the standard deviation for a first-to-die annuity instead of a last-to-die annuity.*
    b. *Using the following incorrect relationship between the variances:*

$$Var\left[\bar{a}_{\overline{T_{\overline{xy}}}|}\right] + Var\left[\bar{a}_{\overline{T_{xy}}|}\right] = Var\left[\bar{a}_{\overline{T_x}|}\right] + Var\left[\bar{a}_{\overline{T_y}|}\right]$$

c)

Let Y be the *PV* of the annuity payments.

$E[Y] = 200{,}000$

$= E\left[(.25X)\bar{a}_{\overline{T_{xy}|}} + (.75X)\bar{a}_{\overline{T_{\overline{xy}}|}}\right]$

$= (.25X)\,\bar{a}_{xy} + (.75X)\,\bar{a}_{\overline{xy}}\,)$

$= (.25X)\,(10.8333) + (.75X)\,(15.5) = 14.3333X$

Alternatively,

$200{,}000 = (.75X)\,\bar{a}_x + (.75X)\,\bar{a}_y - (.5X)\,\bar{a}_{xy} = 14.3333\,X$

$X = 200{,}000/14.3333 = \mathbf{13,953.52}$

Comments:

1. *Performance on this part was mixed.*
2. *The most common error was failing to combine basic annuities with total payments that matched those of the special annuity.*

d)

(i)

$$cov(v^{T_{xy}}, v^{T_{\overline{xy}}}) = E[v^{T_{xy}} v^{T_{\overline{xy}}}] - E[v^{T_{xy}}]E[v^{T_{\overline{xy}}}]$$
$$= E[v^{T_{xy}+T_{\overline{xy}}}] - \bar{A}_{xy}\bar{A}_{\overline{xy}}$$

Since $T_{xy} + T_{\overline{xy}} = T_x + T_y$

$$cov(v^{T_{xy}}, v^{T_{\overline{xy}}}) = E[v^{T_x+T_y}] - \bar{A}_{xy}(\bar{A}_x + \bar{A}_y - \bar{A}_{xy})$$
$$= E[v^{T_x}]E[v^{T_y}] - \bar{A}_{xy}(\bar{A}_x + \bar{A}_y - \bar{A}_{xy}) \quad \text{by independence}$$
$$= (\bar{A}_x - \bar{A}_{xy})(\bar{A}_y - \bar{A}_{xy})$$

(ii)

$$cov\left(\bar{a}_{\overline{T_{xy}|}}, \bar{a}_{\overline{T_{\overline{xy}}|}}\right) = cov\left(\frac{1-v^{T_{xy}}}{\delta}, \frac{1-v^{T_{\overline{xy}}}}{\delta}\right) = cov(v^{T_{xy}}, v^{T_{\overline{xy}}})/\delta^2$$
$$= (\bar{A}_x - \bar{A}_{xy})(\bar{A}_y - \bar{A}_{xy})/\delta^2$$
$$= \left(\frac{1-\bar{A}_x}{\delta} - \frac{1-\bar{A}_{xy}}{\delta}\right)\left(\frac{1-\bar{A}_y}{\delta} - \frac{1-\bar{A}_{xy}}{\delta}\right) = (\bar{a}_x - \bar{a}_{xy})(\bar{a}_y - \bar{a}_{xy})$$

(iii)

$$Var[Y] = Var\left[(.25X)\bar{a}_{\overline{T_{xy}}|} + (.75X)\bar{a}_{\overline{T_{\overline{xy}}}|}\right]$$

$$= X^2\left[(.25)^2 Var\left[\bar{a}_{\overline{T_{xy}}|}\right] + (.75)^2 Var\left[\bar{a}_{\overline{T_{\overline{xy}}}|}\right] + 2(.25)(.75)cov(\bar{a}_{\overline{T_{xy}}|}, \bar{a}_{\overline{T_{\overline{xy}}}|})\right]$$

$$= X^2\left[(.25)^2\left(\frac{{}^2\bar{A}_{xy} - \bar{A}_{xy}^{\,2}}{\delta^2}\right) + (.75)^2\left(\frac{{}^2\bar{A}_{\overline{xy}} - \bar{A}_{\overline{xy}}^{\,2}}{\delta^2}\right) + (.375)\left(\frac{(\bar{A}_x - \bar{A}_{xy})(\bar{A}_y - \bar{A}_{xy}))}{\delta^2}\right)\right]$$

