Long-Term Actuarial Mathematics

Exam LTAM

Date: Friday, October 26, 2018
Time: 8:30 a.m. – 12:45 p.m.

Recognized by the Canadian Institute of Actuaries.

INSTRUCTIONS TO CANDIDATES

General Instructions

1. Write your candidate number here _____. Your name must not appear.

2. Do not break the seal of this book until the supervisor tells you to do so.

3. Tables for this examination will be distributed by the Supervisor.

4. This examination has a total of 96 points. It consists of:

   Section A: 20 multiple-choice questions, each worth 2 points for a total of 40 points, and

   Section B: 6 written-answer questions, worth a total of 56 points. The point value for each written-answer question is indicated at the beginning of the question.

   You may divide your time between the two sections of the examination (written-answer, and multiple-choice) as you choose. You should keep in mind the relative weight of the two sections.

   Your written-answer paper will be graded only if your multiple-choice score is at or above a threshold set after the examination is administered.

5. Failure to stop writing or coding after time is called will result in the disqualification of your answers or further disciplinary action.

6. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Multiple-Choice Instructions

1. A separate answer sheet for the multiple-choice questions is inside the front cover of this book. During the time allotted for this examination, record all your answers on the back of the answer sheet. NO ADDITIONAL TIME WILL BE ALLOWED FOR THIS PURPOSE.

   No credit will be given for anything indicated in the examination book but not transferred to the answer sheet.

2. On the front of the answer sheet, space is provided to write and code candidate information. Complete the information requested by printing in the squares and blackening the circles (one in each column) corresponding to the letters or numbers printed. For each empty box blacken the small circle immediately above the "A" circle. Fill out the boxes titled:

   (a) Name
   (include last name, first name and middle initial)

   (b) Candidate Number
   (Candidate/Eligibility Number, use leading zeros if needed to make it a five digit number)

   (c) Test Site Code
   (The supervisor will supply the number.)

   (d) Examination Part
   (Code the examination that you are taking by blackening the circle to the left of "Exam LTAM")

   (e) Booklet Number
   (The booklet number can be found in the upper right-hand corner of this examination book. Use leading zeros if needed to make it a four digit number.)

   In the box titled "Complete this section only if instructed to do so," fill in the circle to indicate if you are using a calculator and write in the make and model number.

   In the box titled "Signature and Date" sign your name and write today's date. **If the answer sheet is not signed, it will not be graded**.

   Leave the boxes titled "Test Code" and "Form Code" blank.

   On the back of the answer sheet fill in the Booklet Number in the space provided.

CONTINUED ON INSIDE FRONT COVER

475 N. Martingale Road
Schaumburg, IL 60173-2226

Printed in the U.S.A.
Exam LTAM - Front Cover

3. Your score will be based on the number of questions which you answer correctly. No credit will be given for omitted answers and no credit will be lost for wrong answers: hence, you should answer all questions even those for which you have to guess.

4. Five answer choices are given with each multiple-choice question, each answer choice being identified by a key letter (A to E). Answer choices for some questions have been rounded. For each question, blacken the circle on the answer sheet which corresponds to the key letter of the answer choice that you select.

5. Use a soft-lead pencil to mark the answer sheet. To facilitate correct mechanical scoring, be sure that, for each question, your pencil mark is dark and completely fills only the intended circle. Make no stray marks on the answer sheet. If you have to erase, do so completely.

6. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.

7. Clearly indicated answer choices in the test book can be an aid in grading examinations in the unlikely event of a lost answer sheet.

8. After the examination, the supervisor will collect this book and the answer sheet separately. DO NOT ENCLOSE THE ANSWER SHEET IN THE BOOK OR IN THE ESSAY ANSWER ENVELOPE. All books and answer sheets must be returned. THE QUESTIONS ARE CONFIDENTIAL AND MAY NOT BE TAKEN FROM THE EXAMINATION ROOM.

Written-Answer Instructions

1. **Write your candidate number at the top of each sheet. Your name must not appear.**

2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question you are answering. Do not answer more than one question on a single sheet.

