Exam MLC Spring 2018 - MC Solutions

1.

$$p_{[45]} = p_{40} = 0.99722 \quad p_{[45]+1} = p_{43} = 0.99656 \quad p_{[45]+2} = p_{46} = 0.99569 \quad p_{48} = 0.99496$$

$$\Rightarrow {}_4p_{[45]} = 0.9845 \quad \Rightarrow {}_4q_{[45]} = 0.0155$$

Answer Key D

2.

$$0.9 = {}_{0.2}p_{50.6} = \frac{{}_{0.8}p_{50}}{{}_{0.6}p_{50}} = \frac{1 - 0.8\,q_{50}}{1 - 0.6\,q_{50}} \quad \Longrightarrow 0.9 - 0.54\,q_{50} = 1 - 0.8\,q_{50}$$

$$\Longrightarrow 0.1 = 0.26\,q_{50}$$

$$\Longrightarrow q_{50} = 0.385$$

Answer Key E

3.

$$100\,000\,{}_5E_{65:65} = F\,(2\ddot{a}_{65} - \ddot{a}_{65:65})$$

$${}_5E_{65:65} = {}_5E_{65}\,{}_5p_{65} = 0.57629$$

$$F = \frac{57629}{11.9386} = 4827$$

Answer Key B

4.

$$d^{(12)} = 12\left(1 - v^{\frac{1}{12}}\right) = 0.048691$$

$$\ddot{a}_{50}^{(12)} = \frac{1 - A_{50}^{(12)}}{d^{(12)}} = 16.656 \qquad \ddot{a}_{65}^{(12)} = \frac{1 - A_{65}^{(12)}}{d^{(12)}} = 13.267$$

$$\ddot{a}_{50:\overline{15}|}^{(12)} = \ddot{a}_{50}^{(12)} - {}_{15}E_{50}\,\ddot{a}_{65}^{(12)} = 10.53$$

Answer Key D

5.

$$\text{EPV} = 100\left(v(3)\,{}_3p_{80} + v(4)\,{}_4p_{80} + v(5)\,{}_5p_{80} + \cdots\right)$$

$$v(3) = \left((1.04)(1.045)(1.05)\right)^{-1} = 0.87632 \qquad v(4) = v(3)/(1.055) = 0.83063$$

$$v(5) = v(4)\,v_{6\%} \qquad v(6) = v(4)\,v_{6\%}^2 \;\cdots$$

$$\Rightarrow \text{EPV} = 100\left(v(3)\,{}_3p_{80} + v(4)\,{}_4p_{80}\left(1 + v_{6\%}\,p_{84} + v_{6\%}^2\,{}_2p_{84} + \cdots\right)\right)$$

$$= 100\Bigg(0.87632(0.75887) + 0.83063(0.67974)(\ddot{a}_{84})_{6\%})\Bigg) = 344.8$$

Answer Key C

6. Let $T = T_{40}$.

$$L_0 = 100,000v^T - 3144 \quad \text{if } T \leq 10 \qquad (\text{and} = -3144 \quad \text{if } T > 10)$$

$$L_0 \geq 75000 \Leftrightarrow v^T > 0.78144 \Leftrightarrow T \leq 6.3$$

$$\Pr[L_0 \geq 75000] = \Pr[T \leq 6.3] = 1 - \Pr[T > 6.3] = 1 - {}_{6.3}p_{40}$$

$$= 1 - \frac{0.3l_{47} + 0.7\,l_{46}}{l_{40}} = 1 - 0.97879 = 0.0212$$

Answer Key E

7.

$$G\Bigg(0.9\ddot{a}_{45:\overline{10}|} + 0.95\,{}_{10}E_{45}\,\ddot{a}_{55:\overline{10}|} - 0.3\Bigg) = 150\,000\,{}_{20}E_{45}\,\ddot{a}_{65}$$

$$\Longrightarrow\; G = \frac{150\,000(0.081)(7.40)}{0.9 * 6.25 + 0.95 * 0.3 * 6.0 - 0.3} = \frac{89910}{7.035} = 12780$$

Answer Key C

8.

$$P\bar{a}_{75:\,75} = 100\,000\,\bar{A}^{\;2}_{75:\,75} \qquad \bar{A}^{\;2}_{75:\,75} = \bar{A}_{75} - \bar{A}^{\;1}_{75:\,75}$$

$$\bar{A}^{\;1}_{75:\,75} = \frac{1}{2}\,\bar{A}_{75:\,75} = \frac{1}{2}\left(\frac{i}{\delta}(0.70195)\right) = \frac{1}{2}(0.72280)$$

$$\bar{a}_{75:\,75} = \frac{1 - 0.72280}{\delta} = 4.7572 \qquad \bar{A}_{75} = \frac{i}{\delta}A_{75} = 0.60906$$

$$\text{So } P = \frac{100\,000(0.60906 - 0.72280/2)}{4.7572} = 5206$$

Answer Key B

9. NPV per policy is 10 000.

$$0.15 = \frac{10\,000}{P\,\ddot{a}_{45}} \implies P = 4724$$

Answer Key B

10.

$$P = \frac{50\,000\,\bar{A}^{01}_{60} + 50\,000\,\bar{A}^{03}_{60} + 100\,000\,\bar{A}^{02}_{60}}{\bar{a}^{00}_{60}}$$

$$= \frac{50\,000(0.390 + 0.280 + 2(0.181)}{10.989}$$

$$= \frac{51\,600}{10.989} = 4695.6$$

Answer Key B

11. Let $K = K_{55}$ and $T = T_{55}$. Working in '000s.

$$\Pr\left[250v^{K+1} < 5\,\ddot{a}_{\overline{K+1}|}\right] = \Pr\left[250v^{K+1} < 5\left(\frac{1-v^{K+1}}{d}\right)\right], \qquad \text{where } d = 0.056604$$

$$= \Pr\left[v^{K+1}\left(250 + \frac{5}{d}\right) < \frac{5}{d}\right]$$

$$= \Pr\left[v^{K+1} < 0.261\right] = \Pr\left[K + 1 > 23.05\right]$$

$$= \Pr[T > 23] = {}_{23}p_{55} = 0.5243$$

Answer Key A

12.

