Quantitative Finance and Investment Advanced Exam

Exam QFIADV

AFTERNOON SESSION

Date: Thursday, November 1, 2018
Time: 1:30 p.m. – 3:45 p.m.

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This afternoon session consists of 7 questions numbered 10 through 16 for a total of 40 points. The points for each question are indicated at the beginning of the question.

2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.

3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.

2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.

3. The answer should be confined to the question as set.

4. When you are asked to calculate, show all your work including any applicable formulas. When you are asked to recommend, provide proper justification supporting your recommendation.

5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate morning or afternoon session for Exam QFIADV.

6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

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Schaumburg, IL 60173-2226

Printed in the U.S.A.
Exam QFIADV

****BEGINNING OF EXAMINATION****
Afternoon Session
Beginning with Question 10

10. (*5 points*) You are assessing relative "richness" of Lookback Ratchet and Remaining Withdrawal Benefit Base Ratchet features on a variable annuity, in terms of triggering an increase in guaranteed annual withdrawals.

(a) (*0.5 points*) Explain which of these two features is "richer".

You are analyzing a policy with the following parameters:

- Acquisition charge $\Phi^{acq} = 0\%$,
- administration charges $\Phi^{adm} = 0.5\%$,
- guarantee charges $\Phi^{guar} = 0.8\%$,
- guaranteed withdrawal rate $X_{WL} = 10\%$,
- single premium $P = \$50,000$,
- Withdrawals start immediately after inception at time 0.

You assume the underlying fund growth rate from time 0 to time 1 is 10% and the account value evolves according to the formula below:

$$AV_{t+1}^{-} = AV_t^{+} \times \frac{S_{t+1}}{S_t} \times \exp\left(-\Phi^{adm} - \Phi^{guar}\right)$$

(b) (*2 points*) Demonstrate your answer to (a) numerically.

Trigger time is defined as the point in time when the account value drops to zero and the guarantee is triggered.

(c) (*1 point*)

(i) Compare the probability distribution of the trigger time for the guarantee of the 50% Performance Bonus (PB) feature versus the No Ratchet (NR) feature reviewed in the Stochastic Volatility paper by Kling, Ruez, and Russ.

(ii) Explain the shape of the probability distribution of the trigger time for the Performance Bonus in relation to the bonus feature.

10. Continued

The Heston model is often used for modeling the financial market:

$$dS(t) = \mu S(t)dt + \sqrt{V(t)}S(t)\left(\rho dW_1(t) + \sqrt{1-\rho^2}dW_2(t)\right), S(0) \geq 0$$
$$dV(t) = k\left(\theta - V(t)\right)dt + \sigma_v\sqrt{V(t)}dW_1(t), V(0) \geq 0$$

where W_1, and W_2 are independent Brownian motions under the real-world measure P.

You are given the following parameters:

Parameter	Value
θ	$(0.22)^2$
k	4.75
σ_v	0.55
ρ	-0.569
$V(0)$	0.25

(d) (*0.5 points*) Explain the purpose of the equivalent local martingale measure (EMM).

You are given the market-price-of-volatility-risk process $\gamma_1(t) = \lambda\sqrt{V(t)}$.

(e) (*1 point*)

(i) Determine if EMM exists for each of the following market prices of volatility risk λ, assuming underlying model is the Heston Model.

$\lambda_1 = 5, \lambda_2 = 0$, and $\lambda_3 = -10$,

(ii) Determine which λ has the highest corresponding fair guaranteed withdrawal rate (among the λ values for which EMM exists).

11. (*5 points*) You are a pricing actuary for Company XYZ. You are developing a new Single Premium Variable Annuity (SPVA) product. The final product design has GLWB and 100% GMMB guarantees.

You are performing an analysis on the following sample policy:

Initial deposit: $100,000
Annual expense charge: 120 bps (deducted at start of year)
Initial fund allocation (no future rebalancing takes place):

- 50% S&P 500 index
- 50% investment fund earning 4% a year

You apply a dynamic lapse multiplier, calculated as:

$$\max\left(8\%, 1-80\%\left(X-1.1\right)\right) \text{ if } X>1.1, \text{ or 1 otherwise}$$

where

$$X = PV \text{ of GLWB/Account Value}$$

The effective lapse rate in a given year is determined by multiplying the lapse rate by the dynamic lapse multiplier.

