MLC Spring 2017

Model Solutions

Written Answer Questions

Question 1 Model Solution

Learning Outcomes: 1(a), 1(b), 1(d), 3(a), 4(a)

Chapter References: AMLCR Chapter 8.1-8.7

(a)

$$
\begin{aligned}
{}_5p_{40}^{\overline{00}} &= e^{-\int_0^5 \mu_{40+t}^{01} + \mu_{40+t}^{02} dt} \\
&= e^{-\int_0^5 0.04 + 0.01t\, dt} \\
&= e^{-(0.04(5) + 0.01(25/2))} \\
&= 0.7225
\end{aligned}
$$

Comments: This part was done correctly by almost all candidates attempting this question.

(b) Let N_b denote the number of policies for which no benefits are paid during the course of the policy.

$$E[N_b] = 72.25$$

$$V[N_b] = 100 * (0.7225)(0.2775) = 20.0494 = 4.4777^2$$

$$
\begin{aligned}
\implies \Pr[N_b \geq 65] &\approx 1 - \Phi\left(\frac{64.5 - 72.25}{4.4777}\right) \\
&\approx 1 - \Phi(-1.73) \\
&\approx 0.9582
\end{aligned}
$$

Comments:

1. *Many candidates lost some marks by omitting the continuity correction, or by applying it incorrectly (for example, adding 0.5 instead of subtracting).*
2. *The other common error was to use* 100^2 *instead of 100 to calculate the portfolio variance.*

(c)

$$\frac{d}{dt}{}_tp_{40}^{00} = {}_tp_{40}^{01}\,\mu_{40+t}^{10} - {}_tp_{40}^{00}\left(\mu_{40+t}^{01} + \mu_{40+t}^{02}\right)$$

$$\frac{d}{dt}{}_tp_{40}^{01} = {}_tp_{40}^{00}\,\mu_{40+t}^{01} - {}_tp_{40}^{01}\left(\mu_{40+t}^{10} + \mu_{40+t}^{12}\right)$$

$$\frac{d}{dt}{}_tp_{40}^{02} = {}_tp_{40}^{00}\,\mu_{40+t}^{02} + {}_tp_{40}^{01}\,\mu_{40+t}^{12}$$

$$BC: \quad {}_0p_{40}^{00} = 1.0; \quad {}_0p_{40}^{01} = {}_0p_{40}^{02} = 0.0$$

Comments:

1. *This part was done correctly by most candidates.*
2. *Some candidates omitted the* $\frac{d}{dt}$ *terms, and others omitted one or more boundary conditions.*
3. *Boundary conditions were sometimes expressed incorrectly as* ${}_tp_x^{ij}$ *instead of* ${}_0p_x^{ij}$.

(d)

$$\begin{aligned}
{}_{0.1}p_{40}^{01} &\approx {}_0p_{40}^{01} + 0.1\left({}_0p_{40}^{00}\,\mu_{40}^{01} - {}_0p_{40}^{01}(\mu_{40}^{10} + \mu_{40}^{12})\right) \\
&\approx 0 + 0.1(0.03) = 0.003 \\
{}_{0.1}p_{40}^{00} &\approx {}_0p_{40}^{00} + 0.1\left({}_0p_{40}^{01}\,\mu_{40}^{10} - {}_0p_{40}^{00}(\mu_{40}^{01} + \mu_{40}^{02})\right) \\
&\approx 1 + 0.1(-0.04) = 0.9960 \\
{}_{0.2}p_{40}^{01} &\approx {}_{0.1}p_{40}^{01} + 0.1\left({}_{0.1}p_{40}^{00}\,\mu_{40.1}^{01} - {}_{0.1}p_{40}^{01}(\mu_{40.1}^{10} + \mu_{40.1}^{12})\right) \\
&\approx 0.003 + 0.1(0.996(0.03) - 0.003(0.02 + 0.02 * 0.1)) \\
&\approx 0.00598
\end{aligned}$$