Alternatively,

$$Var[Y] = X^2\left[(.25)^2\left(\frac{{}^2\bar{A}_{xy} - \bar{A}_{xy}^{\,2}}{\delta^2}\right) + (.75)^2\left(\frac{{}^2\bar{A}_{\overline{xy}} - \bar{A}_{\overline{xy}}^{\,2}}{\delta^2}\right) + (.375)\left((\bar{a}_x - \bar{a}_{xy})(\bar{a}_y - \bar{a}_{xy})\right)\right]$$

where

$$\frac{(\bar{A}_x - \bar{A}_{xy})(\bar{A}_y - \bar{A}_{xy}))}{\delta^2} = (.23 - .35)(.19 - .35)/0.06^2 = 0.0192/.0036 = 5.3333$$

$$= (\bar{a}_x - \bar{a}_{xy})(\bar{a}_y - \bar{a}_{xy})$$

$$= (12.8333 - 10.8333)(13.5 - 10.8333) = 5.3333$$

$$Var[Y] = 13{,}953.52^2\left[\frac{[(.0625)(.21 - .35^2) + (.5625)(.02 - .07^2)]}{(.06^2)} + (.375)(5.3333)\right]$$

$$= 13{,}953.52^2\ (5.87846)$$

$$SD[Y] = (13{,}953.52)\ (2.424554) = \mathbf{33{,}831.06}$$

Comments:

1. *Showing the results in part (i) proved to be challenging for many candidates. Common errors included not knowing that $T_{xy} + T_{\overline{xy}} = T_x + T_y$ and not using the fact that the future lifetimes T_x and T_y are independent.*
2. *Performance on part (ii) was mixed. Not knowing or incorrectly using the relationship between $\bar{A}_x$ and $\bar{a}_x$ were common errors.*
3. *Only the strongest candidates did well on part (iii). Common errors included ignoring the covariance term, using wrong coefficients and making calculation mistakes.*

Question 3 Model Solution

Learning Outcomes: 3(c), 4(a)

Chapter References: AMLCR Sections 6.6, 7.3, 7.9

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a)

G = P + e

where $P=1000\,(IA)_{75} = 662.46$ and *e* is calculated as follows

$$e\,\ddot{a}_{75} = 500\,A_{75} + 0.08G\,\ddot{a}_{75} + 0.32G + 400 + 45\ddot{a}_{75}$$

$$=> e = (500(.59149) + 0.08G\,(7.217) + 0.32G + 400 + 45(7.217))/7.217$$
$$= (1020.51\; + 0\,.89736\text{G})/7.217$$

So,

$$G = 662.46 + (1020.51\; +\; .89736\text{G})/7.217 \quad \Rightarrow \quad G = \mathbf{918.01}$$

Comments:

1. *Most candidates did very well on this part.*
2. *The most common error was to use incorrect values for the issue or maintenance expenses.*

b)

${}_{0}V^{G} = \mathbf{0}$, by definition

Recursively,

$${}_{1}V^{G} = \frac{({}_{0}V^{G} + 0.6G - 400 - 45)(1.06) - q_{75}(1000 + 500)}{p_{75}}$$

$${}_{1}V^{G} = \frac{(0.6(918.01) - 445)(1.06) - 0.05169(1500)}{0.94831} = \frac{(112.1544\; -\; 77.535)}{0.94831} = \mathbf{36.506}$$

$$ {}_2V^G = \frac{({}_1V^G + 0.92G - 45)(1.06) - q_{76}(2000 + 500)}{p_{76}} $$

$$ {}_2V^G = \frac{(36.506 + 0.92(918.01) - 45)(1.06) - 0.05647(2000 + 500)}{0.94353} = \mathbf{789.657} $$