3. The answer should be confined to the question as set.

4. When you are asked to calculate, show all your work including any applicable formulas.

5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate Exam LTAM.

6. **Sign your essay answer envelope. If it is not signed, your examination will not be graded.**

7. For all parts of all problems, to maximize the credit earned, candidates should show as much work as possible, considering the time allotted for the question. Answers lacking justification will receive no credit. Answers should be organized so that the methods, logic, and formulas used are readily apparent. Candidates should not round their answers excessively; enough precision should be provided so that their answers can be accurately graded.

   In some cases, candidates are asked to show that a calculation results in a particular number. Typically the answer given will be rounded; candidates should provide a greater level of accuracy than the number given in the question. This structure of question is intended to assist the candidate by giving an indication when the calculation has been done incorrectly, providing an opportunity to explore an alternative approach. It also allows a candidate who cannot obtain the correct answer to use the answer given to proceed with subsequent parts of the problem. (Candidates who are able to solve the problem should use their exact answer for subsequent parts.)

   For questions requiring candidates to derive or write down a formula or equation, the resulting expression should be simplified as far as possible, and where numerical values are provided in the problem, they should be used.

Exam LTAM

SECTION A – Multiple-Choice

****BEGINNING OF EXAMINATION****

1. A new entrant to a Continuing Care Retirement Community (CCRC), is offered a choice of three different payment plans: Full Life Care, Modified Life Care, or Fee-for-Service. You are given:

(i) The entry fee is the same for each plan.

(ii) The new entrant is healthy and is entering an Independent Living Unit.

(iii) The initial monthly fee for the Full Life Care plan is F, for the Modified Life Care is M, and for the Fee-for-Service plan is S.

Which of the following is most likely to be correct?

(A) $M > F > S$

(B) $S > M > F$

(C) $S > F > M$

(D) $F > S > M$

(E) $F > M > S$

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

2. You are given:

$$\mu_x = \begin{cases} 0.06 & 30 \le x < 40 \\ 0.03 & 40 \le x \le 60 \end{cases}$$

Calculate ${}_{5|16}q_{30}$.

(A) 0.25

(B) 0.35

(C) 0.45

(D) 0.55

(E) 0.65

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

3. For a double decrement model, you are given:

(i) In its associated single decrement table, decrement 1 is uniformly distributed over each year of age.

(ii) Decrement 2 occurs at exact age $x+0.75$ for integer age x.

(iii) $q'^{(1)}_{50} = 0.02$

(iv) $q'^{(2)}_{50} = 0.10$

Calculate $1000\, q^{(1)}_{50}$.

(A) 19.0

(B) 19.2

(C) 19.5

(D) 19.8

(E) 20.0

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

4. For a three-state Markov chain model, you are given:

(i) The annual transition probability matrix is:

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{pmatrix} 0.2 & 0.3 & 0.5 \\ 0.1 & 0.6 & 0.3 \\ 0.9 & 0.1 & 0.0 \end{pmatrix} \end{array}$$

(ii) The process is in State 1 at the start of year 1.

(iii) Transitions occur only at the end of the year.

Calculate the probability that the process will be in State 1 after the transition at the end of year 3, without having transitioned to State 2.

(A) 0.07

(B) 0.11

(C) 0.15

(D) 0.19

(E) 0.23

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

5. You are given the following information derived from a sample of $n = 20$ lives with respect to a study of time-to-failure.

i	y_i	s_i	b_i
1	1	2	0
2	3	3	1
3	6	5	2
4	8	6	0

Calculate $\hat{S}_{20}(y_3)$, the Kaplan-Meier estimate of the survival probability, $S(6)$.

(A) 0.32

(B) 0.35

(C) 0.48

(D) 0.62

(E) 0.75

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

6. You are given the following mortality rates and mortality improvement factors, $\varphi(x,t)$.

		$\varphi(x,2018+t)$		
x	$q(x,2018)$	$t=1$	$t=2$	$t=3$
60	0.24	0.040	0.035	0.033
61	0.25	0.038	0.035	0.035
62	0.26	0.036	0.035	0.032
63	0.28	0.033	0.032	0.031

Calculate the probability that a life who is age 60 in 2018 survives to age 63.