$\Longrightarrow$ the expected number in State 1 at $t = 0.2$ is 0.598

ALTERNATIVE

$$ {}_hp_x^{ij} \approx h\mu_x^{ij} \; i \neq j \qquad {}_hp_x^{ii} \approx 1 - h\mu_x^{i\cdot} \qquad h = 0.1 $$

$$ {}_{0.2}p_{40}^{01} = {}_{0.1}p_{40}^{00} * {}_{0.1}p_{40.1}^{01} + {}_{0.1}p_{40}^{01} * {}_{0.1}p_{40.1}^{11} $$

$$ {}_{0.1}p_{40}^{00} \approx 1 - 0.1(\mu_{40}^{01} + \mu_{40}^{02}) = 1 - (0.1)(0.03 + 0.01) = 0.996 $$

$$ {}_{0.1}p_{40}^{01} \approx (0.1)\mu_{40}^{01} = (0.01)(0.03) = 0.003 $$

$$ {}_{0.1}p_{40.1}^{01} \approx (0.1)\mu_{40.1}^{01} = 0.003 $$

$$ {}_{0.1}p_{40.1}^{11} \approx 1 - 0.1(\mu_{40}^{10} + \mu_{40}^{12}) = 1 - (0.1)(0.01 + 0.012) = 0.9978 $$

$$ \implies {}_{0.2}p_{40}^{01} \approx (0.996)(0.003) + (0.003)(0.9978) = 0.00598 $$

$\implies$ the expected number in State 1 at $t = 0.2$ is 0.598

Comments:

1. *A common mistake was to use* $\mu_{40.1}^{ij}$ *to calculate* ${}_{0.1}p_{40}^{ij}$ *and* $\mu_{40.2}^{ij}$ *to calculate* ${}_{0.2}p_{40}^{ij}$.
2. *A number of candidates forgot to multiply by 100 for the final answer.*

(e)(i)

$$ \text{EPV Premiums } = P\bar{a}_{40:\overline{5}|}^{00} = 3.887\,P $$

$$ \text{EPV Death Benefit } = 50000\bar{A}_{40:\overline{5}|}^{02} = 7300 $$

$$ \text{EPV Dis. Benefit } = 5000\bar{a}_{40:\overline{5}|}^{01} = 1310 $$

$$ \implies P = 2215.08 $$

(e)(ii)

$$ {}_3V^{(0)} = 50000\bar{A}_{43:\overline{2}|}^{02} + 5000\bar{a}_{43:\overline{2}|}^{01} - P\bar{a}_{43:\overline{2}|}^{00} $$

$$ = 50000(0.092) + 5000(0.050) - 2215.08(1.767) = 935.95 $$

$$ {}_3V^{(1)} = 50000\bar{A}_{43:\overline{2}|}^{12} + 5000\bar{a}_{43:\overline{2}|}^{11} - P\bar{a}_{43:\overline{2}|}^{10} $$

$$ = 50000(0.156) + 5000(1.740) - 2215.08(0.017) = 16\,462.34 $$

$$ \text{Total Reserve } = 80\left({}_3V^{(0)}\right) + 11\left({}_3V^{(1)}\right) = 255\,962 $$

Comments:

1. *A very common mistake was to forget the premium term in* ${}_3V^{(1)}$.
2. *Some candidates included a reserve term for State 2 (the 'Dead' state).*

