MLC Fall 2017

Model Solutions
Written Answer Questions

Question 1 Model Solution

Learning Outcomes: 3(a), 4(a)

Chapter References: AMLCR Chapters 6 (Section 6.6) and 7 (Section 7.9, Example 7.18)

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a)

(i) $P^g\, \ddot{a}_{50:\overline{10|}} = 100{,}000 A_{50:\overline{20|}} + 50\, \ddot{a}_{50:\overline{10|}} + 450 + .04 P^g \ddot{a}_{50:\overline{10|}} + .36 P^g$

$$P^g = \frac{100{,}000 A_{50:\overline{20|}} + 50\, \ddot{a}_{50:\overline{10|}} + 450}{.96\, \ddot{a}_{50:\overline{10|}} - .36}$$

$A_{50:\overline{20|}} = A_{50} + {}_{20}E_{50}\,(1 - A_{70})$ (or $A_{50:\overline{20|}} = 1 - d\ddot{a}_{50:\overline{20|}}$)
= .24905 + .23047(1-.51495) = .360839474

$\ddot{a}_{50:\overline{10|}} = \ddot{a}_{50} - {}_{10}E_{50}\, \ddot{a}_{60}$ (or $\ddot{a}_{50:\overline{10|}} = (1 - A_{50:\overline{10|}})/d$)
= 13.2668 - .51081(11.1454) = 7.573618

$P^g =$ (36083.9474 + 50 (7.573618) + 450) / (.96 (7.573618) - .36)
= 36912.6283 / 6.910673 = 5341.39

(ii) $P^e\, \ddot{a}_{50:\overline{10|}} = P^g\,(.04\, \ddot{a}_{50:\overline{10|}} + .36) + 50\, \ddot{a}_{50:\overline{10|}} + 450$
P^e = 0.08753342 P^g + 109.41678 = 576.967

Comments:

1. *This part was done correctly by almost all candidates.*
2. *Many candidates calculated the net premium (4764.43) and subtracting it from the gross premium to get the expense premium.*

b)

Recursively,

${}_1V^e = ({}_0V^e + P^e – e_0)(1+i) / p_{50}$

$= (0 + 576.967 – (.4)(5341.39) – 500)(1.06) / .99408 = -2196.17$

Alternative

Prospectively,

${}_1V^e = (50 + .04\ P^g - P^e)\ \ddot{a}_{51:\overline{9}|}$ where $\ddot{a}_{51:\overline{9}|} = \frac{\ddot{a}_{50:\overline{10}|}-1}{vp_{50}} = \ddot{a}_{51} - {}_9E_{51}\ddot{a}_{60}$; ${}_9E_{51} = {}_{10}E_{50}/vp_{50}$

${}_1V^e = [\ 50 + (.04)(5341.39) – 576.967] \cdot [13.0803 – (.544683)(11.1454)] = -2196.19$

Comments:

1. *This part was done correctly by many candidates.*
2. *Some candidates lost some marks for not completing the calculations when attempting to get the expense reserve as the difference between the gross reserve and the net reserve.*
3. *Others lost some marks for not completing the calculations when using the prospective method*.
4. *Another common error was to use wrong expenses.*

c)

(Prospectively) The renewal expenses are less than the level expense loading to reimburse the large acquisition expenses of 500 + $.4P^g$. So, at time 1, the present value of future renewal expenses will be less than the present value of the expense premiums. Since the reserve is present value of expenses less present value of expense loading, it will be negative.

Alternative

(Retrospectively) First year expenses are more than the expense loading. So, at time 1, the accumulated value of expenses will exceed the value of the expense premiums collected. Since the reserve is the accumulated value of expense loading received less value of incurred expenses, it will be negative.

