GI ADV Model Solutions
Spring 2018

1.

Learning Objectives:

5. The candidate will understand methodologies for determining an underwriting profit margin.

Learning Outcomes:

(5d) Allocate an underwriting profit margin (risk load) among different accounts.

Source:

An Application of Game Theory: Property Catastrophe Risk Load, Mango

Solution:

(a) Calculate the risk load for this existing contract.

The standard deviation for the existing contract is $\sqrt{1,000,000^2(0.001)(0.999)} = 31,607.$ Multiply by 0.025 to obtain the risk load of 790.

(b) Calculate the risk load for this second contract using the Marginal Surplus method.

The standard deviation for the combined portfolio is $\sqrt{1,500,000^2(0.001)(0.999)} = 47,410.$ Multiply by 0.025 to obtain the total risk load of 1185. The marginal risk load is then 1185 – 790 = 395.

(c) Determine the variance risk load multiplier, λ, that produces the same risk load for the combined portfolio as that obtained using the Marginal Surplus method.

The variance risk load multiplier is the standard deviation multiplier divided by the standard deviation of the combined portfolio. Hence, $\lambda = 0.025 / 47,410 = 0.0000005273.$

(d) Calculate the risk load for each contract using the Marginal Variance method.

The risk load for the initial contract is the variance risk load multiplier times the variance or $0.0000005273(31,607)^2 = 527$. The total risk load is $0.0000005273(47,410)^2 = 1185$. The marginal risk load for the second contract is 1185 – 527 = 658.

1. Continued

(e) Calculate the renewal risk load for each contract using the Marginal Variance method.

The variance of the new contract is $500{,}000^2(0.001)(0.999) = 249{,}750{,}000$. Multiply by the variance risk load multiplier to obtain $0.0000005273(249{,}750{,}000) = 132$. The renewal risk load for the original contract is $1185 - 132 = 1053$. The renewal risk load for the new contract is $1185 - 527 = 658$.

2. Learning Objectives:

4. The candidate will understand how to apply the fundamental techniques of reinsurance pricing.

Learning Outcomes:

(4b) Calculate the price for a property per risk excess treaty.

(4e) Describe considerations involved in pricing property catastrophe covers.

Source:
Basics of Reinsurance Pricing, Clark

Solution:

(a) Calculate the total losses recoverable under each treaty.

	A	B	C	D	E
Surplus Insured Value	0	10,000	6,000	10,000	2,000
Surplus Ceded %	0	50%	75%	80%	50%
Surplus Ceded Loss	0	8,000	2,400	10,000	600
Surplus Retained Loss	400	8,000	800	2,500	600
XS Cession	0	4,000	0	1,500	0
XS Retained	400	4,000	800	1,000	600

Surplus Insured Value = min(10,000, max(0, Insured Value – 2,000))
Surplus Ceded % = Surplus Insured Value/Insured Value
Surplus Ceded Loss = Surplus Ceded % * Loss
Surplus Retained Loss = Loss – Surplus Ceded Loss
XS Cession = min(4,000, max(0, (Surplus Retained Loss – 1,000)))
XS Retained = Surplus Retained Loss – XS Cession

Surplus share treaty recoverable losses = 0 + 8,000 + 2,400 + 10,000 + 600 = 21,000.
Excess of loss treaty recoverable losses = 0 + 4,000 + 0 + 1,500 + 0 = 5,500.
Catastrophe treaty recoverable losses = 400 + 4,000 + 800 + 1,000 + 600 – 6,000 = 800.

(b) Calculate the reinstatement premium for the catastrophe treaty.

Reinstatement premium = premium * (loss in layer / layer) * factor
= 600 (800/8,000)(1.25) = 75.

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2. Continued

(c) Discuss whether a reinstatement pro-rata as to time would be appropriate for this type of cover.

Reinstatement pro-rata as to time is uncommon and usually inappropriate for windstorm coverage, which is seasonal. That is, exposure to risk is not uniform over the coverage period.

(d) Explain with an example why a catastrophe cover is usually written on a losses occurring basis rather than on a risks attaching basis.

A losses occurring basis provides cover for events in the treaty coverage period, regardless of when policies are written. Risks attaching basis provides cover for events covered by policies written during the treaty coverage period. This can lead to the reinsurer paying double for same event.