Question 2 Model Solution

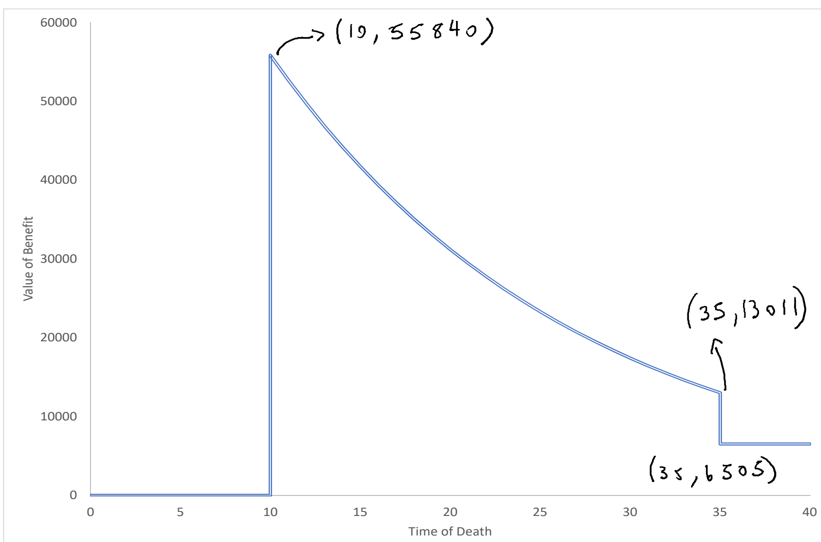
Learning Outcomes: 2(a), 2(c)

Chapter References: AMLCR Chapter 4

(a)

$$Z = \begin{cases} 0 & T_{40} \le 10 \\ 100\,000\, v^{T_{40}} & 10 < T_{40} \le 35 \\ 50\,000\, v^{35} & T_{40} > 35. \end{cases}$$

Comments: Overall, performance was very good on this part, with most candidates getting full credit. The only common mistake was to define $E[Z]$ in EPV symbols, rather than defining Z in terms of T_{40}. This received no credit.



(b)

Comments: Performance was good on this part, with many candidates receiving full credit, and most candidates receiving at least partial credit. Some common mistakes included having the wrong shape curve for the insurance portion of the graph, having a decreasing graph (instead of a straight horizontal line) beyond time 35, not realizing there was a vertical drop at time 35, and miscalculating the numerical values.

(c) The benefit value is below 15 000 if the life dies in the first 10 years, or if the life survives long enough for the discounted value of the death/survivor benefit to be less than 15 000.

First determine when the benefit value falls below 15 000, for $10 < T_{40} \leq 35$.

$$100\,000 v^{T_{40}} < 15\,000 \Leftrightarrow T_{40} > 32.56$$

Next, determine the probability that $T_{40} > 32.56$. UDD means we can use linear interpolation for the l_x functions.

$$\Pr\left[T_{40} > 32.56\right] = {}_{32.56}p_{40} = \frac{0.44\,l_{72} + 0.56\,l_{73}}{l_{40}}$$
$$= 0.64724$$

Then the total probability required is

$$\Pr\left[T_{40} \leq 10\right] + \Pr\left[T_{40} > 32.56\right] = {}_{10}q_{40} + {}_{32.56}p_{40} = 0.68614$$

Comments: Performance was fair on this part, with some candidates receiving full credit, and most candidates receiving at least partial credit. The most common mistake was to neglect the fact that the PV of benefit was sufficiently small when the insured dies in the first ten years. Most candidates were able to get the upper threshold of 32.56 years. Other common mistakes on this part included assuming that the product was discrete (and hence using a value of 33 years instead of 32.56 years) and incorrectly applying the UDD assumption. A small number of candidates used a normal approximation; no credit was given for this approach.

(d) We require τ where

$${}_{10}q_{40} + {}_{\tau}p_{40} = 0.6$$
$$\Rightarrow {}_{\tau}p_{40} = 0.56110 \Rightarrow l_{40+\tau} = 5\,225\,617$$
$$\tau = 35.6$$

If the life survives more than 35 years, then we only need to cover the value of the endowment benefit. Hence, a premium of $50\,000v^{35} = 6505$ is the smallest that will cover the benefits with a probability of at least 60%.