Comment: Few candidates provided a sufficiently complete and correct explanation for full credit.

d)

<u>Time 1</u>: ${}_1V$ = 0 by definition

<u>Time 2</u>:

We need P^{FPT} = 100,000 ${}_9P_{51:\overline{19|}} = \frac{100{,}000\, A_{51:\overline{19|}}}{\ddot{a}_{51:\overline{9|}}}$

where

$A_{51:\overline{19|}} = \frac{A_{50:\overline{20|}} - vq_{50}}{vp_{50}} = \frac{.3608395 - .00592/1.06}{(1 - .00592)/1.06} = .3788124$

and

$\ddot{a}_{51:\overline{9|}} = \frac{\ddot{a}_{50:\overline{10|}} - 1}{vp_{50}}$ = (7.573618 - 1) / ((1-.00592)/1.06) = 7.009532

So, $P^{FPT} =$ 37881.24/ 7.009589 = 5404.247

Recursively,

$\left({}_1V^{FPT} + 100{,}000\ {}_9P_{51:\overline{9|}}\right)(1.06) - 100{,}000 \cdot q_{51} = p_{51} \cdot {}_2V^{FPT}$

⇨ ${}_2V^{FPT}$=5119.323

<u>Time 19</u>:

${}_{19}V$ = $100{,}000 A_{69:\overline{1|}} - 0$ = 100000 v = 94339.62

Comment: Many candidates did not use P^{FPT} to calculate the FPT reserve or did not know how to calculate P^{FPT} .

e)

The FPT method adjusts net premium reserves to approximate gross premium reserves by implicitly assuming that the first year premium is spent on cost of insurance and acquisition expenses. It reduces the initial strain on surplus from new business.

Comment: Many candidates explained how the FPT method works instead of providing a rationale for its use.

Question 2 Model Solution

Learning Outcomes: 1(a), 4(c), 4(e)

Chapter References: AMLCR Chapters 12 and 13

General Comment: Most candidates did very well on this question.

a)

$AV_t = (AV_{t-1} + P_t - E_t - COI_t)(1 + i^c)$

where $COI_t = \frac{ADB_t \cdot q^*}{(1+i_q)}$; $i_q = i^c = .04$;

ADB_t= FA=150,000 (UL Type B)

So,

$AV_3 = (8166 + 4800 - (.05 * 4800) - \frac{150\,(2.62)}{1.04})(1.04)$ = 12,842.04

Total death benefit in year 3: $DB_3 = ADB_3 + AV_3$= 162,842.04

Comment: This part was done correctly by almost all candidates.

b)

$Pr_t = AV_{t-1} + P_t - E_t + I_t - EDB_t - ESB_t - EAV_t;$

$AV_0 = 0$; $Pr_0 = -E_0 = -800$

$E_t = e_t \cdot P_t$ where e_t= .25, .07, .07 for t=1,2,3 respectively

For t=1,2,3:

$I_t = i_t(AV_{t-1} + P_t - E_t)$; i_t=.08

$EDB_t = q_{x+t-1}^{0d}(ADB_t + AV_t + 500)$; q_{x+t-1}^{0d}=.0015

$ESB_t = q_{x+t-1}^{0w}(AV_t - SC_t)$; $q_{x+t-1}^{0w} = (1 - q_{x+t-1}^{0d})q_{x+t-1}^{w}$; $q_{x+t-1}^{w} = \begin{cases} .1 & t = 1 \\ .2 & t = 2 \\ 1 & t = 3 \end{cases}$

$EAV_t = \left(1 - q_{x+t-1}^{0d}\right)(1 - q_{x+t-1}^{w})\, AV_t$

Profit testing table:

t	AV_{t-1}	P_t	E_t	I_t	EDB_t	ESB_t	EAV_t	Pr_t
1	0	4800	1200	288	231.22	114.53	3277.38	264.88
2	3647	4800	336	648.88	238.00	1331.20	6523.00	667.68
3	8166	4800	336	1010.40	245.01	12822.78	0	572.61

Comments:

1. *Most candidates did well on this part.*
2. *Common errors included incorrectly calculating the surrender probabilities, q^{0w}; using wrong probabilities when calculating EDB, ESB or EAV; and calculation mistakes.*

c)