(A) 0.432

(B) 0.438

(C) 0.444

(D) 0.450

(E) 0.456

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

7. For a study of time-to-failure of a certain type of robot, with right-censored data, you are given:

Time	Number of Failures	Number of Robots at Risk
1	2	200
3	2	192
7	3	182
15	17	170

Calculate the Nelson-Åalen estimate of the survival function at time 15.

(A) 0.85

(B) 0.87

(C) 0.89

(D) 0.91

(E) 0.93

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

8. Which of the following relationships is true for any $i > 0$?

(A) $\ddot{a}_x \le \ddot{a}_x^{(4)} \le \bar{a}_x \le a_x^{(4)} \le a_x$

(B) $\ddot{a}_x^{(4)} \le \ddot{a}_x \le \bar{a}_x \le a_x^{(4)} \le a_x$

(C) $a_x \le a_x^{(4)} \le \bar{a}_x \le \ddot{a}_x^{(4)} \le \ddot{a}_x$

(D) $a_x^{(4)} \le a_x \le \bar{a}_x \le \ddot{a}_x^{(4)} \le \ddot{a}_x$

(E) $\bar{a}_x \le a_x \le \ddot{a}_x \le a_x^{(4)} \le \ddot{a}_x^{(4)}$

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

9. You are given:

(i) Mortality and sickness follow the Standard Sickness-Death Model.

(ii) $i = 0.05$

Calculate $\overline{A}^{02}_{60:\overline{10}|}$.

(A) 0.164

(B) 0.181

(C) 0.202

(D) 0.220

(E) 0.241

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

10. For a special 10-year deferred whole life annuity-due on (50) and (60), you are given:

(i) At the end of the deferral period:
- If both lives are alive, the annuity contract terminates with no payment.
- If exactly one is alive, a benefit of 9000 per year is paid until the second death.

(ii) The future lifetimes of (50) and (60) are independent.

(iii) Mortality follows the Standard Ultimate Life Table.

(iv) $i = 0.05$

Calculate the expected present value of this special annuity.

(A) 3610

(B) 4170

(C) 4740

(D) 5320

(E) 5870

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

11. For a special 2-year deferred whole life annuity-due on (40), you are given:

(i) The first annual payment is 1000.

(ii) Payments increase by 4% (compound) each year.

(iii) Level premiums are payable at the beginning of each of the first 2 years.

(iv) The curtate expectation of life for (40), e_{40}, is 11.06.

(v) $p_{40} = 0.99,\ p_{41} = 0.98$

(vi) $i = 0.04$

Calculate the net premium.

(A) 3820

(B) 3970

(C) 4300

(D) 4520

(E) 4770

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

12. An insurer issues a fully discrete whole life insurance on (x), a smoker, with a death benefit of 100. You are given:

(i) Premium rates are based on the equivalence principle.

(ii) Premium rates are different for smokers (sm) and non-smokers (ns).

(iii) $i = 0.05$

(iv) For a smoker, $\ddot{a}_x^{sm} = 12.40$.

(v) For a non-smoker, $\ddot{a}_x^{ns} = 15.50$.

The insurer mistakenly charged the smoker policyholder a non-smoker rate.

Calculate the insurer's expected present value of the loss at issue for this policy.

(A) -20

(B) -10

(C) 0

(D) 10

(E) 20

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

13. An insurer issues a special 10-year deferred whole life annuity-due to (55). You are given:

(i) The annual annuity payment is 50,000.

(ii) There is an additional lump sum benefit of 200,000 payable on survival to age 80.

(iii) Level annual premiums are payable during the deferred period.

(iv) First year commissions are 20% of premium.

(v) Renewal commissions for the second and subsequent years are 5% of premium.

(vi) Mortality follows the Standard Ultimate Life Table.

(vii) Premiums are calculated using the equivalence principle.

(viii) $i = 0.05$

Calculate the gross premium.