Comments: Performance was relatively poor on this part, with a large number of candidates omitting this part entirely. However, there was a significant percentage of candidates who earned full credit. The most common mistakes were similar to part (c): neglecting the fact that no benefit is paid in the first 10 years and assuming the product was discrete. A small number of candidates used a normal approximation; no credit was given for this approach.

(e)

$$\begin{aligned}
\text{EPV} &= 100\,000{}_{10}E_{40}\,\bar{A}^{\,1}_{50:\overline{25|}} + 50\,000\,{}_{35}E_{40} \\
&= 100\,000\frac{i}{\delta}\left({}_{10}E_{40}\,A_{50} - {}_{35}E_{40}\,A_{75}\right) + 50\,000{}_{35}E_{40} \\
&= 12940.7
\end{aligned}$$

Comments: There were several equivalent approaches to calculating the EPV. Performance was relatively good on this part, with a large number of candidates getting full credit or close to full credit. The most common mistake was to omit the i/δ factor, which resulted in a small deduction. Many candidates made various other errors such as using the wrong number of years in the insurance. Many candidates did not show enough work for this part. In the cases where there was a mistake somewhere in the calculations, these candidates were awarded less partial credit than they might have otherwise earned.

Question 3 Model Solution

Learning Outcomes: 2(a), 2(c), 4(a), 4(e)

Chapter References: AMLCR Chapters 9 and 13

Comments: Overall, this question was not well done, even though the topics covered, Type A UL and joint life annuities are not that unusual.

(a) For each year, first calculate the year end AV ignoring the corridor factor, then check whether the corridor factor applies, and adjust as necessary.

$$AV_1 = (50000 * 0.8 - 75 - 0.025(100\,000 - AV_1)v_{4.5\%})\,(1.065) = \frac{39972}{0.97452}$$
$$= 41\,017$$

Check corridor factor: $\dfrac{DB}{AV_1} = \dfrac{100000}{41017} = 2.44 > CF_1$

The corridor factor is exceeded and no adjustment to the DB is required.

$$AV_2 = (41017 + 50000 * 0.92 - 75 - 0.03(100\,000 - AV_2)v_{4.5\%})\,(1.0575)$$
$$= \frac{88906}{0.96964} = 91\,689$$

Check Corridor Factor $\dfrac{DB}{AV_2} = \dfrac{100000}{91689} = 1.09 < CF_2$

So the corridor factor must be applied, increasing the total DB to $AV_2 * 1.4$, so that the ADB is $DB - AV_2 = 0.4AV_2$, and the AV_2 equation becomes

$$AV_2 = (41017 + 50000 * 0.92 - 75 - 0.03(0.4 * AV_2)v_{4.5\%})\,(1.0575)$$
$$= \frac{91941}{1.01214} = 90838$$

Comments: Many students ignored the corridor factor, and others assumed that the corridor factor always applied.

(b) The EPV of the annuity is

$$\text{EPV} = 4Q\left(\ddot{a}^{(4)}_{\overline{10}|} + {}_{10}E_{60}\ddot{a}^{(4)}_{70} + 0.6\,{}_{10}E_{70}\left({}_{10}p_{60}\ddot{a}^{(4)}_{70|80} + (1-{}_{10}p_{60})\ddot{a}^{(4)}_{80}\right)\right)$$

$$= 4Q\left(\ddot{a}^{(4)}_{\overline{10}|} + {}_{10}E_{60}\ddot{a}^{(4)}_{70} + 0.6\,{}_{10}E_{70}\,\ddot{a}^{(4)}_{80} - 0.6\,{}_{10}E_{60:70}\,\ddot{a}^{(4)}_{70:80}\right)$$

$$\ddot{a}^{(4)}_{\overline{10}|} = 7.6341 \qquad \ddot{a}^{(4)}_{70} = 8.5693 - \frac{3}{8} = 8.1943 \qquad \ddot{a}^{(4)}_{80} = 5.9050 - \frac{3}{8} = 5.5300$$

$$\ddot{a}^{(4)}_{70:80} = 5.0014 - \frac{3}{8} = 4.6264$$

$${}_{10}E_{60} = 0.45120 \qquad {}_{10}E_{70} = 0.33037 \qquad {}_{10}E_{60:70} = {}_{10}E_{60}\,{}_{10}p_{70} = 0.266943$$

$$\Rightarrow \text{EPV} = 46.7463\,Q$$

Set this equal to AV_2 to give $Q = 1943.21$.