$\pi_t = {}_{t-1}p_x^{00} \cdot Pr_t; \quad \pi_0 = Pr_0$

$NPV_t = NPV_{t-1} + \pi_t v^t = \sum_{k=0}^{t} \pi_k v^k; \quad NPV_0 = \pi_0; \qquad v = \frac{1}{1.1}$

t	${}_{t-1}p_x^{00}$	Pr_t	π_t	v^t	NPV_t
0		-800	-800	1	-800
1	1	264.88	264.88	.909091	-559.21
2	.89865	667.68	600.01	.826446	-63.33
3	.717842	572.61	411.04	.751315	245.50

Comments:

1. *Most candidates did well on this part.*
2. *A common error was to discount the profits at a rate different than the hurdle rate.*
3. *Another common error was to use ${}_tp_x^{00}$ instead of ${}_{t-1}p_x^{00}$ when calculating π_t.*

d)

$$\text{Profit margin } = \frac{NPV}{EPV\ of\ premiums\ at\ hurdle\ rate}$$

$$= \frac{245.50}{4800\left(1 + \frac{.89865}{1.1} + \frac{.717842}{1.1^2}\right)} = .0212$$

Comment: Many candidates omitted this part.

Question 3 Model Solution

Learning Outcomes: 2(c), 5(c), 5(d)

Chapter References: AMLCR Chapters 5 (Section 5.9) and 10

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a)

Let S_{yy} be the salary earned (or projected) in year yy.

Accrued benefit = $0.02\ x\ Service\ x\ \left(\frac{S_{2015}+S_{2016}+S_{2017}}{3}\right)$

$= 0.02\ x\ (2018 - 1988)\ x\ \left(\frac{90{,}000+92{,}000+95{,}000}{3}\right)$

= (.02)(30)(92,333.33) = 55,400

b)

Under TUC, actuarial liabilities (AL) are valued using salaries as at the valuation date.

(i) Actuarial liability

If retiring on 1/1/2018: $AL_{18} = 55{,}400\ (1 - (.005)(12)(65-60))\ \ddot{a}_{60}^{(12)}$

Woolhouse 2-term: $\ddot{a}_{60}^{(12)} = \ddot{a}_{60} - \frac{11}{24}$ = 11.1454 - .4583 = 10.6871

$AL_{18} = 55{,}400\ (.7)(10.6871)$ = 414,445.74

If retiring on 1/1/2023: $AL_{23} = 55{,}400\ (1-0)v^5\ \ddot{a}_{65}^{(12)}$

Woolhouse 2-term: $\ddot{a}_{65}^{(12)} = \ddot{a}_{65} - \frac{11}{24}$ = 9.8969 - .4583 = 9.4386

$AL_{23} = 55{,}400\ (1)(.747258)(9.4386)$ = 390,740.10

$AL = \Pr(retiring\ in\ 2018) \cdot AL_{18} + \Pr(retiring\ in\ 2023) \cdot AL_{23}$

$= (.5)(414{,}445.74) + (.5)(390{,}740.10) = 402{,}592.90$

(ii) Gain to the plan if Caroline chooses to retire today

Gain = $AL - AL_{18}$= $402{,}592.90 - 414{,}445.74 = -11{,}852.80$

The financial impact to the plan would be a **loss** of 11,852.80

c)

Under PUC, actuarial accrued liabilities are valued using projected salaries.

If retiring on 1/1/2018: accrued benefit = 55,400

If retiring on 1/1/2023:

Projected Benefit= $0.02\ x\ Service\ x\ \left(\frac{S_{2020}+S_{2021}+S_{2022}}{3}\right)$

Projected benefit = $0.02\ x\ (2018-1988)\ x\ 95{,}000\left(\frac{1.02^3+1.02^4+1.02^5}{3}\right)$

= (.02)(30)(102,844.50) = 61,706.70

(i) Actuarial liability

If retiring on 1/1/2018: $AL_{18}\ = 55{,}400\ (1-(.005)(12)(65-60))\ \ddot{a}_{60}^{(12)}$ = 414,445.74