Alternatively, letting $PVFB_t$, $PVFE_t$ and $PVFP_t$ be the present value of future benefits, expenses and gross premiums at time *t*, respectively,

$$ \begin{aligned} {}_1V^G &= PVFB_1 + PVFE_1 - PVFP_1 \\ &= 1000[A_{76} + (IA)_{76}] + (45 + 0.08G)\ddot{a}_{76} + 500A_{76} - G\ddot{a}_{76} \\ &= [P\ddot{a}_{75} - 1000vq_{75}]/vp_{75} + (45 + 0.08G)\ddot{a}_{76} + 500A_{76} - G\ddot{a}_{76} \\ &= [(662.46)(7.217) - 1000(.05169)/1.06]/(.94831/1.06) \\ &\quad + (118.441)(6.9493) + (500)(.60665) - (918.01)(6.9493) \\ &= (4780.974 - 48.764)/.89463 + 823.082 + 303.325 - 6379.527 = 36.45 \end{aligned} $$

and

$$ \begin{aligned} {}_2V^G &= PVFB_2 + PVFE_2 - PVFP_2 \\ &= 2000A_{77} + 1000(IA)_{77} + (45 + 0.08G)\ddot{a}_{77} + 500A_{77} - G\ddot{a}_{77} \\ &= \frac{\left(P\ddot{a}_{75} - 1000vq_{75} - 2000v^2p_{75}q_{76}\right)}{v^2\ {}_2p_{75}} + (45 + 0.08G)\ddot{a}_{77} + 500A_{77} - G\ddot{a}_{77} \\ &= \frac{\left((662.46)(7.217) - 48.764 - \frac{2000}{1.06^2}(.94831).05647)\right)}{\frac{(.94831)(.94353)}{1.06^2}} \\ &\quad + (118.441)(6.6836) + (500)(.62168) - (918.01)(6.6836) \\ &= 5822.800 + 791.612 + 310.84 - 6135.612 = 789.64 \end{aligned} $$

Comments:

1. *Most candidates did very well on this part, especially those using the recursive formula.*
2. *The most common error was valuing the increasing insurances incorrectly.*

c)

First year modified premium under FPT, α :

α = cost of insurance

$\alpha = 1000\ v\ q_{75} = 51.69/1.06 = \mathbf{48.764}$

Comments:

1. *Most candidates did very well on this part.*
2. *Some candidates incorrectly included expenses.*

d)

The level net premium starting in year 2 under FPT, P^{FPT}, is such that *PV* of modified premiums equals *PV* of net premiums

$$P\ddot{a}_{75} = \alpha + P^{FPT}\, v\, p_{75}\, \ddot{a}_{76}$$

$$P^{FPT} = \frac{(P\ddot{a}_{75} - \alpha)(1+i)}{p_{75}\,\ddot{a}_{76}} = \frac{(662.46(7.217) - 48.764)(1.06)}{(0.94831)(6.9493)} = \mathbf{761.164}$$

Alternatively,

$$P^{FPT} = (1000A_{76} + 1000(IA)_{76}) / \ddot{a}_{76}$$

$$P^{FPT} = \frac{(1000(IA)_{75} - 1000vq_{75})/vp_{75}}{\ddot{a}_{76}} = \frac{(P\ddot{a}_{75} - 1000vq_{75})}{v\, p_{75}\, \ddot{a}_{76}}$$

$$P^{FPT} = \frac{\big((662.46)(7.217) - 48.764\big)(1.06)}{(.94831)(6.9493)} = 761.164$$

Comments:

1. *Few candidates did well on this part.*
2. *Taking the increasing insurance benefits into account in the FPT method proved to be a challenge for most candidates.*

e)