(A) 56,250

(B) 57,450

(C) 58,750

(D) 59,950

(E) 60,150

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

14. Joe, age 65, and his wife Lucy, age 55, purchase a special 10-year deferred annuity policy with the following premium and benefit terms:

- Level annual premiums are payable for at most 10 years, while both Joe and Lucy are alive.
- There are no annuity payments during the first 10 years.
- After 10 years, at the start of each year the annuity pays:
  - 100,000 if both Joe and Lucy are alive at the payment date.
  - 55,000 if only one of them is alive at the payment date.

You are given the following assumptions:

(i) Joe and Lucy have independent future lifetimes.

(ii) Mortality follows the Standard Ultimate Life Table.

(iii) $i = 0.05$

Calculate the annual net premium.

(A) 80,400

(B) 83,900

(C) 87,400

(D) 90,900

(E) 94,400

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

15. For a special 20-year term insurance of 200,000 with level monthly premiums issued to (65), you are given:

(i) The death benefit is paid six months after the death of the policyholder.

(ii) Mortality follows the Standard Ultimate Life Table.

(iii) Woolhouse's formula with two terms is used to calculate $\frac{1}{m}$thly and continuous annuities, and to calculate insurance functions from the annuity values.

(iv) The monthly net premium is 274.

(v) $i = 0.05$

Calculate the net premium reserve at time 10.

(A) 18,860

(B) 19,010

(C) 19,390

(D) 19,610

(E) 19,900

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

16. A company issues a special insurance policy to (50) that pays 100,000 at the end of the year of death and doubles the benefit to 200,000 if the insured dies as the result of an accident.

You are given the following table of annual probabilities for this policy, where State 0 is Alive, State 1 is Death due to Accident, and State 2 is Death due to Causes Other Than Accident.

x	p_x^{00}	p_x^{01}	p_x^{02}
54	0.9905	0.0005	0.0090
55	0.9887	0.0005	0.0108
56	0.9866	0.0005	0.0129

Additionally, you are given the following information:

- $i = 0.04$
- Premiums of 1500 are paid annually.
- There are no expenses.
- The gross premium reserve at time *t*, for a life in State *j* at that time, is denoted ${}_tV^{(j)}$.
- ${}_5V^{(0)} = 2480$.

Calculate ${}_6V^{(0)}$.

(A) 2800

(B) 2900

(C) 3000

(D) 3100

(E) 3200

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

17. For a fully discrete 15-pay whole life insurance of 200,000 on (50), you are given:

(i) Mortality follows the Standard Ultimate Life Table.

(ii) $i = 0.05$

Calculate the full preliminary term reserve at the end of year 5.

(A) 16,164

(B) 16,684

(C) 17,214

(D) 17,734

(E) 18,264

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

18. For a profit test of an annual premium whole life insurance of 500,000 on (35), you are given the following information and assumptions:

(i) The annual gross premium is 10,200.

(ii) Renewal expenses are 7% of premium and are paid at the beginning of the year.

(iii) Reserves at the end of the 9th and 10th years are ${}_9V = 69,700$ and ${}_{10}V = 78,120$.

(iv) Interest earned on the insurer's funds is 6% per year.

(v) Death benefits are assumed to be paid in the middle of the year of death.

(vi) There are no withdrawals.

(vii) ${}_9p_{35} = 0.980,\ p_{44} = 0.996$

Calculate Π_{10}, the profit in the 10th policy year, per policy issued.

(A) 3950

(B) 3990

(C) 4090

(D) 4150

(E) 4250

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

19. Myra, (30), is employed at NED, which provides a post-retirement medical benefit to its retirees that covers the first three years following retirement. You are given:

(i) Retirement is assumed to occur at age 62.

(ii) Myra is assumed to remain employed with NED until retirement or earlier death.

(iii) Currently, annual health insurance premiums are 5000 at age 62 and increase by 2.26% with each age year.

(iv) Health insurance premium inflation is assumed to be 2.68% per year.

(v) Mortality follows the Standard Ultimate Life Table.

(vi) $i = 0.05$

Calculate the expected present value today of Myra's benefits under the plan.