Example: Consider a hurricane in August 2017.
2016 Treaty on risks attaching basis will cover this event for all policies written in 2016 that do not expire before August 2017.
2017 Treaty will also cover the event for all policies written in 2017 (up to August).
So the reinsurer can end up paying double the limit for the August hurricane.
If on a losses occurring basis, only the 2017 Treaty will cover the 2017 hurricane.

(e) Calculate the expected losses in the excess layer underwritten by Windy for each of the following properties:

(i) Property A

(ii) Property B

		A	B
(1)	Insured Value	2,000	20,000
(2)=0.05*(1)	Expected Loss	100	1,000
(3)=from (a)	Surplus Share Ceded %	0%	50%
(4)=[1 – (3)]*(2)	Expected Loss after Surplus Share	100	500
(5)=(1)*[1 – (3)]	Insured Value after Surplus Share	2,000	10,000
(6)=[4000 + 1000]/(5)	XSmax/InsValue	250%	50%
(7)=from table	High Factor	100%	70%
(8)=1000/(5)	XSattach/InsValue	50%	10%
(9)=from table	Low Factor	70%	37%
(10)=(7) – (9)	Difference	30%	33%
(11)=(10)*(4)	Expected Loss(layer)	**30**	**165**

3. **Learning Objectives:**

2. The candidate will understand the considerations in selecting a risk margin for unpaid claims.

Learning Outcomes:

(2a) Describe a risk margin analysis framework.

(2b) Identify the sources of uncertainty underlying an estimate of unpaid claims.

(2c) Describe methods to assess this uncertainty.

Source:

A Framework for Assessing Risk Margins, Marshall, et al.

Solution:

(a) Calculate the internal systemic risk coefficient of variation for the home line of business.

First calculate the external systemic risk CoV for both lines combined. It is

$$\sqrt{\left(\frac{3,000}{10,000}\right)^2[2^2+1^2+3^2]+\left(\frac{7,000}{10,000}\right)^2[2^2+1^2+1^2]}=2.05.$$

Let *Y* be the internal systemic risk CoV for both lines combined. Then,

$$9.6=\sqrt{8^2+Y^2+2.05^2}\Rightarrow 92.16=Y^2+68.2025\Rightarrow Y=4.89.$$

Let *X* be the internal systemic risk CoV for the home line. Then,

$$4.89=\sqrt{0.7^2(X^2)+0.3^2(3^2)+2(0.75)(0.7)(0.3)(X)(3)}$$

$$23.9121=0.49X^2+0.945X+0.81$$

$$X=5.97.$$

(b) Propose an approach that can be used if external systemic risk categories are partially correlated within or between valuation classes.

Aggregate the categories into broader categories that are not correlated with other risk categories.

(c) Define hindsight analysis.

Hindsight analysis compares past estimates of outstanding claim liabilities against the latest view of the equivalent liabilities.

(d) Contrast the usefulness of hindsight analysis for short-tail and long-tail portfolios.

Hindsight analysis is more useful for short-tail portfolios because there is less serial correlation between consecutive valuations.

4. Learning Objectives:

4. The candidate will understand how to apply the fundamental techniques of reinsurance pricing.

5. The candidate will understand methodologies for determining an underwriting profit margin.

Learning Outcomes:

(4a) Calculate the price for a proportional treaty.

(5a) Calculate an underwriting profit margin using the target total rate of return model.

Sources:

Basics of Reinsurance Pricing, Clark

Ratemaking: A Financial Economics Approach, D'Arcy and Dyer

Solution:

(a) Explain why owners' equity is difficult to determine.

Commentary on Question:

Either of the following two responses is sufficient.

- Insurers do not set rates in aggregate, but on a by-line and by-state basis. However, equity is normally only calculated in aggregate.
- Statutory surplus is generally lower than actual equity due to ignoring time value of money, excluding some assets, and valuing some assets at other than market value.

(b) Calculate the premium.

$$UPM = -1(0.02) + 1.5(0.04) = 0.04$$

$$0.07 = \frac{P+75}{100}(0.02) + \frac{P}{100}(0.04) = 0.0006P + 0.015$$

$$P = 91.67$$

4. Continued

(c) Calculate the Total Rate of Return under this reinsurance offer.