You are given:

Year	Lapse rate	Surrender charge	S&P 500 index return	PV of GLWB (at end of year)
1	7%	8%	12%	
2	8%	6%	10%	
3	9%	4%	-20%	
4	10%	2%	4%	
5	20%	0%	5%	$124,120.35
6	20%	0%	N/A	N/A

(a) (*2 points*) Calculate the effective lapse rate applicable in year 6, assuming the policy does not lapse during the first 5 years.

11. Continued

Low risk-free interest rates combined with a high level of equity implied volatility have significantly increased the cost of guarantees embedded in the VA product. As a result, your investment department has recommended XYZ consider target volatility funds.

(b) (*1 point*) Describe how a target volatility mechanism works.

To estimate historical equity volatilities, the Exponentially Weighted Moving Average (EWMA) estimator is used. Your company decides to rebalance the portfolio on Wednesday of each week, and assumes an exponential decay rate of 0.9.

(c) (*0.5 points*) Calculate the mean age of the historical data.

Your modeling team would like to more accurately capture the cost of guarantees embedded in the VA product. They have decided to estimate the equity volatility – and hence the target volatility fund return – using the Stochastic Volatility Jump Diffusion (SVJD) model.

Your Chief Actuary is delighted with the model choice, as the S&P 500 Index had performed consistently well in recent years, and experienced some large positive daily movements. He believes that to incorporate these large positive movements would result in a lower cost of guarantees as the Account Value would grow substantially. Hence, the company could enjoy longer fee income and minimal GLWB cost until later years of the product.

(d) (*1.5 points*) Critique your Chief Actuary's opinion.

12. (7 *points*) Your manager is reviewing the interest rate model your team has been using for pricing caps and has the following two requests:

> First request: Make the σ in your model time-dependent.
> Second request: Use a two-factor interest rate model.

You have been using the following interest rate model:

$$dr(t) = \left[\theta(t) - ar(t)\right]dt + \sigma dW(t)$$

You want to calibrate the model per your manager's first request.

(a) (*1 point*) Discuss possible issues in calibrating the model.

(b) (*1 point*)

(i) Discuss how to obtain the implied volatility of a cap.

(ii) Explain how to calibrate your model to cap-volatility market data.

(c) (*0.5 points*) Explain advantages of two-factor models over one-factor models.

(d) (*0.5 points*) Identify two possible situations where one-factor models are as useful as two-factor models.

You now follow your manager's second request and look into two-factor models.

(e) (*2 points*) Compare and contrast the two interest rate scenario models: G2++ versus CIR2++.

12. Continued

Your manager decided to use a G2++ model and requested you to calibrate it to both cap-volatility data and swaption-volatility data, which leads to the following two sets of parameters:

Parameter	Result 1	Result 2
a	0.5437	0.7735
b	0.0756	0.0820
σ	0.0059	0.0223
η	0.0013	0.0104
ρ	-0.9914	-0.7019

(f) (*2 points*) Provide your observations on these results and a recommendation to your manager regarding the model to be used and the calibration method.

13. (*8 points*) The Chief Investment Officer (CFO) at Mortgage Bank A has been tasked with creating a group that governs models used at the bank. He has decided to create a committee with 10 members, all of whom work on the investment team that he manages. He has created the following agenda for the upcoming first meeting of the Modelling Committee.

- Introduction of committee and members – 5 minutes
- Discuss investment policy – 5 minutes
- Discuss potential mortgage loans – 30 minutes

(a) (*1 point*) Identify and describe the sociological conditions of this committee that could impact its decision making.

The committee's discussion moves to potential mortgage loans. You discuss the 2006 subprime mortgage vintage.

(b) (*1 point*) Explain the relationship between borrower's credit score, debt-to-income (DTI) ratio and borrower's default rates.

The CFO has asked you to look at using Principal Component Analysis (PCA) to identify the probability of default in Mortgage Lending for Mortgage Bank A.