Comments: A majority of candidates answering this question used Q instead of $4Q$ *throughout. Many candidates did not include the 60% reversionary benefit for Grant. Many candidates calculated* $\ddot{a}^{(4)}_{\overline{10}|}$ *even though it was given in the question.*

(c) ${}_{10}V = 4Q\ddot{a}^{(4)}_{70} = 63\,693.0$

Comments: Almost all the candidates who attempted this part received full credit, but it was omitted by a large number. Candidates who omitted the '4' term in both (b) and (c) were not penalised again in this part.

One reason for giving candidates a "Show that" type question is to enable them to complete later parts even if they have not managed to find the correct answers to the earlier parts.

(d) **Examples of possible solutions:**

- It is not usual to offer surrender values for annuities, because of adverse selection. The lives whose health is bad and who therefore face higher mortality in the near future would withdraw their funds, reducing the pool available to survivors.
- The pricing of annuities assumes pooling of longevity risk, which means that the reserves which are released when an annuitant dies are required to support the payments for the remaining survivors. Allowing surrenders would undermine this risk sharing as the lives with higher risk of deaths would surrender, and their funds would not be available to offset the costs of the annuities for the survivors.

Comments: Only around 10% of candidates mentioned adverse selection, explicitly or implicitly. Most candidates seemed unaware that (traditional) life annuities cannot be surrendered.

Question 4 Model Solution

Learning Outcomes: 3(a), 3(b), 3(d)

Chapter References: AMLCR Chapter 6

(a) Let G denote the premium. The equation of value is

$$G = 0.04G + 50\ddot{a}_{35} + {}_{30}E_{35}\,(10050\,\ddot{a}_{65})$$

$$= \frac{50(15.3926) + 0.13924(9.8969)(10050)}{0.96}$$

$$= \frac{14619.0}{0.96} = 15\,228.1$$

Comments: Overall, the results for this portion of the question were very good. The common mistakes made were pulling wrong numbers from the tables or incorrectly applying the expenses.

(b) The loss at issue for a given premium is an increasing function of the future lifetime K_{35}. We need to find the smallest k such that

$$\Pr[K_{35} \geq k] > 0.1 \text{ and } \Pr[K_{35} \geq k+1] \leq 0.1$$

The 90% quantile for T_{35} is τ where

$$l_{35+\tau} = 0.1\, l_{35}$$

which gives τ lying between 55 and 56, which means that $k = 55$ and $k + 1 = 56$.

(Check: ${}_{55}p_{35} = 0.112$ and ${}_{56}p_{35} = 0.0911$)

So for the loss to have probability less than 0.1 of being greater than 5000, the premium must be big enough for the loss to be equal to 5000 if (35) survives to age 90, which is a term of 56 years ($K_{35} + 1$) for the expense annuity-due, and a term of 26 years ($K_{35} + 1 - 30$) for the deferred annuity-due.

If (35) dies between age 90 and 91, the loss at issue is

$${}_0L\big|\,[K_{35}\!=\!55] = 0.04G + 50\ddot{a}_{\overline{56|}} + 10050v^{30}\ddot{a}_{\overline{26|}} - G$$

$$= 24967.7 - 0.96G$$

To assure a loss no greater than 5000 when $K_{35} = 55$, we set G such that

$$24967.7 - 0.96G = 5000$$

$$\implies G = 20,800$$

Comments:

1. *In general, the candidates struggled with the portion of the question.*
2. *Many tried to do question using the normal approximation. This is not appropriate and received no credit.*
3. *Many students omitted this portion of the question.*

Please be advised that the solution for written answer 4, part b, was originally incorrect. The original solution was based on the premise that the time of death that would meet this condition is age 91, whereas it should be between age 90 and 91. All candidates who answered this correctly, but who were not originally given credit, were given that credit. Those candidates who originally were unsuccessful on the examination but who scored additional points after the correction were notified that they passed the examination.

