SOCIETY OF ACTUARIES
Quantitative Finance and Investment Advanced Exam

Exam QFIADV

AFTERNOON SESSION

Date: Thursday, April 26, 2018
Time: 1:30 p.m. – 3:45 p.m.

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This afternoon session consists of 6 questions <u>numbered 10 through 15</u> for a total of 40 points. The points for each question are indicated at the beginning of the question.

2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.

3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.

2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.

3. The answer should be confined to the question as set.

4. When you are asked to calculate, show all your work including any applicable formulas. When you are asked to recommend, provide proper justification supporting your recommendation.

5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate morning or afternoon session for Exam QFIADV.

6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d'examen pour la version française.

10. (5 *points*) You are hired as a quantitative analyst at a hedge fund company. You have been asked to study and model stock returns.

    (a)   (*1.5 points*) Compare characteristics of the three types of factor models:

        (i)   Macroeconomic factor models

        (ii)   Fundamental factor models

        (iii)   Statistical factor models

    (b)   (*1 point*) Compare the two approaches to fundamental factor models:

        (i)   BARRA approach
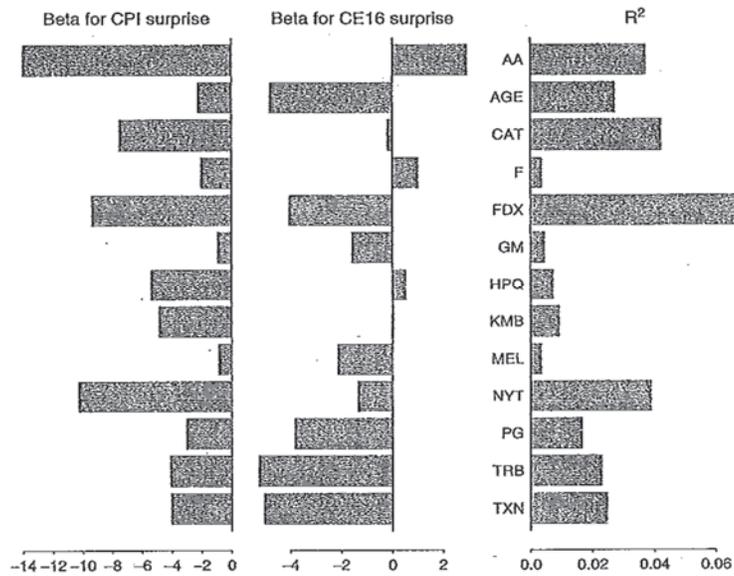
        (ii)   Fama-French approach

Your manager has picked 13 stocks from a variety of industry sectors to construct a portfolio. You have decided to use the macroeconomic factor model to study excess returns for the 13 stocks. The factors used consist of unexpected changes or surprises of two variables:

- Consumer price index (CPI) for all urban consumers
- Civilian employment numbers 16 years and over (CE16)

Both CPI and CE16 data span from 1975 to 2003 and are seasonally adjusted.

10. Continued

Chart below shows the bar plot of the beta estimates and R^2 for the 13 stocks:

The correlation matrix of the returns is very close to the identity matrix.

(c)   (*1.5 points*) Interpret the information shown above.

(d)   (*1 point*) Outline and justify an approach for further analyzing these 13 stock returns.

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11. (*5 points*) You are working on the hedging of a GLWB product that uses an actively managed equity mutual fund as its underlying investment.

(a) (*1 point*) Describe the major steps of simulating a Greeks-based hedging strategy.

You have decided to hedge equity risk and interest rate risk using a delta and rho hedging strategy.

(b) (*2 points*)

    (i) Identify three risks that cannot be managed by such a strategy.

    (ii) Propose a way to reduce exposure to each of these three risks.

The table below shows the unhedged, market-consistent balance sheet positions at current time and one year in the future under five different scenarios. You assume that the GLWB product is the only liability and statutory capital is based on the conditional tail expectation (CTE) measure.

	Asset	Liability	Statutory Capital	Surplus
Current Balance Sheet	500,000	350,000	28,000	122,000
Balance Sheet at 1 Year				
Scenario 1	518,271	348,877	27,690	141,704
Scenario 2	461,810	332,519	28,087	101,204
Scenario 3	512,010	368,317	28,276	115,417
Scenario 4	465,192	298,179	27,817	139,196
Scenario 5	526,170	374,988	26,981	124,201

(c) (*1 point*) Describe the simulations required to obtain the values in the table above.

(d) (*1 point*) Describe the likely impacts that a highly effective hedging program would have on the projections above.

12. (*7 points*) You are an actuary in the pricing department of ABC Insurance. The company is in the initial stages of pricing a new variable annuity product with a guaranteed minimum accumulation benefit (GMAB). The accumulation benefit is based on the maximum return of four domestic and foreign stock and bond indices. You have decided that hedging activities will be solely focused on delta and rho exposures.