You are given the following:

Let r_1, r_2 and r_3 be the explanatory variables for the Principal Component Analysis:

- r_1 - Debt-to-income (DTI) ratio
- r_2 - 1/100 of borrower's credit score
- r_3 - Employment transfer
- $r = (r_1, r_2, r_3)$

The random variables r_1, r_2 and r_3 have the covariance matrix:

$$Q = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

13. Continued

- $\lambda_1, \lambda_2, \lambda_3$ are the eigenvalues corresponding to the principal components PC1, PC2 and PC3, respectively using covariance matrix.
- $\lambda_1 > \lambda_2 > \lambda_3 > 0$
- $\lambda_2 = 2$
- The determinant $\left|Q - (I)\ \lambda\right| = -\lambda^3 + 8\lambda^2 - 13\lambda + 2,$ where (*I*) is the identity matrix
- The normalized eigenvectors are:
  - $e_1^{'} = \mathrm{k}_1 * \left(-\sqrt{2} + 1, 1, 0\right)$
  - $e_2^{'} = (0, 0, 1)$
  - $e_3^{'} = \mathrm{k}_3 * \left(\sqrt{2} + 1, 1, 0\right)$

    where $k_1 < 0,\ k_3 > 0$
- The i-th principal component of r is $y_i = e_i^{'} r$, where i =1, 2, 3

(c) (*2 points*) Demonstrate that

(i) $$\mathrm{k}_1 = -\sqrt{\frac{1}{\left(4 - 2\sqrt{2}\right)}} = 0.92388$$

(ii) Variance of $y_1 = \lambda_1$

(iii) Covariance of y_1 and $y_2 = 0$

(d) (*1.5 points*) Describe:

(i) the shortcomings of performing PCA on data that has a covariance matrix like Q;

(ii) how to overcome those shortcomings; and

(iii) any general requirements that the data should meet for PCA to be successful.

Question 13 continued on the next page.

13. Continued

In addition, Mortgage Bank A is considering the following two models for assessing interest rate risk:

I. Model 1 for the short rate:

$$d\ln\left(r(t)\right)=\left[\theta(t)-a\ln\left(r(t)\right)\right]dt+\sigma dW(t), r(0)=r_0$$

where r_0 is a positive constant and $\theta(t)$ is deterministic function of time t

II. Model 2 for the short rate:

$$dr(t)=k\left[\theta(t)-r(t)\right]dt+\sigma dW(t), r(0)=r_0$$

where r_0, k and σ are positive constants

Model Selection Criteria:

- Model displays an affine term structure.
- Model does not share the explosion of bank account.
- Model is analytically tractable.

Considering a general risk-neutral dynamic for the short rate,

$$dr(t)=b\left(t,r(t)\right)dt+\sigma\left(t,r(t)\right)dW(t)$$

(e) (*1.5 points*) Assess, for each of Models 1 and 2, whether there exist conditions on $b\left(t,r(t)\right)$ and $\sigma\left(t,r(t)\right)$ such that it displays an affine term structure.

(f) (*1 point*) Assess, for each of Models 1 and 2, whether they satisfy the other two model selection criteria.

14. (*5 points*) You are an investment actuary for NOP Life Insurance. You are preparing a commentary on the most recent monthly performance attribution of NOP's investment portfolio.

(a) (*2 points*) Describe each step of a performance attribution algorithm.

NOP Life invests exclusively in fixed income securities. Its performance attribution model calculates outperformance due to yield curve, implied volatility, mortgage, and residual as common factors, using bottom-up aggregation. Outperformance due to spread duration is calculated using the absolute allocation algorithm.

You are given the following Portfolio Outperformance Details Report:

Portfolio Outperformance Details Report (bps)	
Yield Curve	33.2
Implied Volatility	0.2
Asset Allocation	-3.4
Security Selection	-5.0
Mortgage	-2.0
Residual	2.0
Total	25.0

NOP Life is now considering changing its performance attribution model from the fully analytical sector-level model reported above to a fully analytical, top-level model that treats spread duration as a common factor.

(b) (*1 point*) Discuss the reasons why NOP Life might want to make this change to its attribution model.

The following information regarding spread duration is extracted from the performance attribution results:

NOP Life Investment Portfolio Spread Duration Report	
Portfolio Option Adjusted Spread Duration	5.1
Benchmark Option Adjusted Spread Duration	4.9
Portfolio Change in Option Adjusted Spread	12 bps
Benchmark Change in Option Adjusted Spread	13 bps

(c) (*2 points*) Show how the Portfolio Outperformance Details Report for NOP's fully analytical sector-level model above changes if outperformance due to spread duration is calculated using the relative allocation algorithm.

15. (*5 points*) You have been asked to review the proposed new RBC formula and its possible impact on the company investments and have been provided some information from the ERM, Operations and Valuation teams.