(c) The EPV of the return of premium benefit, given a premium G^*, is

$$G^* \left(q_{35} v(1.06) + {}_{1|1}q_{35}\, v^2\, (1.06)^2 + \cdots + {}_{29|1}q_{35}\, v^{30}\, (1.06)^{30} \right)$$
$$= {}_{30}q_{35}\, G^*$$
$$= 0.200272\, G^*$$

So the new equation of value is

$$G^* = 0.04G^* + 0.200272G^* + 13849.4 + 769.63$$
$$= 19,242$$

Comments: The students who attempted this part generally did well. However, there were many omits. The most common mistake was to omit the interest accumulation in the return of premium benefit.

(d)(i) Suppose $G = 20800$ as above. Then the loss from death before age 65 is at most $0.04(20800) + 50\ddot{a}_{\overline{30|}} + 20800(1.06)^{30}v^{30} - 20800 = 1,562$, so there is no possibility that ${}_0L > 5000$ when $K_{35} < 30$.

When $K_{35} \geq 30$ we have the worst 10% of losses when $K_{35} > 55$, exactly as in (b) above, and so the premium for a loss of 5000 is still 20800.

(d)(ii) The premium is the same because it is determined by the worst 10% outcome for the L_0 random variable. The return of premium benefit does not change the worst case losses, and therefore does not affect the quantile premium.

Comments: There were other ways of answering (d)(i), and graders were flexible in interpreting the candidates' answers. However, the majority of candidates omitted this part, and candidates who did attempt it mostly focussed on the 'explain' part and left out the 'show' part.

Question 5 Model Solution

Learning Outcomes: 4(c), 4(g)

Chapter References: AMLCR Chapter 12

Comments: Overall, this question was done very well by those who attempted it.

(a) We have a profit test table (using notation from AMLCR):

t	${}_{t-1}V$	P	E	I	EDB	E_tV	Pr_t
0*	0	0	200	0	0	0	-200
1	0	770	20	30	592	99.4	88.6
2	100	770	20	34	642	159.0	83.0

Or, line by line:

$Pr_0 = -200$ from the pre-contract expenses.

$$Pr_1 = (770 - 20)(1.04) - q_{50}(100\,000) - p_{50}(100) = 88.6$$

$$Pr_2 = (100 + 770 - 20)(1.04) - q_{51}(100\,000) - p_{51}(160) = 83.0$$

The partial NPV at time 2 is

$$\begin{aligned} NPV(2) & Pr_0 + v_{6\%}\, Pr_1 + v_{6\%}^2\, p_{50}\, Pr_2 \\ &= -200 + 88.6 v_{6\%} + (0.99408) v_{6\%}^2 (83.0) \\ &= -42.98 \end{aligned}$$

(b) The DPP is greater than 2 years, as $NPV(2) < 0$.

Calculate $NPV(3)$:

$$Pr_3 = (160 + 770 - 20)(1.04) - q_{52}(100\,000) - p_{52}(170) = 80.59$$

$$NPV(3) = NPV(2) + {}_2p_{50}\, v_{6\%}^3\, (80.59) = 23.85$$

As $NPV(3) > 0$ and $NPV(2) < 0$, the discounted payback period is 3 years.

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Comments: Most candidates achieved full credit for parts (a) and (b).

