SOCIETY OF ACTUARIES

Quantitative Finance and Investment Advanced Exam

Exam QFIADV

MORNING SESSION

Date: Thursday, November 1, 2018
Time: 8:30 a.m. – 11:45 a.m.

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has a total of 100 points. It consists of a morning session (worth 60 points) and an afternoon session (worth 40 points).

   a) The morning session consists of 9 questions numbered 1 through 9.

   b) The afternoon session consists of 7 questions numbered 10 through 16.

   The points for each question are indicated at the beginning of the question.

2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.

3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.

2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.

3. The answer should be confined to the question as set.

4. When you are asked to calculate, show all your work including any applicable formulas. When you are asked to recommend, provide proper justification supporting your recommendation.

5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate morning or afternoon session for Exam QFIADV.

6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Recognized by the Canadian Institute of Actuaries.

Tournez le cahier d'examen pour la version française.

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Exam QFIADV

****BEGINNING OF EXAMINATION***
Morning Session

1. (*7 points*) Company XYZ has a portfolio of 5 bonds. The Chief Investment Officer (CIO) of XYZ has asked you to evaluate the liquidity risk of the portfolio. First, she asks you to take a look at Bonds A, B and C.

	Bid Price Indication	Ask Price Indication	Trader Bid Spread Indication	Trader Ask Spread Indication	Option-Adjusted Spread Duration	Trading Volume Percentile
Bond A	\$80	\$85	n/a	n/a	12 years	90th
Bond B	n/a	n/a	185 bps	225 bps	14 years	60th
Bond C	n/a	n/a	100 bps	200 bps	3 years	40th
Bond Z	n/a	n/a	n/a	n/a	25 years	0th

Bonds A and B are off-the-run issues that are quoted several times a month. Bond C is an on-the run issue that has not been quoted in the past few months. You also identified a Bond Z but research indicated that Bond Z has never been traded since its issue 10 years ago.

XYZ uses a non-benchmark adjustment factor of 1.40 and a non-quoted adjustment factor of 1.05.

In order to reduce liquidity risk, the CIO is considering implementing a new policy where all bonds in the portfolio must have a Liquidity Cost Score™ (LCS) of 7% or better.

(a) (*1.5 points*) Determine whether each of Bonds A, B and C is compliant with the new policy.

You colleague has determined that the LCS for Bond D is 7%. Your CIO has provided you with the following information for Bonds D and E.

	Trading Volume	Issue Date	Coupon	Option Adjusted Spread (OAS)	Amount Outstanding
Bond D	\$20M	1/1/2010	5%	100 bps	\$100M
Bond E	\$85M	1/1/2015	3%	90 bps	\$200M

(b) (*2 points*) Evaluate whether Bond E is likely to be compliant with the new policy.

1. Continued

Your CIO is interested in understanding the impact of the underlying liquidity risk and default risk using only spread decomposition techniques on the OAS for Bonds F, G and H. Using default swap spread indications and LCS, your colleague has calculated the OAS decomposition level for these bonds. A partial summary of this decomposition is provided in the table below. All bonds are zero-coupon bonds which mature in 5 years.

	Yield	Risk Premium Component	Default Component
Treasury	2.2%		
Bond F	2.6%	0 bps	50%
Bond G	3.0%	40 bps	40%
Bond H	3.2%	70 bps	25%

(c) (*1.5 points*)

(i) Demonstrate which bond likely has the highest default risk.

(ii) Demonstrate which bond likely has the highest liquidity risk.

Your CIO would like to buy $100 million of Bond F, but is unwilling to take on any additional default risk as a result of this trade. Your colleague has indicated that the Credit Spread DV01 of a representative credit default swap is $5,410 per contract.

(d) (*2 points*) Calculate the number of credit default swap contracts necessary to fully hedge the default risk of the CIO's desired trade, including whether the contracts must be bought or sold.

2. (*8 points*) You are helping your manager to prepare a proposal for hedging a single premium fixed indexed annuity product. The company credits interest to the policyholder's account of this product at each policy anniversary, based on a price return of a major equity index, with the formula as below:

$$Credited\ Rate_t = Max\left(Min\left(\frac{S_t}{S_{t-1}} - 1, 0.05 \right), 0 \right), \text{ where}$$

S_t represents the value of the equity index at time *t* applicable to this product:

(a) (*0.5 points*) Draw a payoff diagram for the credited rate profile.