Prior to the reinsurance:

$$UPM = 1 - \frac{L}{P} - \frac{E}{P};\ 0.04 = 1 - \frac{L}{91.67} - \frac{25}{91.67};\ L = 0.96(91.67) - 25 = 63.$$

With reinsurance:

Equity = 100(1 – 0.4) = 60,

Expenses = 25 – 0.35(0.40)91.67 = 12.17,

Retained premium = 91.67(0.6) = 55, $UPM = 1 - \frac{0.6(63)}{55} - \frac{12.17}{55} = 0.0915$,

$$TRR = \frac{55 - 12.17 + 60}{60}(0.02) + \frac{55}{60}(0.0915) = 0.118 = 11.8\%.$$

5. Learning Objectives:

1. The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:

(1b) Test for the validity of these assumptions.

(1d) Estimate the standard deviation of a chain ladder estimator of unpaid claims.

Sources:

Measuring the Variability of Chain Ladder Reserve Estimates, Mack
Testing the Assumptions of Age-to-Age Factors, Venter

Solution:

(a) Demonstrate that the value of α_4^2 was correctly calculated. (Your calculation need not match to all four decimal places.)

$$\alpha_4^2 = \frac{1}{2}\left[18,546\left(\frac{18,128}{18,546} - 0.95408\right)^2 + 23,304\left(\frac{22,887}{23,304} - 0.95408\right)^2 + 22,854\left(\frac{20,718}{22,854} - 0.95408\right)^2\right] = 40.0504$$

(b) Calculate the standard error of the reserve estimator for accident years 4 and 5 combined.

The standard error is the square root of

$$1,761^2 + 1,514^2 + 42,644(27,507)(2)\left(\frac{40.0504/0.95408^2}{18,546 + 23,304 + 22,854} + \frac{0.00098/1.02128^2}{18,128 + 22,887} + \frac{0.000000024/1.02004^2}{18,517}\right),$$

which is 2,644.

(c) Calculate the test statistic suggested by Venter to test the significance of this correlation.

$$0.574\left(\frac{5-2}{1-0.574^2}\right)^{0.5} = 1.214$$

5. Continued

(d) Determine whether this correlation is significant.

The test statistic has a *t* distribution with three degrees of freedom. At any reasonable significance level the null hypothesis of no correlation is not rejected. There is no evidence of significant correlation.

(e) Demonstrate that the test statistic suggested by Mack to test for a calendar year effect is equal to 1.

The calculation is in the following table:

j	S_j	L_j	Z_j
2	0	2	0
3	2	0	0
4	1	2	1
5	0	5	0
6	5	0	0
Total			1

(f) Determine whether there is a significant calendar year effect and what this indicates about the use of the chain ladder method in this case.

The test statistic is (1 – 4.875)/1.196 = –3.24 standard deviations below the mean. It is significant at any reasonable significance level and thus there is a significant calendar year effect. The chain ladder method may not be appropriate.

6. Learning Objectives:

1. The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:

(1e) Apply a parametric model of loss development.

(1f) Estimate the standard deviation of a parametric estimator of unpaid claims.

Source:

LDF Curve Fitting and Stochastic Reserving: A Maximum Likelihood Approach, Clark

Solution:

(a) Provide two examples of situations where this assumption might not hold.

Commentary on Question:

Any two of the following are sufficient.

- There may be positive correlation if all periods are equally affected by a change in loss inflation.
- There may be negative correlation if a large settlement in one period replaces a stream of payments in later periods.
- Different risks and mixes of business may have been written in each period with possibly different claims handling and settlement strategies, resulting in different emergence patterns.

(b) Explain why the variance estimates are an approximation.

Commentary on Question:

Any two of the following are sufficient.

- The variance/mean scale parameter is estimated.
- The functions are nonlinear, so the lower bound does not provide exact variance estimates.
- The true lower bound is based on the expected value of the matrix of second derivatives, but Clark approximates it with the observed information matrix.

(c) Calculate the maximum likelihood estimate of *ELR*.

$G(6)=1-e^{-6/6.689}=0.5922$, $G(18) = 0.9322$, $G(30) = 0.9887$.