(c) (i) Let * denote the updated values.

$$Pr_2^* = (770 - 20)(1.04) - 642 - 159 = -21.00$$

(ii) The emerging profit is changed in years 1 and 2 only. The NPV before the change is given as $NPV = 144.56$. We need to adjust this for the changed emerging profit at times 1 and 2 as follows.

$$Pr_1^* = (770 - 20)(1.04) - 592 = 188.0$$

$$\Rightarrow NPV^* = NPV + (Pr_1^* - Pr_1)v_{6\%} + (Pr_2^* - Pr_2)\,p_{50}\,v_{6\%}^2$$

$$= 146.33$$

Comments: Most candidates received full credit for the revised Pr_2 in part (c) but only partial or no credit for the NPV. A common error was to ignore the change in Pr_1.

(d)(i)

$$({}_1V + 770 - 20)(1.04) - 642 - 159.0 = 0$$

$$\Longrightarrow {}_1V = 20.19$$

(d)(ii) Negative emerging profit indicates that the insurer must raise money in the course of the term of the policy. This is inconsistent with prudent financial management, as it may not be practical for the insurer to raise money at that time. Reserves should be established such that policies are expected to be self financing after the initial outlay associated with acquiring the new business.

Comments: Most candidates received full credit for the minimum ${}_1V$ in part (d). Very few candidates received full credit for explaining why a negative Pr_2 is undesirable. The most common explanation provided essentially stated that insurers want to make a profit. No credit was awarded for this explanation.

Question 6 Model Solution

Learning Outcomes: 5(b), 5(c), 5(d), 5(f)

Chapter References: AMLCR Chapter 10

Other than part (a), this question was not well answered. This may be partly due to time constraints, but may also reflect that it is a more complex question than previous pension questions as it incorporated mid-year exits. Many candidates appeared to have memorized formulas without understanding them, and were not able to adjust for the modifications required.

Candidates who present their work clearly and coherently tend to receive more partial credit than those who do not, as the graders are able to identify the parts of the answer that are correct.

Candidates are strongly advised not to pretend that they have reproduced a result in a 'Show that...' question. Graders read every line. Candidates are more likely to get partial credit if they produce a straightforward solution which does not get to the given answer, than if they produce a fudged answer which is designed to hide the fact that the solution does not give the required result.

Some candidates were confused as to when to use the Illustrative Service Table (before retirement) and when to use the Illustrative Life Table (after retirement).

(a) The benefit payable on retirement at age 64.5 is the product of the accrual rate, the years of service at exit, final 1-year salary and benefit reduction factor.

The final 1-year salary is

$$40,000\frac{1.03^{28}+1.03^{29}}{2} = 92,889.96$$

The annual benefit paid on exit at age 64.5 is

$$0.02 \times 29.5 \times 92,890 \times (1-6(0.005))$$
$$= 53160.9$$

The replacement rate is the first year's pension divided by the final year's salary:

$$RR = \frac{53160.87}{92889.96} = 57.2\%$$

This part was fairly well done. The most common error was using the salary at retirement, rather than the final year's salary in the replacement rate denominator.

(b)

$$\ddot{a}_{65} = 9.8969 \quad \Rightarrow \quad \ddot{a}_{65}^{(2)} = \alpha(2)\ddot{a}_{65} - \beta(2) = 9.6416$$

$$\ddot{a}_{64.5}^{(2)} = 0.5 + v^{0.5}\,{}_{0.5}p_{64.5}\;\ddot{a}_{65}^{(2)}$$

$$({}_{0.5}p_{64})\;({}_{0.5}p_{64.5}) = p_{64} = 0.98048$$

$$\text{and } \;{}_{0.5}p_{64} = 1 - 0.5(q_{64}) = 0.99024 \implies {}_{0.5}p_{64.5} = 0.99014$$

$$\implies \ddot{a}_{64.5}^{(2)} = 9.77245$$

There were many other ways to do this part, including starting from $\ddot{a}_{64}$ from the Illustrative Life Table, or from $\ddot{a}_{63.5}^{(2)}$ given in the question.