(b) (*1 point*) Specify the embedded call option(s) underlying the product.

Your manager wants to know the equity Delta of this product based on the company's practice as follows:

- Use the Black-Scholes model to price call options embedded in the product
- Assume a sticky volatility smile
- Vega $= S \times \sqrt{T-t} \times \frac{e^{-\frac{d_1^2}{2}}}{\sqrt{2\pi}}$

(c) (*1 point*) Explain the presence of the "volatility smile".

2. Continued

You are also given the following information:

- Initial premium is $10,000
- $S_0 = 100$
- The continuously compounded risk-free interest rate = 3%
- Ignore dividend yields on the index
- Ignore any lapses, deaths or other decrements
- The 1-year implied volatilities for various levels of S_t and Strike Levels *K* are as follows:

		Strike *K*				
Implied Volatility		90	95	100	105	110
$S(t)$	95	20.68%	18.12%	16.25%	15.50%	15.98%
	100	23.75%	20.62%	18.12%	16.29%	15.57%
	105	27.90%	23.63%	20.57%	18.12%	16.33%

(d) (*2 points*) Calculate the equity Delta of the product immediately after issue.

Your manager suggests switching the volatility smile assumption to a floating smile (also known as a "sticky Delta" approach).

(e) (*1 point*) Explain why you need to change your calculations because of this suggestion.

(f) (*2.5 points*) Recalculate the equity Delta of the product immediately after issue.

3. (*5 points*) You are the portfolio manager responsible for managing an alternative investment portfolio for an insurance company, focusing on real estate and other alternative investments. You have been asked by the chief risk officer about specific considerations of appraisal-based returns.

(a) (*1 point*) Describe two primary reasons why smooth series returns from the real estate market may not be easily unsmoothed by arbitrageurs.

You are considering a first order autocorrelation model to apply to these returns such that the most current return of the smoothed return series depends partially on the new market information contained in the true return, and partially on the smoothed return of the previous time period.

(b) (*1 point*) Assess why your choice of model is appropriate for a real estate price index.

You are given the following first-order autocorrelation model:

$$R_t^{\text{reported}} = (1-\rho) R_t^{\text{true}} + \rho R_{t-1,\text{ reported}}$$

You are given a smoothed return series (0%, 0%, 7%, 4%, 3%).

(c) (*1.5 points*) Estimate the parameter ρ using the model above.

(d) (*0.5 points*) Calculate the unsmoothed return at time 3, using the parameter ρ estimated in part (c).

You are given the corresponding true unsmoothed returns in following table. The true value of ρ is 0.2.

T	True returns
1	0.0%
2	0.0%
3	8.75%
4	3.25%
5	2.75%

(e) (*1 point*) Discuss a possible reason why the unsmoothed return you calculated in part (d) above is different from the true unsmoothed return given in the table.

4. (7 *points*) You have been provided with historical prices for 5 large-cap stocks in the US over a long period of time from t_1 to t_2 .

(a) (*2 points*) List all steps necessary to compute the related principal components.

Your analyst provides the following additional information about the covariance matrix of daily returns (which are assumed to follow a stationary process) across the 5 stocks:

	λ_1	λ_2	λ_3	λ_4	λ_5
Eigenvalues	0.001853	0.00053	0.000221	0.000121	0.000109

Eigenvectors	$\boldsymbol{W_1}$	$\boldsymbol{W_2}$	$\boldsymbol{W_3}$	$\boldsymbol{W_4}$	$\boldsymbol{W_5}$
	-0.142	-0.480	-0.586	0.630	-0.098
	-0.964	0.261	0.030	0.003	-0.049
	-0.105	-0.389	-0.414	-0.754	-0.313
	-0.132	-0.305	-0.060	-0.175	0.925
	-0.151	-0.676	0.694	0.067	-0.187

(b) (*1 point*) Determine the fewest number of principal components needed to explain at least 80% of the variation.

(c) (*3 points*) Calculate the correlation between the returns of the first and fifth stocks using the first 3 principal components.

Assume that data is missing for the first stock prior to t_1.

(d) (*1 point*) Describe how the principal components analysis on the 5 stocks between t_1 and t_2 can be used to generate price data for the first stock prior to t_1 (assuming returns follow a stationary process).