(A) 3030

(B) 4030

(C) 5030

(D) 6030

(E) 7030

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

20. Katherine, (35), has just joined a firm offering a defined benefit plan with a benefit based on career average earnings. You are given:

(i) The benefit accrual rate is 2%.

(ii) Katherine expects her salary to increase by 4% per year at the start of each year beginning at age 36.

(iii) Katherine will retire at age 65.

Calculate Katherine's projected replacement ratio.

(A) 0.36

(B) 0.42

(C) 0.48

(D) 0.54

(E) 0.60

Exam LTAM

SECTION B – Written-Answer

1. (*10 points*) You are performing a profit test on a 50,000, fully discrete 10-year term life insurance policy issued to a healthy life age 60.

The annual premium is waived when the insured is sick.

You are given the following information regarding the reserve basis:

(i) A Markov model with three states: Healthy (0), Sick (1), and Dead (2) is used to value the policy.

(ii) The annual probability transition matrix for an insured age $60+k, k=0,1,\cdots,9$ is:

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{pmatrix} 0.90-0.01k & 0.05 & 0.05+0.01k \\ 0.70-0.01k & 0.20 & 0.10+0.01k \\ 0 & 0 & 1 \end{pmatrix} \end{array}$$

(iii) The annual gross premium is 5000 from the start of the third year.

(iv) Premiums in the first two years are lower.

(v) Reserves are calculated using the gross premium reserve method.

(vi) Issue expenses are 300 per policy.

(vii) Maintenance expenses are 150 incurred at the start of each year, including the first, for all policies in force.

(viii) $i=6\%$

(ix) The following actuarial present value functions, calculated at $i=6\%$:

k	$A^{02}_{60+k:\overline{10-k}\rceil}$	$A^{12}_{60+k:\overline{10-k}\rceil}$	$\ddot{a}^{00}_{60+k:\overline{10-k}\rceil}$	$\ddot{a}^{01}_{60+k:\overline{10-k}\rceil}$	$\ddot{a}^{10}_{60+k:\overline{10-k}\rceil}$	$\ddot{a}^{11}_{60+k:\overline{10-k}\rceil}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
2	0.46667	0.49680	4.7328	0.2533	3.3340	1.4060
⋮	⋮	⋮	⋮	⋮	⋮	⋮

(a) (*2 points*) Calculate ${}_2p^{01}_{60}$.

1. Continued

(b) (*4 points*)

(i) Show that ${}_2V^{(0)}$ is 420 to the nearest 10. You should calculate the value to the nearest 1.

(ii) Show that ${}_2V^{(1)}$ is 8880 to the nearest 10. You should calculate the value to the nearest 1.

(iii) You are given that ${}_3V^{(0)} = 1788$. Calculate ${}_3V^{(1)}$.

You are given the following additional information regarding the profit test for this policy:

- The earned rate is 5.7%.
- The hurdle rate is 8%.
- Pre-contract expenses are 200.
- Maintenance expenses are 60 at the start of each year, including the first, for all policies in force.
- Mortality and morbidity are the same as in the reserve basis.
- There are no withdrawals.
- The profit signature values for $t = 1, 2$ are $\Pi_1 = 84.74$ and $\Pi_2 = 80.35$

(c) (*4 points*)

(i) Show that Π_3 is 70 to the nearest 10. You should calculate the value to the nearest 0.1.

(ii) Calculate the Discounted Payback Period.

2. (*9 points*) You are given the following definition for a present value random variable,

$$Y = \begin{cases} 0 & T_x \le 5 \\ 1000\, v^5\, \overline{a}_{\overline{T_{x+5}}|} & T_x > 5 \end{cases}$$

(a) (*1 point*) Describe in words the benefit being valued.

(b) (*1 point*) Write down an expression in terms of actuarial symbols for the expected value of *Y*.