${}_1V^{FPT} = \mathbf{0}$ by definition

Recursively,

$${}_2V^{FPT} = \frac{({}_1V^{FPT} + 761.164)(1.06) - q_{76}(2000)}{p_{76}} = \frac{806.8338 - (.05647)(2000)}{.94353} = \mathbf{735.423}$$

Alternatively,

$${}_2V^{FPT} = (2000A_{77} + 1000(IA)_{77}) - P^{FPT}\;\ddot{a}_{77}$$

$$= \frac{(P\ddot{a}_{75} - 1000vq_{75} - 2000v^2 p_{75} q_{76})}{v^2\;{}_2p_{75}} - P^{FPT}\;\ddot{a}_{77}$$

$$= \frac{\left((662.46)(7.217) - 48.764 - \frac{2000}{1.06^2}(.94831)(.05647)\right)}{(.94831)(.94353)/1.06^2} - 761.164(6.6836) = 735.484$$

Comments:

1. *Few candidates did well on this part.*
2. *As in part d), taking the increasing insurance benefits into account in the FPT reserves proved to be a challenge for most candidates.*

f)

The FPT method adjusts net premium reserves to approximate gross premium reserves by implicitly assuming that the first year premium is spent on cost of insurance and acquisition expenses. The company would use the FPT method because it produces lower reserves than the level net premium reserves. It therefore reduces the initial strain on surplus from new business.

Comment:

Only well-prepared candidates were able to provide a good rationale for why the lower FPT reserves compared to the level net premium reserves are good for the insurance company.

Question 4 Model Solution

Learning Outcomes: 4(a), 4(c)

Chapter References: AMLCR Sections 12.3-12.5, 13.3

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a)

$${}_2V = 100{,}000\, A_{82} - P\, \ddot{a}_{82}$$

$$P = \frac{100{,}000\, A_{80}}{\ddot{a}_{80}} = \frac{66{,}575}{5.905} = 11{,}274.34$$

$${}_2V = 69{,}398 - (11{,}274.34)(5.4063) = \mathbf{8{,}445.54}$$

Alternatively,

$${}_2V = \frac{({}_1V + P)(1.06) - q_{81}(100{,}000)}{p_{81}}$$

$$= \frac{(4{,}263 + 11{,}274.34)(1.06) - 0.08764(100{,}000)}{0.91236} = \frac{7{,}705.58}{0.91236} = \mathbf{8{,}445.77}$$

Comments:

1. *Candidates did very well on this part.*
2. *Some candidates incorrectly used the gross premium (13,000) instead of P.*
3. *Using a wrong probability of death (e.g. q_{80} or q_{82}) was another common mistake.*

b)

$$Pr_2 = Pr_{2-} - Div_2$$
$$Pr_{2-} = {}_1V + G - E + I_2 - EDB_2 - ECV_2 - E_2V$$
$$Div_2 = (0.7)Pr_{2-}$$

$$Pr_2 = (0.3)\left[({}_1V + (0.95)G - 75)(1.07) - p_{81}^{0d} \cdot S - p_{81}^{0w} \cdot CV_2 - p_{81}^{00} \cdot {}_2V\right]$$

where

$$p_{81}^{0d} = (0.85)q_{81}^{ILT} = (0.85)(0.08764) = 0.074494$$
$$p_{81}^{0w} = \left(1 - p_{81}^{0d}\right)q_{81}^{*w} = (1 - 0.074494)(0.05) = 0.0462753$$
$$p_{81}^{00} = 1 - p_{81}^{0d} - p_{81}^{0w} = 0.879231$$

$$Pr_2 = (0.3)[(4{,}263 + (0.95)13{,}000 - 75)(1.07) - (0.074494)(100{,}000)$$
$$-(0.0462753)(0.2)(8{,}445.54) - (0.879231)(8{,}445.54)] = \mathbf{822.75}$$

Comments:

1. *Most candidates did very well on this part.*
2. *Some candidates calculated some but not all of the terms in the expressions for Pr_2.*
3. *A few candidates used incorrect values for one or more of the probabilities, p_{81}^{0*}.*
4. *Other common errors included ignoring the dividend, using a wrong premium or wrong reserves.*
5. *Calculation mistakes were also common. Candidates not writing down the formulas or clearly showing their work may have missed out on some partial credit.*

c)

$\pi_t = {}_{t-1}p_x^{00} \cdot Pr_t;$ with $\pi_0 = Pr_0$

$NPV_t = NPV_{t-1} + \pi_t v^t = \sum_{k=0}^{t} \pi_k v^k; \quad NPV_0 = \pi_0; \qquad v = v_{.1} = \frac{1}{1.1}$

$$NPV(3) = -1000 + 265.72\, v_{.1} + 822.69\, v_{.1}^2\ p_{80}^{00} + 894.99\, v_{.1}^3\ {}_2p_{80}^{00}$$

$$NPV(3) = -1000 + \frac{265.72}{1.1} + \frac{822.69}{1.1^2}(0.8)\big(1-(0.85)(.0803)\big) + \frac{894.99}{1.1^3}(0.8)\big(1-(0.85)(.0803)\big)\cdot(.95)(1-(.85)(0.08764))$$

$$NPV(3) = -1000 + 241.564 + 506.802 + 440.687 = \mathbf{189.05}$$

Comments:

1. *Many candidates did well on this part.*
2. *The most common errors were using incorrect probabilities (e.g. ignoring lapses) and discounting at a rate other than the hurdle rate.*

d)

(i) $Div_1 = (0.7)Pr_{1-} = (0.7)\frac{Pr_1}{0.3} = \frac{(0.7)(265.72)}{0.3} = 620.013$

$ADB_1 = \frac{Div_1}{A_{81}} = \frac{620.013}{0.680} = \mathbf{911.784}$

(ii) Using * to denote the revised values, we get

$$Pr_2^* = (0.3)\big[({}_1V^* + (0.95)G - 75)(1.07) - p_{81}^{0d}\cdot S^* - p_{81}^{0w}\cdot CV^* - p_{81}^{00}\cdot {}_2V^*\big]$$

where

$${}_1V^* = {}_1V + 911.784\, A_{81} = {}_1V + Div_1 = 4263 + 620.013 = 4883.013$$

$$S^* = 100{,}911.784$$

$$ {}_2V^* = {}_2V + 911.784\, A_{82} = 8445.767 + 911.784(.69398) \;=\; 9078.527 $$

$$ CV^* = (0.2)\, {}_2V^* = 1815.705 $$

$$ \begin{aligned} Pr_2^* = (0.3)[&(4883.013 + \;(0.95)13{,}000 - 75)(1.07) - (0.074494)(100{,}911.784) \\ &-(0.0462753)(1815.705) - (0.879231)(9078.527)] \\ = (0.3)[&18{,}359.074 - 7517.322 - 84.022 - 7982.122] = \;\mathbf{832.68} \end{aligned} $$

Alternatively, $Pr_{2-}^* = Pr_{2-} + \Delta_1V + \Delta I_2 - \Delta EDB_2 - \Delta ECV_2 - \Delta E_2V$
where

$$ \begin{aligned} \Delta_1V &= Div_1 = ADB_1\, A_{81} = 620.01 \\ \Delta I_2 &= (.07)\Delta_1V = 43.40 \\ \Delta EDB_2 &= 911.784\,(.85)(.08764) = 67.92 \\ \Delta ECV_2 &= (.2)(911.784\, A_{82})(.05)\big(1 - (.85)(.08764)\big) = 5.86 \\ \Delta E_2V &= (911.784\, A_{82})(.95)\big(1 - (.85)(.08764)\big) = 556.34 \\ Pr_{2-}^* &= 2742.3 + 620.01 + 43.4 - 67.92 - 5.86 - 556.34 = 2775.59 \\ Pr_2^* &= (0.3)(2775.59) = \mathbf{832.68} \end{aligned} $$

Comments:

1. *Many candidates did not attempt this part. Few achieved full credit.*
2. *Common errors included incorrectly calculating the dividend as* $(0.7)Pr_1$ *instead of* $(0.7)Pr_{1-}$ *in part (i) and only considering the change due to the revised death benefit in part (ii).*

e)

If the dividend is used to purchase an additional death benefit instead of paying it in cash immediately, the insurer will generate an expected profit due to

- the interest spread (additional benefit determined based on a pricing interest of 6% when the insurer assumes 7% in profit testing)
- the reduced payouts in case of withdrawal in future years (since *CV* is 20% of the reserve, a withdrawal will release some reserve and increase profit).

Comments:

1. *Only top candidates were able to provide a good rationale as to why* Pr_2 *increases when the dividend is converted into an ADB by identifying the sources of additional profit.*
2. *Many candidates did not attempt this part and most of those who did received little or no credit for an insufficient rationale, e.g. the company doesn't have to pay the dividend, or the company keeps more money, or the company pays the amount later.*

Question 5 Model Solution

Learning Outcomes: 1(a), 4(a), 4(b)

Chapter References: AMLCR Sections 4.4, 5.5, 11.4

General comment:
Overall performance on this question was poor. Many candidates did not attempt this question.

a)

(i) Let L_j denote the loss for policy *j* and *L* the total loss. Then, $L = \sum_{j=1}^{N} L_j$

$Var[L] = N \cdot Var[L_j]$ since the lives are independent

$$L_j = S\, v^{T_{80}} - P\, \bar{a}_{\overline{T_{80}|}} = \left(S + \frac{P}{\delta}\right) v^{T_{80}} - \frac{P}{\delta}$$

$$Var[L_j] = \left(S + \frac{P}{\delta}\right)^2 \left({}^2\bar{A}_{80} - (\bar{A}_{80})^2\right)$$

where $\bar{A}_{80} = 1 - \delta \bar{a}_{80} = 0.72025$

$${}^2\bar{A}_{80} = 1 - 2\delta\, {}^2\bar{a}_{80} = 1 - (0.1)(4.544) = 0.5456$$

$$Var[L_j] = \left(10{,}000 + \frac{P}{0.05}\right)^2 (0.0268399)$$

$$SD[L] = \sqrt{N\, Var[L_j]} = \sqrt{N}(1638.29 + 3.27658\, P)$$

So, $\underline{\mathbf{a = 1638.29}}$ and $\underline{\mathbf{b = 3.27658}}$

(ii) The risk is diversifiable if $\lim_{N\to\infty} \frac{SD[L]}{N} = 0$

Since $\lim_{N\to\infty} \frac{\sqrt{N}(1638.29 + 3.27658\, P)}{N} = 0$, the risk is **diversifiable.**

Comments:

1. *Candidates who attempted this question did well on this part.*
2. *The most common error was not using a proper definition of a diversifiable risk in part (ii).*

b) Let $I = \begin{cases} 0 & \text{if the drug doesn't work} \\ 1 & \text{if the drug works} \end{cases}$.

(i) From a)

$$Var[L_j|I=0] = (1638.29 + 3.27658\,P)^2$$
$$= (1638.29 + 3.27658(1300))^2 = 34{,}784{,}563.85 = \mathbf{5897.844}^2$$

(ii) Let * indicate that the value is based on the reduced mortality, i.e. I =1

$$\mu_x^* = \mu_x - 0.01 \;\; \forall x$$

$$\bar{a}_x^* = \int_0^\infty e^{-\delta t}\, {}_tp_x^*\; dt = \int_0^\infty e^{-\delta t}\, e^{-\int_0^t \mu_{x+s}^*\,ds}\; dt = \int_0^\infty e^{-\delta t}\, e^{-\int_0^t (\mu_{x+s}-0.01)\,ds}\; dt$$

$\bar{a}_x^* = \int_0^\infty e^{-(\delta-0.01)t}\, e^{-\int_0^t \mu_{x+s}\,ds}\; dt = \bar{a}_x$ valued at $(\delta - 0.01)$

