GI ADV Model Solutions
Spring 2017

1. **Learning Objectives:**

4. The candidate will understand how to apply the fundamental techniques of reinsurance pricing.

Learning Outcomes:

(4c) Calculate the price for a casualty per occurrence excess treaty.

Source:

Basics of Reinsurance Pricing, Clark

Solution:

(a) Estimate the experience rating loss and ALAE cost as a percentage of the subject premium.

Commentary on Question:

For Loss 4, when allocating ALAE to the layer, it is the policy limit of 1,000,000 that is used and not the trended loss of 1,191,016.

Loss 1: Trended loss is 200,000 x 1.06^4 = 242,495. It is not in the layer and thus loss and ALAE in the layer are 0.
Loss 2: Trended loss and ALAE are 400,000 x 1.06^4 = 504,991 and 200,000 x 1.06^4 = 252,495. Loss in layer is 104,991 and ALAE is 252,495 x 104,991/504,991 = 52,495.
Loss 3: Trended loss is 550,000 x 1.06^3 = 655,059. Loss in layer is 200,000 and ALAE is 0.
Loss 4: Trended loss and ALAE are 1,000,000 x 1.06^3 = 1,191,016 and 500,000 x 1.06^3 = 595,508. Loss in layer is 200,000 and ALAE is 595,508 x 200,000/1,000,000 = 119,102.
Loss 5: Trended loss and ALAE are 600,000 x 1.06^2 = 674,160 and 300,000 x 1.06^2 = 337,080. Loss in layer is 200,000 and ALAE is 337,080 x 200,000/674,160 = 100,000.
Loss 6: Trended loss is 450,000 x 1.06^2 = 505,620. Loss in layer is 105,620 and ALAE is 0.

1. **Continued**

Developed loss and ALAE in layer are:
2014: (104,991 + 52,495) x 1.10 = 173,235
2015: (200,000 + 200,000 + 119,102) x 1.50 = 778,653
2016: (200,000 + 100,000 + 105,620) x 2.00 = 811,240
Total is 1,763,128

The percentage of subject premium is 1,763,128/30,000,000 = 5.9%.

(b) Explain what additional information you would need, if any, to experience rate each layer.

Commentary on Question:
Not all candidates understood that different information would be needed for the two cases.

For both cases revised (likely smaller) development factors would be needed. For case (ii) information about untrended losses below 200,000 would be needed. Assuming the 6% trend factor applies to these losses, untrended losses of 200,000/1.06^4 = 158,419 (likely rounded to 150,000) or larger would be required.

2. Learning Objectives:

2. The candidate will understand the considerations in selecting a risk margin for unpaid claims.

Learning Outcomes:

(2a) Describe a risk margin analysis framework.

(2b) Identify the sources of uncertainty underlying an estimate of unpaid claims.

(2c) Describe methods to assess this uncertainty.

Source:
A Framework for Assessing Risk Margins, Marshall, et al.

Solution:

(a) Calculate the internal systemic risk coefficient of variation for outstanding claim liabilities for both lines combined.

Commentary on Question:
The major challenge for candidates was matching risk indicators to sources of internal systemic risk.

Specification error – Risk Indicators 1 and 3
Parameter selection error – Risk Indicators 2 and 6
Data error – Risk Indicators 4 and 5

Motor score: 0.3(3+2)/2 + 0.5(5+3)/2 + 0.2(4+7)/2 = 3.85
Home score: 0.3(5+3)/2 + 0.5(3+4)/2 + 0.2(4+6)/2 = 3.95

Both scores are in the range 3-4 and hence CoV = 8.5% for each.

Combined CoV = $\sqrt{8.5^2[0.4^2+0.6^2+2(0.5)(0.4)(0.6)]}=7.41\%$ where the factors 0.4 and 0.6 are the relative proportions of outstanding claim liabilities for each line.

(b) Provide a situation where this a priori assumption may not hold.

If there is unclosed or contractually bound future business then premium liabilities will have additional uncertainty due to needing to estimate the associated premium or exposure.