(c) (*3 points*) You are given the following indicator random variable:

$$I = \begin{cases} 0 & T_x \le 5 \\ 1 & T_x > 5 \end{cases}$$

(i) Show that the variance of *Y* given $I = 1$ is equal to $1000^2\, v^{10}\, \dfrac{\left({}^2\overline{A}_{x+5} - \overline{A}^2_{x+5}\right)}{\delta^2}$

(ii) Show that the variance of *Y* unconditionally is given by

$$\mathrm{Var}(Y) = 1000^2 \left({}_5p_x\, v^{10}\, \frac{\left({}^2\overline{A}_{x+5} - \overline{A}^2_{x+5}\right)}{\delta^2} + \left(v^5\, \overline{a}_{x+5}\right)^2\, {}_5p_x\, {}_5q_x \right)$$

Assume now that

- $x = 65$
- Mortality follows the Standard Ultimate Life Table,
- Deaths are uniformly distributed between integer ages.
- $i = 0.05$

(d) (*1 point*) Show that $\mathrm{E}[Y] = 8700$ to the nearest 100. You should calculate the value to the nearest 1.

(e) (*3 points*)

(i) Show that $\Pr\left[Y > \mathrm{E}[Y]\right] = 0.6$ to the nearest 0.1. You should calculate the probability to the nearest 0.001.

(ii) Explain why the Normal Approximation would not be appropriate to evaluate $\Pr\left[Y > \mathrm{E}[Y]\right]$.

3. (*9 points*) The Lee-Carter stochastic mortality model assumes that the natural log of the central death rate, *m(x,t)*, follows the discrete stochastic process:

$$\log m(x,t) = \alpha_x + \beta_x K_t$$

where x is the attained age, t is the calendar year, and α_x and β_x are age-dependent parameters.

You are given that:

(i) $K_t = K_{t-1} + c + \sigma_k Z_t$ where Z_t are i.i.d. N(0,1) random variables.

(ii) For a lognormal random variable Y with parameters μ and σ,
$$\mathrm{E}\left[Y^k\right] = e^{k\mu + k^2\sigma^2/2}$$

(a) (*1 point*) Define the central death rate, m_x, in terms of the force of mortality and survival probabilities.

(b) (*3 points*) You are given that:

$$\alpha_{80} = -2.4, \quad \beta_{80} = 0.05, \quad K_{2017} = -4, \quad c = -0.2, \text{ and } \sigma_k = 1.5$$

(i) Calculate the mean of $m(80,2018)$.

(ii) Calculate the standard deviation of $m(80,2018)$.

(iii) Calculate the 10% quantile of $m(80,2018)$.

(c) (*3 points*) Assume uniform distribution of deaths between integer ages.

(i) Show that $\dfrac{1-0.5m_x}{1+0.5m_x} = p_x$ for integer x

(ii) Calculate the median of $p(80,2018)$, the one-year survival probability for (80) in 2018.

(d) (*2 points*) A criticism of the Lee-Carter model is that it can give a poor fit to population data. State two common features of population mortality that are not captured in the Lee-Carter model.

4. (*9 points*) An insurer issues a special fully discrete whole life insurance of 100,000 to (60), under which premiums cease when the policyholder reaches age 80.

You are given the following assumptions for pricing and reserving:

(i) Mortality follows the Standard Ultimate Life Table.

(ii) Commissions are 40% of the first premium, 10% thereafter.

(iii) Maintenance expenses are 500 at issue and 50 at the beginning of each subsequent year while the policy is in force.

(iv) $i = 0.05$

(a) (*1 point*) Explain why insurance policies often include a maximum age for paying premiums.

(b) (*3 points*)

(i) Show that the gross premium calculated using the equivalence principle is 2790 to the nearest 10. You should calculate the premium to the nearest 0.1.

(ii) Calculate the gross premium reserve at the end of the second policy year.

The insurer holds reserves on a modified net premium reserve basis defined by these rules:

- The net premium payment term is the same as the gross premium payment term.
- The first year modified net premium is 50% of the renewal modified net premium.
- The renewal modified net premiums are level.

(c) (*3 points*)

(i) Show that the first year modified net premium using this method is 1220 to the nearest 10. You should calculate the premium to the nearest 0.1.