$\bar{a}_{80}^* = \bar{a}_x$ at 4% = 5.855 and $\bar{A}_{80}^* = 1 - (0.05)\bar{a}_{80}^* = 0.70725$

${}^2\bar{a}_{80}^* = \bar{a}_x$ at 9% i.e. 2(0.05)-0.01 ; ${}^2\bar{a}_{80}^* = 4.725$

${}^2\bar{A}_{80}^* = 1 - (0.1)\,{}^2\bar{a}_{80}^* = 1 - (0.1)(4.725) = 0.5275$

$$Var[L_j|I=1] = \left(S + \frac{P}{\delta}\right)^2 \left({}^2\bar{A}_{80}^* - (\bar{A}_{80}^*)^2\right) = \left(10{,}000 + \tfrac{1300}{0.05}\right)^2 (0.0272974)$$
$$= \mathbf{5947.90}^2$$

(iii) $N = 400$

$$Var[L] = E\big[Var[L|I]\big] + Var\big[E[L|I]\big]$$

$$E[L_j|I=0] = S\,\bar{A}_{80} - P\,\bar{a}_{80} = 7202.5 - 1300(5.595) = -71$$
$$E[L_j|I=1] = S\,\bar{A}_{80}^* - P\,\bar{a}_{80}^* = 7072.5 - 1300(5.855) = -539$$

$$Var\big[E[L|I]\big] = (0.6)(0.4)(539-71)^2\,N^2 = 52{,}565.76\;\; N^2 = 91{,}708.9^2$$

$$E\big[Var[L|I]\big] = (0.6)N(5897.844)^2 + (0.4)N(5947.9)^2$$
$$= 35{,}021{,}744.07\,(400) = 118{,}358.34^2$$

$Var[L] = 149{,}730.49^2$ and $SD[L] = \mathbf{149{,}730.49}$

Comments:

1. *Most candidates did well on part (i).*
2. *Some candidates did recognize that pricing the annuity if the drug worked could be done by appropriately adjusting the force of interest. Even when they did, most used a wrong force of interest to calculate ${}^2\bar{a}_{80}^*$ and ${}^2\bar{A}_{80}^*$.*
3. *Few candidates completed all of part b).*

c)

$$Var[L] = 35{,}021{,}744.07\,N + 52{,}565.76\,N^2$$

$$\lim_{N\to\infty}\frac{SD[L]}{N} = \lim_{N\to\infty}\frac{\sqrt{35{,}021{,}744.07\,N + 52{,}565.76\,N^2}}{N} = \sqrt{52{,}565.76} > 0$$

⇨ The risk is **<u>not diversifiable</u>**.

Comments:

1. *Most candidates did not attempt this part.*
2. *A number of those who did answer it explained why the risk is no longer diversifiable instead of providing a proof based on the definition of a diversifiable risk.*

d)

With the new drug, the risk is not diversifiable because the drug would affect the mortality of all lives. The underlying mortality will be lower for everyone if the drug works, or stay the same for everyone if the drug doesn't work.

Comments:

1. *Many candidates did not attempt this part.*
2. *Only well-prepared candidates were able to provide a satisfactory rationale as to why the introduction of the new drug has created a non-diversifiable risk.*
3. *A common error made by those answering both part c) and part d) was to repeat essentially the same answer. For example, writing in part d) that the risk is non-diversifiable because the limit in c) is not 0; or providing the same explanation for both parts.*

Question 6 Model Solution

Learning Outcomes: 5(a), 5(b), 5(c), 5(d), 5(e), 5(f)

Chapter References: AMLCR Chapter 10

General Comment:
Most candidates omitted this question entirely or only answered parts a) and b).

a)

Finn:

$$RR = \frac{S_{25}(1.02)^{39}(0.017)(65-25)}{S_{25}(1.02)^{39}} = \mathbf{0.68}$$

Oscar:

$$RR = \frac{S_{64}(0.017)(29+1)}{S_{64}} = \mathbf{0.51}$$

Comment: Most candidates did very well on this part.

b)

Finn:

$AL = \mathbf{0}$ since Finn has no service, so no accrued liability.