- The ERM team has disclosed that investment strategies that are approved must not exceed a maximum permitted VaR but it is very complex how the VaR changes as investment mix changes.
- The valuation team has indicated that the RBC ratio must not be less than 250% of available capital and the current strategy allows the company to meet this requirement comfortably.
- The operations team indicates that a yield rate on fixed income assets of 5.25% would be strongly preferable than 5%.

The company has never performed an optimization of its investment strategy to account for all of the constraints and has been told by consultants that most of the industry has not done so either.

Here is a table that illustrates the current allocation of assets along with their yield, the current RBC factors, and the proposed RBC factors.

Asset Classification	**Current RBC Factor % of value of assets**	**Proposed RBC Factor % of value of assets**	**Average Yield of assets for classification**	**Amount of assets**
NAIC 1A	0.3	0.21	4.5	40
NAIC 1B-1G	0.3	0.6	4.75	20
NAIC 2	1	1.2	5.25	20
NAIC 3	4	3	6	10
NAIC 4	9	6	7	10

(a) (*2 points*) Outline the most relevant considerations of the current and proposed RBC factors as they impact the ERM, Operations and Valuation teams.

(b) (*1 point*) Recommend how your company might change its investments from its current allocation of fixed income assets based on the RBC proposed factors in respect to regulatory capital arbitrage.

(c) (*2 points*) Explain how the proposed RBC factors encourage, discourage, or do not change the incentives for the use of securitization for the Company.

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16. (*5 points*) This question is concerned with the modeling of mortgage defaults.

(a) (*0.5 points*) State two necessary conditions for a mortgage default to have occurred.

To model home price appreciation (HPA) as a percentage value (denoted by *r%*), you use the following equation:

$$dr = \mu dt + \sigma dW_t$$

where W_t is a standard Brownian motion, and $t = 0$ corresponds to year 2010. Calibration of the model was done using historical data.

An investor obtained a zero-amortizing (interest only) mortgage January 1st, 2010.

You are given:

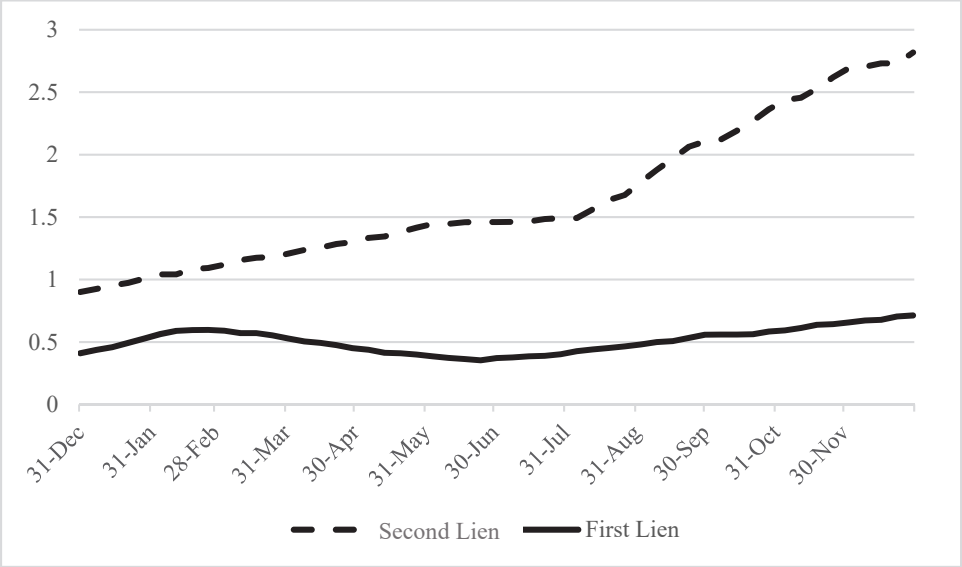
LTV (Loan to Value)	80%
μ	2.1%
σ	3.5%.

(b) (*1.5 points*) Calculate the probability that the investor has negative equity on their property on January 1st, 2018.

(c) (*1 point*) Identify a challenge to predictive models such as the one above.

16. Continued

Recent data from 2017 regarding default rates on first-lien mortgages and second-lien mortgages are as follows:



Your colleague claims that a certain aspect of the above graph reminds her of the lead up to the 2008 mortgage credit crisis.

(d) (*0.5 points*) Evaluate your colleague's claim.

(e) (*1.5 points*) Describe three borrowing and loan characteristics that are key factors in assessing default risk.

****END OF EXAMINATION****
Afternoon Session

USE THIS PAGE FOR YOUR SCRATCH WORK