Many candidates could not correctly calculate ${}_{0.5}p_{64.5}$ or ${}_{0.5}p_{63.5}$ using the UDD assumption.

(c)(i) **12 Grading Points**

Let S_x denote the salary earned in the year of age x to $x+1$, assuming the employee is in force throughout that period.

Working in \$000s

$$AL_0 = \left\{ \frac{d_{63}^{(r)}}{l_{63}^{(\tau)}} v^{0.5}\;(0.02)\;(28)\;S_{62.5}\;\ddot{a}_{63.5}^{(2)}\;(1 - 18(0.005)) \right.$$

$$+ \quad \frac{d_{64}^{(r)}}{l_{63}^{(\tau)}} v^{1.5}\;(0.02)\;(28)\;S_{63.5}\;\ddot{a}_{64.5}^{(2)}\;(1 - 6(0.005))$$

$$\left. + \quad \frac{l_{65}^{(\tau)}}{l_{63}^{(\tau)}} v^{2}\;(0.02)\;(28)\;S_{64}\;\ddot{a}_{65}^{(2)} \right\}$$

$$= \frac{0.02\,(28)}{l_{63}^{(\tau)}} \left\{ d_{63}^{(r)} v^{0.5}\;S_{62.5}\;\ddot{a}_{63.5}^{(2)}\;(0.91) + d_{64}^{(r)} v^{1.5} S_{63.5}\;\ddot{a}_{64.5}^{(2)}\;(0.97) \right.$$

$$\left. + \quad l_{65}^{(\tau)} v^{2}\;S_{64}\;\ddot{a}_{65}^{(2)} \right\}$$

$$S_{62.5} = 40\frac{(1.03)^{27} + (1.03)^{28}}{2} = 90.184$$

$$S_{63.5} = 92.890 \quad \text{from (a)}$$

$$S_{64} = 40(1.03)^{29} = 94.262$$

$$\Rightarrow AL_0 = 436.530 \text{ or } \$436,530$$

(c)(ii) Following equation 10.3 of AMLCR, the Normal Contribution is

$$NC = v\, p_{63}^{(\tau)} AL_1 + \text{EPV [Benefits from mid-year exits]} - AL_0$$

Where

$$vp_{63}^{(\tau)}\ AL_1 = \frac{0.02\,(29)}{l_{63}^{(\tau)}}\left\{ d_{64}^{(r)} v^{1.5} S_{63.5}\ \ddot{a}_{64.5}^{(2)}\ (0.97) + l_{65}^{(\tau)} v^2\ S_{64}\ \ddot{a}_{65}^{(2)} \right\}$$

$$= \frac{(0.02)(29)}{15130}\left(2006v^{1.5}\ (92.890)(9.77245)(0.97) + 11246v^2(94.262)(9.6416) \right)$$

$$= 410.752$$

$$\text{EPV [Benefits from mid-year exits]} = \frac{0.02\,(28.5)}{l_{63}^{(\tau)}}\left\{ d_{63}^{(r)} v^{0.5}\ S_{62.5}\ \ddot{a}_{63.5}^{(2)}\ (0.91) \right\}$$

$$= \frac{(0.02)(28.5)}{15130}\left(1350v^{0.5}(90.184)(10.0282)(0.91) \right)$$

$$= 40.654$$

$$\Longrightarrow\ NC = 451.407 - 436.530 = 14.877 \text{ or } \$14,877$$

Note that we need to deal with the two cases separately because for the lives still in force at age 64 the contribution must fund one full year of accrual (increasing from 28 to 29), while for the lives who exit at age 63.5 we only add one-half of a year of additional accrual (28 to 28.5).

Few candidates achieved full credit for (c)(i), and an even smaller number for (c)(ii). Most candidates who did not omit the question ignored the midy-year exits in (c)(ii), and simply divided the AL by 28 for the normal contribution.