Oscar:

$$AL = 29(0.017)\, S_{63}\left((0.5)\,{}_{.5}p_{64}\, v^{0.5}\ \ddot{a}^{(12)}_{64.5} + (0.5)\, p_{64}\, v\ \ddot{a}^{(12)}_{65}\right)$$

$${}_{.5}p_{64} = 1 - (0.5)\, q_{64} = 1 - 0.5 \times 0.01952 = 0.99024$$

$$\ddot{a}^{(12)}_{65} = \alpha(12)\ddot{a}_{65} - \beta(12) = 1.00028 \times 9.8969 - 0.46812 = 9.431551$$

$$AL = 29(0.017)(95{,}000)[(0.5)(0.99024)1.06^{-0.5}(9.5613) + (0.5)(0.98048)1.06^{-1}(9.431551)] = \mathbf{419{,}644.49}$$

Comments:

1. *Most candidates did well on this part.*
2. *The most common error was to recognize future service when calculating the accrued liabilities.*
3. *The mid-year exits were a challenge for some candidates.*

c)

(i) Finn:

$$NC = (0.017)\, S_{25} \left[(0.5)\,{}_{39.5}p_{25}\, v^{39.5}\, \ddot{a}^{(12)}_{64.5} \;+\; (0.5)\,{}_{40}p_{25}\, v^{40}\, \ddot{a}^{(12)}_{65}\right]$$

$${}_{39.5}p_{25} = \frac{(.5)[7{,}683{,}979+7{,}533{,}964]}{9{,}565{,}017} = 0.795500 \quad \text{and} \quad {}_{40}p_{25} = \frac{7{,}533{,}964}{9{,}565{,}017} = 0.787658$$

$$NC = (0.017)(60{,}000)[(0.5)(0.795500)(1.06)^{-39.5}(9.5613) + (0.5)(0.787658)(1.06)^{-40}(9.431551)] = \mathbf{756.63}$$

(ii) Oscar:

$$NC = EPV\ of\ benefits\ for\ mid\text{-}year\ exits + E_0[AL_1] - AL_0$$

$$AL_0 = 419{,}644.49$$

$$EPV\ of\ benefits\ for\ mid\text{-}year\ exits = (0.017)(29.5)\, S_{63.5} \left((0.5)\,{}_{.5}p_{64}\, v^{0.5}\ \ddot{a}^{(12)}_{64.5}\right)$$
$$= (0.5015)(97{,}500)(4.5980584) = 224{,}827.81$$

$$E_0[AL_1] = (0.017)(30)\, S_{64} \left((0.5)\, p_{64}\, v\, \ddot{a}^{(12)}_{65}\right)$$
$$= (0.51)(100{,}000)(4.3620034) = 222{,}462.17$$

$$NC = 224{,}827.81 + 222{,}462.17 - 419{,}644.49 = \mathbf{27{,}645.49}$$

Comments:

1. *Most candidates did poorly on this part.*
2. *Mid-year exits proved to be challenging for most candidates who answered this part.*
3. *Another common error was to use PUC instead of TUC.*

d)

(i) The PUC would be more expensive for Finn as the NC includes the impact of future salary increases in the pension cost. Because Finn has many years of service ahead, prefunding salary increases will have a significant impact.

(ii) The TUC would be more expensive for Oscar. The TUC requires all past accrued liability to be adjusted for current salary increases. As Oscar is near retirement, this is a significant cost. The PUC prefunds future salary increases, but since Oscar has little time left in employment, this cost is small compared with the salary upgrade in TUC.

Comments:

1. *Most candidates omitted this part entirely or did not provide a satisfactory reason why PUC was more expensive for Finn and TUC more expensive for Oscar.*
2. *Only a few candidates provided complete and correct explanations for both cases.*