$$12P\ddot{a}^{(12)}_{40:\overline{20}|} = 1\,000\,000A_{40:\overline{20}|}; \qquad A_{40:\overline{20}|} = A_{40} + {}_{20}E_{40}\,(1 - A_{60}) = 0.33427$$

$$\ddot{a}_{40:\overline{20}|} = \frac{1 - 0.33427}{d} = 11.7613$$

$$\ddot{a}^{(12)}_{40:\overline{20}|} = \alpha(12)\,11.7613 - \beta(12)\,(1 - {}_{20}E_{40}) = 11.4248$$

So $P = 2438$

Answer Key C

13.

$$AV_{11} = (66\,600 + 0.95(10\,000) - 120)\,(1.04) - 0.015\,(500\,000 - AV_{11})$$

$$= \frac{71\,519.2}{0.985} = 72\,608$$

$$ADB = 500\,000 - AV_{11} = 427\,391$$

Answer Key A

14. Let $S = 1\,000\,000$ and $P = 3489$.

$${}_{10}V = SA^{\,1}_{45:\overline{10}|} - P\ddot{a}_{45:\overline{10}|}; \qquad \ddot{a}_{45:\overline{10}|} = 7.6486 \qquad A^{\,1}_{45:\overline{10}|} = 0.04054$$

$$\Longrightarrow {}_{10}V = 13\,852$$

$${}_{10.4}V = \frac{({}_{10}V + P)\,(1.06)^{0.4} - {}_{0.4}q_{45}\,S(1.06)^{-0.6}}{{}_{0.4}p_{45}} \qquad {}_{0.4}p_{45} = 0.9984 \quad {}_{0.4}q_{45} = 0.0016$$

$${}_{10.4}V = 16,232.95$$

Answer Key E

15. Let $S = 100\,000$ and $G = 2700$.

$$\frac{d}{dt}\,{}_tV = \delta\,{}_tV + 0.95G - \mu_{50+t}\,(1.025S - {}_tV)$$

at $t = 10:$ $\dfrac{d}{dt}\,{}_tV = 0.04 * 18\,700 + 0.95 * 2700 - 0.013 * (105\,500 - 18\,700) = 2223.6$

Answer Key B

16. Let P denote the level net premium and P^* denote the FPT net premium after the first year.

$${}_{20}V - {}_{20}V^{FPT} = (1000A_{60} - P\ddot{a}_{60}) - (1000A_{60} - P^*\ddot{a}_{60}) = (P^* - P)\,\ddot{a}_{60}$$

$$P = \frac{1000A_{40}}{\ddot{a}_{40}} = 18.868; \qquad P^* = \frac{1000A_{41}}{\ddot{a}_{41}} = \frac{1000(1 - d\ddot{a}_{41})}{\ddot{a}_{41}} = 20.083$$

$$\Rightarrow {}_{20}V - {}_{20}V^{FPT} = 11.8$$

Answer Key A

17. Let $S = 100\,000$

$$\left({}_{10.5}V^g + 0.9(900)\right)(1.05)^{0.5} = {}_{0.5}q_{55.5}S + {}_{0.5}p_{55.5}\,{}_{11}V^g$$

$${}_{0.5}p_{55.5} = \frac{p_{55}}{{}_{0.5}p_{55.5}} = 0.99550 \Rightarrow {}_{0.5}q_{55.5} = 0.0045$$

$${}_{11}V^g = \frac{(16\,074 + 0.9(900))(1.05)^{0.5} - 0.0045S}{0.9955} = 16\,927$$

Answer Key A

18.

$$\frac{d}{dt}\,{}_tp_x^{00} + \frac{d}{dt}\,{}_tp_x^{01} + \frac{d}{dt}\,{}_tp_x^{02} = 0 \Rightarrow \frac{d}{dt}\,{}_tp_x^{02} = 0.0018$$

also $\frac{d}{dt}\,{}_tp_x^{02} = {}_tp_x^{00}\,\mu_{x+t}^{02} + {}_tp_x^{01}\,\mu_{x+t}^{12} = ({}_tp_x^{00} + {}_tp_x^{01})(0.0020) = (1 - {}_tp_x^{02})(0.0020)$

So ${}_tp_x^{02} = 1 - \frac{0.0018}{0.0020} = 0.1$

Answer Key B

19. Let B denote the regular annual pension and B^* the annual pension allowing for a 10-year guarantee.

$$B\,\ddot{a}_{65} = B^*\ddot{a}_{\overline{65:\overline{10|}}}$$

$$\ddot{a}_{\overline{65:\overline{10|}}} = \ddot{a}_{\overline{10|}} + {}_{10}E_{65}\,\ddot{a}_{75} = 7.8017 + 2.8864 = 10.6881$$

$$B = 0.02 * 100\,000\left(\frac{1 + v_{3\%} + v_{3\%}^2}{3}\right) * 30 = 58\,269$$

$$B^* = \frac{58\,269\,\ddot{a}_{65}}{10.6881} = 53\,955$$

Answer Key E

20. The normal cost is the present value of a single year's accrual:

$$NC = \frac{10\,000\,d_{64}^{(d)}v^{14.5} + 15\,000\,d_{65}^{(d)}v^{15.5}}{l_{50}} = 90.3$$

Answer Key D