(a)　　(*2 points*) Describe how the binomial lattice and Monte Carlo methods can be used to price derivative securities.

(b)　　(*1 point*) Recommend one of the above methods to use to value the GMAB at ABC Insurance.

Your manager is interested in being able to simulate a stochastic model for regulatory capital purposes and to demonstrate the effectiveness of the hedging activities over the lifetime of the product. She wishes to be able to run the model on a daily basis.

(c)　　(*1 point*) Describe why the use of a nested stochastic model is preferred in this context and the challenges of doing so.

(d)　　(*1 point*) Describe how proxy functions can be used as an alternative to nested simulations.

(e)　　(*2 points*) Outline one approach to fit the proxy function to Monte Carlo estimates using the least squares approach.

13. (*8 points*) Adam, Caroline, and David are 6th grade elementary students at NuSchool. Eric is in the 3rd grade at the same school.

Adam lent to Eric 1,000 marbles until the end of the semester (in 20 weeks) at the usual weekly fee for 3rd graders of 5 marbles per 100 each week, payable at the end of the week. The full amount of 1,000 marbles is to be returned at the end of 20 weeks and the periodic fee is payable each week. Adam is nervous about the prospect of getting his 1,000 marbles back. Caroline, Eric's big sister, offered Adam the opportunity to guarantee the loan. The agreement is that if at any point Eric fails to pay the weekly charge, or return the 1,000 marbles at the end, Caroline will top up Adam's shortfall, up to 1,000.

Of course, nothing being free, Caroline is asking Adam to give her 5 marbles at the beginning of every week as compensation for this guarantee. Also, Eric will then deal with Caroline instead of Adam in the event he is unable to pay Adam.

Eric's ability to pay will be assessed at the end of every week, at the payment date.

(a)   (*1 point*)

　　(i)   Describe a financial instrument that the above agreement resembles.

　　(ii)   Sketch a graph showing each participant and indicate their respective roles within this financial agreement.

The fee usually charged on a loan to 6th graders like David, whom are considered riskless, is usually 3 marbles per 100 marbles per week. There is nothing known that would justify asking Eric a different fee for the loan than for any other 3rd graders.

(b)   (*1 point*) Recommend a deal that David could propose to either side for his own benefit.

The scenarios below for parts c), d) and e) are independent of each other.

Scenario 1: Assume after 15 weeks of poor results Eric decides to leave the marbles game. In this scenario Eric has 600 marbles that Adam is entitled to receive.

(c)   (*2 points*) Calculate the profit and loss for Adam and Caroline showing the flow of marbles for all participants.

13. Continued

Scenario 2: Assume at the end of 18 weeks, Eric leaves the game and hands all of his marbles to Adam. Caroline covers the difference as promised, after doing so her net profit is zero.

(d) (*1 point*) Calculate how many marbles Eric left to Adam when he left the game.

Scenario 3: Assume at the end of the 17th week, before Adam makes his payment he wants to terminate the deal. Eric owns 900 marbles at that moment and both Caroline and Adam assume they are likely to get that number of marbles back from him. To be fair, Caroline thinks that she should be compensated for the payment forfeited. She also calculates the amount of compensation if the agreement is terminated at the end of the 18th or 19th week.

Compensation to pay by Adam to Caroline if termination occurs at the end of the week and before the next weekly payment:

  Week 17: 6 marbles
  Week 18: 5 marbles
  Week 19: 3 marbles

To reduce calculations, you are given the probability of default in year 18

$$q_{18} = 0.03780952$$

(e) (*3 points*) Calculate, from this information, the probability that Eric will fail on his commitment sometime from the end of week 17 to end of week 20.

14. (*9 points*) You are an investment actuary responsible for managing credit risk.

Your colleague approaches you to peer review his work in credit risk modelling. He uses a two-state Markov-chain (the two states being survival and default) to model the credit risk of a company's zero-coupon bonds. He presents you with the following assumptions:

- The bond recovery rate is 65%.
- The current value of a 6-month zero-coupon bond with face value of $100 is $88.50.
- The annual continuously compounded risk-free interest rate is 2%.

(a) (*1.5 points*) Calculate the implied risk-neutral default intensity based on the assumptions above.

You want to model the term structure of default probability. Your chief actuary suggested to use a Merton-style approach as described in "An Introduction to Credit Risk Modeling" by *Bluhm, Overbeck, and Wagner* with the following assumptions:

- The current market value of the company's assets is $100 million.
- The company's default point is $75 million.
- The asset volatility is 30%
- The annual continuously compounded expected return of the company's assets is 7%.
- The annual continuously compounded risk-free interest rate is 2%.