If retiring on 1/1/2023: $AL_{23}\ = 61706.70\ (1-0)v^5\ \ddot{a}_{65}^{(12)}$

$AL_{23}\ = 61{,}706.70\ (1)(.747258)(9.4386)$ = 435,221.70

$AL\ = (.5)(414{,}445.74\,) + \ (.5)(435{,}221.70) =\ 424{,}833.70$

(ii) Gain to the plan if Caroline chooses to retire today

Gain = $AL - AL_{18}$= $424{,}833.70\ \ - 414{,}445.74\ =\ 10{,}387.99$

The financial impact to the plan would be a **gain** of 10,387.99

Comments:

1. *Many candidates omitted this question or only answered part (a).*
2. *The candidates who answered this question more fully did fairly well.*
3. *The specific decrement model (no exits before retirement and retirements at 60 or 65) proved to be challenging for some candidates.*
4. *Analyzing the financial impact to the plan of an early retirement was another challenge.*

Question 4 Model Solution

Learning Outcomes: 2(a), 3(a), 3(c), 4(a), 4(b)

Chapter References: AMLCR Sections 4.4, 5.4, 6.4 and 6.5

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a)

$L_5 = 10000\ v^{K_{40}+1} - P \cdot \ddot{a}_{\overline{\min(5, K_{40}+1)}|},\ K_{40} = 0,1, \ldots$

Comments:

1. *Candidates either did well or did poorly on this part.*
2. *Many candidates provided an expression with EPV's instead of random variables.*
3. *Another common error was to use K_{40} instead of $K_{40} + 1$.*

b)

Let ${}_5V = A_{40} - P \cdot \ddot{a}_{40:\overline{5}|}$

$\ddot{a}_{40:\overline{5}|} = \ddot{a}_{40} - {}_5E_{40} \cdot \ddot{a}_{45}$ = 14.8166 - .73529(14.1121) = 4.44011

$A_{40} = 0.16132$

Net premium policy value: 10,000 ${}_5V$ = 1613.20 – 166.58 (4.44011) = 873.57

Comment: Most candidates did very well on this part.

c)

(i) $Z = 10{,}000\ v^{K_{40}+1}$, $K_{40} = 0,1, \ldots$;

$Var(Z)$ =10,000^2 (${}^2A_{40} - A_{40}^2$) = 10,000^2 (.04863 - .16132^2) = 2,260,585.76

The standard deviation of Z is 1503.52

(ii) Let $Y = P \cdot \left(\frac{1-Z_2}{d}\right)$;

where $Z_2 = \begin{cases} v^{K_{40}+1} & K_{40} = 0,1, \ldots, 4 \\ v^5 & K_{40} = 5,6, \ldots \end{cases} = v^{\min(5, K_{40}+1)}$, $K_{40} = 0,1, \ldots$

and d=.06/1.06 = .0566038

So, $Var(Y) = \left(\frac{P}{d}\right)^2 V(Z_2) = \left(\frac{P}{d}\right)^2 \left({}^2A_{40:\overline{5}|} - A_{40:\overline{5}|}{}^2\right)$

Then,

$E(Z_2) = A_{40:\overline{5}|}$ = $A_{40} + {}_5E_{40}\,(1-A_{45})$

= (.16132 + (.73529)(1-.2012)) = 0.74867

$E(Z_2^2) = {}^2A_{40:\overline{5}|}$ = ${}^2A_{40} + v^5\,{}_5E_{40}\,(1-{}^2A_{45})$

= .04863 + 1.06^{-5} (.73529) (1-.06802) = .5607078

and

$Var(Y) = \left(\frac{P}{d}\right)^2 \left({}^2A_{40:\overline{5}|} - A_{40:\overline{5}|}{}^2\right)$

= $(166.58/.0566038)^2$ [.5607078 – $.74867^2$]

= (8,660,738.80) [.5607078 - .56050625] = 1745.361

The standard deviation of Y is 41.78

Comments:

1. *Most candidates did well on part (i) but only a minority of stronger candidates correctly completed part (ii).*
2. *Some candidates worked with a six-point distribution for all possible values for Y with their respective probabilities. This received full credit if done correctly.*
3. *A common error was to work with a term insurance contract for Z_2 instead of an endowment contract.*
4. *Other common errors were to use incorrect ages, probabilities, or using a 10-year contract instead of a 5-year one for Z_2 .*

d)