5. (*5 points*) Your company's investment committee purchased one share of each of two stocks – Stock A and Stock B, a while ago. Your manager asks you to prepare a recommendation for each stock for one of the following two actions:

I. Hold the stock for an additional year or
II. Sell the stock and invest the proceeds in a risk-free asset earning 5% per year compounded annually.

Your manager asks for your analysis based only upon Prospect Theory considerations using the following utility function $v(X)$, where X represents gains and losses:

$$v(X) = \begin{cases} X & for \quad X > 0 \\ 2.25X & \quad X < 0 \end{cases}$$

(a) (*1 point*) Critique the investment committee's choice of utility function $v(X)$.

The following table shows the historical prices of the stocks and their two possible future price outcomes:

	Initial Purchase Price	Current Price	Expected Price Next Year Scenario 1 (50% probability)	Expected Price Next Year Scenario 2 (50% probability)
Stock A	50	52	60	50
Stock B	25	20	30	17

(b) (*2 points*) Calculate for each of the two stocks the expected utility for both actions above, without assuming any mental accounting effects.

(c) (*1 point*) Calculate for each of the two stocks the expected utility for both actions, after taking into account the effect of mental accounting.

(d) (*1 point*) Recommend for each of the two stocks whether to take action I or II and justify your answer.

6. (*5 points*) ATL Pension Plan, a defined benefit pension plan, is considering adding infrastructure investments to its portfolio. ATL is currently concerned about the funding needs of the pension plan over the next 12 months and is also concerned with the J-curve effect. You are the pension actuary for ATL.

(a) (*1 point*) Describe four key features of alternative asset classes.

(b) (*1 point*) Explain four challenges of asset allocation involving infrastructure investments.

(c) (*1 point*) Explain why the J-curve effect could dissuade ATL from adding infrastructure assets to its portfolio.

ATL is now choosing one of the following three asset allocations for the portfolio:

Asset allocation	Expected return	Standard deviation	12-month cash flow
A	4.25%	8.52%	12.25 million
B	4.45%	8.61%	11.30 million
C	4.80%	9.00%	10.40 million

The chosen asset allocation for the portfolio must satisfy each of the following:

1. Portfolio cash flows must cover benefit payments
2. The expected return of the portfolio must exceed that of the liabilities
3. An asset allocation with the greatest excess return relative to risk is preferred

You have determined the following decision parameters.

- ATL has pension liabilities of 200 million and pays out 5.50% of the liabilities in benefit payments annually.
- The expected return on the pension liabilities is 4.10%.
- The risk-free rate is 1.20%.

(d) (*2 points*) Recommend the optimal asset allocation for ATL's portfolio.

7. (*8 points*) BESTLife is a popular permanent life insurance product offered by BEST Financial that provides guaranteed cash values and guaranteed death benefits with index linked options. You are an investment actuary at BEST Financial, managing a portfolio of assets that back this block of insurance liabilities characterized by very long duration cashflows. Your portfolio consists of equities, bonds, and real estate properties. You apply a buy-and-hold strategy.

You are concerned with the liquidity needs of BESTLife liabilities due to the generous cash value guarantees. You have reviewed a memo outlining the current company's approach for liquidity management which includes:

"Exposure to liquidity risk will be measured using a one-month internal liquidity coverage metric under a Run-on-the-Bank stress test scenario. This stress scenario assumes an immediate and unforeseen elevation of customer withdrawals. This situation could be caused by BEST Financial being downgraded by rating agencies and loss of consumer confidence."

(a) (*1.5 points*) Critique the cited approach including recommending improvements for deficiencies.

Corporate Risk Management is impressed by your knowledge of liquidity risk management and has asked you to join a working group developing a new Liquidity Policy.

(b) (*1.5 points*) Describe what the Liquidity Policy should include with respect to liquidity adequacy and liquidity crisis planning.

Corporate Risk Management is concerned with being able to recognize "Black Swans" in liquidity management. The current Liquidity Policy utilizes Monte Carlo modeling as calibrated to industry data.

(c) (*1.5 points*)

(i) (*1 point*) Evaluate the appropriateness of this approach, including consideration of the Black Swan problem.

(ii) (*0.5 points*) Describe one recommended improvement.