4,369 = ELR*[10,000(1 – 0.9887) + 8,500(1 – 0.9322) + 12,000(1 – 0.5922)] = 5,583(ELR). ELR = 78.26%.

6. **Continued**

(d) Estimate the expected payments in 2018 for accident year 2017.

Ultimate = 12,000(0.7826) = 9,391.
Expected in 2018 = 9,391(0.9322 – 0.5922) = 3,193.

7. **Learning Objectives:**

3. The candidate will understand excess of loss coverages and retrospective rating.

Learning Outcomes:

(3e) Explain Table M and Table L construction in graphical terms.

(3f) Explain the limiting case in retrospective rating.

Source:

The Mathematics of Excess of Loss Coverages and Retrospective Rating – A Graphical Approach, Lee

Solution:

(a) Identify the areas on the graph that correspond to each of the following:

(i) k, the loss elimination ratio

(ii) $1-k$

(iii) $\psi(r_1)$, the Table M savings at entry ratio r_1

(iv) $\psi^*(r_1)$, the Table L savings at entry ratio r_1

(v) $\varphi(r_2)$, the Table M charge at entry ratio r_2

(vi) $\varphi^*(r_2)$, the Table L charge at entry ratio r_2

Commentary on Question:

This graph matches Figure 18 in Lee.

(i) k is areas II, V, and VIII

(ii) $1-k$ is areas III, VI, and IX

(iii) $\psi(r_1)$ is area VII

(iv) $\psi^*(r_1)$ is areas VII and VIII

(v) $\varphi(r_2)$ is areas II and III

(vi) $\varphi^*(r_2)$ is areas II, III, V, and VIII

7. **Continued**

(b) Determine each of the following:

(i) The limit of the Table M charge as the entry ratio goes to infinity

(ii) The limit of the Table L charge as the entry ratio goes to infinity

(i) The Table M charge is areas II and III. As r_2 goes to infinity, both regions shrink to have an area of zero and thus the limit is zero.

(ii) The Table L charge is areas II, III, V, and VIII. Areas II, V, and VIII are equal to k. Because area III goes to zero, the limiting charge is k.

8. Learning Objectives:

4. The candidate will understand how to apply the fundamental techniques of reinsurance pricing.

Learning Outcomes:

(4d) Apply an aggregate distribution model to a reinsurance pricing scenario.

Source:

Basics of Reinsurance Pricing, Clark

Solution:

(a) Calculate the probability that aggregate flood losses will be:

(i) 4 billion

(ii) 8 billion

Commentary on Question:

The probabilities can also be calculated using the recursive formula. That solution is not presented here, but would earn full credit.

Aggregate losses of 8 can occur only if there are two claims and both are for 4. The probability that this happens is
$\Pr(N = 2)\Pr(S = 4)\Pr(S = 4) = 0.25(0.25)(0.25) = 0.015625.$
The only possible aggregate losses are 0, 1, 2, …, 8. Thus the probability that aggregate losses are 4 is 1 minus the sum of the other probabilities, which is 0.171875.

Alternatively, the probability of 4 can be calculated directly. It can arise from a single claim of 4 (probability = 0.5(0.25) = 0.125), two claims with values 1 and 3 (probability = 0.25(0.25)(0.25) = 0.015625), two claims with values 2 and 2 (probability = 0.015625), or two claims with values 3 and 1 (probability = 0.015625). The total probability is 0.171875.

(b) Explain the advantages of using a recursive formula.

Commentary on Question:

Both advantages are required for full credit.

- The formula is simple to work with.
- The formula is efficient when the frequency is low.

8. **Continued**

(c) Calculate the mean and coefficient of variation of aggregate flood losses.

$$E(N) = 2(0.5) = 1$$
$$Var(N) = 2(0.5)(0.5) = 0.5$$
$$E(S) = 0.25(1+2+3+4) = 2.5$$
$$Var(S) = 0.25(1+4+9+16) - 2.5^2 = 1.25$$
$$E(A) = 1(2.5) = 2.5$$
$$Var(A) = 1(1.25) + 0.5(2.5^2) = 4.375$$
$$CoV(A) = 4.375^{1/2} / 2.5 = 0.837.$$