$cov(Z,Y) = -\frac{P}{d}cov(Z,Z_2)$ where $cov(Z,Z_2) = E(Z\cdot Z_2) - E(Z)E(Z_2)$

Since $Z\,Z_2 = \begin{cases} v^{2(K_{40}+1)} & K_{40} = 0,1,\ldots,4 \\ v^5 v^{K_{40}+1} & K_{40} = 5,6,\ldots \end{cases}$

$E(Z\,Z_2) = {}^2A_{\overset{1}{40:\overline{5}|}} + v^5\ {}_{5|}A_{40}$

= [.04863 - 1.06^{-5} (.73529) (.06802)] + 1.06^{-5} (.73529)(.2012)

= .0112563 + .1105496 = .1218059

So,

$cov(Z,Y) = -\frac{P}{d}\left({}^2A_{\overset{1}{40:\overline{5}|}} + v^5\ {}_{5|}A_{40} - A_{40}\ \cdot A_{40:\overline{5}|}\right)$

$cov(Z,Y) = -(\frac{166.58}{.0566038})$ [.1218059 – (.16132)(.16132 + (.73529)(1-.2012))] = -30328.41

Comments:

1. *Many candidates found this part challenging. Few achieved full credit.*
2. *A few candidates worked with a six-point distribution for $Z \cdot Y$ but most used incorrect probabilities and/or an incorrect (expected) value for $Z \cdot Y$ in case of survival to age 45.*

e)

Let $L_5 = Z - Y$

where

$Z = 10000\, v^{K_{40}+1} \quad K_{40} = 0,1,\ldots \quad ;$

and

$Y = P \cdot \left(\frac{1-Z_2}{d}\right); \qquad Z_2 = \begin{cases} v^{K_{40}+1} & K_{40} = 0,1,\ldots,4 \\ v^5 & K_{40} = 5,6,\ldots \end{cases}$

Then,

$Var[L_5] = Var[Z] + Var[Y] - 2cov(Z,Y)$

$$= \begin{array}{l} 10{,}000^2\left({}^2A_{40} - A_{40}^{\;2}\right) + \left(\tfrac{P}{d}\right)^2\left({}^2A_{40:\overline{5}|} - A_{40:\overline{5}|}^{\;2}\right) + \\ \quad - 2\tfrac{P}{d}\left({}^2A^{\;1}_{40:\overline{5}|} + v^5 \cdot ({}_{5|}A_{40}) - A_{40} \cdot A_{40:\overline{5}|}\right) \end{array}$$

= 2,260,585.76 + 1745.361 + 2(30328.41) = 2,322,988

The standard deviation of L_5 is 1524.14

Comments:

1. *Most candidates failed to recognize that $L_5 = Z - Y$.*
2. *Among those who did, many ignored the covariance term.*

f)

Z and Y move in opposite direction as a function of the year of death. An early death implies a large value of Z and a small value of Y; and a death at an older age implies a small value of Z and a large value of Y.

Comments:

1. *Candidates either did well or did poorly on this part.*
2. *Only a few candidates provided a complete and correct explanation of the relationship between Z and Y as a function of the year of death.*

Question 5 Model Solution

Learning Outcomes: 1(a), 1(b), 1(d), 2(a), 3(a), 3(b), 4(a), 4(b)

Chapter References: AMLCR Section 9.6; Examples 9.10, 9.11; Exercises 9.15, 9.16

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a)

$${}_tp_{xy}^{01} = \int_0^t {}_sp_{xy}^{00} \cdot \mu_{x+s:y+s}^{01} \cdot {}_{t-s}p_{x+s:y+s}^{11} ds$$

where ${}_sp_{xy}^{00} = e^{-(.01+.014+.001)s}$ and ${}_{t-s}p_{x+s:y+s}^{11} = e^{-.012(t-s)}$