3. Learning Objectives:

5. The candidate will understand methodologies for determining an underwriting profit margin.

Learning Outcomes:

(5b) Calculate an underwriting profit margin using the capital asset pricing model.

Source:

Ratemaking: A Financial Economics Approach, D'Arcy and Dyer

Solution:

(a) State four assumptions of the Capital Asset Pricing Model.

Commentary on Question:

Any four of the listed items were sufficient for full credit.

- Investors are risk averse.
- Investors are price takers.
- Investors have identical expectations.
- Investors have no transaction costs.
- Investors can borrow or invest at the risk-free rate.
- Assets are infinitely divisible.

(b) Calculate the underwriting profit margin.

The slope of the Security Market Line = market risk premium = 3%

$$
\begin{aligned}
T_A &= 0.20(0\%) + 0.20(0.30)(35\%) + 0.60(35\%) = 23.1\% \\
\beta_u &= Cov(R_u, R_m) / Var(R_m) = 0.50 \\
UPM &= -kR_f(1 - T_A)/(1 - T) + \beta_u[E(R_m) - R_f] + (S/P)R_f[T_A/(1 - T)] \\
&= -0.75(2\%)(1 - 0.231)/(1 - 0.35) + 0.5(3\%) + 2(2\%)(0.231)/(1 - 0.35) = 1.15\%.
\end{aligned}
$$

4. Learning Objectives:

1. The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:

(1a) Identify the assumptions underlying the chain ladder estimation method.

(1b) Test for the validity of these assumptions.

(1c) Identify alternative models that should be considered depending on the results of the tests.

(1d) Estimate the standard deviation of a chain ladder estimator of unpaid claims.

Sources:

Measuring the Variability of Chain Ladder Reserve Estimates, Mack
Testing the Assumptions of Age-to-Age Factors, Venter

Solution:

(a) Demonstrate that the value of α_4^2 was correctly calculated. (Your calculation need not match to all three decimal places.)

$$
\begin{aligned}
\alpha_4^2 &= \frac{1}{7-4-1}\sum_{j=1}^{7-4} C_{j,4}\left(\frac{C_{j,5}}{C_{j,4}} - f_4\right)^2 \\
&= \frac{1}{2}\left[C_{1,4}\left(\frac{C_{1,5}}{C_{1,4}} - f_4\right)^2 + C_{2,4}\left(\frac{C_{2,5}}{C_{2,4}} - f_4\right)^2 + C_{3,4}\left(\frac{C_{3,5}}{C_{3,4}} - f_4\right)^2\right] \\
&= \frac{1}{2}\left[35{,}957\left(\frac{36{,}328}{35{,}957} - 1.04517\right)^2 + 36{,}752\left(\frac{38{,}103}{36{,}752} - 1.04517\right)^2 + 45{,}157\left(\frac{48{,}759}{45{,}157} - 1.04517\right)^2\right] \\
&= 50.162.
\end{aligned}
$$

4. **Continued**

(b) Demonstrate that the standard error for accident year 5 was correctly calculated.

$$C_{5,7}^2 \sum_{k=7+1-5=3}^{7-1=6} \frac{\alpha_k^2}{f_k^2}\left(\frac{1}{C_{5,k}}+\frac{1}{\sum_{j=1}^{7-k} C_{j,k}}\right) = C_{5,7}^2\left[\frac{\alpha_3^2}{f_3^2}\left(\frac{1}{C_{5,3}}+\frac{1}{\sum_{j=1}^{7-3=4} C_{j,3}}\right)+\frac{\alpha_4^2}{f_4^2}\left(\frac{1}{C_{5,4}}+\frac{1}{\sum_{j=1}^{7-4=3} C_{j,4}}\right)\right.$$

$$\left.+\frac{\alpha_5^2}{f_5^2}\left(\frac{1}{C_{5,5}}+\frac{1}{\sum_{j=1}^{7-5=2} C_{j,5}}\right)+\frac{\alpha_6^2}{f_6^2}\left(\frac{1}{C_{5,6}}+\frac{1}{\sum_{j=1}^{7-6=1} C_{j,6}}\right)\right]$$