(ii) Calculate the modified net premium reserve at the end of the second policy year.

4. Continued

(d) (*2 points*)

(i) Calculate the first year modified net premium using the Full Preliminary Term (FPT) method.

(ii) Without further calculation, state with reasons whether the FPT reserve at the end of the second policy year will be higher, lower, or the same as the modified net premium reserve in part (c).

5. (*9 points*) Matt, currently age 35, takes out a mortgage of 250,000 to purchase a home at a loan interest rate of 5%. This loan will be repaid with annual payments of X at the end of each year for 20 years.

NED Life offers Matt a mortgage life insurance policy with a 20-year term which will pay off the outstanding balance on the mortgage at the end of the year of his death, including the annual payment then due. This policy will have level annual net premiums to be paid at the beginning of each year that Matt is alive during the 20-year term.

You are given:

(i) Mortality follows the Standard Ultimate Life Table.

(ii) $i = 0.05$

(iii) K is the curtate future lifetime of Matt.

(a) (*3 points*)

(i) Show that $X = 20,060$ to the nearest 10. You should calculate X to the nearest 1.

(ii) Show that the outstanding loan balance <u>immediately before</u> the loan payment at time 5 is 228,280 to the nearest 10. You should calculate the value to the nearest 1.

(iii) Express the outstanding loan balance at the end of the year of Matt's death in terms of K and interest rate functions.

(b) (*3 points*) Let Z denote the present value of the death benefit at the issue date.

(i) Write down an expression for Z in terms of K and interest rate functions.

(ii) Show that $\mathrm{E}[Z] = \frac{X}{d}\left(A^{1}_{35:\overline{20}|} - v^{21}\,{}_{20}q_{35}\right)$.

(c) (*2 points*) Calculate the level annual net premium.

(d) (*1 point*) Explain why NED Life might expect a high lapse rate at later durations.

THIS PAGE INTENTIONALLY LEFT BLANK

6. (*10 points*) ABC Corporation offers its employees a Final Salary Defined Benefit pension with an accrual rate of 2% per year of service. The Final Pensionable Salary is the salary earned in the final year of employment. The pension is paid as a single life monthly annuity-due with a 10-year guarantee.

Employees contribute 6% of pay at the end of each month throughout their period of employment.

Riley is an employee of ABC who is about to retire at age 65. She was hired at age 30 and has worked continuously since then. Her starting salary was 40,000 per year.

You are given the following additional information:

(i) Salaries increase monthly at a rate of 3.6% per year compounded monthly.

(ii) The two term Woolhouse Formula is used for valuing the retirement pension.

(iii) Mortality is assumed to follow the Standard Ultimate Life Table.

(iv) The valuation interest rate is 5%.

(v) Members have the option of taking a lump sum at retirement equal to the expected present value of their retirement pension, using the valuation assumptions.

(a) (*2 points*) Show that Riley's monthly pension is 8050 to the nearest 10. You should calculate the value to the nearest 0.1.

(b) (*2 points*) Calculate the expected present value of Riley's retirement pension at age 65.

(c) (*2 points*) Show that the accumulated value of Riley's contributions at retirement, assuming interest of 9.6% per year compounded monthly, is 995,500 to the nearest 100. You should calculate the value to the nearest 10.

(d) (*1 point*) Assume that Riley takes the lump sum option. State with reasons whether the internal rate of return (IRR) that Riley has earned on her contributions is more than or less than 9.6% per year compounded monthly.

6. Continued

(e) (*1 point*) Employees have the option to take a higher monthly pension at retirement without a guarantee period. The revised benefit has the same expected present value at age 65 as the standard benefit.

Calculate the revised monthly payment.

(f) (*2 points*)

(i) Define adverse selection.

(ii) Explain how adverse selection might impact the pension plan's costs based on the employees' selection of annuity with guarantee, annuity without guarantee, and lump sum.

****END OF EXAMINATION****

USE THIS PAGE FOR YOUR SCRATCH WORK

USE THIS PAGE FOR YOUR SCRATCH WORK

USE THIS PAGE FOR YOUR SCRATCH WORK