So, ${}_tp_{xy}^{01} = \int_0^t e^{-(.01+.014+.001)s} (.01) e^{-.012(t-s)} ds$

$$= (.01)(e^{-.012t}) \frac{(1-e^{-.013t})}{.013} = \frac{.01}{.013}(e^{-.012t} - e^{-.025t})$$

Comment: This part was done correctly by most candidates.

b)

(i)- Simultaneous deaths:

$$\text{EPV} = 100{,}000 \int_0^{10} v^t \, {}_tp_{xy}^{00} \cdot \mu_{x+t:y+t}^{03} \, dt$$

$$= 100{,}000 \int_0^{10} e^{-.06t} \, e^{-(.01+.001+.014)t} \, (.001) \, dt$$

$$= (100{,}000)(.001)\,(1 - e^{-.085(10)})/.085 = 673.63$$

(ii)- Fred dying after Molly:

$\text{EPV} = 100{,}000 \int_0^{10} v^t \, {}_tp_{xy}^{01} \, \mu_{x+t:y+t}^{13} \, dt$ where ${}_tp_{xy}^{01}$ is given in (a)

$$\text{EPV} = 100{,}000 \int_0^{10} e^{-.06t} \left[\frac{.01}{.013}(e^{-.012t} - e^{-.025t})\right] (.012) \, dt$$

$$= 100{,}000 \, \frac{(.012)(.01)}{.013} \left[\frac{(1-e^{-.072(10)})}{.072} - \frac{(1-e^{-.085(10)})}{.085}\right] = 361.981$$

Comments:

1. *Most candidates received full credit for part (i).*
2. *However, part (ii) proved to be more challenging.*

c)

EPV of benefit: 673.63 + 361.98 = 1035.61

EPV of premiums

EPV of annuity per \$1 of premium = $\bar{a}^{00} + .75\,\bar{a}^{01}$

$$= \int_0^{10} v^t\ {}_tp_{xy}^{00}\,dt + (.75)\int_0^{10} v^t\ {}_tp_{xy}^{01}\,dt$$

$$= \int_0^{10} e^{-.06t}\ e^{-(.01+.001+.014)t}\,dt + (.75)\int_0^{10} e^{-.06t}\left[\int_0^t e^{-.025s}\,(.01)e^{-.012(t-s)}ds\right]dt$$

$$= (1 - e^{-.085(10)})/.085 \quad + (.75)\ \frac{(.01)}{.013}\frac{(1-e^{-.072(10)})}{.072} - \frac{(1-e^{-.085(10)})}{.085}$$

$$= 6.7363 + .22624 = 6.96254$$

⇨ P = 1035.61 / 6.96254 = 148.74

Comments:

1. *Many candidates did not attempt this part.*
2. *For those who did answer it, the changing premium rate proved to be challenging.*

d)

$${}_5V^{(1)} = (100{,}000)\int_0^{10-5} v^t\ {}_tp_{x+5:y+5}^{11}\ \mu_{x+5+t:y+5+t}^{13}\,dt\ -(.75)P\ \cdot \int_0^{10-5} v^t\ {}_tp_{x+5:y+5}^{11}\,dt$$

$$= (100{,}000)\int_0^{5} e^{-.06t}\ e^{-.012t}\,(.012)dt - (.75)(148.74)\cdot\int_0^5 e^{-.06t}\ e^{-.012t}\,dt$$

$$= (100{,}000)\,(.050387) - (.75)(148.74)(4.19894) = 4570.29$$

Alternatively, <u>given State 1</u>, we have a single constant force of decrement.

So, with $\mu = \mu_{x+5+t:y+5+t}^{13} = .012$ and $\delta = 0.06$, we get

$\bar{A}^{\ 1}_{x+5:\overline{5}|} = \frac{\mu}{\mu+\delta}(1-e^{-5(\mu+\delta)})$ = 0.050387 and $\bar{a}_{x+5:\overline{5}|} = \frac{1}{\mu+\delta}(1 - e^{-5(\mu+\delta)})$= 4.19894

${}_5V^{(1)} = (100{,}000)\bar{A}^{\ 1}_{x+5:\overline{5}|} - (.75)P\ \bar{a}_{x+5:\overline{5}|}$ = 4570.29

Comment: Most candidates did not attempt this part.

e)

The EPV of death benefit would increase.
A transition to State 2 would be less likely, therefore increasing the probability of moving to State 3, directly or via State 1, and resulting in the death benefit being paid with a higher probability.