$$=45,317^2\left[\frac{122.133}{1.09150^2}\left(\frac{1}{38,140}+\frac{1}{33,208+30,971+42,282+46,113}\right)\right.$$

$$+\frac{50.162}{1.04517^2}\left(\frac{1}{41,630}+\frac{1}{35,957+36,752+45,157}\right)+\frac{0.0596}{1.02119^2}\left(\frac{1}{43,510}+\frac{1}{36,328+38,103}\right)$$

$$\left.+\frac{0.000071}{1.01993^2}\left(\frac{1}{44,432}+\frac{1}{37,131}\right)\right]$$

$$=3,157^2.$$

(c) Indicate whether or not this observation provides support for the underlying assumptions of Mack's model. Justify your response.

No, Mack assumes that expected development is a multiple of the previous value. It does not assume that the factor is greater than one.

(d) Indicate whether or not this observation provides support for the underlying assumptions of Mack's model. Justify your response.

No, Mack makes no assumption about the observed ratios, only about their expected values and variances.

(e) Indicate whether or not this result confirms that the model's assumptions hold. Justify your response.

The results cannot confirm that the assumptions hold because there may be other tests that would reveal ways in which the assumptions do not hold.

4. **Continued**

(f) Indicate whether or not this result confirms that the model's assumptions do not hold. Justify your response.

The results cannot confirm that the assumptions do not hold. When conducting multiple tests in an environment where the assumptions do hold, it is possible that due to chance a few of the tests may yield an adverse result.

5. **Learning Objectives:**

1. The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:

(1e) Apply a parametric model of loss development.

(1f) Estimate the standard deviation of a parametric estimator of unpaid claims.

Source:

LDF Curve Fitting and Stochastic Reserving: A Maximum Likelihood Approach, Clark

Solution:

(a) State one advantage and one disadvantage of using a parametric distribution function to model loss development.

Commentary on Question:

Any one of the three listed advantages was sufficient for full credit.

Advantages:

- Provides smoothing
- Small number of parameters to estimate
- Does not require equal spacing of data points

Disadvantage:

- Development pattern must be increasing

(b) Estimate the expected payments in 2017 for accident year 2015.

$F(18) = 1 - e^{-18/8.858} = 0.8689,\ F(30) = 1 - e^{-30/8.858} = 0.9662$

Solution is $12,000(0.752)(0.9662 - 0.8689) = 878.$

(c) Estimate ultimate losses for accident year 2016.

$F(6) = 1 - e^{-6/8.858} = 0.4920$

Solution is $6,000 + 12,000(0.752)(1 - 0.4920) = 10,584.$

(d) Estimate the process standard deviation of the accident year 2016 reserve.

Reserve = 10,584 – 6,000 = 4,584.

Process standard deviation = $\sqrt{4,584(813)} = 1,930$.

5. **Continued**

(e) Calculate the number of degrees of freedom in the estimate of σ^2 if the LDF method were used.

Degrees of freedom = 6 – 4 = 2.

(f) Indicate which of the LDF and Cape Cod methods is likely to have a smaller standard deviation of the total reserve. Justify your response.

Cape Cod likely has a smaller standard deviation because of the additional information the exposure base provides.

6. Learning Objectives:

5. The candidate will understand methodologies for determining an underwriting profit margin.

Learning Outcomes:

(5d) Allocate an underwriting profit margin (risk load) among different accounts.

Source:

An Application of Game Theory: Property Catastrophe Risk Load, Mango

Solution:

(a) Calculate the risk load for the combined portfolio of X and Y.

$$\mathrm{SD}(X) = 0.3(2,000) = 600$$
$$\mathrm{SD}(Y) = 0.4(450) = 180$$
$$\mathrm{Var}(X+Y) = 600^2 + 180^2 + 2(0.2)(600)(180) = 435,600$$
$$\mathrm{SD}(X+Y) = \sqrt{435,600} = 660$$
$$\lambda = 0.1(2.33)/(1+0.1) = 0.2118$$
$$\text{Risk load} = 0.2118(660) = 140$$

(b) Calculate the renewal risk loads for accounts X and Y.