(b) (*2 points*) Calculate:

(i) The real cumulative default probability from time 0 to time 2

(ii) The risk-neutral cumulative default probability from time 0 to time 2

14. Continued

In addition, you want to establish a relationship between the risk-neutral default probability and the real cumulative default probability using the Merton-style approach and the continuous time CAPM. Let:

σ_m = the volatility of the market

$\rho_{a,m}$ = the correlation between the asset and market returns

π = the market risk premium

PD_t^{Real} = the real cumulative default probability from time 0 to time t

PD_t^{RN} = the risk-neutral cumulative default probability from time 0 to time t

(c)   (*2 points*) Show that:

$$PD_t^{RN} = N\left(N^{-1}\left(PD_t^{Real}\right) + \rho_{a,m}\frac{\pi}{\sigma_m}\sqrt{t} \right)$$

Your chief actuary also asked you to develop an internal Cox-Ingersoll-Ross (CIR) stochastic intensity model to estimate default probabilities. The following defines the process used in the CIR stochastic intensity model:

$$d\lambda_t = k\left(\theta - \lambda_t\right)dt + \sigma\sqrt{\lambda_t}dB_t$$

where B_t is a standard Brownian motion under the risk-neutral measure. The coefficients θ and σ are all positive constants.

You are given:

- The long-term mean of λ_t as t tends to infinity is 2%.
- The long-term variance of λ_t as t tends to infinity is 0.0125%.
- The mean of λ_1 as seen from time 0 is 2.4493%.
- $k = 0.8$

(d)   (*1.5 points*) Calculate the variance of λ_1 as seen from time 0.

(e)   (*1 point*) Explain the impact of k and σ on the variance of λ_t.

(f)   (*1 point*) Evaluate the use of the CIR model for estimating the default probabilities.

15. (*6 points*) ABC Company holds a fixed-income portfolio of 10 bonds with total market value of 100 million. ABC Company has an investment policy that allows only fixed income investments. Your manager, the Chief Investment Officer, suggests using CreditMetrics to analyze the credit risk of the portfolio.

(a) (*2 points*) Describe the CreditMetrics approach and its key assumptions.

The following are the results from Monte Carlo simulation with 10,000 simulations.

Portfolio Value with coupon (million)	Frequency
95	128
100	313
105	9,146
110	299
115	114

(b) (*2 points*) Verify that   (units in millions)

      (i)   the expected portfolio value is   105
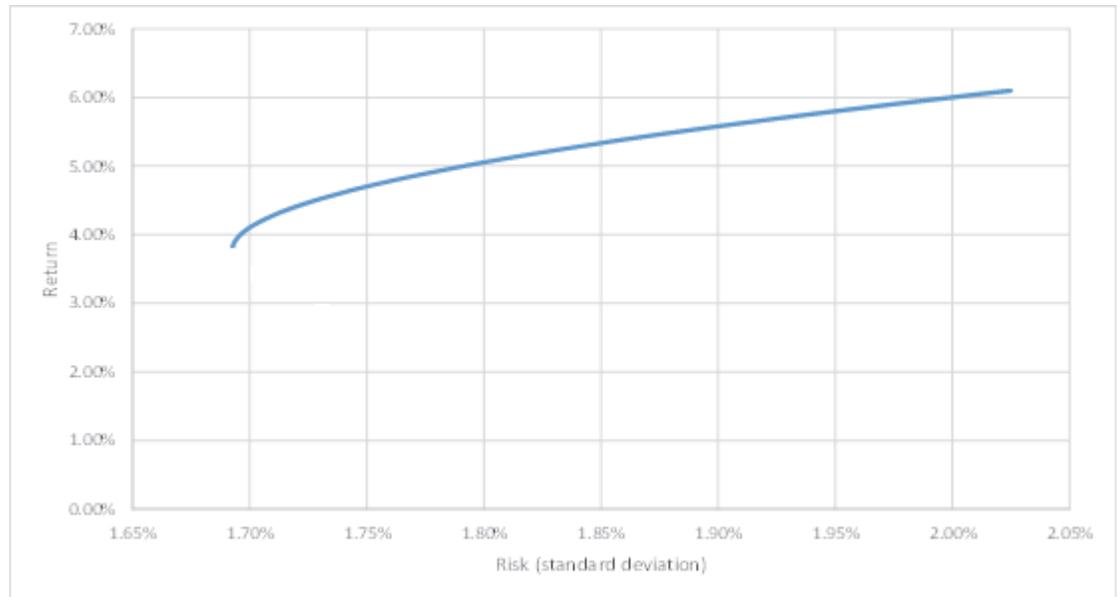
      (ii)   the standard deviation is   2

      (iii)   the 1% value is   10

15. Continued

One of your colleagues has developed the efficient frontier of the 10 bonds.

(c)   (*1 point*)

   (i)   Define the efficient frontier.

   (ii)   Assess if the current portfolio is efficient.

(d)   (*1 point*) Recommend two approaches that could be applied to improve the current fixed income portfolio.

END OF EXAMINATION
Afternoon Session

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