Comments:

1. *Many candidates did not understand the effect of the change.*
2. *For full credit, the candidate's explanation needed to be coherent, and to demonstrate that the candidate understood the effect of the change in terms of the multiple state model transitions. Few candidates achieved full credit.*

Question 6 Model Solution

Learning Outcomes: 1(a), 1(b), 1(d)

Chapter References: AMLCR Sections 8.4, 8.9 – 8.13

General Comments:

1. *Most candidates omitted or received no points for this question.*
2. *Adapting multiple decrement concepts to non-standard fractional age assumptions proved to be challenging for candidates.*
3. *Many candidates seemed to rely on formulas that did not apply to this question.*
4. *Many candidates also struggled with the distinction between dependent and independent decrement probabilities.*

a)

The general Kolmogorov forward differential equation is:

$$\frac{d}{dt}\,{}_tp_x^{ij} = \sum_{k=0,k\neq j}^{n}\left({}_tp_x^{ik}\,\mu_{x+t}^{kj} - {}_tp_x^{ij}\,\mu_{x+t}^{jk}\right)$$

In this case, *i* = 0, so that

$$\frac{d}{dt}\,{}_tp_x^{0j} = \sum_{k=0,k\neq j}^{n}\left({}_tp_x^{0k}\,\mu_{x+t}^{kj} - {}_tp_x^{0j}\,\mu_{x+t}^{jk}\right)$$

Now $\mu_{x+t}^{jk} = 0, \forall k$ and $\mu_{x+t}^{kj} = 0$ for k≠0, so we have

$$\frac{d}{dt}\,{}_tp_x^{0j} = {}_tp_x^{00}\,\mu_{x+t}^{0j}$$

Alternatively, using traditional multiple decrement actuarial notation we have

$$\frac{d}{dt}\,{}_tq_x^{(j)} = {}_tp_x^{(\tau)}\,\mu_{x+t}^{(j)}$$

Comments:

1. *Most candidates were able to write down Kolmogorov's Forward Differential Equations for a general multi-state model.*
2. *Only a small number of candidates were able to properly simplify the equation and get full credit for this part.*

b)

(i) $p_x^{(\tau)} = 1 - (q_x^{(1)} + q_x^{(2)} + q_x^{(3)})$

$p_{60}^{(\tau)} = 1 - (.05 + .10 + .08) = 0.77$ and $p_{61}^{(\tau)} = 1 - (0 + .14 + .12) = 0.74$

${}_2p_{60}^{(\tau)} = p_{60}^{(\tau)}\, p_{61}^{(\tau)} = 0.77 * 0.74 = 0.5698$

(ii) ${}_tp_x^{(\tau)} = 1 - ({}_tq_x^{(1)} + {}_tq_x^{(2)} + {}_tq_x^{(3)})$

${}_{0.8}p_{60}^{(\tau)} = 1 - ({}_{0.8}q_{60}^{(1)} + {}_{0.8}q_{60}^{(2)} + {}_{0.8}q_{60}^{(3)})$

${}_{0.8}q_{60}^{(1)} = 0.05$ (decrement happens at t=0.25)

${}_{0.8}q_{60}^{(2)} = (0.8)(0.10) = .08$ (UDD)