Renewal of X: 0.2118(660 – 180) = 102
Renewal of Y: 0.2118(660 – 600) = 13

(c) State a problem with using the Marginal Surplus method to calculate renewal risk loads.

Individual risk loads will add to less than the total risk load.

(d) Calculate the renewal risk loads for accounts X and Y using the Shapley method.

The multiplier is 140/435,600 = 0.0003214 (an alternative derivation is 0.2118/660)
Covariance = 0.2(600)(180) = 21,600
Risk load for X: $0.0003214(600^2 + 21,600) = 123$
Risk load for Y: $0.0003214(180^2 + 21,600) = 17$

7. Learning Objectives:

3. The candidate will understand excess of loss coverages and retrospective rating.

Learning Outcomes:

(3d) Explain retrospective rating in graphical terms.

(3e) Explain Table M and Table L construction in graphical terms.

Source:

The Mathematics of Excess of Loss Coverages and Retrospective Rating – A Graphical Approach, Lee

Solution:

(a) Provide an equation that demonstrates the relationship among all five of the quantities above.

$$b + C \times (E - I) = e + E$$

(b) Provide an expression for the minimum premium *H* in terms of *b*, *E*, *C* and r_H, the entry ratio corresponding to the minimum premium.

$$H = b + C \times E \times r_H$$

(c) Provide an expression for the maximum premium *G* in terms of *b*, *E*, *C* and r_G, the entry ratio corresponding to the maximum premium.

$$G = b + C \times E \times r_G$$

(d) Provide an expression for *I* in terms of *E*, the Table M savings $\psi(r_H)$, and the Table M charge $\phi(r_G)$.

$$I = E \times [\phi(r_G) - \psi(r_H)]$$

8. Learning Objectives:

4. The candidate will understand how to apply the fundamental techniques of reinsurance pricing.

Learning Outcomes:

(4d) Apply an aggregate distribution model to a reinsurance pricing scenario.

Source:

Basics of Reinsurance Pricing, Clark

Solution:

(a) Calculate the probability that aggregate catastrophe losses will be 10 billion.

Commentary on Question:

The only efficient way to calculate the probability is using the recursive formula. Candidates who knew this but were unable to execute the formula correctly received partial credit.

The probability is

$$\frac{2}{10}(1\times 0.4\times 0.0318+2\times 0.3\times 0.0453+3\times 0.2\times 0.0621+4\times 0.1\times 0.0804)=0.0219\,.$$

(b) Calculate the mean and coefficient of variation of aggregate catastrophe losses.

$$\text{Mean loss} = 1\times 0.4+2\times 0.3+3\times 0.2+4\times 0.1=2.0$$

$$\text{Second moment of loss} = 1^2\times 0.4+2^2\times 0.3+3^2\times 0.2+4^2\times 0.1=5.0$$

$$\text{Aggregate mean} = 2(2.0)=4.0 \text{ billion}$$

$$\text{Aggregate variance} = 2(5.0)=10.0 \text{ billion}^2$$

$$\text{CoV} = \sqrt{10.0}/4.0=0.7906$$

(c) Calculate the parameters of the lognormal distribution using your answer to part (b).

Commentary on Question:

To receive full credit, all calculations had to be in billions.

The aggregate second moment is 10 + 4^2 = 26 billion^2. The equations to solve are:

$$4\times 10^9 = e^{\mu+\sigma^2/2}$$

$$26\times (10^9)^2 = e^{2\mu+2\sigma^2}.$$

The solutions are:

$$\sigma^2 = \ln(0.7906^2+1)=0.4855$$

$$\mu = \ln(4\times 10^9)-0.4855/2=21.87.$$