7. Continued

You are reviewing the investment strategy for your portfolio for possible expansion to alternative investments. Your manager has asked you to consider adding unlisted infrastructure investments to your portfolio, given each of the following two considerations:

I. Inflation protection
II. Diversification

(d) (*2 points*) Assess whether or not to add unlisted infrastructure investments to your portfolio for each of your manager's stated considerations.

You hear the following comment by a colleague in a meeting to discuss how to identify alternative asset types appropriate for your portfolio:

"We have long liabilities and already invest in commercial properties under a buy-and-hold strategy. Compared to the private property market, REIT valuation is more informationally-efficient and reflects correct property valuations. It also provides greater liquidity to the portfolio. Therefore, going forward we should invest in REITs instead of purchasing properties."

(e) (*1.5 points*) Critique the comment made by your colleague.

8. (*8 points*) You are a credit risk manager working at the Delta Bank of America, you are asked by the Chief Risk Officer (CRO) to analyze a credit risk portfolio. The portfolio contains three borrowers, and for each of them there exist credit lines structured with the total credit line and cash withdrawn allowance. You are also told that all three borrowers have the same amount outstanding, $10 million USD.

You are provided the following information for each of these three borrowers.

Borrower	Total Credit Line ($million)	Cash Withdrawn Allowance ($million)
A	30	30
B	30	15
C	30	10

Borrower	Draw Down Factor (DDF) – Cash	Draw Down Factor (DDF) – Contingent Liabilities	Cash Equivalent Exposure Factor (CEEF)
A	50%	40%	60%
B	80%	60%	70%
C	95%	80%	80%

Borrower	Probability of Default (PD)
A	5%
B	10%
C	20%

(a) (*1.5 points*)

(i) Identify the contingent liabilities for each of these three borrowers.

(ii) Explain the possible random effects for the contingent liabilities.

(b) (*1.5 points*) Calculate the expected exposure at default (EAD) for each of the three borrowers.

You are given the following formula for expected loss (EL)

$$EL = E(EAD) \times E(LGD) \times PD$$

where LGD is the loss given default.

(c) (*1 point*) Explain why the EL formula is not realistic in real life.

8. Continued

Unexpected loss is defined to be the standard deviation of losses under the following assumptions:

- EAD is deterministic
- LGD and the default event are independent
- V[LGD] and E[LGD] represent the variance and expected value of LGD, respectively.

(d) (*2.5 points*) Prove that the unexpected loss (UL) can be calculated as following:

$$UL = EAD \times \sqrt{V[LGD] \times PD + E[LGD]^2 \times PD(1-PD)}$$

Assume:

- LGD = 100%
- EAD = $10 million for each of the three borrowers.
- Default correlations between the borrowers are:

	A	**B**	**C**
A	1	0.5	0
B	0.5	1	-0.5
C	0	-0.5	1

(e) (*1.5 points*) Calculate the unexpected loss of this portfolio

9. (*7 points*) You are the manager of a credit hedge fund. Your analyst showed you the following two cases involving the bonds and CDS contracts, where the maturity date of the CDS contract is equal to its underlying bond's maturity date.

Case	I	II
Bond Issuer	Mcline Inc.	The Gene Company
Bond LIBOR spread (bps)	130	85
CDS Par spread (bps)	100	100
CDS Coupon rate (bps)	100	100

(a) (*0.5 points*) Calculate the CDS basis for each case.

(b) (*2.5 points*) Propose an arbitrage trading strategy for each case, showing all the steps required from inception to end.

(c) (*1.5 points*) Outline the reasons that your proposed strategy may not work.

You are given the following information about a CDS contract:

Trade Date	June 14, 2018
Settlement Date	June 19, 2018
Maturity	June 20, 2023
Notional	$5,000,000
Buy/Sell	Buy Protection
Quoted Spread	50 bps
Deal Spread	100 bps
3-year swap rate	3.5%
Recovery rate	40%
Coupon Frequency	Quarterly, occurring on the 20th of March, June, September, December
Days between Coupons in 2018	90 ending March 20 92 ending June 20 92 ending September 20 91 ending December 20

(d) (*1 point*) Calculate the accrued coupon payment as of the trade date.

(e) (*1.5 points*) Estimate the upfront payment to enter the contract as of the trade date.

****END OF EXAMINATION****
Morning Session

USE THIS PAGE FOR YOUR SCRATCH WORK

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