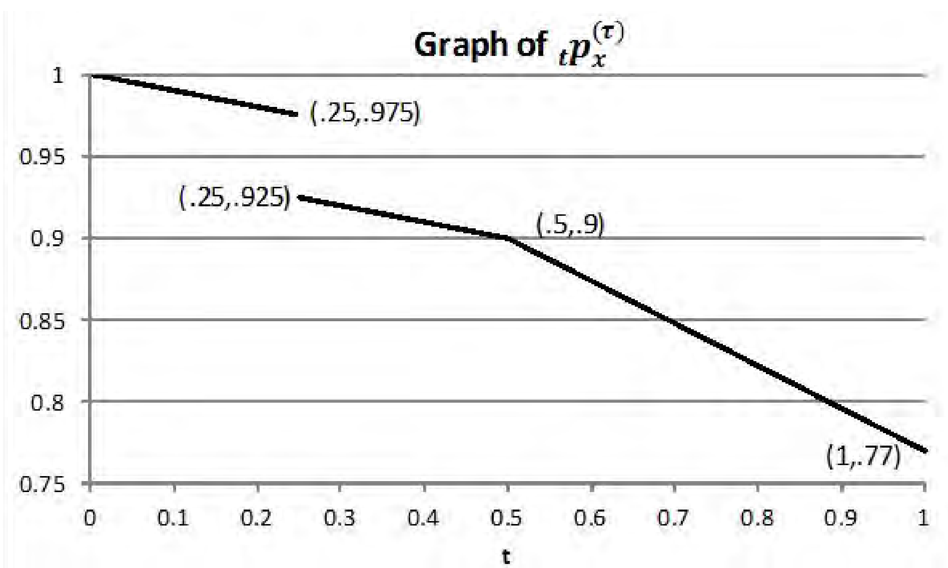
${}_{0.8}q_{60}^{(3)} = 2(0.8-0.5)(0.08) = 0.048$

${}_{0.8}p_{60}^{(\tau)} = 1 - (.05 + .08 + .048) = 0.822$

Comments:

1. *Most candidates did well on part (i), which did not require using the given fractional age assumptions.*
2. *Most candidates did poorly on part (ii). The most common error was to use the given dependent probabilities as if they were independent probabilities.*

c)



Comments:

1. *Performance on this part was mixed. Most candidates who attempted this part received partial credit, with some of the better candidates receiving full credit.*
2. *The most common errors were to draw one or more segments as curved rather than linear, or to ignore the discontinuity at time 0.25*

d)

From (a), we have

$$\frac{d}{dt}{}_tq_x^{(2)} = {}_tp_x^{(\tau)}\,\mu_{x+t}^{(2)}$$

For this model, $\frac{\mathrm{d}}{\mathrm{dt}}{}_{\mathrm{t}}\mathrm{q}_{\mathrm{x}}^{(2)} = \frac{\mathrm{d}}{\mathrm{dt}}\mathrm{t}\cdot\,\mathrm{q}_{\mathrm{x}}^{(2)} = \mathrm{q}_{\mathrm{x}}^{(2)}\,,\ \ 0 \le \mathrm{t} \le 1$

$$\Rightarrow \quad \mu_{x+t}^{(2)} = \frac{q_x^{(2)}}{{}_tp_x^{(\tau)}}$$

$$\mu_{60.8}^{(2)} = \frac{q_{60}^{(2)}}{{}_{0.8}p_{60}^{(\tau)}} = \frac{0.1}{.822} = 0.121655$$

Similarly,

$$\frac{d}{dt}{}_tq_x^{(3)} = {}_tp_x^{(\tau)}\,\mu_{x+t}^{(3)}$$

and

$$\frac{d}{dt}{}_tq_x^{(3)} = \begin{cases} 0 & 0 \le t < 0.5 \\ \frac{d}{dt}2(t-.5)\,q_x^{(3)} = 2\cdot q_x^{(3)} & 0.5 \le t < 1 \end{cases}$$

$$\Rightarrow \quad \mu_{x+t}^{(3)} = \frac{2\,q_x^{(3)}}{{}_tp_x^{(\tau)}}$$

$$\mu_{60.8}^{(3)} = \frac{2\,q_{60}^{(3)}}{{}_{0.8}p_{60}^{(\tau)}} = \frac{2(.08)}{.822} = 0.194647$$

Comments:

1. *Although a small number of strong candidates received full credit, most candidates either omitted or received little or no credit for this part.*
2. *Some candidates used memorized formulas that do not apply in this situation.*